

Histogram Equalization

Histogram equalization is a technique which consists of adjusting the gray scale of the image so that the graylevel histogram of the input image is mapped on to a uniform histogram. The histogram equalization technique is based on a transformation using the histogram of a complete image in histogram equalization the goal is to obtain a uniform histogram for the output image. Let the variable r represents a random variable which indicates the gray level of an image. Initially we can assume that r is continuous and lies within the closed interval $[0:1]$ with $r=0$ representing black and $r=1$ representing white. For any r in the specified interval let us consider a transformation of the form: $s = T(r)$

The transformation produces a level s for every pixel value r in the original image.

To find the uniform histogram apply the following steps:

1. normalized the graylevels of the image by:

$$r_k = \frac{g_k}{L} \quad \text{where: } g_k: \text{represent graylevel}, L: \text{maximum of graylevel}$$

2. find the probability of each gray level of the image by calculate the occurrence of each one using the following equation

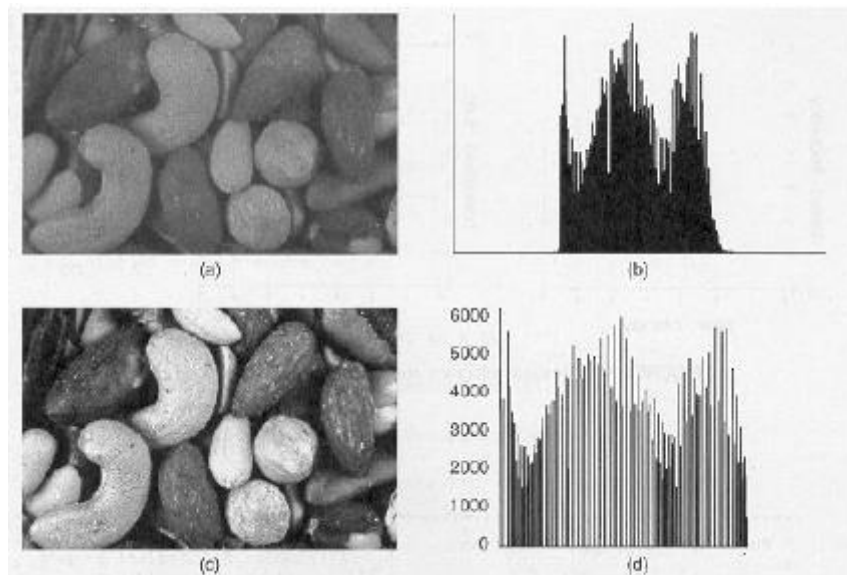
$$P_r(r_k) = \frac{n_k}{n} \quad 0 \leq r_k \leq 1 \quad k=0,1,\dots,L-1$$

n_k : represent occurrence of each r_k .
 n : number of total pixel in the image.

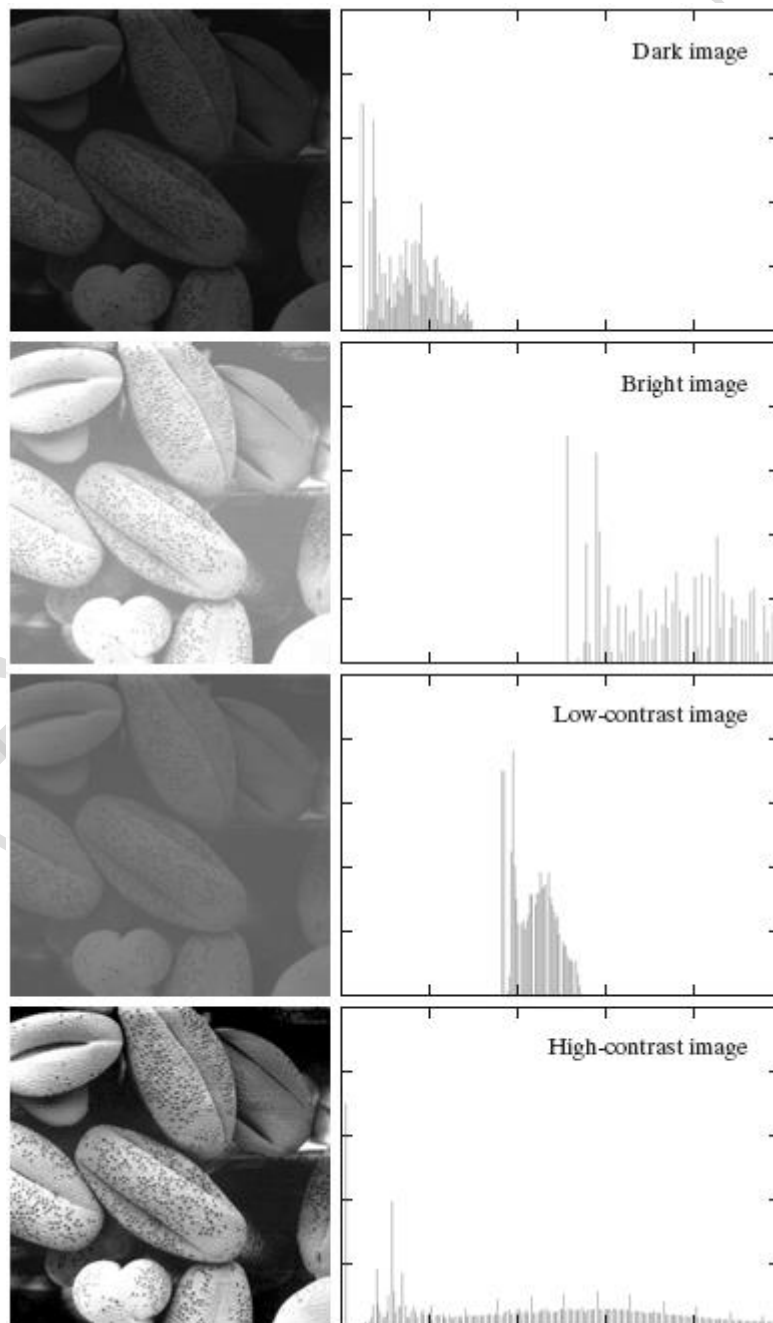
3. to find the new graylevel using the following equation:

$$S_k = T(r_k) = \sum_{j=0}^k n_j / n = \sum_{j=0}^k P_r(r_j)$$





(a) Original image; (b) Histogram of original image; (c) Equalized image; (d) Histogram of equalized image.



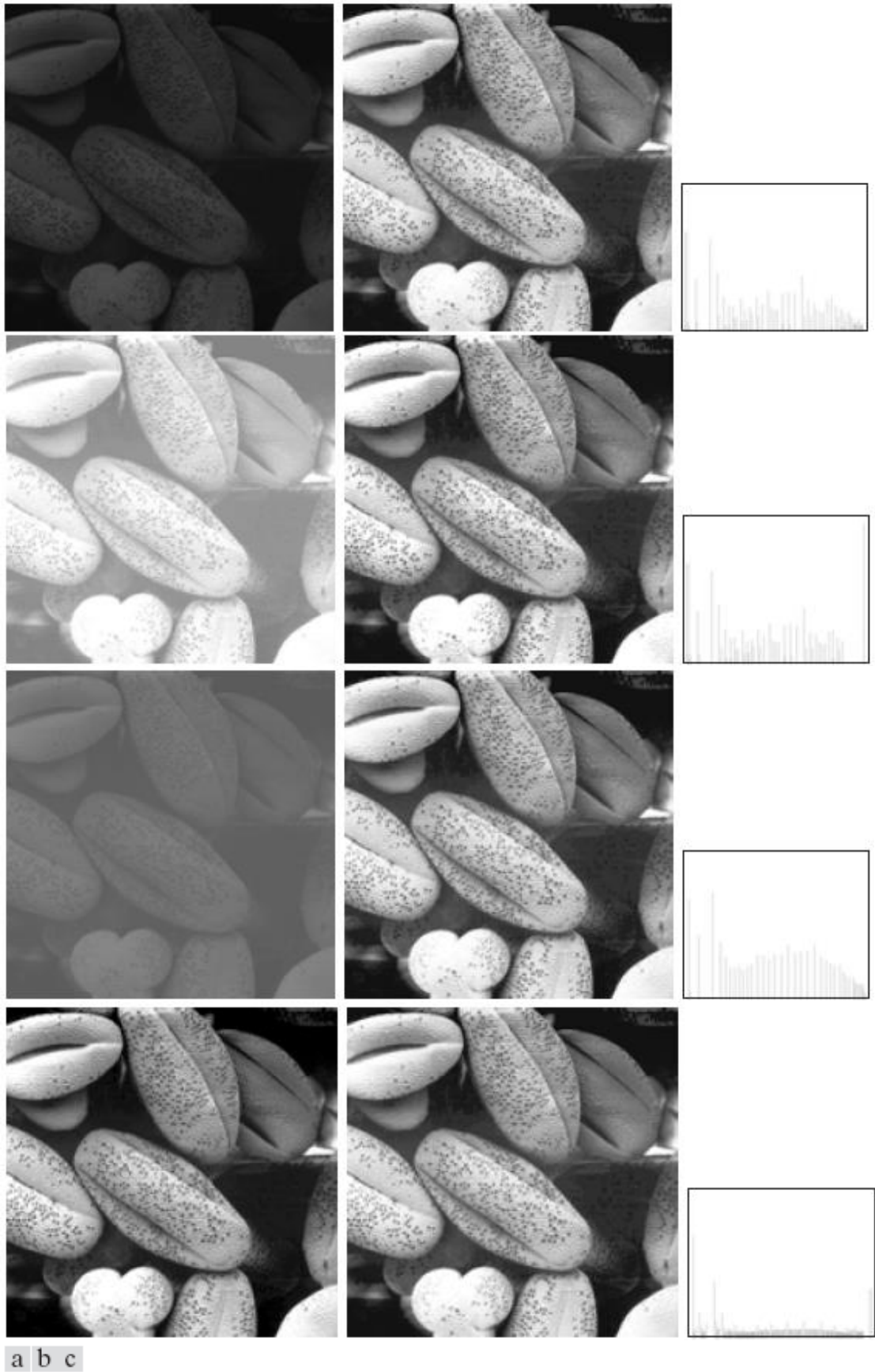


Figure: (a) Original Images (b) Results of histogram equalization (c) Corresponding histograms

Example: Suppose that a 64*64 , 8-level image has the gray-level distribution:

S on 8level				
r_k	n_k	$P_r(r_k) = n_k/n$	$S_k = \sum_{j=0}^k P_r(r_j)$	closest valid level(s_k*L)
$r_0=0$	790	0.19	0.19	$0.19*7=1.33=1/7$
$r_1=1/7$	1023	0.25	0.44	$0.44*7=3.08=3/7$
$r_2=2/7$	850	0.21	0.65	$0.65*7=4.55=5/7$
$r_3=3/7$	656	0.16	0.81	$0.81*7=5.67=6/7$
$r_4=4/7$	329	0.08	0.89	$0.89*7=6.23=6/7$
$r_5=5/7$	245	0.06	0.95	$0.95*7=6.65=7/7$
$r_6=6/7$	122	0.03	0.98	$0.98*7=6.86=7/7$
$r_7=1$	81	0.02	1.00	$1*7=7=7/7$

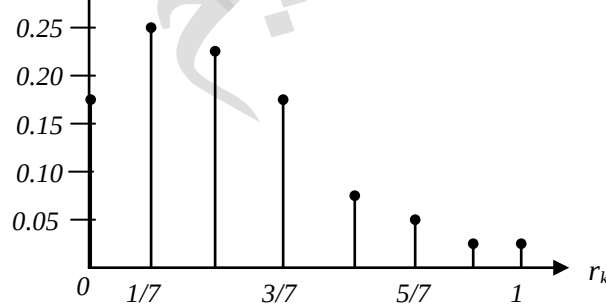
 $n=4096$ 1.00

$$S_0 = T(r_0) = \sum_{j=0}^0 P_r(r_j) = p_r(r_0) = 0.19$$

$$S_1 = T(r_1) = \sum_{j=0}^1 P_r(r_j) = p_r(r_0) + p_r(r_1) = 0.19 + 0.25 = 0.44$$

s_k*L	n_k	$p_s(s_k)$
1/7	790	0.19
3/7	1023	0.25
5/7	850	0.21
6/7	656+329=985	0.24
7/7	245+122+81=448	0.11

$P_r(r_k)$ Original histogram



$P_s(s_k)$ Equalization histogram

