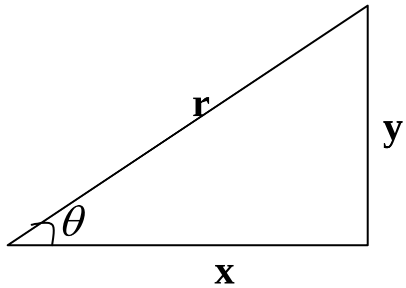
There is relation between Cartesian coordinates (x,y) and the polar coordinates (r,θ) **and there are from the following bases :**

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x}$$



EXAM :

1) Find the rectangular (Cartesian) coordinates of the point P whose polar coordinates are $(6,\frac{2\pi}{3})$

$$x = r\cos\theta = 6\cos\frac{2\pi}{3} = 6\cos(\pi - \frac{\pi}{3})$$
$$= -6\cos\frac{\pi}{3} = -6(\frac{1}{2}) = -3$$
$$y = r\sin\theta = 6\sin\frac{2\pi}{3} = 6\sin(\pi - \frac{\pi}{3})$$
$$= 6\sin\frac{\pi}{3} = 6(\frac{\sqrt{3}}{2}) = 3\sqrt{3}$$
$$\therefore \textit{the cartesian coor. is} \quad (-3, 3\sqrt{3})$$

2)Find the polar coordinates of the point P whose rectangular coordinates $(-2, 2\sqrt{3})$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$$
$$\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \therefore \theta = \frac{\pi}{3}$$

بما ان النقطة تقع في الربع الثاني ، اذن الزاوية هي $\pi - \theta$

$$\therefore \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$
$$\therefore (r,\theta) = (4, \frac{2\pi}{3})$$

3)Find the polar coordinates of the point P whose rectangular coordinates (6,-6) and check the result .

$$r = \sqrt{x^2 + y^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$
$$\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = \frac{-6}{6} = -1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\therefore\ \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$\therefore\ (r,\theta) = (6\sqrt{2}, \frac{7\pi}{4})$$

CHECK :

$$x = r\cos\theta = 6\sqrt{2}\cos\frac{7\pi}{4} = 6\sqrt{2}\cos(2\pi - \frac{\pi}{4})$$

$$= 6\sqrt{2}\cos\frac{\pi}{4} = 6\sqrt{2}(\frac{1}{\sqrt{2}}) = 6$$

$$y = r\sin\theta = 6\sqrt{2}\sin\frac{7\pi}{4} = 6\sqrt{2}\sin(2\pi - \frac{\pi}{4})$$

$$= -6\sqrt{2}\sin\frac{\pi}{4} = -6\sqrt{2}\,\frac{1}{\sqrt{2}} = -6$$

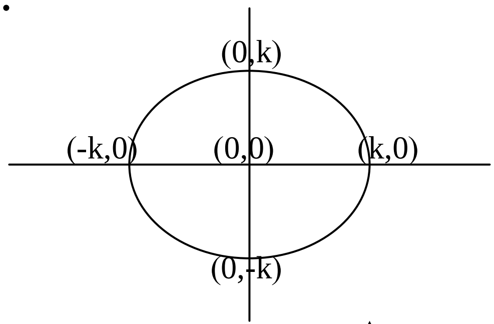
$\therefore$ *the cartesian coor. is* $(6,-6)$

GRAPHS OF POLAR FUNCTIONS

1 $r = k$, k *is cons.*

represent circle with center (0,0) and radius k .

Proof:

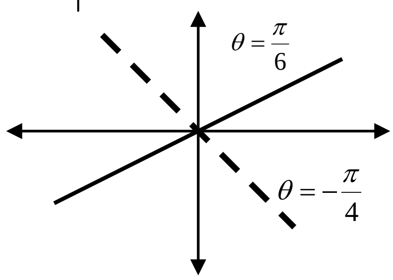


$$r = \sqrt{x^2 + y^2}$$

$$\therefore \sqrt{x^2 + y^2} = k$$

$$x^2 + y^2 = k^2$$

EXAM :Sketch r = 4 .



2 $\theta = k$, k *is cons.*

represent straight line .

EXAM :Sketch $\theta = \frac{\pi}{6}$, $\theta = -\frac{\pi}{4}$

3 $r = \dfrac{k}{\sin\theta \pm \cos\theta}$, k *is cons.*

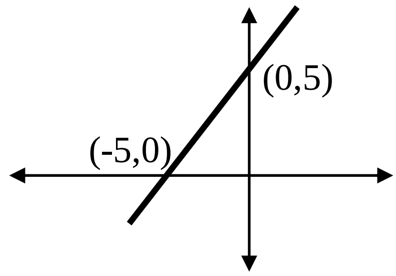
represent straight line.

Proof:

$$r\sin\theta \pm r\cos\theta = k$$

$$y \pm x = k \Rightarrow y = \pm x + k$$

EXAM **:Sketch** $r = \dfrac{5}{\sin\theta - \cos\theta}$



$r\sin\theta - r\cos\theta = 5$

$y - x = 5 \Rightarrow y = x + 5$

$\boxed{4} - \boxed{a}$ $r = 2a\sin\theta$, a *is cons.*

represent circle with center (0,a) and radius is "a"

proof :

$r^2 = 2ar\sin\theta$

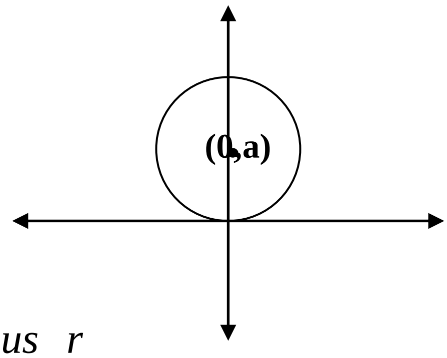
$x^2 + y^2 = 2ay \Rightarrow x^2 + y^2 - 2ay = 0$

$x^2 + y^2 - 2ay + a^2 = a^2 \Rightarrow x^2 + (y - a)^2 = a^2$

By comparing with circle equation

$(x - h)^2 + (y - k)^2 = r^2$

then the graph is circle with center $(0, a)$ *and radius* r



EXAM **:Sketch** $r = 10\sin\theta$

$\boxed{4} - \boxed{b}$ $r = -2a\sin\theta$, a *is cons.*

represent circle with center (0,-a) and radius is "a"

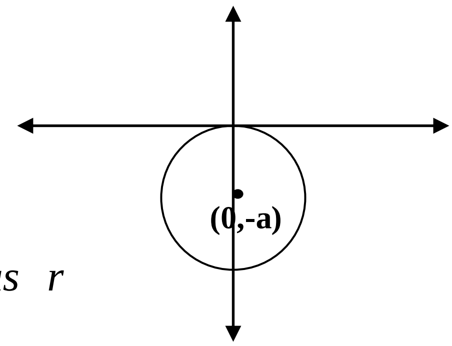
proof :

$r^2 = -2ar\sin\theta$

$x^2 + y^2 = -2ay \Rightarrow x^2 + y^2 + 2ay = 0$

$x^2 + y^2 + 2ay + a^2 = a^2 \Rightarrow x^2 + (y + a)^2 = a^2$

then the graph is circle with center $(0, -a)$ *and radius* r



EXAM **:Sketch** $r = -14\sin\theta$

$\boxed{4} - \boxed{c}$ $r = 2a\cos\theta$, a *is cons.*

represent circle with center (a,0) and radius is "a"

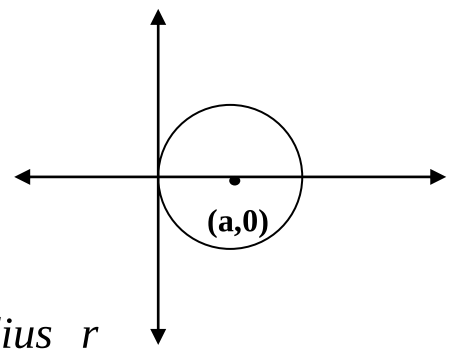
proof :

$r^2 = 2ar\cos\theta$

$x^2 + y^2 = 2ax \Rightarrow x^2 - 2ax + y^2 = 0$

$x^2 - 2ax + y^2 + a^2 = a^2 \Rightarrow (x - a)^2 + y^2 = a^2$

then the graph is circle with center $(a, 0)$ *and radius* r



EXAM **:Sketch** $r = 10\cos\theta$

$\boxed{4}-\boxed{d}$ $r=-2a\cos\theta$, a is cons.

represent circle with center (-a,0) and radius is "a"

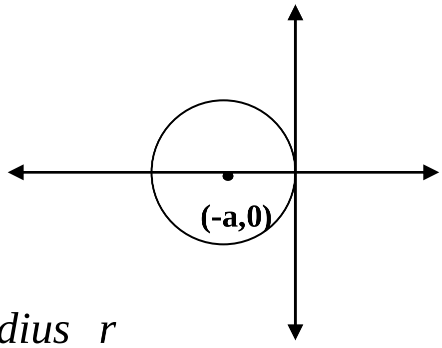
proof :

$r^2=-2ar\cos\theta$

$x^2+y^2=-2ax \Rightarrow x^2+2ax+y^2=0$

$x^2+2ax+y^2+a^2=a^2 \Rightarrow (x+a)^2+y^2=a^2$

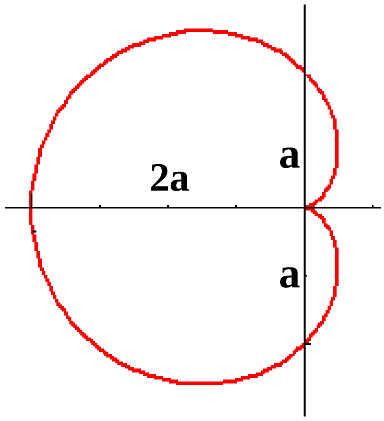
then the graph is circle with center $(-a,0)$ *and radius* r



<u>EXAM</u> :Sketch $r=-8\cos\theta$

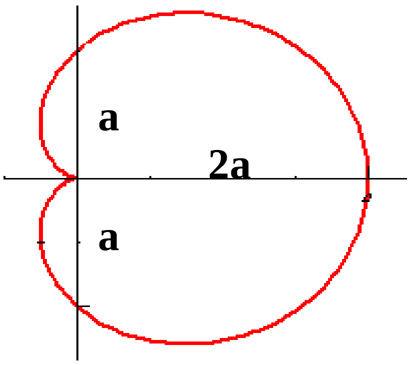
$\boxed{5}-\boxed{a}$ $r=a(1-\cos\theta)$, a is cons.

represent Cardioid



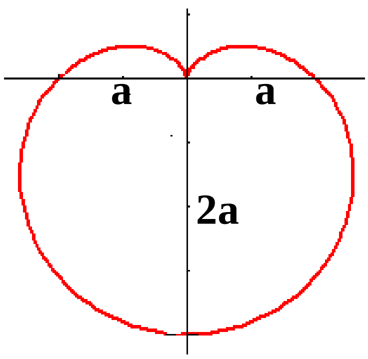
$\boxed{5}-\boxed{b}$ $r=a(1+\cos\theta)$, a is cons.

represent Cardioid



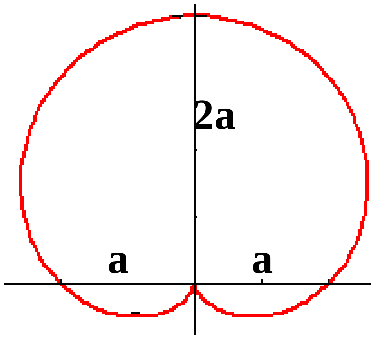
$\boxed{5}-\boxed{c}$ $r=a(1-\sin\theta)$, a is cons.

represent Cardioid



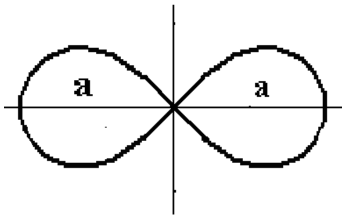
$\boxed{5}-\boxed{d}$ $r = a(1+\sin\theta)$, a is cons.

represent Cardioid



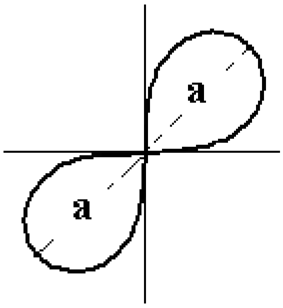
$\boxed{6}-\boxed{a}$ $r^2 = a^2\cos 2\theta$, a is cons.

represent two leaf rose



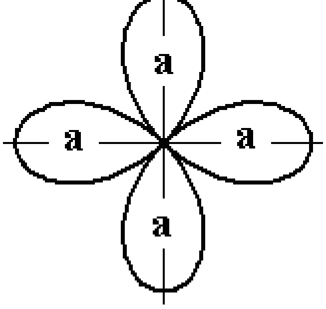
$\boxed{6}-\boxed{b}$ $r^2 = a^2\sin 2\theta$, a is cons.

represent two leaf rose



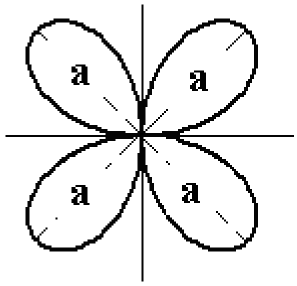
$\boxed{7}-\boxed{a}$ $r = a\cos 2\theta$, a is cons.

represent four leaf rose



$\boxed{7}-\boxed{b}$ $r = a\sin 2\theta$, a is cons.

represent four leaf rose



THE SYMMETRIC التناظر

1) symmetric with x-axis (polar) .

a function f is symmetric with x-axis iff the polar equation doesn't change if we are put $(r,-\theta)$ instead of (r,θ) :-

$$f \quad is \quad symm.about\ x-axis \Leftrightarrow f\ (r,-\theta) = f\ (r,\theta)$$

2) symmetric with y-axis ($\theta = \pi/2$) .

a function f is symmetric with y-axis iff the polar equation doesn't change if we are put $(-r,-\theta)$ instead of (r,θ) <u>or</u> put $(r,\pi-\theta)$ instead of (r,θ) : -

$$f \quad is \quad symm.about\ y-axis \Leftrightarrow f\ (-r,-\theta) = f\ (r,\theta)\ or$$
$$f\ (r,\pi-\theta) = f\ (r,\theta)$$

3) symmetric with origin (pole) .

a function f is symmetric with origin iff the polar equation doesn't change if we are put $(-r,\theta)$ instead of (r,θ) <u>or</u> put $(r,\pi+\theta)$ instead of (r,θ) : -

$$f \quad is \quad symm.about\ origin \Leftrightarrow f\ (-r,\theta) = f\ (r,\theta)\ or$$
$$f\ (r,\pi+\theta) = f\ (r,\theta)$$

4) symmetric with line y=x ($\theta = \pi/4$) .

a function f is symmetric with line y=x iff the polar equation doesn't change if we are put $(r,\dfrac{\pi}{2}-\theta)$ instead of (r,θ) :-

$$f \quad is \quad symm.about\ Line\ \ y = x \Leftrightarrow f\ (r,\frac{\pi}{2}-\theta) = f\ (r,\theta)$$

<u>EXAM</u> :

Test the symmetry of the following :

$\boxed{1}$ $r = 5$

$*\ symm.of\ \ x-axis$

$$f\ (r,-\theta) \overset{?}{=} f\ (r,\theta)$$

$r=5=f(r,\theta)$

$\therefore f(r,-\theta)=f(r,\theta) \Rightarrow \therefore symm.with\ x-axis$

* $symm.of\ \ y-axis$

$f(-r,-\theta) \overset{?}{=} f(r,\theta)$

$-r=5 \Rightarrow r=-5 \neq f(r,\theta)$

$f(r,\pi-\theta) \overset{?}{=} f(r,\theta)$

$r=5=f(r,\theta) \Rightarrow \therefore symm.with\ y-axis$

* $symm.of\ \ origin$

$f(-r,\theta) \overset{?}{=} f(r,\theta)$

$-r=5 \Rightarrow r=-5 \neq f(r,\theta)$

$f(r,\pi+\theta) \overset{?}{=} f(r,\theta)$

$r=5=f(r,\theta) \Rightarrow \therefore symm.with\ origin$

* $symm.of\ \ line\ y=x$

$f(r,\frac{\pi}{2}-\theta) \overset{?}{=} f(r,\theta)$

$r=5=f(r,\theta) \Rightarrow \therefore symm.with\ line\ y=x$

$\boxed{2}\ \ \theta=4$

* $symm.of\ \ x-axis$

$f(r,-\theta) \overset{?}{=} f(r,\theta)$

$\theta=-4 \neq f(r,\theta)$

$\therefore f(r,-\theta) \neq f(r,\theta) \Rightarrow \therefore not\ symm.with\ x-axis$

* $symm.of\ \ y-axis$

$f(-r,-\theta) \overset{?}{=} f(r,\theta)$

$-\theta=4 \Rightarrow \theta=-4 \neq f(r,\theta)$

$f(r,\pi-\theta) \overset{?}{=} f(r,\theta)$

$\pi-\theta=4 \Rightarrow \theta=\pi-4 \neq f(r,\theta) \Rightarrow \therefore not\ symm.with\ y-axis$

$*\ symm.of\ \ origin$

$f\ (-r,\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$\theta = 4 = f\ (r,\theta) \Rightarrow \therefore symm.with\ origin$

$*\ symm.of\ \ line\ \ y = x$

$f\ (r,\frac{\pi}{2}-\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$\frac{\pi}{2} - \theta = 4 \Rightarrow \theta = \frac{\pi}{2} - 4 \neq f\ (r,\theta) \Rightarrow \therefore not\ symm.with\ line\ y = x$

$\boxed{3}\ \ r = 2a\sin\theta$

$*\ symm.of\ \ x-axis$

$f\ (r,-\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$r = 2a\sin(-\theta) \Rightarrow r = -2a\sin\theta \neq f\ (r,\theta)$

$\therefore not\ symm.with\ x-axis$

$*\ symm.of\ \ y-axis$

$f\ (-r,-\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$-r = 2a\sin(-\theta) \Rightarrow -r = -2a\sin\theta \Rightarrow r = 2a\sin\theta = f\ (r,\theta)$

$\therefore f\ \ is\ symm.about\ \ y-axis$

$*\ symm.of\ \ origin$

$f\ (-r,\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$-r = 2a\sin\theta \Rightarrow r = -2a\sin\theta \neq f\ (r,\theta)$

$f\ (r,\pi+\theta) \overset{?}{=} f\ (r,\theta)$

$r = 2a\sin(\pi+\theta) = 2a(-\sin\theta) = -2a\sin\theta \neq f\ (r,\theta)$

$\therefore not\ symm.with\ origin$

$*\ symm.of\ \ line\ \ y = x$

$f\ (r,\frac{\pi}{2}-\theta)\ \overset{?}{=}\ f\ (r,\theta)$

$r = 2a\sin(\frac{\pi}{2}-\theta) = 2a\cos\theta \neq f\ (r,\theta)$

$\therefore not\ symm.with\ line\ \ y = x$

$\boxed{4}\ r = a(1-\cos\theta)$

$*\ symm.of\ \ x-axis$

$f\ (r,-\theta) \overset{?}{=} f\ (r,\theta)$

$r = a(1-\cos(-\theta)) \Rightarrow r = a(1-\cos\theta) = f\ (r,\theta)$

$\therefore\ symm.with\ x-axis$

$*\ symm.of\ \ y-axis$

$f\ (-r,-\theta) \overset{?}{=} f\ (r,\theta)$

$-r = a(1-\cos(-\theta)) \Rightarrow -r = a(1-\cos\theta) \neq f\ (r,\theta)$

$f\ (r,\pi-\theta) \overset{?}{=} f\ (r,\theta)$

$r = a(1-\cos(\pi-\theta)) \Rightarrow r = a(1+\cos\theta) \neq f\ (r,\theta)$

$\therefore not\ symm.with\ y-axis$

$*\ symm.of\ \ origin$

$f\ (-r,\theta) \overset{?}{=} f\ (r,\theta)$

$-r = a(1-\cos\theta) \Rightarrow r = -a(1-\cos\theta) \neq f\ (r,\theta)$

$f\ (r,\pi+\theta) \overset{?}{=} f\ (r,\theta)$

$r = a(1-\cos(\pi+\theta)) = a(1+\cos\theta) \neq f\ (r,\theta)$

$\therefore not\ symm.with\ origin$

$*\ symm.of\ \ line\ \ y = x$

$f\ (r,\frac{\pi}{2}-\theta) \overset{?}{=} f\ (r,\theta)$

$r = a(1-\cos(\frac{\pi}{2}-\theta)) \Rightarrow r = a(1-\sin\theta) \neq f\ (r,\theta)$

$\therefore not\ symm.with\ line\ \ y = x$

$\boxed{5}\ r^2 = a^2\cos 2\theta$

$*\ symm.of\ \ x-axis$

$f\ (r,-\theta) \overset{?}{=} f\ (r,\theta)$

$r^2 = a^2\cos 2(-\theta) \Rightarrow r^2 = a^2\cos 2\theta = f\ (r,\theta)$

$\therefore\ symm.with\ x-axis$

$* \ symm.of \ \ y-axis$

$$f(-r,-\theta) \overset{?}{=} f(r,\theta)$$

$$(-r)^2 = a^2\cos 2(-\theta) \Rightarrow r^2 = a^2\cos 2\theta = f(r,\theta)$$

$\therefore symm.with \ y-axis$

$* \ symm.of \ \ origin$

$$f(-r,\theta) \overset{?}{=} f(r,\theta)$$

$$(-r)^2 = a^2\cos 2\theta \Rightarrow r^2 = a^2\cos 2\theta = f(r,\theta)$$

$\therefore symm.with \ origin$

$* \ symm.of \ \ line \ y = x$

$$f(r,\frac{\pi}{2}-\theta) \overset{?}{=} f(r,\theta)$$

$$r^2 = a^2\cos 2(\frac{\pi}{2}-\theta) \Rightarrow r^2 = a^2\cos(\pi - 2\theta) = -a^2\cos 2\theta \neq f(r,\theta)$$

$\therefore not \ symm.with \ line \ y = x$

HOME WORK

1) Sketch the graph and test the symmetry of the following :

$r = 10$	$r = -14\sin\theta$	$r^2 = 9\sin 2\theta$
$\theta = 12$	$r = -8\cos\theta$	$r = 4\cos 2\theta$
$r = \dfrac{10}{\sin\theta + \cos\theta}$	$r = 4(1-\sin\theta)$	$r = 4\sin 2\theta$

2) Find the Cartesian coordinates of the following :

$(5,\pi)$, $(10,\frac{\pi}{2})$, $(3,\frac{\pi}{4})$, $(-2,\frac{3\pi}{2})$, $(4,-\pi)$

$(10,\frac{7\pi}{6})$, $(12,\frac{5\pi}{3})$, $(1,\frac{\pi}{6})$, $(1,\frac{\pi}{3})$, $(3,\frac{2\pi}{3})$

3) Find the polar coordinates of the following :

$(-5,-5\sqrt{3})$, $(2,2)$, $(-2,-2)$, $(\sqrt{3},-1)$

MOHAMED SABAH AL TAEE
M.SC / MATHEMATICS
E-MAIL : msmt_80@yahoo.com
2013 -2014