[1] $Sinh(x)=\frac{e^{x}-e^{-x}}{2}$, $D_f=\mathbb{R}$, $R_f=\mathbb{R}$

[2] $Cosh(x)=\frac{e^{x}+e^{-x}}{2}$, $D_f=\mathbb{R}$, $R_f=[1,\infty)$

[3] $\tan h(x)=\frac{Sinh(x)}{Cosh(x)}=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$, $D_f=\mathbb{R}$, $R_f=(-1,1)$

[4] $\cot h(x)=\frac{Cosh(x)}{Sinh(x)}=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$, $D_f=\mathbb{R}-\{0\}$, $R_f=\mathbb{R}-\{0\}$

[5] $Sech(x)=\frac{1}{Cosh(x)}=\frac{2}{e^{x}+e^{-x}}$, $D_f=\mathbb{R}$, $R_f=(0,1]$

[6] $\csc h(x)=\frac{1}{Sinh(x)}=\frac{2}{e^{x}-e^{-x}}$, $D_f=\mathbb{R}-\{0\}$, $R_f=\mathbb{R}-[-1,1]$

Some rules :

$\langle 1\rangle$ $\cosh(x)+\sinh(x)=e^{x}$ $\langle 2\rangle$ $\cosh(x)-\sinh(x)=e^{-x}$

$\langle 3\rangle$ $\cosh^{2}(x)-\sinh^{2}(x)=1$ $(main\ rule)$

$\langle 4\rangle$ $1-\tanh^{2}(x)=\sec h^{2}(x)$ $\langle 5\rangle$ $\coth^{2}(x)-1=\csc h^{2}(x)$

$\langle 6\rangle$ $\cosh^{2}(x)=\frac{1}{2}(\cosh 2x+1)$ $\langle 7\rangle$ $\sinh^{2}(x)=\frac{1}{2}(\cosh 2x-1)$

$\langle 8\rangle$ $\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x\Rightarrow\sinh(2x)=2\sinh x\cosh y$

$\langle 9\rangle$ $\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y\Rightarrow\cosh(2x)=\cosh^{2}x+\sinh^{2}x$

$\langle 10\rangle$ $\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\cdot\tanh y}\Rightarrow\tanh(2x)=\frac{2\tanh x}{1+\tanh^{2}x}$

The proof:

1) $\cosh(x)+\sinh(x)=\frac{e^{x}+e^{-x}}{2}+\frac{e^{x}-e^{-x}}{2}$

$$=\frac{2e^{x}}{2}=e^{x}$$

2) $\cosh(x)-\sinh(x)=\frac{e^{x}+e^{-x}}{2}-\frac{e^{x}-e^{-x}}{2}$

$$=\frac{2e^{-x}}{2}=e^{-x}$$

3) $\cosh^2(x)-\sinh^2(x)=[\cosh(x)+\sinh(x)][\cosh(x)-\sinh(x)]$

$$=e^{x}\cdot e^{-x}=e^{0}=1$$

<u>Ex:</u> Find the value of cosh(0) , sinh(0)

$$\cosh(0)=\frac{e^{0}+e^{-0}}{2}=1$$

$$\sinh(0)=\frac{e^{0}-e^{-0}}{2}=0$$

HYPERBOLIC TRIG. FUN. DERIVATIVE :

$\langle 1\rangle\ \frac{d}{dx}\sinh x=\cosh x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\sinh u=\cosh u\cdot du$

$\langle 2\rangle\ \frac{d}{dx}\cosh x=\sinh x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\cosh u=\sinh u\cdot du$

$\langle 3\rangle\ \frac{d}{dx}\tanh x=\text{sech}^2 x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\tanh u=\text{sech}^2 u\cdot du$

$\langle 4\rangle\ \frac{d}{dx}\coth x=-\text{csch}^2 x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\coth u=-\text{csch}^2 u\cdot du$

$\langle 5\rangle\ \frac{d}{dx}\text{sech}\, x=-\sec hx\ \tanh x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\text{sech}\, u=-\sec hu\ \tanh u\cdot du$

$\langle 6\rangle\ \frac{d}{dx}\text{csch}\, x=-\csc hx\ \coth x \quad \Rightarrow Gen.\Rightarrow \frac{d}{dx}\text{csch}\, u=-\csc hu\ \coth u\cdot du$

<u>PROOF:</u>

$$\langle 1\rangle\ \frac{d}{dx}\sinh x=\frac{d}{dx}\frac{e^{x}-e^{-x}}{2}=\frac{e^{x}+e^{-x}}{2}=\cosh x$$

$$\langle 3\rangle\ \frac{d}{dx}\tanh x=\frac{d}{dx}\left[\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right]=\frac{(e^{x}+e^{-x})(e^{x}+e^{-x})-(e^{x}-e^{-x})(e^{x}-e^{-x})}{(e^{x}+e^{-x})^2}$$

$$=\frac{(e^{2x}+2+e^{-2x})-(e^{2x}+2+e^{-2x})}{(e^{x}+e^{-x})^2}$$

$$=\frac{4}{(e^{x}+e^{-x})^2}=\left[\frac{2}{e^{x}+e^{-x}}\right]^2$$

$$=\sec h^2 x$$

Exam :Find $\frac{dy}{dx}$ to the functions :

[1] $y = \tanh(x^2)$

$y' = 2x \operatorname{sec}h^2(x^2)$

[2] $y = e^{\sinh 3x}$

$y' = e^{\sinh 3x} \cdot \cosh 3x \cdot 3$

[3] $y = Ln(1+\cosh 4x)$

$y' = \frac{4\sinh 4x}{1+\cosh 4x}$

[4] $y = (\operatorname{csc} hx)^{Lnx}$

$Lny = Ln(\operatorname{csc} hx)^{Lnx} = Lnx \cdot Ln(\operatorname{csc} hx)$

$\frac{1}{y} y' = Lnx \cdot \frac{-\operatorname{csc} hx \coth x}{\operatorname{csc} hx} + \frac{Ln(\operatorname{csc} hx)}{x}$

$y' = y[Lnx \cdot -\coth x + \frac{Ln(\operatorname{csc} hx)}{x}]$

$y' = (\operatorname{csc} hx)^{Lnx}[-\coth x \cdot Lnx + \frac{Ln(\operatorname{csc} hx)}{x}]$

HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1 \rangle \int \sinh u \cdot du = \cosh u + C$

$\langle 2 \rangle \int \cosh u \cdot du = \sinh u + C$

$\langle 3 \rangle \int \operatorname{sech}^2 u \cdot du = \tanh u + C$

$\langle 4 \rangle \int \operatorname{csch}^2 u \cdot du = -\coth u + C$

$\langle 5 \rangle \int \operatorname{sec} hu \tanh u \cdot du = -\operatorname{sech} u + C$

$\langle 6 \rangle \int \operatorname{csc} hu \coth u \cdot du = -\operatorname{csch} u + C$

$\lfloor 1 \rfloor \int \cosh 7x dx = \frac{1}{7} \sinh 7x + c$

$\lfloor 2 \rfloor \int \tanh x \operatorname{sec} h^2 x dx = \frac{1}{2} \tanh^2 x + c$

$\lfloor 3 \rfloor \int \frac{\sinh(Lnx)}{x} dx = \cosh(Lnx) + c$

INVERSE HYPERBOLIC TRIG. FUN.

$\langle 1\rangle \ \sinh^{-1}(x) = Ln(x + \sqrt{x^2+1})$

$\langle 2\rangle \ \cosh^{-1}(x) = Ln(x + \sqrt{x^2-1})$

$\langle 3\rangle \ \tanh^{-1}(x) = \frac{1}{2} Ln(\frac{1+x}{1-x})$

$\langle 4\rangle \ \coth^{-1}(x) = \frac{1}{2} Ln(\frac{x+1}{x-1})$

$\langle 5\rangle \ \text{sech}^{-1}(x) = Ln(\frac{1+\sqrt{1-x^2}}{x})$

$\langle 6\rangle \ cs\,\text{ch}^{-1}(x) = Ln(\frac{1+\sqrt{1+x^2}}{x})$

<u>Some rules :</u>

$\text{csch}^{-1}(x) = \sinh^{-1}(\frac{1}{x})$

$sec\,\text{h}^{-1}(x) = \cosh^{-1}(\frac{1}{x})$

$\coth^{-1}(x) = \tanh^{-1}(\frac{1}{x})$

<u>INVERSE HYPERBOLIC TRIG. FUN. DERIVATIVE :</u>

$\langle 1\rangle \ \frac{d}{dx}\sinh^{-1}(u) = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$

$\langle 2\rangle \ \frac{d}{dx}\cosh^{-1}(u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$

$\langle 3\rangle \ \frac{d}{dx}\tanh^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$

$\langle 4\rangle \ \frac{d}{dx}\coth^{-1}(u) = \frac{1}{1-u^2}\frac{du}{dx}$

$\langle 5\rangle \ \frac{d}{dx}\text{sech}^{-1}(u) = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$

$\langle 6\rangle \ \frac{d}{dx}cs\,\text{ch}^{-1}(u) = \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$

exam :

1) $y = \sinh^{-1}(3x)$

$$y' = \frac{3}{\sqrt{1+9x^2}}$$

2) $y = \cosh^{-1}(e^x)$

$$y' = \frac{e^x}{\sqrt{e^{2x}-1}}$$

3) $y = \operatorname{sech}^{-1}(\ln x)$

$$y' = \frac{-1}{x \ln x \sqrt{1-\ln^2 x}}$$

4) $y = (\coth^{-1}(e^{-x}))^5$

$$y' = 5(\coth^{-1}(e^{-x}))^4[\frac{-e^{-x}}{1-e^{-2x}}]$$

INVERSE HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1\rangle \int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a}) + C$

$\langle 2\rangle \int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + C$

$$\langle 3,4\rangle \int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) \end{cases} + C$$

$\langle 5\rangle \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left|\frac{u}{a}\right| + C$

$\langle 6\rangle \int \frac{du}{|u|\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\frac{u}{a} + C$

exam :

1) $\int \frac{dx}{\sqrt{1+4x^2}} = \frac{1}{2}\sinh^{-1}(2x) + C$

2) $\int \frac{dx}{\sqrt{9+16x^2}} = \frac{1}{4}\sinh^{-1}(\frac{4x}{3}) + C$

3) $\int \frac{e^{x}}{1-e^{2x}}\,dx = \tanh^{-1}(e^{x})+C$

4) $\int \frac{dx}{9-x^{2}} = \frac{1}{3}\tanh^{-1}\frac{x}{3}+C$

HOMEWORK

$\boxed{1}$ $if \quad \sinh(x)=\frac{3}{4}\ ,\ Find \quad \tanh(4x)$

$\boxed{2}$ $if \quad \tanh(x)=\frac{-4}{5}\ ,\ show\ that:\ \sinh(x)+\cosh(x)=\frac{1}{3}$

$\boxed{3}$ $Find\ the\ value\ of \quad \cosh(\ln 4)\ ?$

$\boxed{4}$ $Evaluate:$

$$\int \frac{\cosh 2x}{\sinh^{3} 2x}dx \quad , \int \frac{\cosh(\tan^{-1}x)}{1+x^{2}}dx \quad , \int \frac{\operatorname{sec}h^{2}(e^{x})}{e^{x}}dx \quad , \int \frac{\operatorname{csc}h^{2}(\sin^{-1}x)}{\sqrt{1-x^{2}}}dx$$

$$\int \frac{\operatorname{csc}h\sqrt{x}\ \coth\sqrt{x}}{\sqrt{x}}\,dx \quad , \quad \int \frac{\operatorname{sec}h(\frac{x+1}{x-1})\tanh(\frac{x+1}{x-1})}{(x-1)^{2}}\,dx$$

$\boxed{5}$ $Find\ y'\ to:$

$$y=(10)^{\tanh(\ln x)} \quad , \quad y=\operatorname{sec}h^{3}(\tan x) \quad , \quad y=e^{\tan(\operatorname{sec}hx^{2})}$$

$\boxed{6}$ $Evaluate:$

$$\int \frac{dx}{4-x^{2}} \quad , \quad \int \frac{dx}{x\sqrt{4-x^{4}}}$$

MOHAMED SABAH AL TAEE
M.SC / MATHEMATICS
E-MAIL : msmt_80@yahoo.com
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