Properties of exp. fun.

$Lne = 1 \quad , \quad e^0 = 1 \quad , \quad e = 2.718...$

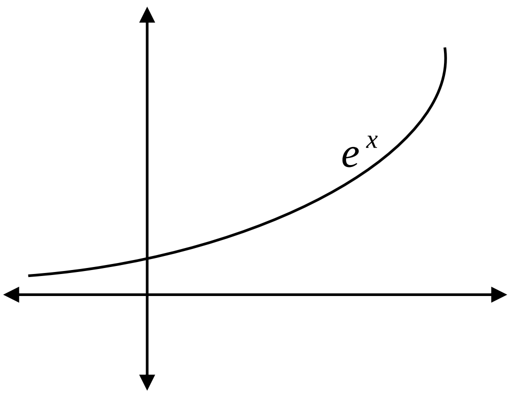
$e^{a+b} = e^a . e^b$

$e^{ax} = (e^a)^x = (e^x)^a$

its domain is all real no.

its range are $y > 0$

the graph of $y = e^x$ *is*



أشتقاق الدوال الاسية :

$$\boxed{\frac{d}{dx}e^x = e^x}$$

Proof:

let $y = e^x$

$Lny = Lne^x = xLne = x$

$\frac{1}{y}\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = y = e^x$

and in general form

$$\boxed{\frac{d}{dx}e^u = e^u \frac{du}{dx}}$$

EXAM: Find $\frac{dy}{dx}$ **to :**

$y = e^{3x} \Rightarrow \frac{dy}{dx} = e^{3x}(3) = 3e^{3x}$

$y = e^{-x} \Rightarrow \frac{dy}{dx} = e^{-x}(-1) = -e^{-x}$

$y = xe^x \Rightarrow \frac{dy}{dx} = xe^x + e^x(1) = xe^x + e^x$

$y = e^{x^3} + LnLnx + x^3 + e \Rightarrow \frac{dy}{dx} = e^{x^3}(3x^2) + \frac{1}{xLnx} + 3x^2 + 0$

EXAM : Simplify

$y = e^{2Lnx} \Rightarrow e^{Lnx^2} = x^2$

$y = e^{2Ln\frac{x}{y}} \Rightarrow e^{Ln(\frac{x}{y})^2} = (\frac{x}{y})^2$

تكاملات الدوال الاسية :

$$\int e^x\,dx = e^x + C$$

and in general form :

$$\int e^u\,du = e^u + C$$

EXAM : Find

$$\int e^{4x}dx = \frac{1}{4}\int 4e^{4x}dx = \frac{1}{4}e^{4x} + C$$

$$\int e^{-2x}dx = \frac{-1}{2}\int -2e^{-2x}dx = \frac{-1}{2}e^{-2x} + C$$

$$\int e^{2Lnx}dx = \int e^{Lnx^2}dx = \int x^2dx = \frac{x^3}{3} + C$$

$$\int x^2e^{-2x^3}dx = \frac{-1}{6}\int -6x^2e^{-2x^3}dx = \frac{-1}{6}e^{-2x^3} + C$$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}dx = Ln\left|e^x - e^{-x}\right| + C$$

<u>Exponetial fun. of type</u> $a^x \quad , \quad a^u$

$$a^x = e^{xLna} = e^{Lna^x} = a^x$$

اشتقاق الدالة الاسية من نوع a^x

$$\frac{d}{dx}a^x = a^x Lna$$

and in general form :

$$\frac{d}{dx}a^u = a^u Lna\frac{du}{dx}$$

EXAM: Find $\frac{dy}{dx}$ **to :**

$$y = 3^{x^4} \Rightarrow \frac{dy}{dx} = 3^{x^4}.Ln3.(4x^3)$$

$$y = 7^{x^2} \Rightarrow \frac{dy}{dx} = 7^{x^2}.Ln7.(2x)$$

$$y = 10^{Lnx} \Rightarrow \frac{dy}{dx} = 10^{Lnx}.Ln10.(\frac{1}{x})$$

تكامل الدالة الاسية من نوع a^x

$$\int a^u du = \frac{a^u}{Lna} + C \quad , \quad a > 0 \ , \ a \neq 1$$

EXAM : Find

$$\int 3^x dx = \frac{3^x}{Ln3} + C$$

$$\int 7^{x^2} x \ dx = \frac{1}{2}\int 7^{x^2} 2x \ dx = \frac{1}{2}\frac{7^{x^2}}{Ln7} + C$$

EXAM : a) Find the area bounded by the curve $y = e^{-4x}$ and the x-axis from x=-1 , 1
b) The area in (a) rotated about the x-axis , find the volume generated ?

$$(a) \ A = \int_a^b y dx = \int_{-1}^{1} e^{-4x} dx = \frac{-1}{4} e^{-4x} \Big|_{-1}^{1}$$

$$= \frac{-1}{4}\left(e^{-4} - e^{4}\right) \ unit^2$$

$$(b) \ V_x = \pi \int_a^b y^2 dx = \pi \int_{-1}^{1} (e^{-4x})^2 dx = \pi \int_{-1}^{1} e^{-8x} dx$$

$$= \frac{-\pi}{8} e^{-8x} \Big|_{-1}^{1} = ? \quad unit^3$$

HOME WORK

1) Show that

$(a) \ y = xe^{-x}$ satisfies the eq. $xy' = (1-x)y$

$(b) \ y = xe^{-x^2/2}$ satisfies the eq. $xy' = (1-x^2)y$

2) Find $\frac{dy}{dx}$ to :

$$y = Ln(1 - xe^{-x}) \quad , \quad y = \frac{e^x}{Lnx} \quad , \quad y = x^3 e^{-x}$$

3) Find

$$\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}} dx \quad , \quad \int \sqrt{e^{x^2}} dx$$

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