Clustering Method

Cluster techniques may be more appropriate than histogram-oriented ones in segmenting images, where each pixel has several attributes and is represented by a vector. Cluster analysis has attracted much attention since the 1960's and has been applied in many fields such as *OCR* (optical character Recognition) system, fingerprint identification, remote sensing, biological image segmentation, and so on.

K-means is a popular cluster based segmentation method where each pixel is iteratively assigned to the nearest cluster and the cluster center position is recalculated. After each iteration the cost will decrease until the cluster configuration converges at a stable state, at which point the cost is at a local minimum.

K-means clustering Algorithm

Step1: Given *K=no.* of class let $z_1(1), z_2(1), z_3(1) \ldots z_k(1)$ at iteration one be the centers of the given classes.

Where *no.(1)* represent iteration one

Where:

$z_1(1) = x_1$

$z_2(1) = x_2$

$\vdots$

$z_k(1) = x_k$

step2: at each iteration compute the distance between the input vectors and the means of the classes *(z)*.

step3: Assign x to the set *si(k)* where *k* is the iteration no.

so that $\|x - zi(k)\| < \|x - zj(k)\|$

for j= 1,2,….k and $j \neq i$

i=1

$\|x - z1(k)\| < \|x - z2(k)\|$

$\|x - z1(k)\| < \|x - z3(k)\|$

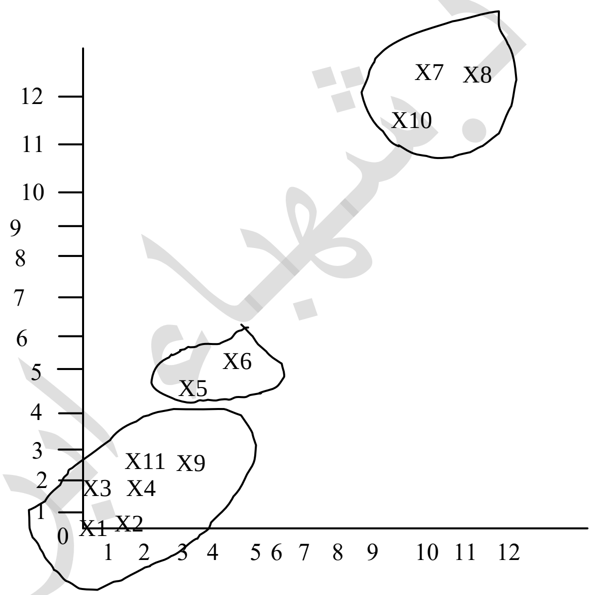
step4: calculate the new means

$$zi(k+1) = \frac{1}{Ni} \sum_{j=1}^{N} xj \qquad x \in si(k), (s) \text{ set}$$

step5: check if *Zi(k) = Zi (k+1) for all i=1,2,…..k*
then the algorithm converges.
Else
Repeat from **step 2**.

<u>Ex.</u>

X1=(0 0)
X2=(1 0)
X3 = (0 1)
X4 =(1 1)
X5 =(3 4)
X6= (5 5)
X7 = (10 12)
X8 = (12 12)
X9 = (3 2)
X10 = (10 11)
X11 = (2 2)



<u>SoL.</u> *K=3*
Z1(1)= (0 0)
Z2(1)= (1 0)
Z3(1)= (0 1)

Compute x4

$$d14 = \|x4 - z1(1)\| = \sqrt{\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]} = \sqrt{(1\ \ 1)\begin{pmatrix}1\\1\end{pmatrix}} = \sqrt{2} = 1.4$$

$$d24 = \|x4 - z2(1)\| = \sqrt{\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]} = \sqrt{(0\ \ 1)\begin{pmatrix}0\\1\end{pmatrix}} = \sqrt{1} = 1$$

$$d34 = \|x4 - z3(1)\| = \sqrt{\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]} = \sqrt{(1\ 0)\begin{pmatrix}1\\0\end{pmatrix}} = \sqrt{1} = 1$$

$\therefore x4 \in z2$

Compute x5

$$d15 = \|x5 - z1(1)\| = \sqrt{\left[\begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]} = \sqrt{(3\ 4)\begin{pmatrix}3\\4\end{pmatrix}} = \sqrt{25} = 5$$

$$d25=\|x5-z2(1)\|=\sqrt{\left[\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]}=\sqrt{(2\ 4)\begin{pmatrix}2\\4\end{pmatrix}}=\sqrt{20}=4.4$$

$$d35=\|x5-z3(1)\|=\sqrt{\left[\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]}=\sqrt{(3\ 3)\begin{pmatrix}3\\3\end{pmatrix}}=\sqrt{18}=4.2$$

$\therefore x5\in z3$

Compute x6

$$d16=\|x6-z1(1)\|=\sqrt{\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]}=\sqrt{(5\ 5)\begin{pmatrix}5\\5\end{pmatrix}}=\sqrt{50}=7.07$$

$$d26=\|x6-z2(1)\|=\sqrt{\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]}=\sqrt{(4\ 5)\begin{pmatrix}4\\5\end{pmatrix}}=\sqrt{41}=6.40$$

$$d36=\|x6-z3(1)\|=\sqrt{\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}5\\5\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]}=\sqrt{(5\ 4)\begin{pmatrix}5\\4\end{pmatrix}}=\sqrt{41}=6.40$$

$\therefore x6\in z2$

Compute x7

$$d17=\|x7-z1(1)\|=\sqrt{\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]}=\sqrt{(10\ 12)\begin{pmatrix}10\\12\end{pmatrix}}=\sqrt{244}=15.62$$

$$d27=\|x7-z2(1)\|=\sqrt{\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]}=\sqrt{(9\ 12)\begin{pmatrix}9\\12\end{pmatrix}}=\sqrt{255}=15$$

$$d37=\|x7-z3(1)\|=\sqrt{\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}10\\12\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]}=\sqrt{(10\ 11)\begin{pmatrix}10\\11\end{pmatrix}}=\sqrt{221}=14.8$$

$\therefore x7\in z3$

Compute x8

$$d18=\|x8-z1(1)\|=\sqrt{\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right]}=\sqrt{(12\ 12)\begin{pmatrix}12\\12\end{pmatrix}}=\sqrt{288}=16.97$$

$$d28=\|x8-z2(1)\|=\sqrt{\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\right]}=\sqrt{(11\ 12)\begin{pmatrix}11\\12\end{pmatrix}}=\sqrt{265}=16.2$$

$$d38=\|x8-z3(1)\|=\sqrt{\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}12\\12\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right]}=\sqrt{(12\ 11)\begin{pmatrix}12\\11\end{pmatrix}}=\sqrt{265}=16.2$$

$\therefore x8\in z2$

Compute x9

$$d19 = \|x9 - z1(1)\| = \sqrt{\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]} = \sqrt{(3\ 2)\begin{pmatrix}3\\2\end{pmatrix}} = \sqrt{13} = 3.6$$

$$d29 = \|x9 - z2(1)\| = \sqrt{\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]} = \sqrt{(2\ 2)\begin{pmatrix}2\\2\end{pmatrix}} = \sqrt{8} = 2.8$$

$$d39 = \|x9 - z3(1)\| = \sqrt{\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]} = \sqrt{(3\ 1)\begin{pmatrix}3\\1\end{pmatrix}} = \sqrt{10} = 3.1$$

$\therefore x9 \in z2$

Compute x10

$$d1{,}10 = \|x10 - z1(1)\| = \sqrt{\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]} = \sqrt{(10\ 11)\begin{pmatrix}10\\11\end{pmatrix}} = \sqrt{221} = 14.8$$

$$d2{,}10 = \|x10 - z2(1)\| = \sqrt{\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]} = \sqrt{(9\ 11)\begin{pmatrix}9\\11\end{pmatrix}} = \sqrt{202} = 14.2$$

$$d3{,}10 = \|x10 - z3(1)\| = \sqrt{\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}10\\11\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]} = \sqrt{(10\ 10)\begin{pmatrix}10\\10\end{pmatrix}} = \sqrt{200} = 14.1$$

$\therefore x10 \in z3$

Compute x11

$$d1{,}11 = \|x11 - z1(1)\| = \sqrt{\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right]} = \sqrt{(2\ 2)\begin{pmatrix}2\\2\end{pmatrix}} = \sqrt{8} = 2.82$$

$$d2{,}11 = \|x11 - z2(1)\| = \sqrt{\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right]} = \sqrt{(1\ 2)\begin{pmatrix}1\\2\end{pmatrix}} = \sqrt{5} = 2.23$$

$$d3{,}11 = \|x11 - z3(1)\| = \sqrt{\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]\left[\begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\right]} = \sqrt{(2\ 1)\begin{pmatrix}2\\1\end{pmatrix}} = \sqrt{5} = 2.23$$

$\therefore x11 \in z2$

$S1 = \{X1\}\ \{(0\ 0\}$
$S2 = \{X2, X4, X6, X8, X9, X11\}$
$S3 = \{X3, X5, X7, X10\}$

now compute new means

$Z1 = X1 = (0\ \ 0)$

$$Z2=\frac{1}{6}\begin{pmatrix}1+1+5+12+3+2\\0+1+5+12+2+2\end{pmatrix}=\begin{pmatrix}4\\3.6\end{pmatrix}$$

$$Z3=\frac{1}{4}\begin{pmatrix}0+3+10+10\\1+4+12+11\end{pmatrix}=\begin{pmatrix}5.75\\7\end{pmatrix}$$