properties of the two-dimensional Fourier transform

discrete correlation

The correlation of two continuous functions *f(x)* and *g(x)* denoted by $f(x) \circ g(x)$ is defined by the relation :

$$f(x) \circ g(x) = \int_{-\infty}^{\infty} f(\alpha) g(x+\alpha) d\alpha$$

This form is similar to convolution the only difference being that the function *g(x)* is not folded about the origin. Thus to perform correlation we simply slide *g(x)* by *f(x)* and integrate the product from $-\infty$ to ∞ for each value of displacement *x*.

The ***discrete*** equivalent of convolution eq. is defined as :

$$f_e(x) \circ g_e(x) = \sum_{m=0}^{M-1} f_e(m) g_e(x+m)$$

for x=0,1,2, ... , M-1. The correlation function is a discrete, periodic array of length *M* , with the values *x=0,1,2 , ... , M-1* describing a full period of $f_e(x) \circ g_e(x)$.

<u>Example</u>: Find the correlation function of the following functions:

f = { 3 , 2.5 , 1 , 0.5 }
g = { 2 , 5 , 7 , 9 }

<u>solution:</u>
f contains 4 elements
g contains 4 elements
M = A+ B – 1 = 4 + 4 – 1 = 7 the interval 0 . . . 6

$$h_e(x) = f_e(x) \circ g_e(x) = \sum_{m=0}^{M-1} f_e(m) g_e(x+m)$$

$h_e(0) = f_e(0).g_e(0) + f_e(1).g_e(1) + f_e(2).g_e(2) + f_e(3).g_e(3) + f_e(4).g_e(4) + f_e(5).g_e(5) + f_e(6).g_e(6)$
$= 3 * 2 + 2.5 * 5 + 1 * 7 + 0.5 * 9 = 6 + 12.5 + 7 + 4.5 = \mathbf{30}$

$h_e(1) = f_e(0).g_e(1) + f_e(1).g_e(2) + f_e(2).g_e(3) + f_e(3).g_e(4) + f_e(4).g_e(5) + f_e(5).g_e(6) + f_e(6).g_e(0)$
$= 3 * 5 + 2.5 * 7 + 1 * 9 + 0.5 * 0 = 15 + 17.5 + 9 = \mathbf{41.5}$

$h_e(2) = f_e(0).g_e(2) + f_e(1).g_e(3) + f_e(2).g_e(4) + f_e(3).g_e(5) + f_e(4).g_e(6) + f_e(5).g_e(0) + f_e(6).g_e(1)$
$= 3 * 7 + 2.5 * 9 + 1 * 0 = 21 + 22.5 = \mathbf{43.5}$

$$h_e(3) = f_e(0).g_e(3) + f_e(1).g_e(4) + f_e(2).g_e(5) + f_e(3).g_e(6)$$
$$+ f_e(4).g_e(0) + f_e(5).g_e(1) + f_e(6).g_e(2)$$
$$= 3 * 9 + 2.5 * 0 + 1 * 0 + 0.5 * 0 + 0*2 + 0*5 + 0*7 = \mathbf{27}$$

$$h_e(4) = f_e(0).g_e(4) + f_e(1).g_e(5) + f_e(2).g_e(6) + f_e(3).g_e(0)$$
$$+ f_e(4).g_e(1) + f_e(5).g_e(2) + f_e(6).g_e(3)$$
$$= 3 * 0 + 2.5 * 0 + 1 * 0 + 0.5 * 2 + 0 * 5 + 0 * 7 + 0*9 = \mathbf{1.0}$$

$$h_e(5) = f_e(0).g_e(5) + f_e(1).g_e(6) + f_e(2).g_e(0) + f_e(3).g_e(1)$$
$$+ f_e(4).g_e(2) + f_e(5).g_e(3) + f_e(6).g_e(4)$$
$$= 3 * 0 + 2.5 * 0 + 1 * 2 + 0.5 * 5 + 0 * 7 + 0*9 = 2 + 2.5 = \mathbf{4.5}$$

$$h_e(6) = f_e(0).g_e(6) + f_e(1).g_e(0) + f_e(2).g_e(1) + f_e(3).g_e(2)$$
$$+ f_e(4).g_e(3) + f_e(5).g_e(4) + f_e(6).g_e(5)$$
$$= 3*0 + 2.5 * 2 + 1 * 5 + 0.5 * 7 + 0 * 9 = 5.0 + 5 + 3.5 = \mathbf{13.5}$$

-discrete correlation by matrix

$$\begin{pmatrix}
g_e(0) & g_e(1) & g_e(2) & g_e(3) & \cdots & g_e(M-1) \\
g_e(1) & g_e(2) & g_e(3) & g_e(4) & \cdots & g_e(0) \\
g_e(2) & g_e(3) & g_e(4) & g_e(5) & \cdots & g_e(1) \\
g_e(3) & g_e(4) & g_e(5) & g_e(6) & \cdots & g_e(2) \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
g_e(M-1) & g_e(0) & g_e(1) & g_e(2) & \cdots & g_e(M-2)
\end{pmatrix}$$

- **_solution by matrix:_**

$A=4 \qquad B=4 \qquad \text{then} \quad M = A + B - 1 = 7$

$$
\begin{bmatrix} h_e(0) \\ h_e(1) \\ h_e(2) \\ h_e(3) \\ h_e(4) \\ h_e(5) \\ h_e(6) \end{bmatrix}
=
\begin{pmatrix}
g_e(0) & g_e(1) & g_e(2) & g_e(3) & g_e(4) & g_e(5) & g_e(6) \\
g_e(1) & g_e(2) & g_e(3) & g_e(4) & g_e(5) & g_e(6) & g_e(0) \\
g_e(2) & g_e(3) & g_e(4) & g_e(5) & g_e(6) & g_e(0) & g_e(1) \\
g_e(3) & g_e(4) & g_e(5) & g_e(6) & g_e(0) & g_e(1) & g_e(2) \\
g_e(4) & g_e(5) & g_e(6) & g_e(0) & g_e(1) & g_e(2) & g_e(3) \\
g_e(5) & g_e(6) & g_e(0) & g_e(1) & g_e(2) & g_e(3) & g_e(4) \\
g_e(6) & g_e(0) & g_e(1) & g_e(2) & g_e(3) & g_e(4) & g_e(5)
\end{pmatrix}
\begin{bmatrix} f_e(0) \\ f_e(1) \\ f_e(2) \\ f_e(3) \\ f_e(4) \\ f_e(5) \\ f_e(6) \end{bmatrix}
$$

$$
=
\begin{pmatrix}
2 & 5 & 7 & 9 & 0 & 0 & 0 \\
5 & 7 & 9 & 0 & 0 & 0 & 2 \\
7 & 9 & 0 & 0 & 0 & 2 & 5 \\
9 & 0 & 0 & 0 & 2 & 5 & 7 \\
0 & 0 & 0 & 2 & 5 & 7 & 9 \\
0 & 0 & 2 & 5 & 7 & 9 & 0 \\
0 & 2 & 5 & 7 & 9 & 0 & 0
\end{pmatrix}
\begin{bmatrix} 3 \\ 2.5 \\ 1 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix} \mathbf{30} \\ \mathbf{41.5} \\ \mathbf{43.5} \\ \mathbf{27} \\ \mathbf{1} \\ \mathbf{4.5} \\ \mathbf{13.5} \end{bmatrix}
$$