$\boxed{1}$ $Sinh(x) = \dfrac{e^x - e^{-x}}{2}$, $D_f = \mathbb{R}$, $R_f = \mathbb{R}$

$\boxed{2}$ $Cosh(x) = \dfrac{e^x + e^{-x}}{2}$, $D_f = \mathbb{R}$, $R_f = [1, \infty)$

$\boxed{3}$ $\tan h(x) = \dfrac{Sinh(x)}{Cosh(x)} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, $D_f = \mathbb{R}$, $R_f = (-1, 1)$

$\boxed{4}$ $\cot h(x) = \dfrac{Cosh(x)}{Sinh(x)} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$, $D_f = \mathbb{R} - \{0\}$, $R_f = \mathbb{R} - \{0\}$

$\boxed{5}$ $Sech(x) = \dfrac{1}{Cosh(x)} = \dfrac{2}{e^x + e^{-x}}$, $D_f = \mathbb{R}$, $R_f = (0, 1]$

$\boxed{6}$ $\csc h(x) = \dfrac{1}{Sinh(x)} = \dfrac{2}{e^x - e^{-x}}$, $D_f = \mathbb{R} - \{0\}$, $R_f = \mathbb{R} - [-1, 1]$

Some rules :

$\langle 1 \rangle$ $\cosh(x) + \sinh(x) = e^x$ $\quad$ $\langle 2 \rangle$ $\cosh(x) - \sinh(x) = e^{-x}$

$\langle 3 \rangle$ $\cosh^2(x) - \sinh^2(x) = 1$ $\quad (main\ rule)$

$\langle 4 \rangle$ $1 - \tanh^2(x) = \sec h^2(x)$ $\quad$ $\langle 5 \rangle$ $\coth^2(x) - 1 = \csc h^2(x)$

$\langle 6 \rangle$ $\cosh^2(x) = \dfrac{1}{2}(\cosh 2x + 1)$ $\quad$ $\langle 7 \rangle$ $\sinh^2(x) = \dfrac{1}{2}(\cosh 2x - 1)$

$\langle 8 \rangle$ $\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x \Rightarrow \sinh(2x) = 2 \sinh x \cosh y$

$\langle 9 \rangle$ $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \Rightarrow \cosh(2x) = \cosh^2 x + \sinh^2 x$

$\langle 10 \rangle$ $\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \cdot \tanh y} \Rightarrow \tanh(2x) = \dfrac{2 \tanh x}{1 + \tanh^2 x}$

The proof:

1) $\cosh(x)+\sinh(x)=\dfrac{e^{x}+e^{-x}}{2}+\dfrac{e^{x}-e^{-x}}{2}$

$$=\frac{2e^{x}}{2}=e^{x}$$

2) $\cosh(x)-\sinh(x)=\dfrac{e^{x}+e^{-x}}{2}-\dfrac{e^{x}-e^{-x}}{2}$

$$=\frac{2e^{-x}}{2}=e^{-x}$$

3) $\cosh^{2}(x)-\sinh^{2}(x)=[\cosh(x)+\sinh(x)][\cosh(x)-\sinh(x)]$

$$=e^{x}\cdot e^{-x}=e^{0}=1$$

Ex: Find the value of cosh(0) , sinh(0)

$$\cosh(0)=\frac{e^{0}+e^{-0}}{2}=1$$

$$\sinh(0)=\frac{e^{0}-e^{-0}}{2}=0$$

HYPERBOLIC TRIG. FUN. DERIVATIVE :

$\langle 1\rangle \ \frac{d}{dx}\sinh x = \cosh x \qquad \Rightarrow Gen. \Rightarrow \frac{d}{dx}\sinh u = \cosh u \cdot du$

$\langle 2\rangle \ \frac{d}{dx}\cosh x = \sinh x \qquad \Rightarrow Gen. \Rightarrow \frac{d}{dx}\cosh u = \sinh u \cdot du$

$\langle 3\rangle \ \frac{d}{dx}\tanh x = \text{sech}^2 x \qquad \Rightarrow Gen. \Rightarrow \frac{d}{dx}\tanh u = \text{sech}^2 u \cdot du$

$\langle 4\rangle \ \frac{d}{dx}\coth x = -\text{csch}^2 x \qquad \Rightarrow Gen. \Rightarrow \frac{d}{dx}\coth u = -\text{csch}^2 u \cdot du$

$\langle 5\rangle \ \frac{d}{dx}\text{sech}\, x = -\sec hx \tanh x \Rightarrow Gen. \Rightarrow \frac{d}{dx}\text{sech}\, u = -\sec hu \tanh u \cdot du$

$\langle 6\rangle \ \frac{d}{dx}\text{csch}\, x = -\csc hx \coth x \Rightarrow Gen. \Rightarrow \frac{d}{dx}\text{csch}\, u = -\csc hu \coth u \cdot du$

PROOF:

$$\langle 1\rangle \ \frac{d}{dx}\sinh x = \frac{d}{dx}\frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$\langle 3\rangle \ \frac{d}{dx}\tanh x = \frac{d}{dx}[\frac{e^x - e^{-x}}{e^x + e^{-x}}] = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} + 2 + e^{-2x})}{(e^x + e^{-x})^2}$$

$$= \frac{4}{(e^x + e^{-x})^2} = [\frac{2}{e^x + e^{-x}}]^2$$

$$= \sec h^2 x$$

Exam :**Find** $\frac{dy}{dx}$ **to the functions :**

$\boxed{1}$ $y = \tanh(x^2)$

$$y' = 2x \sec h^2(x^2)$$

$\boxed{2}$ $y = e^{\sinh 3x}$

$$y' = e^{\sinh 3x} \cdot \cosh 3x \cdot 3$$

$\boxed{3}$ $y = Ln(1+\cosh 4x)$

$$y' = \frac{4\sinh 4x}{1+\cosh 4x}$$

$\boxed{4}$ $y = (\csc hx)^{Lnx}$

$$Lny = Ln(\csc hx)^{Lnx} = Lnx \cdot Ln(\csc hx)$$

$$\frac{1}{y} y' = Lnx \cdot \frac{-\csc hx \coth x}{\csc hx} + \frac{Ln(\csc hx)}{x}$$

$$y' = y[Lnx \cdot -\coth x + \frac{Ln(\csc hx)}{x}]$$

$$y' = (\csc hx)^{Lnx}[-\coth x \cdot Lnx + \frac{Ln(\csc hx)}{x}]$$

HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1 \rangle$ $\int \sinh u \cdot du = \cosh u + C$

$\langle 2 \rangle$ $\int \cosh u \cdot du = \sinh u + C$

$\langle 3 \rangle$ $\int \text{sech}^2 u \cdot du = \tanh u + C$

$\langle 4 \rangle$ $\int \text{csch}^2 u \cdot du = -\coth u + C$

$\langle 5 \rangle$ $\int \sec hu \tanh u \cdot du = -\text{sech} u + C$

$\langle 6 \rangle$ $\int \csc hu \coth u \cdot du = -\text{csch} u + C$

$\lfloor 1 \rfloor \int \cosh 7x\,dx = \frac{1}{7}\sinh 7x + c$

$\lfloor 2 \rfloor \int \tanh x \operatorname{sec}h^2 x\,dx = \frac{1}{2}\tanh^2 x + c$

$\lfloor 3 \rfloor \int \frac{\sinh(Lnx)}{x}dx = \cosh(Lnx) + c$

INVERSE HYPERBOLIC TRIG. FUN.

$\langle 1 \rangle\ \sinh^{-1}(x) = Ln(x + \sqrt{x^2+1})$

$\langle 2 \rangle\ \cosh^{-1}(x) = Ln(x + \sqrt{x^2-1})$

$\langle 3 \rangle\ \tanh^{-1}(x) = \frac{1}{2}Ln(\frac{1+x}{1-x})$

$\langle 4 \rangle\ \coth^{-1}(x) = \frac{1}{2}Ln(\frac{x+1}{x-1})$

$\langle 5 \rangle\ \operatorname{sech}^{-1}(x) = Ln(\frac{1+\sqrt{1-x^2}}{x})$

$\langle 6 \rangle\ cs\,\mathrm{ch}^{-1}(x) = Ln(\frac{1+\sqrt{1+x^2}}{x})$

Proof :

1) Let $w = sinh^{-1}x$

$x = sinhw$

$x = \frac{e^w - e^{-w}}{2}$

$2x = e^w - e^{-w}$, By multiply two sides by e^w

$e^{2w} - 2xe^w - 1 = 0$

$$e^w = \frac{2x \mp \sqrt{4x^2 - 4(1)(-1)}}{2(1)}$$

$$e^w = \frac{2x \mp 2\sqrt{x^2+1}}{2}$$

$e^w = x \mp \sqrt{x^2+1}$

$w = ln(x + \sqrt{x^2+1})$

$$\therefore sinh^{-1}x = ln(x + \sqrt{x^2+1})$$

3) Let $w = tanh^{-1}x$

$x = tanhw$

$x = \frac{e^w - e^{-w}}{e^w + e^{-w}}$

$xe^w + xe^{-w} = e^w - e^{-w}$, By multiply two sides by e^w

$xe^{2w} + x = e^{2w} - 1$

$xe^{2w} - e^{2w} = -x - 1 \rightarrow e^{2w}(x-1) = -x - 1$

$e^{2w} = \frac{-x-1}{x-1}$

$e^{2w} = \frac{1+x}{1-x}$

$2w = ln\left(\frac{1+x}{1-x}\right) \rightarrow w = \frac{1}{2} ln\left(\frac{1+x}{1-x}\right)$

$$\therefore tanh^{-1}x = \frac{1}{2} ln\left(\frac{1+x}{1-x}\right)$$

Some rules :

$\text{csch}^{-1}(x) = \sinh^{-1}(\frac{1}{x})$

$\sec h^{-1}(x) = \cosh^{-1}(\frac{1}{x})$

$\coth^{-1}(x) = \tanh^{-1}(\frac{1}{x})$

Proof :

1) Let $y = sinh^{-1}\frac{1}{x}$

$sinhy = \frac{1}{x}$

$cschy = x$

$y = csch^{-1}x$

$\therefore \; csch^{-1}x = sinh^{-1}\frac{1}{x}$

INVERSE HYPERBOLIC TRIG. FUN. DERIVATIVE :

$\langle 1\rangle \; \frac{d}{dx}\sinh^{-1}(u) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$

$\langle 2\rangle \; \frac{d}{dx}\cosh^{-1}(u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$

$\langle 3\rangle \; \frac{d}{dx}\tanh^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$

$\langle 4\rangle \; \frac{d}{dx}\coth^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$

$\langle 5\rangle \; \frac{d}{dx}\mathrm{sech}^{-1}(u) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$

$\langle 6\rangle \; \frac{d}{dx}cs\,\mathrm{ch}^{-1}(u) = \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$

Proof :

1) $sinh^{-1}x = ln(x + \sqrt{x^2 + 1})$

$$(sinh^{-1}x)' = \frac{1}{x + \sqrt{x^2 + 1}} (1 + \frac{2x}{2\sqrt{x^2 + 1}})$$

$$(sinh^{-1}x)' = \frac{1}{x + \sqrt{x^2 + 1}} (\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}})$$

$$\therefore (sinh^{-1}x)' = \frac{1}{\sqrt{x^2 + 1}}$$

The proof of 2 , 3 , 4 , 5 and 6 is Home Work

exam :

1) $y = \sinh^{-1}(3x)$

$$y' = \frac{3}{\sqrt{1 + 9x^2}}$$

2) $y = \cosh^{-1}(e^x)$

$$y' = \frac{e^x}{\sqrt{e^{2x} - 1}}$$

3) $y = \operatorname{sech}^{-1}(\ln x)$

$$y' = \frac{-1}{x \ln x \sqrt{1 - \ln^2 x}}$$

4) $y = (\coth^{-1}(e^{-x}))^5$

$$y' = 5(\coth^{-1}(e^{-x}))^4[\frac{-e^{-x}}{1 - e^{-2x}}]$$

INVERSE HYPERBOLIC TRIG. FUN. INTEGRATION :

$$\langle 1\rangle \int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a})+C$$

$$\langle 2\rangle \int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a})+C$$

$$\langle 3,4\rangle \int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) \end{cases} +C$$

$$\langle 5\rangle \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left|\frac{u}{a}\right|+C$$

$$\langle 6\rangle \int \frac{du}{|u|\sqrt{a^2+u^2}} = -\frac{1}{a}cs\,\mathrm{ch}^{-1}\frac{u}{a}+C$$

exam :

$$1)\ \int \frac{dx}{\sqrt{1+4x^2}} = \frac{1}{2}\sinh^{-1}(2x)+C$$

$$2)\ \int \frac{dx}{\sqrt{9+16x^2}} = \frac{1}{4}\sinh^{-1}(\frac{4x}{3})+C$$

$$3)\ \int \frac{e^x}{1-e^{2x}}\,dx = \tanh^{-1}(e^x)+C$$

$$4)\ \int \frac{dx}{9-x^2} = \frac{1}{3}\tanh^{-1}\frac{x}{3}+C$$

HOME WORK

$\boxed{1}$ $if \quad \sinh(x)=\dfrac{3}{4} \;,\; Find \quad \tanh(4x)$

$\boxed{2}$ $if \quad \tanh(x)=\dfrac{-4}{5} \;,\; show \;\; that : \; \sinh(x)+\cosh(x)=\dfrac{1}{3}$

$\boxed{3}$ $Find \; the \; value \; of \quad \cosh(\ln 4)\,?$

$\boxed{4}$ $Evaluate :$

$$\int \frac{\cosh 2x}{\sinh^3 2x}dx \quad , \quad \int \frac{\cosh(\tan^{-1}x)}{1+x^2}dx \quad , \quad \int \frac{\operatorname{sec}h^2(e^x)}{e^x}dx \quad , \quad \int \frac{\operatorname{csc}h^2(\sin^{-1}x)}{\sqrt{1-x^2}}dx$$

$$\int \frac{\operatorname{csc}h\sqrt{x}\,\coth\sqrt{x}}{\sqrt{x}}\,dx \quad , \quad \int \frac{\operatorname{sec}h\left(\frac{x+1}{x-1}\right)\tanh\left(\frac{x+1}{x-1}\right)}{(x-1)^2}\,dx$$

$\boxed{5}$ $Find \;\; y' \;\; to \;\; :$

$$y=(10)^{\tanh(\ln x)} \quad , \quad y=\operatorname{sec}h^3(\tan x) \quad , \quad y=e^{\tan(\operatorname{sec}hx^2)}$$

$\boxed{6}$ $Evaluate :$

$$\int \frac{dx}{4-x^2} \quad , \quad \int \frac{dx}{x\sqrt{4-x^4}}$$

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