Integral of form $\sqrt[n]{ax+b}$

$Let\ \ u=\sqrt[n]{ax+b}\ \Rightarrow u^n=ax+b$

$ex:\ \ u=\sqrt{x}\Rightarrow u^2=x \quad , \quad u=\sqrt[3]{x-1}\Rightarrow x=u^3+1$

EXAM :

1 $\int\frac{\sqrt{x}}{1+\sqrt{x}}dx$

$u=\sqrt{x}\Rightarrow u^2=x\Rightarrow dx=2udu$

$\int\frac{u}{1+u}2udu=\int\frac{2u^2}{1+u}du=\int(2u-2+\frac{2}{u+1})du$

$u^2-2u+2Ln|u+1|=x-2\sqrt{x}+2Ln\left|\sqrt{x}+1\right|+C$

$$\begin{array}{r|l} & 2u-2 \\ u+1 & 2u^2 \\ & \underline{2u^2+2u} \\ & -2u \\ & \underline{-2u-2} \\ & 2 \end{array}$$

2 $\int\frac{dx}{1+\sqrt{x}}$

$u=\sqrt{x}\Rightarrow u^2=x\Rightarrow dx=2udu$

$\int\frac{2u}{1+u}du=2\int\frac{u}{1+u}du=2\int(1-\frac{1}{u+1})du$

$2\left(u-Ln|u+1|\right)=2\left(\sqrt{x}-Ln\left|\sqrt{x}+1\right|\right)+C$

3 $\int\sqrt{x}\sqrt{1+x\sqrt{x}}\,dx$

$u=\sqrt{x}\Rightarrow u^2=x\Rightarrow dx=2udu$

$\int\sqrt{x}\sqrt{1+x\sqrt{x}}\,dx=\int u\sqrt{1+u^2.u}\,.2udu$

$=\int 2u^2\sqrt{1+u^3}du=\frac{2}{3}\frac{(1+u^3)^{3/2}}{3/2}=\frac{4}{9}(1+u^3)^{3/2}=\frac{4}{9}(1+x\sqrt{x})^{3/2}+C$

4 $\int\frac{dx}{x\sqrt{\sqrt{x}+\sqrt[4]{x}}}$

$u=\sqrt[4]{x}\Rightarrow u^4=x\Rightarrow dx=4u^3du$

$\int\frac{dx}{x\sqrt{\sqrt{x}+\sqrt[4]{x}}}=\int\frac{4u^3du}{u^4\sqrt{u^2+u}}=4\int\frac{du}{u\sqrt{u^2(1+u^{-1})}}=4\int\frac{du}{u^2\sqrt{(1+u^{-1})}}$

$4\int u^{-2}(1+u^{-1})^{-1/2}du=-4\frac{(1+u^{-1})^{1/2}}{1/2}=-8\sqrt{1+\frac{1}{\sqrt[4]{x}}}+C$

5 $\int \frac{\sqrt{x}}{1+\sqrt[4]{x}}dx$

$u=\sqrt[4]{x} \Rightarrow u^4 = x \Rightarrow dx = 4u^3du$

$\int \frac{\sqrt{x}}{1+\sqrt[4]{x}}dx = \int \frac{u^2}{1+u}4u^3du = \int \frac{4u^5}{1+u}du$

$4\int (u^4 - u^3 + u^2 - u + 1 - \frac{1}{u+1})du$

$4\left[\frac{u^5}{5} - \frac{u^4}{4} + \frac{u^3}{3} - \frac{u^2}{2} + u - Ln\left|u+1\right|\right]$

$4\left[\frac{x\sqrt[4]{x}}{5} - \frac{x}{4} + \frac{x^{3/4}}{3} - \frac{\sqrt{x}}{2} + \sqrt[4]{x} - Ln\left|\sqrt[4]{x}+1\right|\right] + C$

$$
\begin{array}{r|l}
 & u^4 - u^3 + u^2 - u + 1 \\
u+1 & u^5 \\
 & \underline{u^5 + u^4} \\
 & -u^4 \\
 & \underline{-u^4 - u^3} \\
 & u^3 \\
 & \underline{u^3 + u^2} \\
 & -u^2 \\
 & \underline{-u^2 - u} \\
 & u \\
 & \underline{u + 1} \\
 & -1
\end{array}
$$

6 $\int \frac{x}{\sqrt{x-1}+2}dx$

$u=\sqrt{x-1} \Rightarrow u^2 = x-1 \Rightarrow dx = 2udu$

$\int \frac{u^2+1}{u+2}2udu = 2\int \frac{u^3+u}{u+2}du$

$2\int (u^2 - 2u + 5 - \frac{10}{u+2})du = 2(\frac{u^3}{3} - u^2 + 5u - 10Ln\left|u+2\right|)$

$= 2[\frac{(x-1)\sqrt{x-1}}{3} - (x-1) + 5\sqrt{x-1} - 10Ln\left|\sqrt{x-1}+2\right|] + C$

$$
\begin{array}{r|l}
 & u^2 - 2u + 5 \\
u+2 & u^3 + u \\
 & \underline{u^3 + 2u^2} \\
 & -2u^2 + u \\
 & \underline{-2u^2 - 4u} \\
 & 5u \\
 & \underline{5u + 10} \\
 & -10
\end{array}
$$

7 $\int x(x-4)^{2/3}dx$

$= \int x(\sqrt[3]{x-4})^2dx$

$u = \sqrt[3]{x-4} \Rightarrow u^3 = x-4 \Rightarrow x = u^3+4 \Rightarrow dx = 3u^2du$

$= \int (u^3+4)u^2 3u^2du = 3\int (u^7 + 4u^4)du = 3(\frac{u^8}{8} + \frac{4u^5}{5})$

$= \frac{3(x-4)^{8/3}}{8} + \frac{12(x-4)^{5/3}}{5} + C$

HOME WORK

1 *FIND* :

$$\int \frac{dx}{2\sqrt{x}+3} \qquad \int \frac{x}{\sqrt{2x+7}}dx$$

$$\int 4x\sqrt{5x+1}\,dx \qquad \int x^2\sqrt{3x-1}\,dx$$

$$\int \cos(\sqrt{2x})\,dx \qquad \int \frac{\sqrt{x}}{\sqrt[3]{x}+1}dx$$

$$\int \frac{\sqrt{x}}{x+x^{4/3}}dx \qquad \int \frac{dx}{x^{1/4}(1+x^{1/6})}$$

2 *Find the area of reigon bounded by the curve* $y=\frac{x}{\sqrt{x+1}}$ *and the line* $y=0$ *and* $x=3$ *, about the* $x-axis$.

3 *Find the area of reigon bounded by the curve* $y=\frac{1}{1+\sqrt{x}}$ *and the line* $y=0$, $x=0$ *and* $x=4$ *, about the* $x-axis$.

MOHAMED SABAH AL TAEE
M.SC / MATHEMATICS
E-MAIL : msmt_80@yahoo.com
2013 -2014