Image Sharpening

The principle objective of image sharpening is to highlight fine details and edges in a given image or to enhance details that has been blurred, either in an error or as a natural effect of a particular method of image acquisition.

A sharpening by differentiation

Is a simple sharpening method in spatial domain that use the differentiation of a given image.
For a digital image, the derivative of an image can be approximated by differences as following:

$$G[f(x,y)] \approx \sqrt{[f(x,y)-f(x+1,y)]^2+[f(x,y)-f(x,y+1)]^2}$$

where : *G[f(x,y)]* it is called gradient.

Note that the value of *G(gradient)* is proportial to the difference in graylevel between adjacent pixels.
So, the value of *G* assumes relatively large values for prominent edges in an image and small values in regions that are smooth.

The Gradient can be used in numerous ways to generating a gradient image *g(x,y)*.

1- Let the value of *G* at coordinate *(x,y)* be equal to the gradient of *f* at that point. *g(x,y)=G[f(x,y)]*
In this method, all smooth regions in the image appear dark in *g(x,y)* because of the small values of the gradient in that region.

2- $g(x,y)=\begin{cases} G[f(x,y)] & \text{if } G[f(x,y)] \ge T \\ f(x,y) & \text{otherwise} \end{cases}$

Where *T* is a non-negative threshold value. Using this form , it is possible to emphasite significant edges without destroying the characteristics of smooth background.

3- Another approach can be used, where the edges are set to a specified graylevel *LG*.

$$g(x,y)=\begin{cases} LG & \text{if } G[f(x,y)] \ge T \\ f(x,y) & \text{otherwise} \end{cases}$$

4- The following gradient form can be used if it is desirable to study graylevel variation of edges without interference from the background

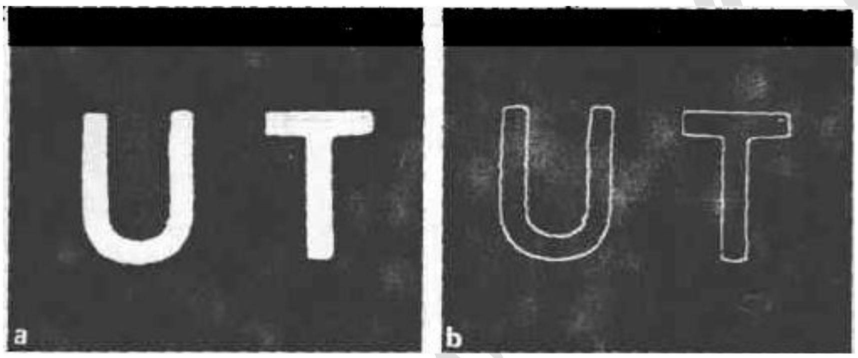
$$g(x,y) = \begin{cases} G[f(x,y)] & if \ G[f(x,y)] \geq T \\ LB & otherwise \end{cases}$$

Where *LB* is a specified graylevel for the background

5- When only the location of edges is of interest, the gradient can be accomplished as:

$$g(x,y) = \begin{cases} LG & if \ G[f(x,y)] \geq T \\ LB & otherwise \end{cases}$$

Using this method will give a binary gradient picture where edges & background are displayed in any two specified graylevels.



High pass filter

Since edges and other abrupt changes in graylevels are associated with high frequency components. Image sharpening in frequency domain can be achieved by a high pass filtering process which attenuates the low frequency components without disturbing high frequency information in the fourier transform.

Also the basic model for frequency domain filtering of *f(x,y)* is given by the equation:

$$G(u,v)= H(u,v)\ F(u,v)$$

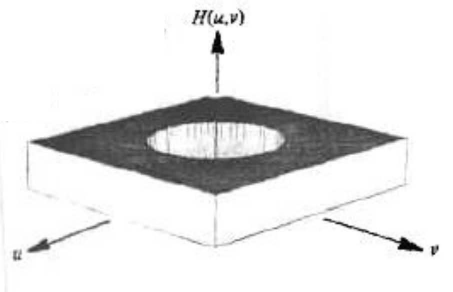
Where *F(u,v)* is the transform of the image we wish to sharp. The problem is to select a function *H(u,v)* which yields *G(u,v)* by attenuating the low frequency components of *F(u,v)*. The inverse transform of *G(u,v)* will then yield the desired sharp image *g(x,y)*. The function *H(u,v)* is referred to filter transfer function.

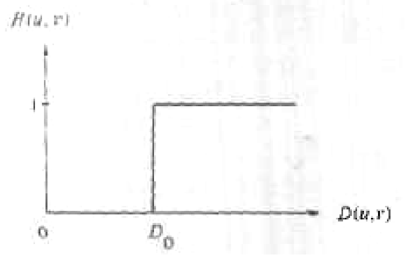
1. Ideal Filter

A two dimensional ideal highpass filter *(IHPF)* is one whose transfer function satisfies the relation:

$$H(u,v) = \begin{cases} 0 & if \ \ D(u,v) \leq D_0 \\ 1 & if \ \ D(u,v) > D_0 \end{cases}$$

where D_0 is the cut off distance measured from the origin of the frequency plane. $D(u,v) = \{u^2 + v^2\}^{1/2}$





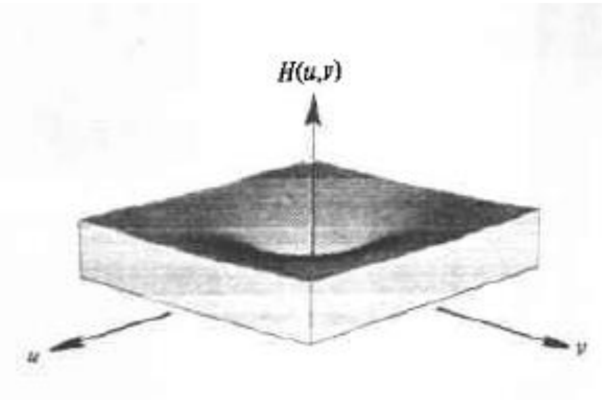
2. *Butterworth Filter*

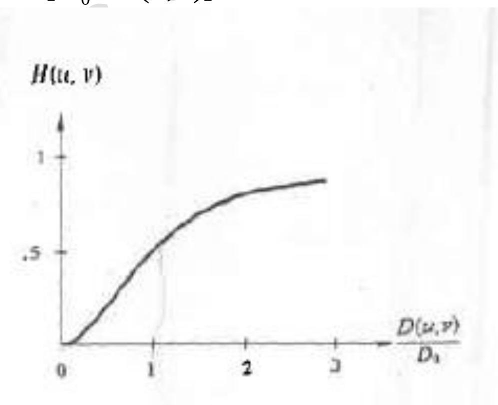
The transfer function of the butterworth highpass filter *(BHPF)* of order n and with cutoff frequency locus as a distance D_0 from the origin is defined by the relation:

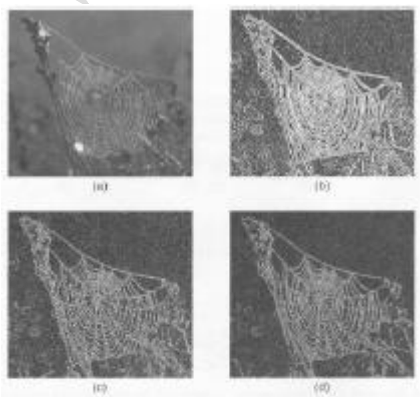
$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

And modified *BHPF* is:

$$H(u,v) = \frac{1}{1 + 0.414[D_0 / D(u,v)]^{2n}}$$



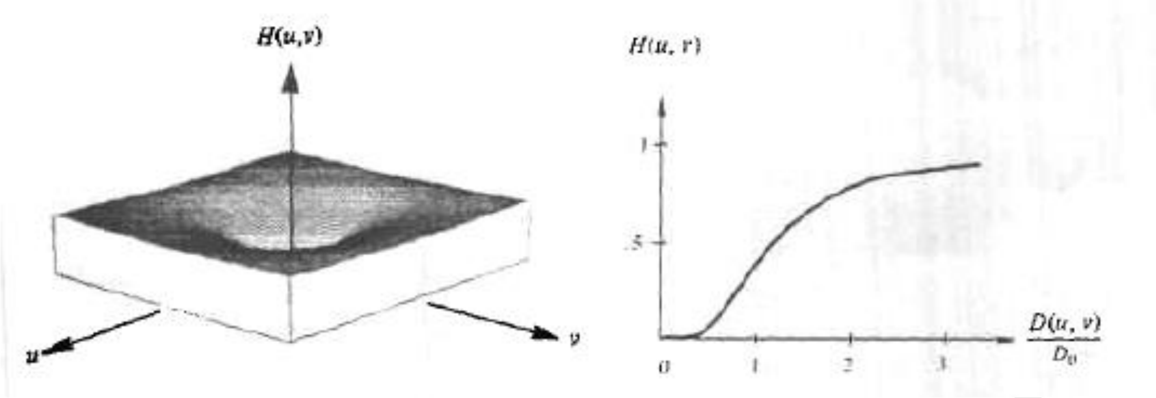




Effect of second order (n=2) Butterworth filter: (a) Original image (512 x 512); (b) high pass filtered D_0=64; (c) high pass filtered D_0=128; (d) high pass filtered D_0=192.

3. Exponential Filter

$H(u,v) = e^{-[D_0 / D(u,v)]^n}$ *and the modified EHPF:* $H(u,v) = e^{-0.347[D_0 / D(u,v)]^n}$



4. Trapezoidal Filter

A trapezoidal highpass filter *(THPF)* can be defined by the relation:

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) < D_1 \\ \dfrac{1}{[D_0 - D_1]}[D(u,v) - D_1] & \text{if } D_1 \le D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0, D_1 are specified and it is assumed that $D_0 > D_1$