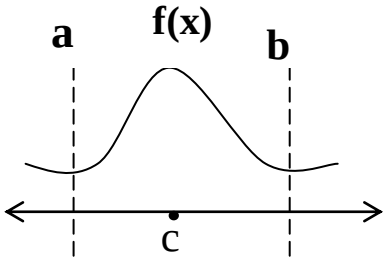
PROPERTIES OF INTEGRAL : -



$$\int_a^b dx = b-a \Rightarrow \int_a^b c.dx = c(b-a)$$

$$\int_a^b c.f(x)dx = c.\int_a^b f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \ , c \in [a,b]$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad , n \neq -1$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$if \ f(x) \geq 0 \Rightarrow \int_a^b f(x)dx \geq 0$$

$$if \ f(x) \leq g(x) \Rightarrow \int_a^b f(x)dx \leq \int_a^b g(x)dx$$

$$\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad , n \neq -1$$

EXAM : -

$$\int_6^{10} dx = 10-6 = 4$$

$$\int_6^{10} 5.dx = 5(10-6) = 5.4 = 20$$

$$\int_1^3 x.dx = \frac{x^2}{2}\Big|_1^3 = \frac{1}{2}(9-1) = 4$$

$$\int_2^5 (x+5)dx = \frac{x^2}{2} + 5x \Big|_2^5 = 25.5$$

$$\int_0^1 (x^2 + 3x)dx = \frac{x^3}{3} + \frac{3}{2}x^2 \Big|_0^1 = \frac{11}{6}$$

$$\int_1^3 (2x^2 + 5x)^2 (4x+5)dx = \frac{(2x^2+5x)^3}{3} \Big|_1^3 = ?$$

$$\int_1^3 (2x^2 + 6x)^2 (2x+3)dx = \frac{1}{2}\int_1^3 (2x^2 + 6x)^2 (4x+6)dx =$$

$$= \frac{(2x^2+6x)^3}{3} \Big|_1^3 = ?$$

$$\int_0^1 (x^3+1)^2 dx = \int_0^1 (x^6 + 2x^3 + 1)dx = \frac{x^7}{7} + \frac{x^4}{2} + x \Big|_0^1 = ?$$

EQUATION OF STRAIGHT LINE : معادلـــة الخـط المستـقيـم

The general form of straight line equation is :-

$$ax + by + c = 0$$

OR

$$y = mx + b$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is 2x+1 and passing through the point (1,3) .

$$m = \frac{dy}{dx} = 2x + 1$$

$$y = \int \frac{dy}{dx} dx = \int (2x+1)dx \Rightarrow \boxed{y = x^2 + x + C} \Leftarrow general\ curves$$

$$(1,3) \in curves \Rightarrow 3 = 1^2 + 1 + C \Rightarrow C = 1$$

$$\boxed{y = x^2 + x + 1} \Leftarrow special\ curve$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is

$$4x^3 + 18x^2 + 8x + 3$$

and passing through the point (1,11) .

$$m=\frac{dy}{dx}=4x^3+18x^2+8x+3$$

$$y=\int(4x^3+18x^2+8x+3)dx$$

$$y=x^4+6x^3+4x^2+3x+C$$

$$11=1+6+4+3+C\Rightarrow C=-3$$

$$y=x^4+6x^3+4x^2+3x-3$$

DOUBLE INTEGRATION :

EXAM :

$$\int_0^1\int_0^x(3-x-y)dydx$$

$$=\int_0^1 3y-xy-\frac{1}{2}y^2\Big|_0^x dx=\int_0^1(3x-x^2-\frac{1}{2}x^2)dx=\int_0^1(3x-\frac{3}{2}x^2)dx$$

$$=\frac{3}{2}x^2-\frac{1}{2}x^3\Big|_0^1=1$$

EXAM :

$$\int_1^2\int_y^{y^2}dxdy$$

$$=\int_1^2 x\Big|_y^{y^2}dy=\int_1^2(y^2-y)dy=\frac{1}{3}y^3-\frac{1}{2}y^2\Big|_1^2=\frac{5}{6}$$

EXAM :

$$\int_0^{\sqrt{2}}\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}}ydxdy$$

$$=\int_0^{\sqrt{2}} yx\Big|_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}}dx=\int_0^{\sqrt{2}} y\sqrt{4-2y^2}+y\sqrt{4-2y^2}dx=2\int_0^{\sqrt{2}} y\sqrt{4-2y^2}dx$$

$$\frac{2}{-4}\frac{(4-2y^2)^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^{\sqrt{2}}=\frac{8}{3}$$

TRIPLE INTEGRATION :

EXAM :

$$\int_0^1\int_0^x\int_{-y^2}^{x^2}(x+1)\,dz\,dy\,dx$$

$$=\int_0^1\int_0^x (x+1)z\Big|_{-y^2}^{x^2}\,dy\,dx=\int_0^1\int_0^x (x+1)x^2+(x+1)y^2\,dy\,dx$$

$$=\int_0^1\int_0^x x^3+x^2+xy^2+y^2\,dy\,dx=\int_0^1 x^3y+x^2y+\frac{1}{3}xy^3+\frac{1}{3}y^3\Big|_0^x\,dx$$

$$=\int_0^1 x^4+x^3+\frac{1}{3}x^4+\frac{1}{3}x^3\,dx=\int_0^1\frac{4}{3}x^4+\frac{4}{3}x^3\,dx=\frac{4}{15}x^5+\frac{1}{3}x^4\Big|_0^1=\frac{9}{15}$$

EXAM :

$$\int_0^1\int_0^x\int_{-y^2}^{x^2}dz\,dy\,dx$$

$$=\int_0^1\int_0^x z\Big|_{-y^2}^{x^2}\,dy\,dx=\int_0^1\int_0^x x^2+y^2\,dy\,dx$$

$$=\int_0^1 x^2y+\frac{1}{3}y^3\Big|_0^x\,dx=\int_0^1 x^3+\frac{1}{3}x^3\,dx$$

$$=\int_0^1\frac{4}{3}x^3\,dx=\frac{1}{3}x^4\Big|_0^1=\frac{1}{3}$$

$$=\int_0^1 x^4+x^3+\frac{1}{3}x^4+\frac{1}{3}x^3\,dx=\int_0^1\frac{4}{3}x^4+\frac{4}{3}x^3\,dx=\frac{4}{15}x^5+\frac{1}{3}x^4\Big|_0^1=\frac{9}{15}$$

HOME WORK

1) Find the equation of the curve whose slope at any point p(x,y) is
$m = (x^4 + 16x + 4)^2 (x^3 + 4)$
and passing through the point (2,1) .

2) Find the equation of the curve whose slope at any point p(x,y) is
$m = x\,(x + 5)^2$
and passing through the point (2,1) .

3) Find :

$$\int_0^1 \int_0^3 x\sqrt{x^2 + y}\, dy dx$$

$$\int_0^1 \int_{-1}^{\sqrt{y}} y\, dx dy$$

$$\int_0^1 \int_x^{x^2} \int_{x-y}^{x+y} (x + 2y + 4z)\, dz dy dx$$

$$\int_0^1 \int_0^2 \int_0^3 (z^3 y^2 x)\, dx dy dz$$

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