

Fidelity criteria

When lossy compression techniques are employed, the decompressed image will not be identical to the original image. In such cases, we can define fidelity criteria that measure the difference between these two images. Two general classes of criteria are used: (1) objective fidelity criteria and (2) subjective fidelity criteria. Subjective: based on human observers. Objective: mathematically defined criteria.

A good example for (1) objective fidelity criteria is root-mean square (RMS) error between an input and output image. For any value of x , and y , the error $e(x,y)$ can be defined as:

$$e(x,y) = f'(x,y) - f(x,y)$$

The total error between two images is:
$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f'(x,y) - f(x,y)]$$

The root mean square error, e_{rms} or (RMS) is:
$$e_{rms} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f'(x,y) - f(x,y)]^2 \right]^{\frac{1}{2}}$$

or

$$e_{rms} = \sqrt{\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x,y) - f(x,y))^2}$$

- Mean-square signal-to-noise ratio (SNR): A closely related objective fidelity criterion is the mean square signal to noise ratio of the compressed-decompressed image.

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x, y))^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (\hat{f}(x, y) - f(x, y))^2}$$

$$SNR_{peak} = 10 \log_{10} \frac{(L-1)^2}{\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]^2}$$

Where L is number of gray levels.

A larger number of SNR_{peak} implies a better image.

Subjective fidelity criteria

These criteria measure image quality by the subjective evaluations of a human observer. This can be done by showing a decompressed image to a group of viewers and averaging their evaluations. The evaluations may be made using an absolute rating scale, for example {Excellent, Fine, Passable, Marginal, Inferior, and Unusable}.

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Compression Ratio

$$\text{Compression Ratio} = \frac{\text{Uncompressed File Size}}{\text{Compressed File Size}} = C_R$$

Ex. : Image 256 * 256 pixels, 256 level grayscale can be compressed file size 6554 byte.

$$\begin{aligned}\text{Original Image Size} &= 256 * 256 \text{ (pixels)} * 1 \text{ (byte/pixel)} \\ &= 65536 \text{ bytes}\end{aligned}$$

$$\text{compression Ratio} = \frac{65536}{6554} \approx 10$$

Bits per pixel (*bpp*): denote the number of bits required to represent an image pixel

$$\text{Bits per Pixel} = \frac{\text{Number of Bits}}{\text{Number of Pixels}}$$

$$\text{bpp} = \frac{\text{Number of Bits}}{\text{Number of Pixels}} = \frac{8(\text{Number of Bytes})}{N \times N}$$

Ex. : Image 256 * 256 pixels, 256 level grayscale can be compressed file size 6554 byte.

$$\begin{aligned}\text{Original Image Size} &= 256 * 256 \text{ (pixels)} * 1 \text{ (byte/pixel)} \\ &= 65536 \text{ bytes}\end{aligned}$$

$$\begin{aligned}\text{Compressed file} &= 6554 \text{ (bytes)} * 8 \text{ (bits/pixel)} \\ &= 52432 \text{ bits}\end{aligned}$$

$$\text{Bits per Pixel} = \frac{52432}{65536} \approx 0.8$$