Image Smoothing

Type of image enhancement, smoothing operations are used primarily for diminishing spurious effects that may be present in a digital image as a result of poor sampling system or transmission channel. Smoothing techniques are used for blurring and for noise reduction.

Lowpass Filtering

Edges and other sharp transitions (such as noise) in the graylevels of an image contribute heavily to the high frequency content of its fourier transform. The blurring can be achieved in the frequency domain by attenuating a specified range of high frequency components in the transform of a given image. Basic model for frequency domain filtering of *f(x,y)* is given by the equation:

$$G(u,v) = H(u,v)F(u,v)$$

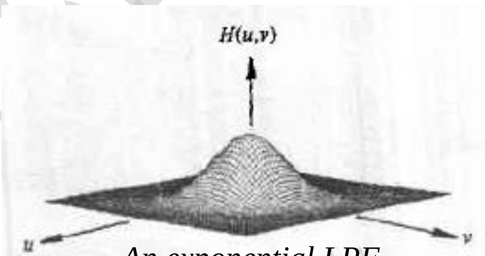
where *F(u,v)* is the transform of the image we wish to smooth. The problem is to select a function *H(u,v)* which yields *G(u,v)* by attenuating the high frequency components of *F(u,v)*. The inverse transform of *G(u,v)* will then yield the desired smoothed image *g(x,y)*. Since high frequency components are *"filtered out"* and information in the lowfrequency range *"passed"* without attenuation, this method is commonly referred to as lowpass filtering. The function *H(u,v)* is referred to filter transform function.

Exponential Filter

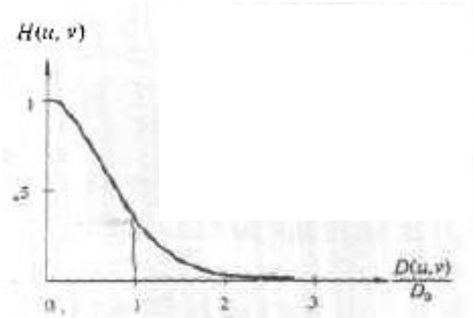
The exponential lowpass filter *(ELPF)* is another smooth filter commonly used in image processing. The *ELPF* with cut off frequency locus at a distance D_0 from the origin has a transfer function given by the relation :

$$H(u,v) = e^{-[D(u,v)/D_0]^n}$$

where: $D(u,v) = \left\{u^2 + v^2\right\}^{1/2}$, and *n* controls the rate of decay of the exponential function. The plot and cross section of the *ELPF* are shown in the following figure.



An exponential LPF



Radial cross sections for n=2

When $D(u,v)=D_0$ we have from above equation that *H(u,v)=1/e* , a simple modification given by:

$$H(u,v) = e^{-0.347[D(u,v)/D_0]^n}$$

forces *H(u,v)* to be equal to $1/\sqrt{2}$ of its maximum value at frequencies in the cut off locus.

Example: Design exponential lowpass filter (normal):

$D_0=3$ *size = 5*5* $n=1$

D	H(D)
0	1
1	0.7165
2	0.5134
3	0.3679
4	0.2636

	0	1	2	3	4
0	1	0.7165	0.5134	0.3679	0.2636
1	0.7165	0.6241	0.4746	0.3485	0.2530
2	0.5134	0.4746	0.3895	0.3006	0.2252
3	0.3679	0.3485	0.3006	0.2431	0.1889
4	0.2636	0.2530	0.2252	0.1889	0.1517

Example: Design exponential lowpass filter (modified):

$D_0=3$ *size = 5*5* $n=2$

D	H(D)
0	1
1	0.9622
2	0.8571
3	0.7068
4	0.5396

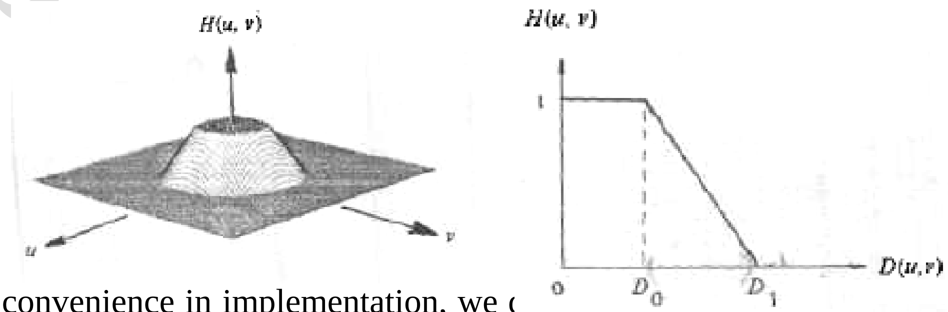
	0	1	2	3	4
0	1	0.9622	0.8571	0.7068	0.5396
1	0.9622	0.9258	0.8247	0.6801	0.5192
2	0.8571	0.8247	0.7346	0.6058	0.4625
3	0.7068	0.6801	0.6058	0.4996	0.3814
4	0.5396	0.5192	0.4625	0.3814	0.2912

Trapezoidal Filter

A trapezoidal lowpass filter *(TLPF)* is a compromise between the *ILPF* and a completely smooth filter. *TLPFs* can be defined by the relation:

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0\\ \dfrac{1}{[D_0-D_1]}[D(u,v)-D_1] & \text{if } D_0\le D(u,v)\le D_1\\ 0 & \text{if } D(u,v)>D_1\end{cases}$$

where $D(u,v)=\{u^2+v^2\}^{1/2}$, D_0 and D_1 are specified , and it is assumed that $D_0 < D_1$. a plot and cross section of a typical *TLPF* transfer function are shown below.



Plot and cross section of a typical TLPF

For convenience in implementation, we c[illegible] the first breakpoint (D_0) of the transfer function. The second variable D_1 is arbitrary as long as it is greater than D_0.

Example: Design a trapezoidal lowpass filter:

*$D_0=5$ $D_1=7$ size=9*9*

	D	H(D)
	0	1
	1	1
	2	1
	3	1
	4	1
D_0 →	5	1
	6	0.5
D_1 →	7	0
	8	0

	0	1	2	3	4	5	6	7	8
0	1	1	1	1	1	1	0.5	0	0
1	1	1	1	1	1	0.9505	0.4586	0	0
2	1	1	1	1	1	0.8074	0.3377	0	0
3	1	1	1	1	1	0.5845	0.1459	0	0
4	1	1	1	1	0.6716	0.2984	0	0	0
5	1	0.9505	0.8074	0.5845	0.2984	0	0	0	0
6	0.5	0.4586	0.3377	0.1459	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0