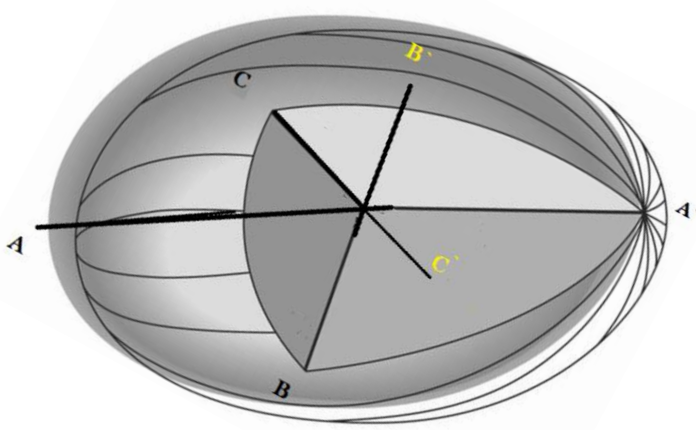
# Relation of rupture to strain



A convenient way of visualizing deformation is to imagine the change in shape of an imaginary sphere in the rocks.
For example, imagine a sphere in a body of granite. If the granite were compressed from the top and bottom , the imaginary sphere would become deformed into an oblate spheroid مفلطح كروي and the short axis of which would be vertical.
the most general solid resulting from the deformation of a sphere is an ellipsoid.
This imaginary figure may be called the strain ellipsoid or deformation ellipsoid.

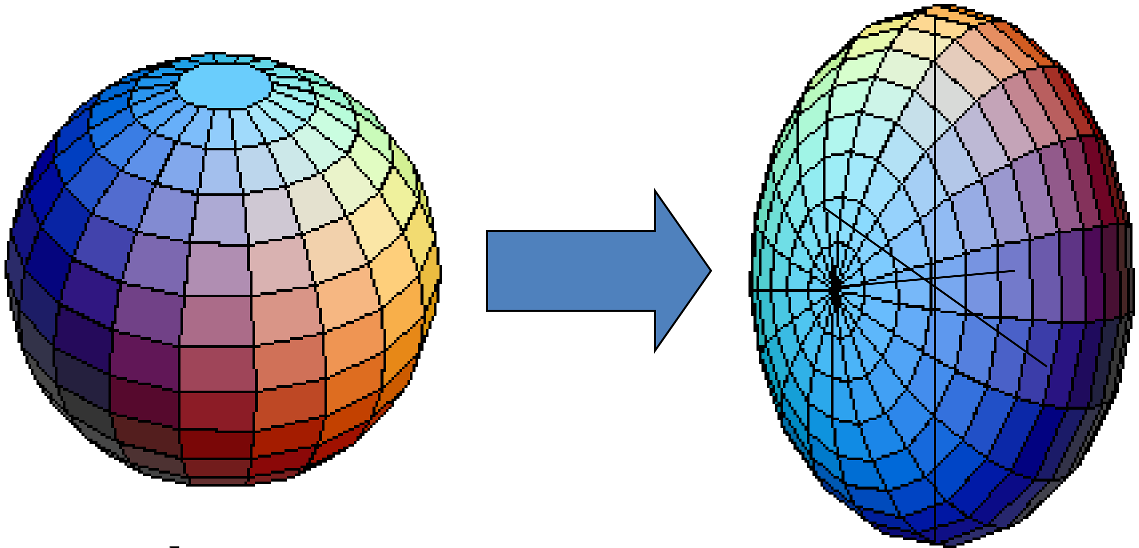



AA' is the largest strain axis

BB‘ is the intermediate strain axis

CC‘ is the least strain axis

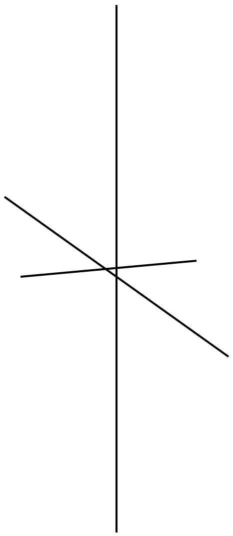
# Principal stretches

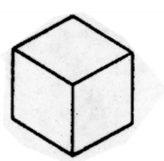


Sphere

Ellipsoid

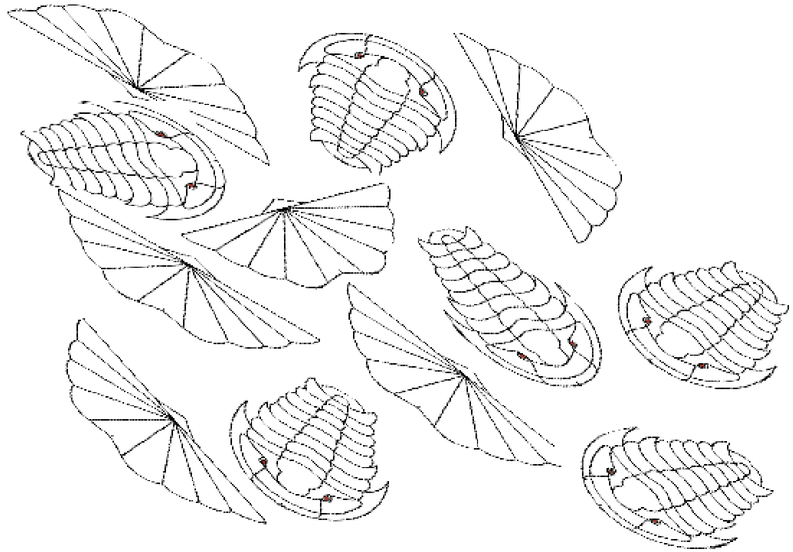
In 3D, $S_1$ is max, $S_3$ is min, and $S_2$ is intermediate



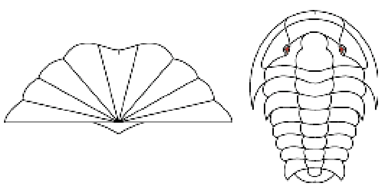


Cube

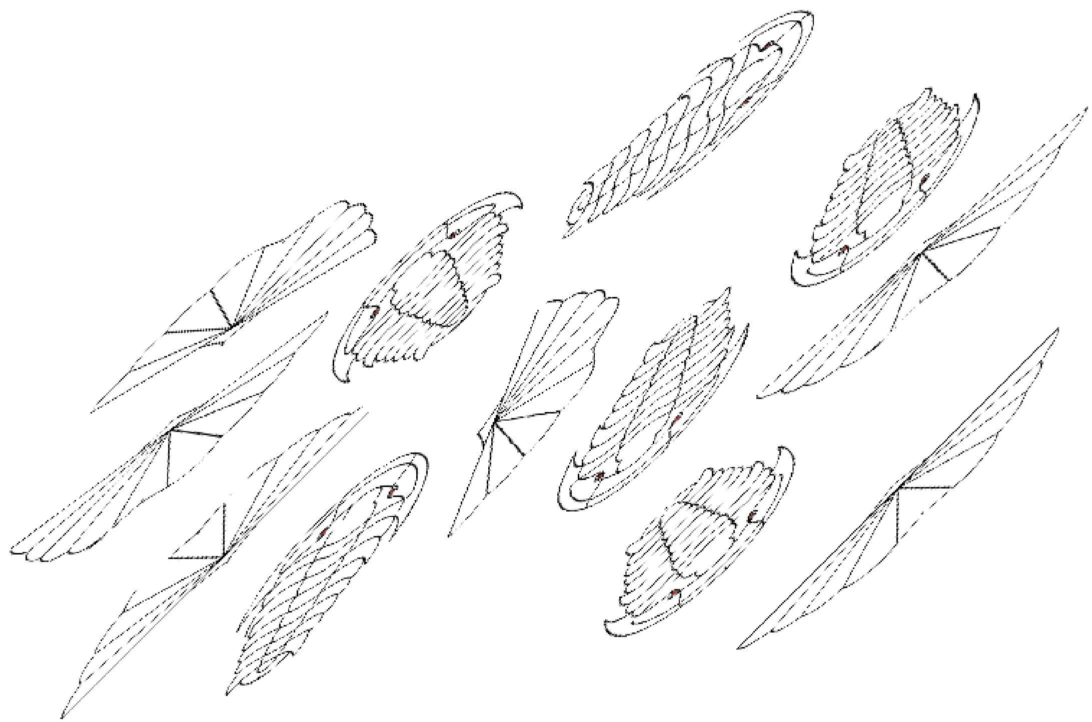
Prism



Deformed trilobites do exist!



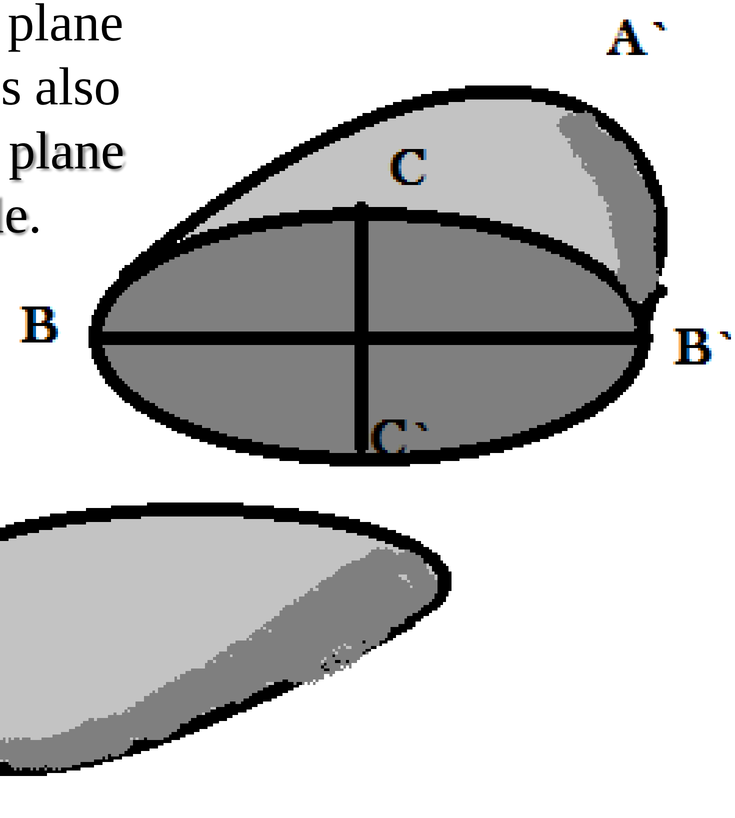
Draw the stretch ellipse for each



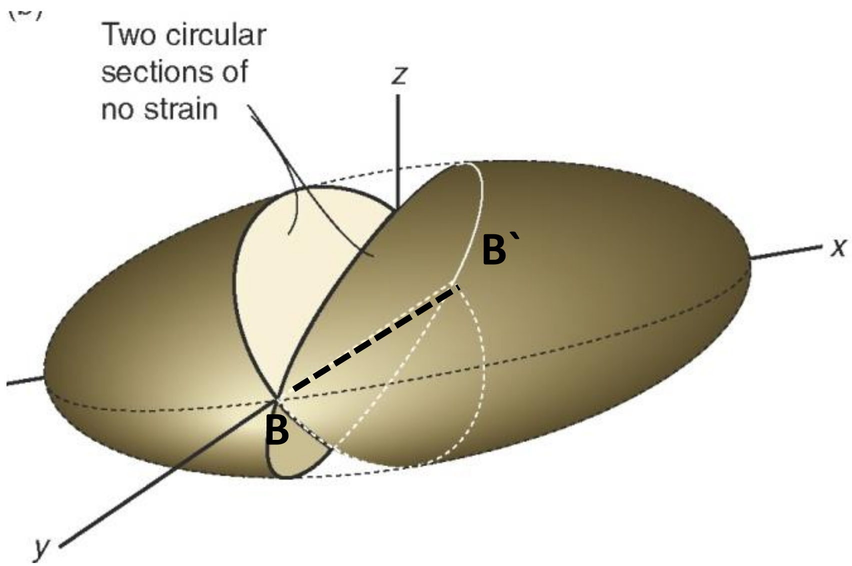
# Uses of strain ellipse in structural geology

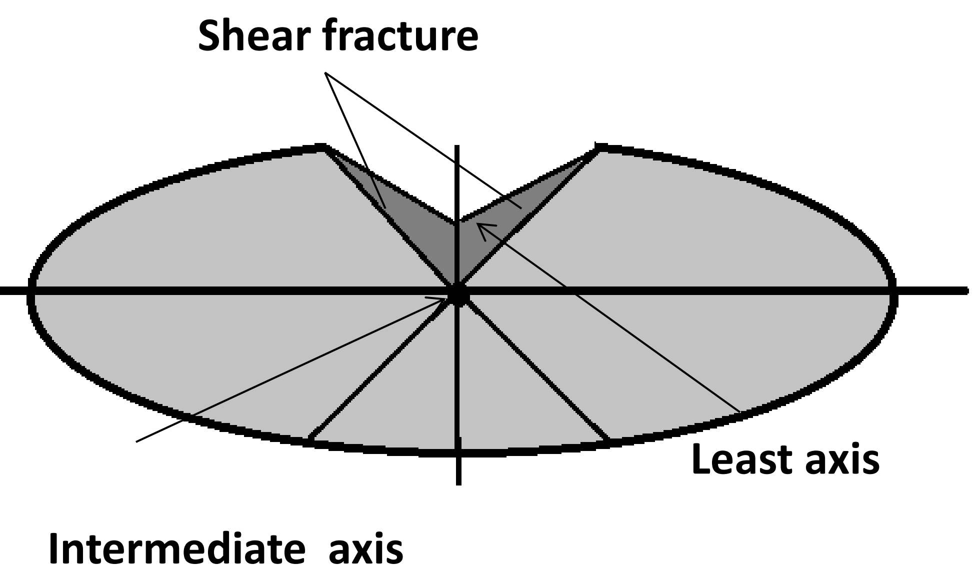
1- If tension fractures form , they are parallel to the plane that contains the Intermediate and Least Strain Axes. B B` C C`. That is ,tension fractures form at right angles to the Greatest Strain Axis

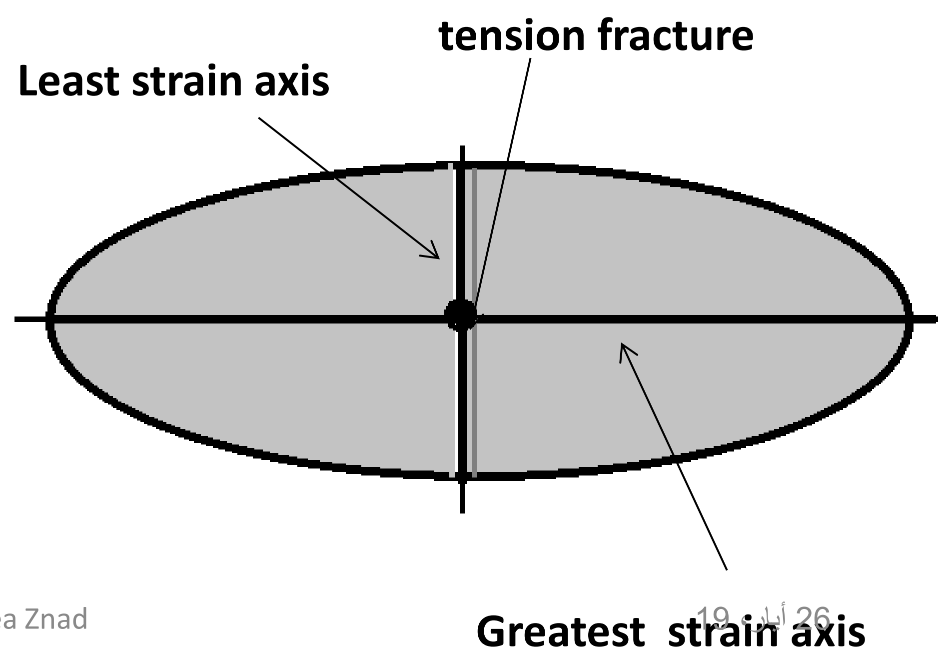
If the attitude of the strain ellipsoid is known , the position of the tension fractures may be predicted. Conversely, if fractures can be identified as of tension origin ,the greatest strain axis is readily determined; the plane Containing the least and intermediate strain axes is also Defined, but the position of these axes within this plane Can be determined only if additional data available.



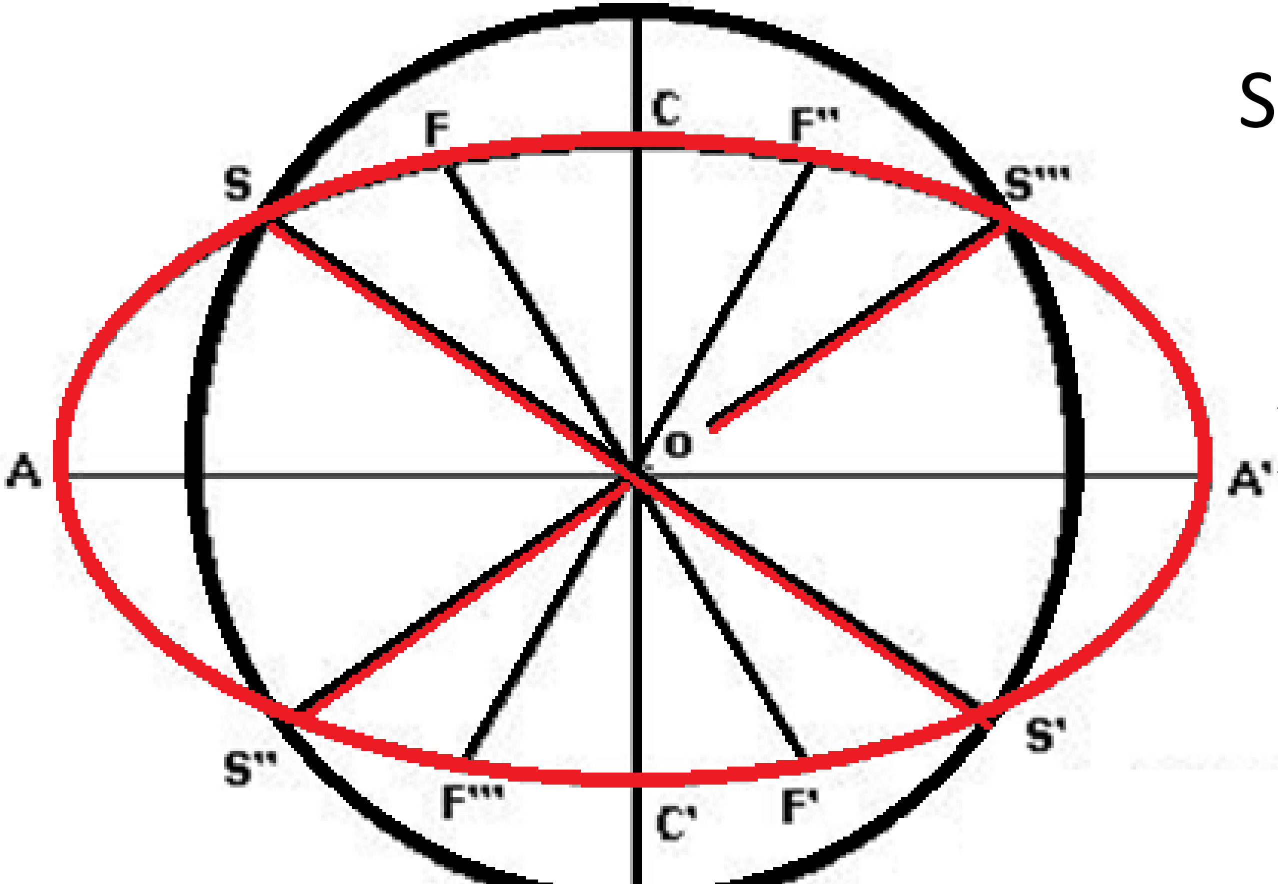
2-Most section through ellipsoids are ellipses. Two of the sections Are circular. These circular section pass through the intermediate axis B B` ..NO strain circles



# Strain ellipse




Experiments show that shear fractures are closer to least strain axis than the circular sections.

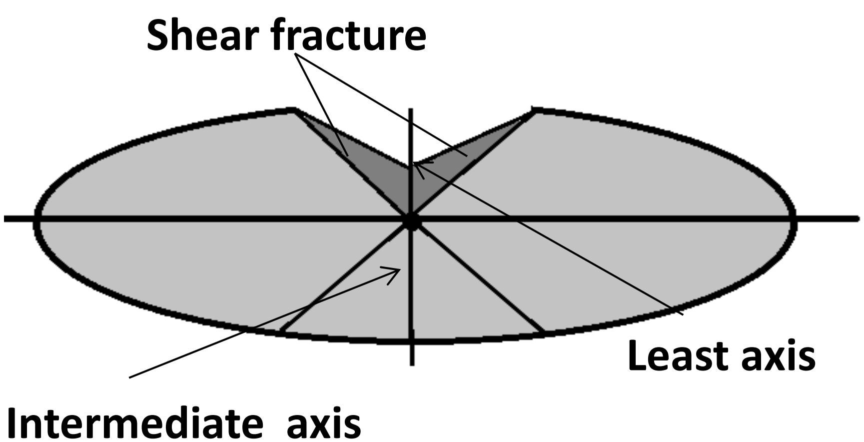
***A A'*** is the greatest strain axis

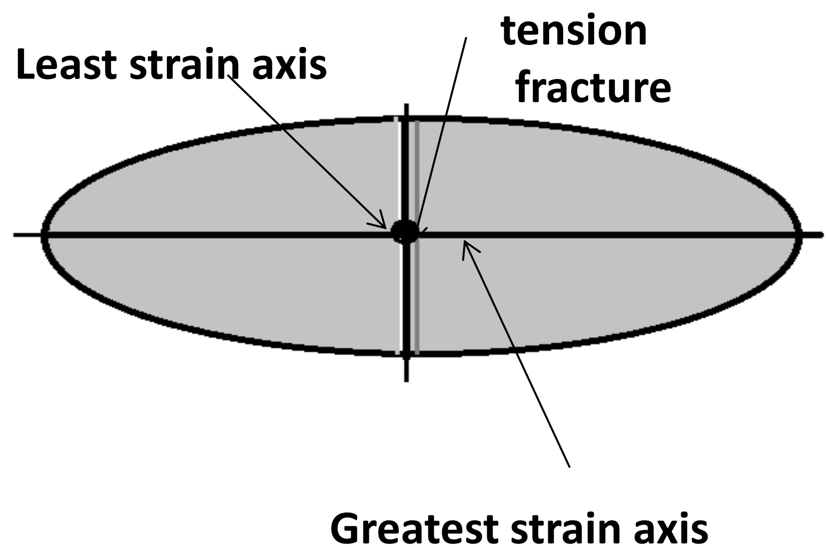
***CC'*** is the least strain axis

***SS' and S" S'''*** are the traces of the Circular Section of the ellipsoid. **الزاوية بينها وبين CC دائما اكبر من 45°**

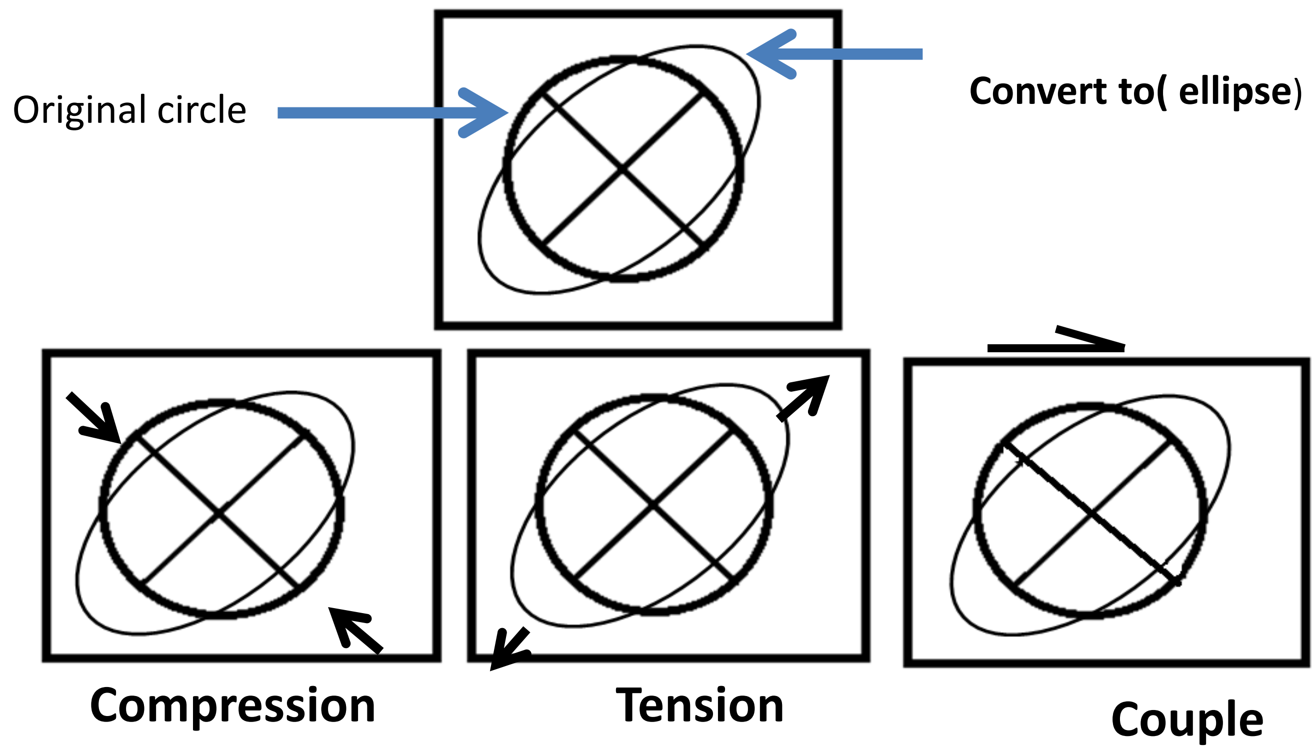
***FF' and F" F'''*** are the trace of the planes parallel to which shear fracture form. **الزاوية بينها وبين CC تقريبا 30°**

We  may utilize this concept as follows.
If two sets of shear fractures are present and are product of the same deformation . the line formed by their intersection is parallel to the intermediate axis of the strain ellipsoid.
Moreover .the least strain axis C C`  bisects the acute angle between the shear fractures.



3-A point of fundamental importance is that the strain gives us no direct evidence of external forces that cause the deformation . An ellipsoid may be formed from a sphere by Simple Compression, by Tension, or by a Couple.




Deformation of circle into an ellipse

Thus even the field geologist may accurately describe the strain ,he cannot directly deduce the forces without some additional evidence.

وعلى ذلك فان جيولوجي الحقل يمكنه وصف الانفعال بصورة دقيقة الا انه لايستطيع استنباط القوى الخارجية مباشرة من دون ادلة اخرى

# Use Of Strain Ellipse In A Structural Problem.

A simple example may serve to illustrate the use of the concept of the strain ellipsoid in relation to ruptures .Fig. below, is a cross section through a fault—that is a fracture along which the blocks on opposite sides have been displaced relative to each other. Scratches on the surface of the fault indicate that the movement was parallel to the dip of the fault.

The *problem is to decide whether the eastern block moved up or down relative to the western block*---That is which arrows, those at **a** or those at **b**, represent the movement.


Tension cracks

If the couple along acted as shown by the arrows at (b )the long axis of the ellipse would be horizontal.

# The Measurement of Strain

Change in line length

$$e \equiv \frac{\Delta l}{l_i} = \frac{l_f - l_i}{l_i}$$

**Example:**

If a belemnite of an original length (**$l_o$**) of 10 cm is now 12 cm (i.e., l´=12 cm), the longitudinal strain is positive, and
e = (12-10)/10 * 100% which gives an extension, e = 20%

Change in volume $e_v$

$$\equiv \frac{V_f - V_i}{V_i}$$

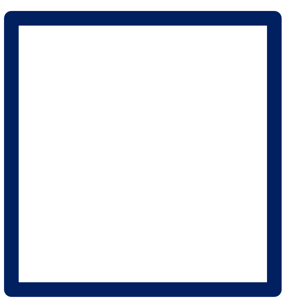
Change in angle between lines that were perpendicular.

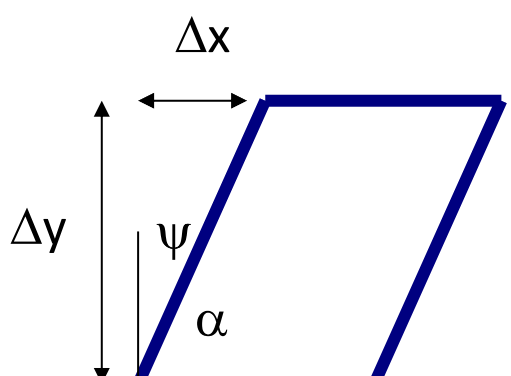
Angular shear ψ) psi.

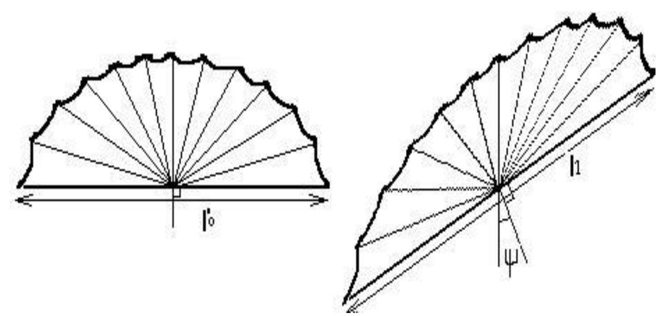
Shear strain ( γ) gamma

$$\text{angular shear} \equiv 90 - \alpha = \psi$$

$$\gamma = \tan\psi = \frac{\Delta x}{\Delta y} =$$





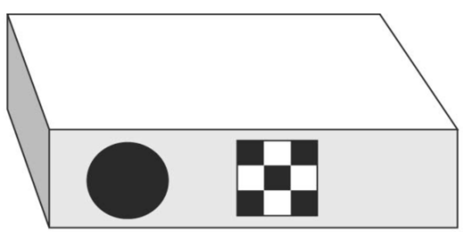



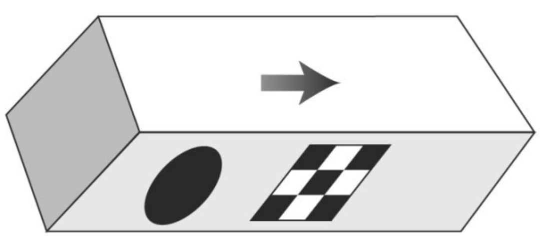
# Homogenous  and Inhomogeneous Strain

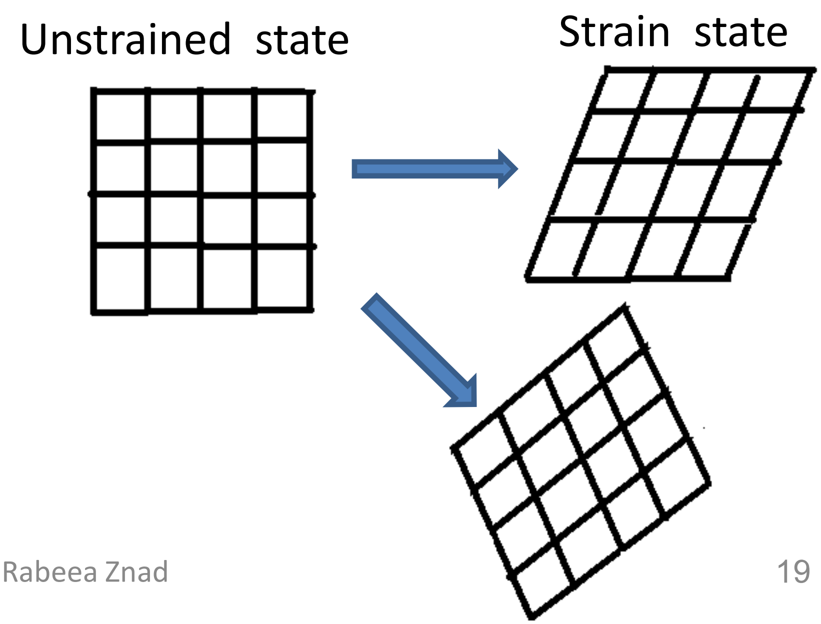
The type of strain shown by a distorted body can be classified as homogenous  or inhomogeneous Based on the following geometrical criteria:

## Homogenous deformation:-

1- Straight lines remain straight after deformation.
2- Parallel lines remain parallel after deformation.
3- flat planes remain flat after deformation.
4-  All lines in the same direction in the strained body have
   constant values of **e, γ & ψ**.
5-  Circles become ellipses :    in three dimension,  sphere become   ellipsoids.


(ɐ)

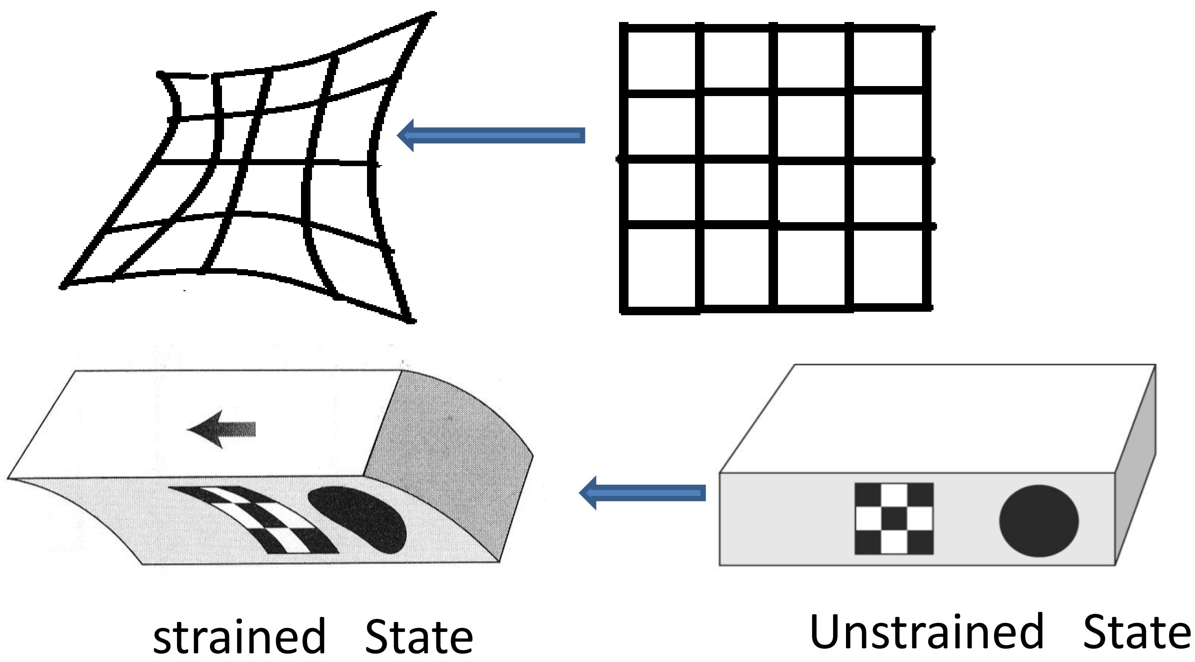

(d)



# Inhomogeneous (Heterogeneous ):-

1- Straight lines become curved after deformation.
2- Parallel lines lose their parallelism after deformation.
3-For any given direction in the body after deformation, the values of **e, γ & ψ** are variable.
4-Circles and squares or their three-dimensional counter parts, cubes and spheres, are distorted into complex forms.

any heterogeneously strained rock body can be subdivided into small areas that exhibit the characteristics of homogeneous strain (these areas are called ***domain)***



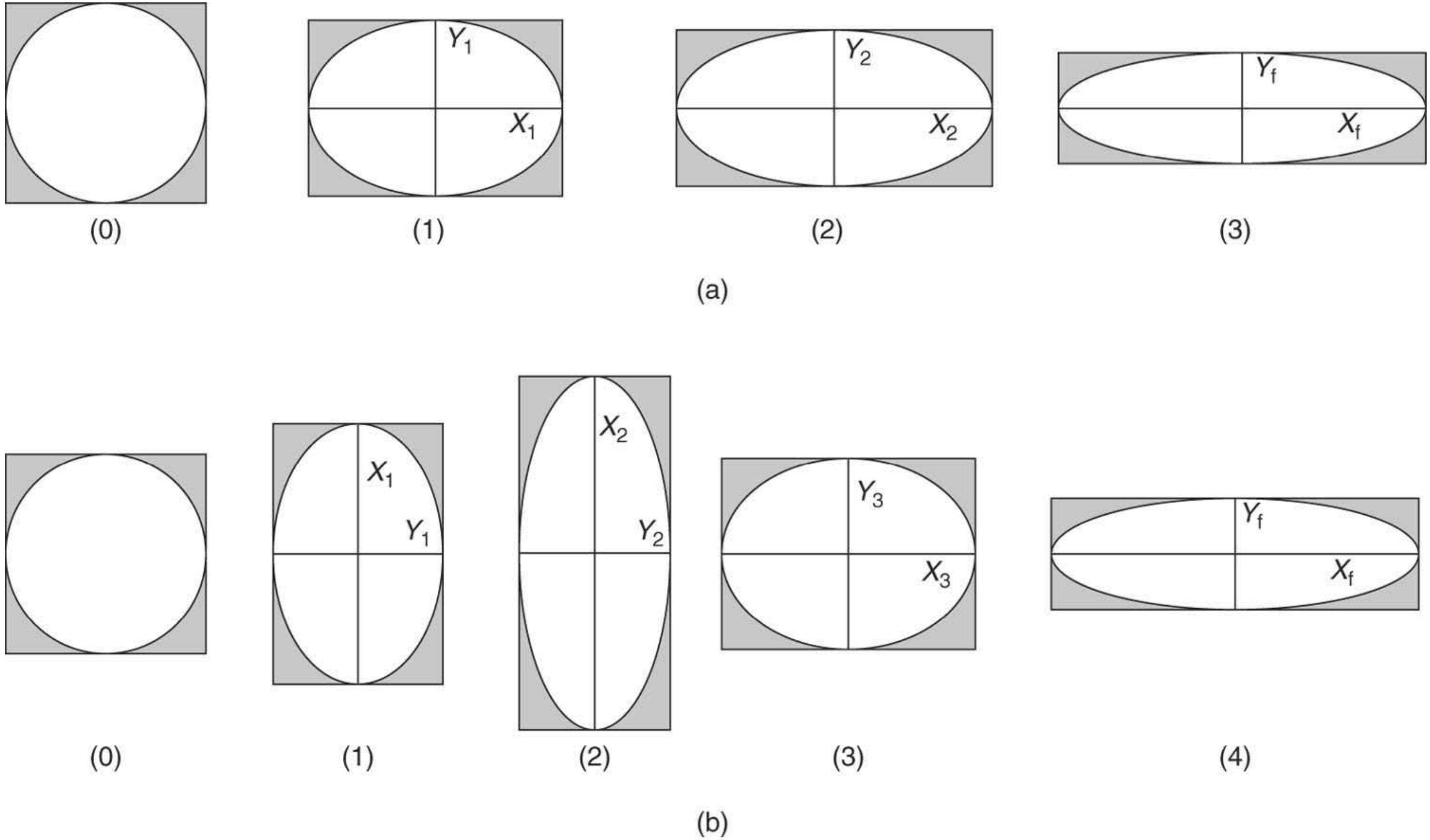
# Strain path

Any deformed substance has an involved history, for it passes from its initial condition through a whole Series of deformed states before it eventually arrives its final states.
This process is known as ***progressive deformation*** .The geologist investigating rocks deformed by natural tectonic processes sees only the end product of the deformation processes known as ***the finite state of strain*** of the material ,or ***finite strain*** or ***total strain***. which is independent of the details of the steps toward the final configuration.
When these intermediate strain steps are determined they are called ***incremental strains***, ***Incremental strain or infinitesmal strains*** is a Strain state of one step in a progressive strain history. The summation of all incremental strains (that is, their product), therefore, is the ***finite strain***

ان اية مادة متعرضة للتشوه سوف تمتلك تاريخ لمرور المادة من الحالة الاولية وعبر سلسلة من مراحل التشوه وصولا الى حالة التشوه النهائية, وتدعى هذه العملية بالتشوه التقدمي progressive deformation . او مسار الانفعال strain path. ان الجيولوجي يلاحظ في الحقل المرحلة النهائية والتي تدعى حالة التشوه النهائية finite states of strain او finite strain ويرمز له *f* واحيانا يدعى total strain و ان هذه الهيئة النهائية ا (كما سنلاحظ) لاتعتمد على تفاصيل الخطوات الوسطية والتي تتضمن مراحل تشوه صغيرة والتي تدعى infinitesmal strains or incremental strains ويرمزله *i*

There are many ways to measure finite strain in a rock, but measurement of strain increments is more difficult. Yet, incremental strain may be more crucial for unraveling the deformation history of a rock or region than finite strain. Let us explore this with a simple example

هناك عدة طرق لقياس الانفعال النهائي المسجل في الصخور بينما من الصعب متابعة الخطوات المرحلية على الرغم من اهميتها في دراسة تاريخ التشوة للصخور او الاقاليم مقارنة بدراسة بالتشوه النهائي اذ ان مسارات انفعال مختلفة يمكن ان تؤول الى شكل انفعال نهائي متشابه كما نلاحظ في الشكل التالي.




The finite strains, Xf and Yf, in (a) and (b) are the same, but the strain path by which each was reached is different. This illustrates the importance of understanding the incremental strain history (here, Xi and Yi ) of rocks and regions and inherent limitation of finite strain analysis.

# Lines of no finite elongation - lnfe

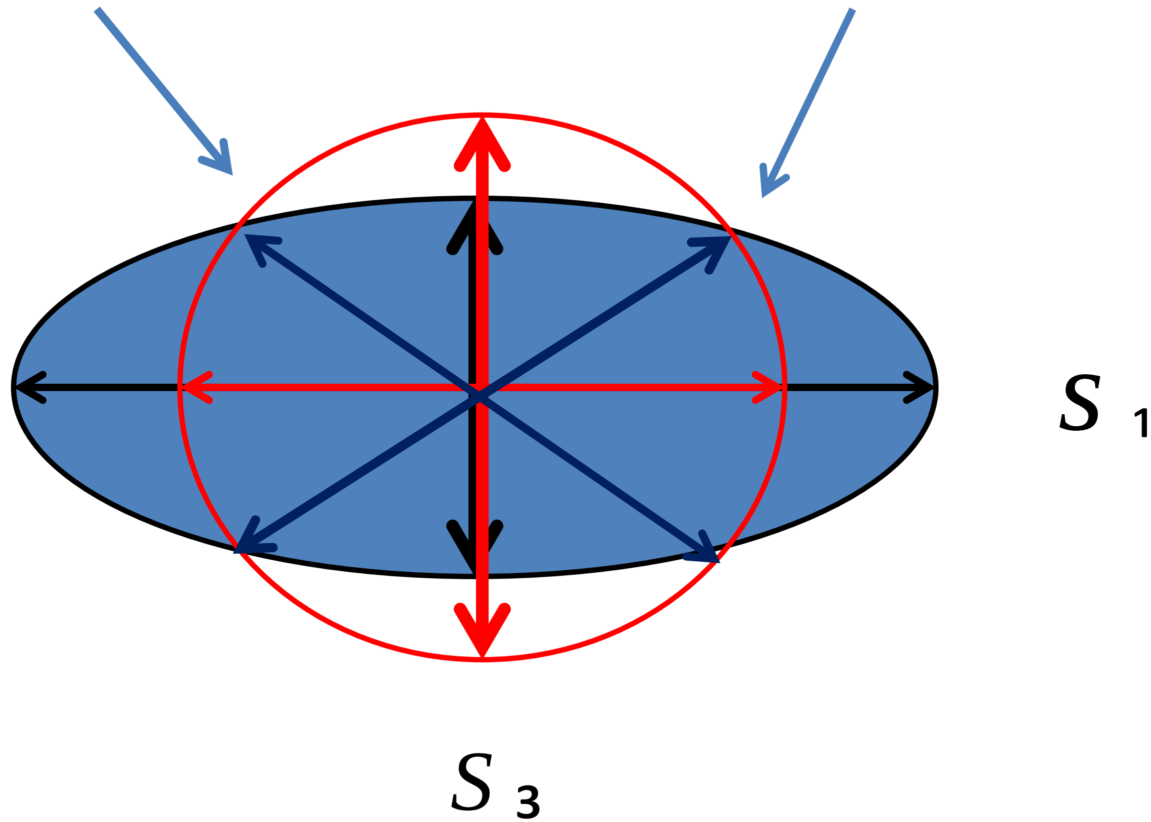
If we draw a circle on a deck of card, and deform the deck, the **strain ellipse** will intersect the original circle (if we redraw it) along the two lines of **lnfe**

**If we draw a new circle in the deformed state (in a different color, say red) and restore the deck to its original unstrained configuration, the red circle becomes an ellipse**

**This ellipse which has long axis perpendicular to the strain ellipse is called the reciprocal strain ellipse**

اهليج الانفعال العكسي

# Lnfe – lines of no finite elongation




$S_1$ long axis of ellipse

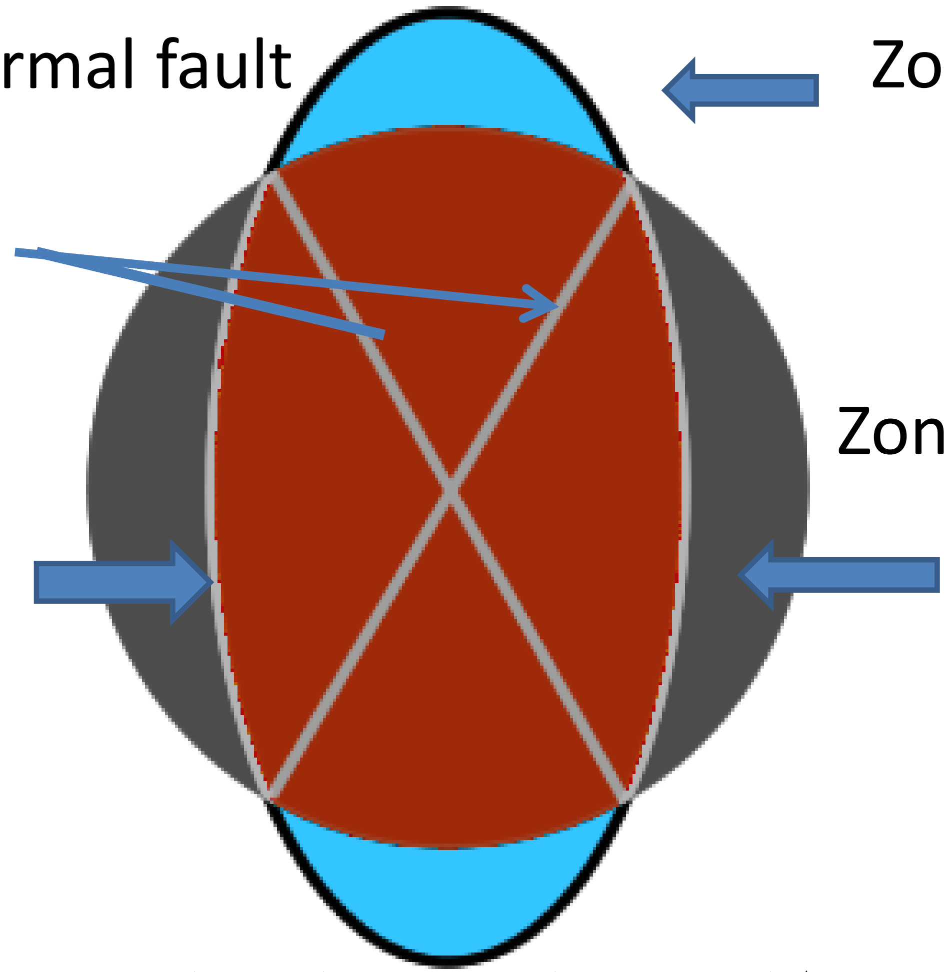
$S_3$ short axis of ellipse

# Zones of Extension & Shortening

boudinage&normal fault

Zone of extension
نطاق الاستطالة

Infe

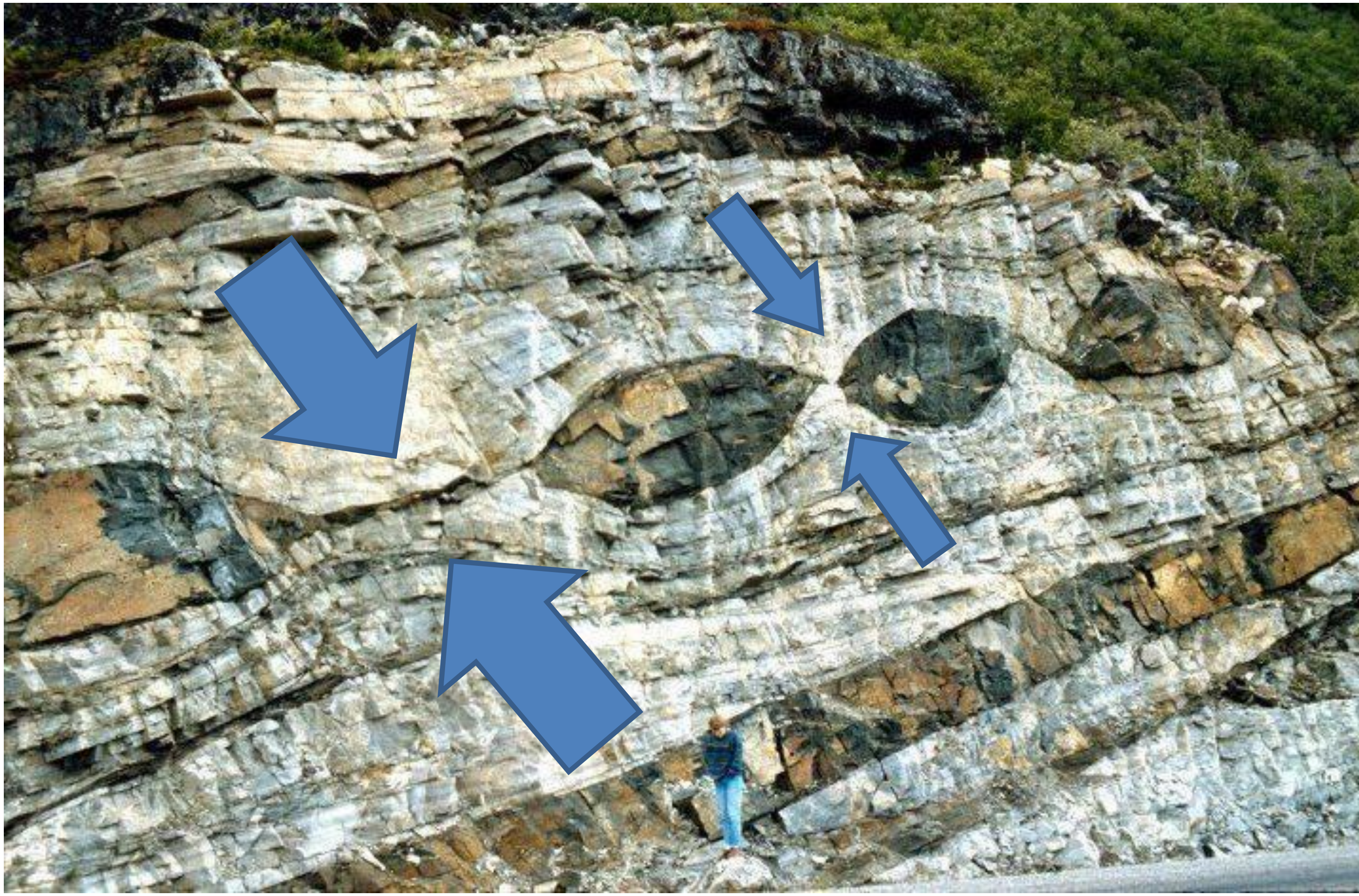


Zone of shortening
نطاق التقصير

Folding
&reverse and
thrust fault

تحديد انطقة الاستطالة والتقصير على اهليج الانفعال مع ذكر التراكيب المرافقة لكل نطاق.

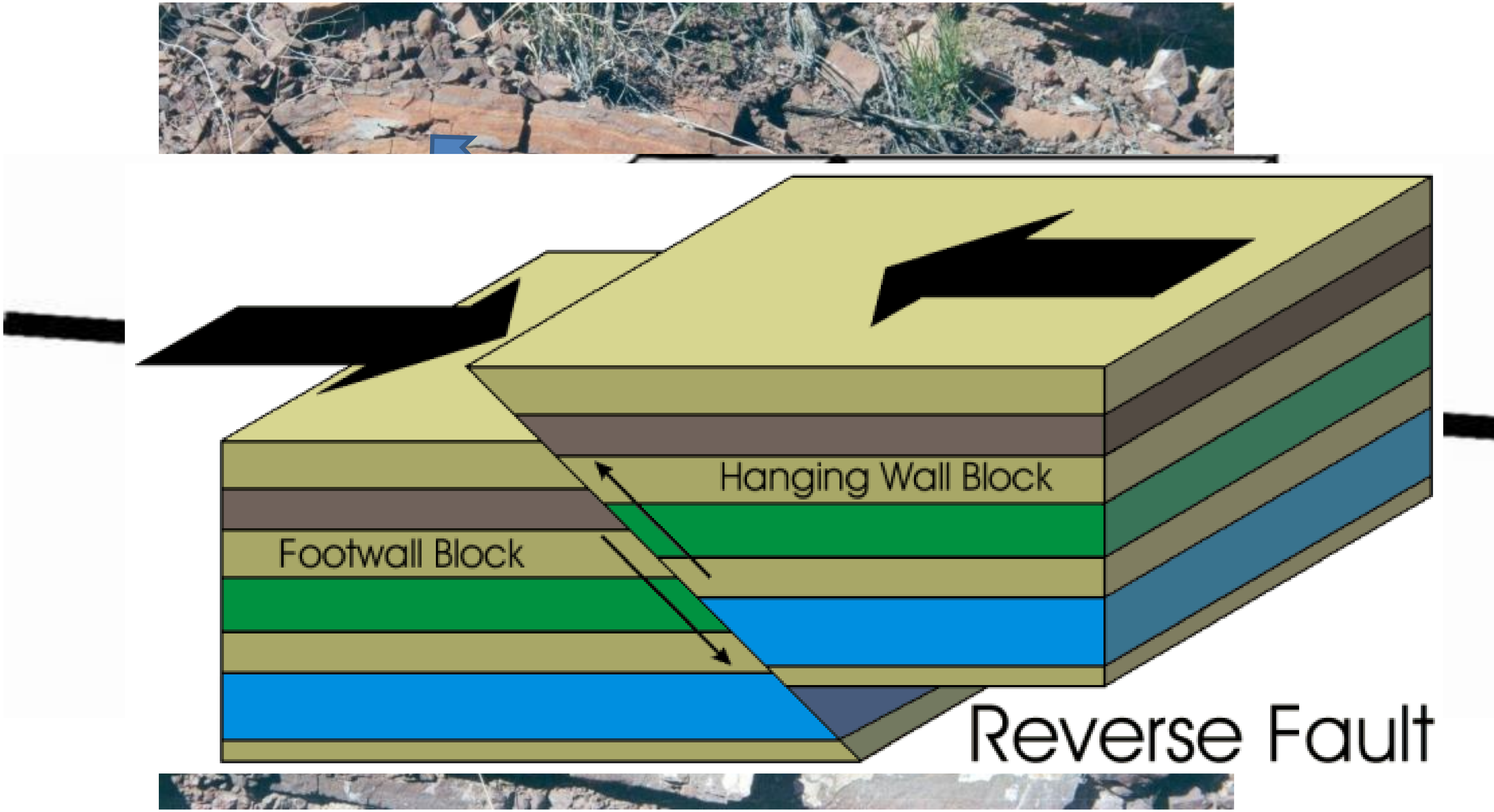
# udinage



**Boudinage** is a geological term for structures formed by extension, where a rigid tabular body such as Hornfels, is stretched and deformed amidst less competent surroundings. The competent bed The thickest boudins are about 20 m (65 feet) thick, and the thinnest about 1 cm (0.39 inch).

# Normal fault

fold




Reverse fault

# Rotational and Irrotational Strain

## (Coaxial and non-coaxial strain accumulation)

**يعتبر الانفعال دورانياً إذا تغيرت اتجاهات محاور الانفعال الرئيسة بعد الانفعال.**

**ويعتبر الانفعال غير دورانياً إذا لم تتغير اتجاهات محاور الانفعال الرئيسة بعد الانفعال**

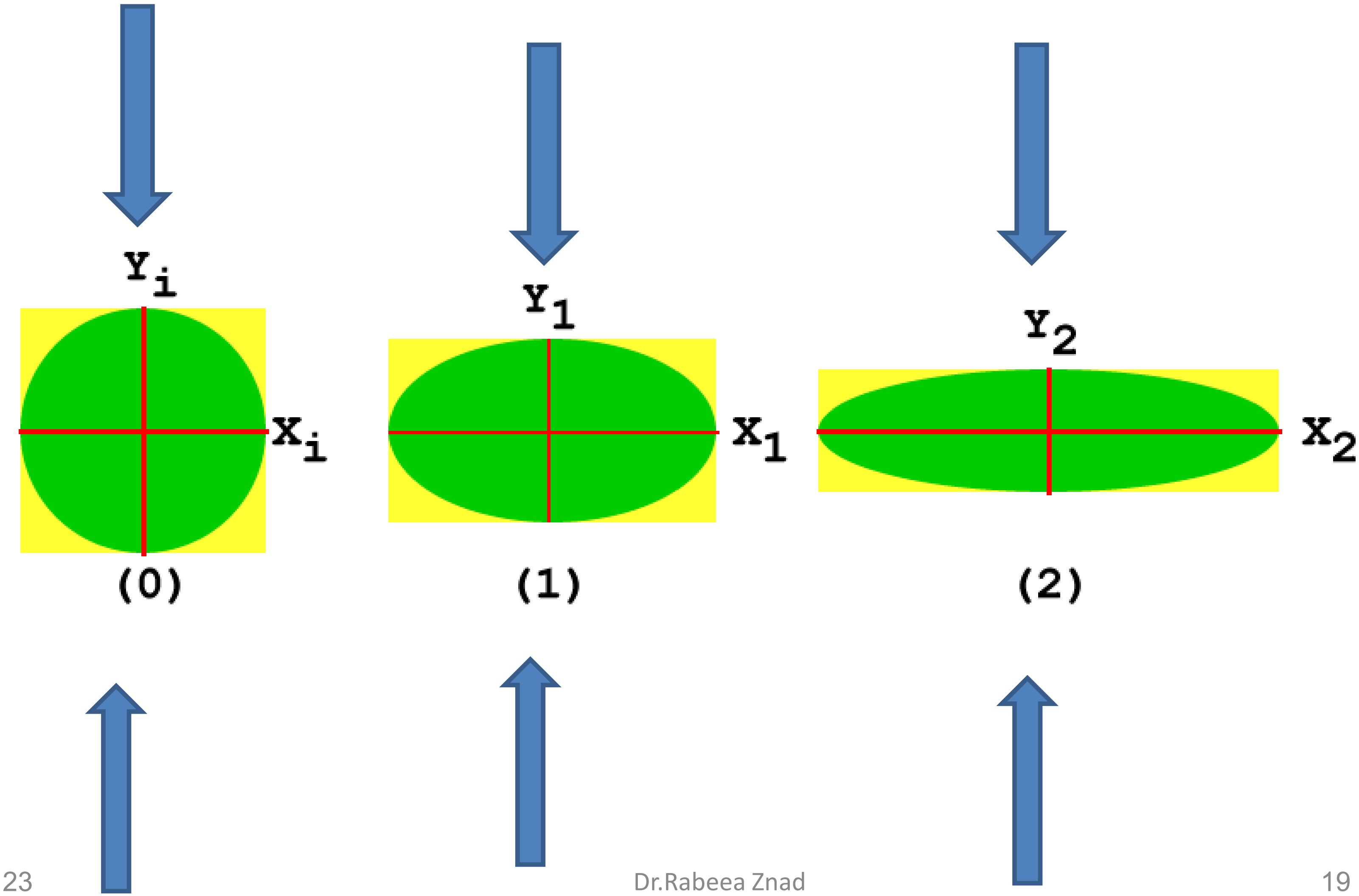
If the strain axes have the same orientation in the deformed as in undeformed state we describe irrotational the strain as a non-rotational or Coaxial

If the strain axes end up in a rotated position, then the strain is rotational or non coaxial

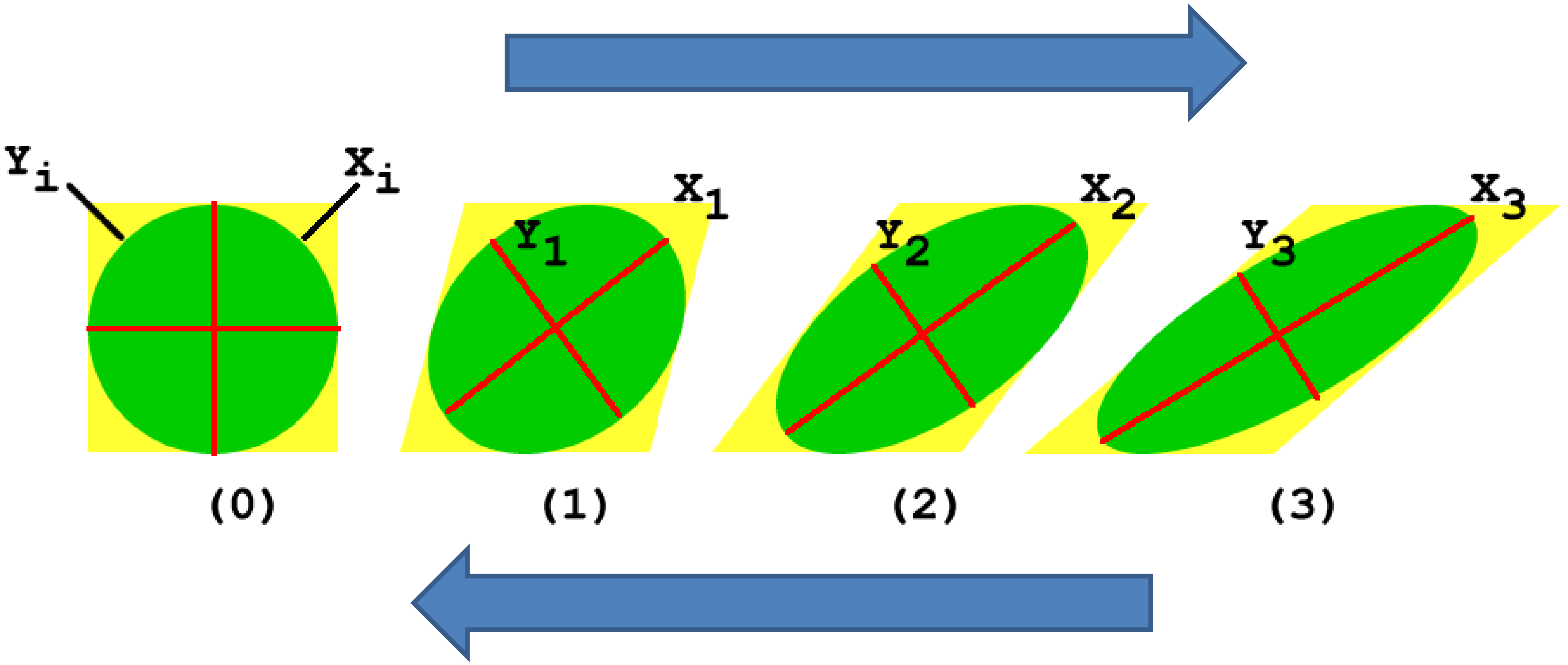
the principal incremental strain axes rotate relative to the finite strain axes, a scenario that is called **non-coaxial strain**

# An example of a non-rotational strain is **pure shear** - it's a pure strain with no dilation of the area of the plane

Pure means lack of rotation



# An example of a rotational strain is a **simple shear**



# Graphic representation of finite strain ellipse in two dimension

A common goal in strain analysis is to compare results obtained in one place with those obtained elsewhere in an area or in an outcrop, or even to compare data from several different regions.

Strain ellipses may occur in a variety of shapes. In Fig. 14.4 there are seven circles of radius 1, and the strain ellipse that has developed from each. In Fig. 14.5 is a graph in which $1 + e2$ is plotted against $1 + e1$. The undeformed circle is shown at $1 + e2 = 1$ and $1 + e1 = 1$. Before reading further, plot the letter of each of the seven strain ellipses of Fig. 14.4 onto its appropriate position on Fig. 14.5. You will probably have difficulty understanding the following discussion if you do not take the time to do this.

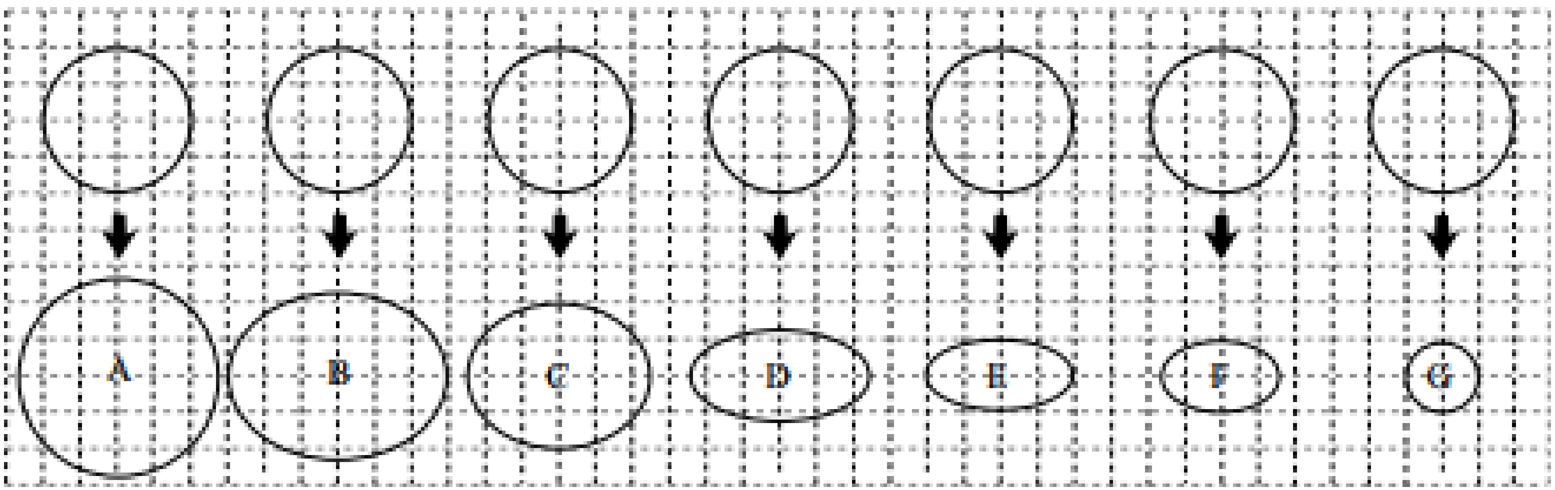



Fig. 14.4 Seven circles and their corresponding strain ellipses. The seven strain ellipses should be plotted on the graph in Fig. 14.5.

The seven strain ellipses that you have plotted on Fig. 14.5 represent seven generalized classes. Notice that no ellipse can ever be plotted above the diagonal line on the graph, why ?.

The diagonal line is the locus of all strain ellipses that are not ellipses at all; they are circles. ‘‘Ellipse’’ A, which exhibits equal elongation in all directions, and ‘‘ellipse’’ G, which exhibits equal contraction in all directions, both plot on this line

The graph of Fig. 14.5 can be divided into three fields with the e1=0 and e2 = 0 lines acting as dividers. Field 1 includes all ellipses in which both principal strains have positive extensions, such as ellipse B in Fig. 14.4. Field 2 includes ellipses in which e1 is positive and e2 is negative, such as ellipse D in Fig. 14.4. And field 3 includes ellipses in which both e1 and e2 are negative, such as ellipse F in Fig. 14.4. Figure 14.6 summarizes the characteristics of each of the three fields

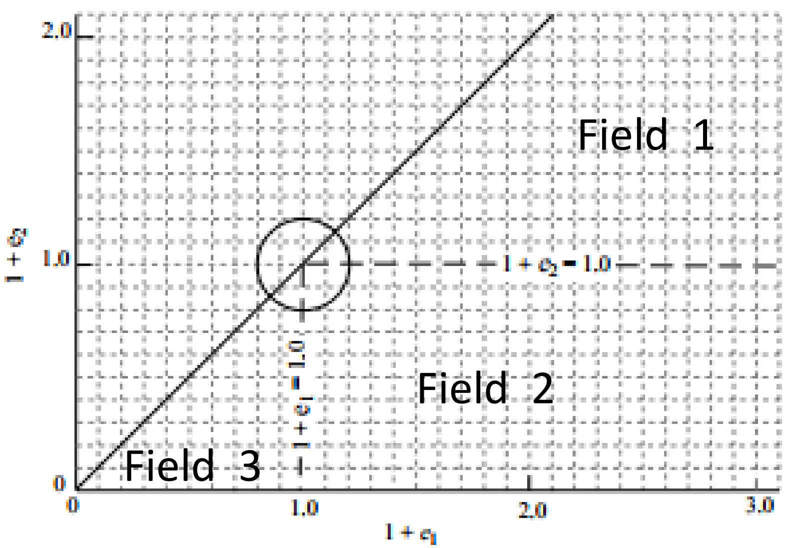



Fig. 14.5 Graph on which 1 + e2 is plotted against 1 + e1 for a given strain ellipse

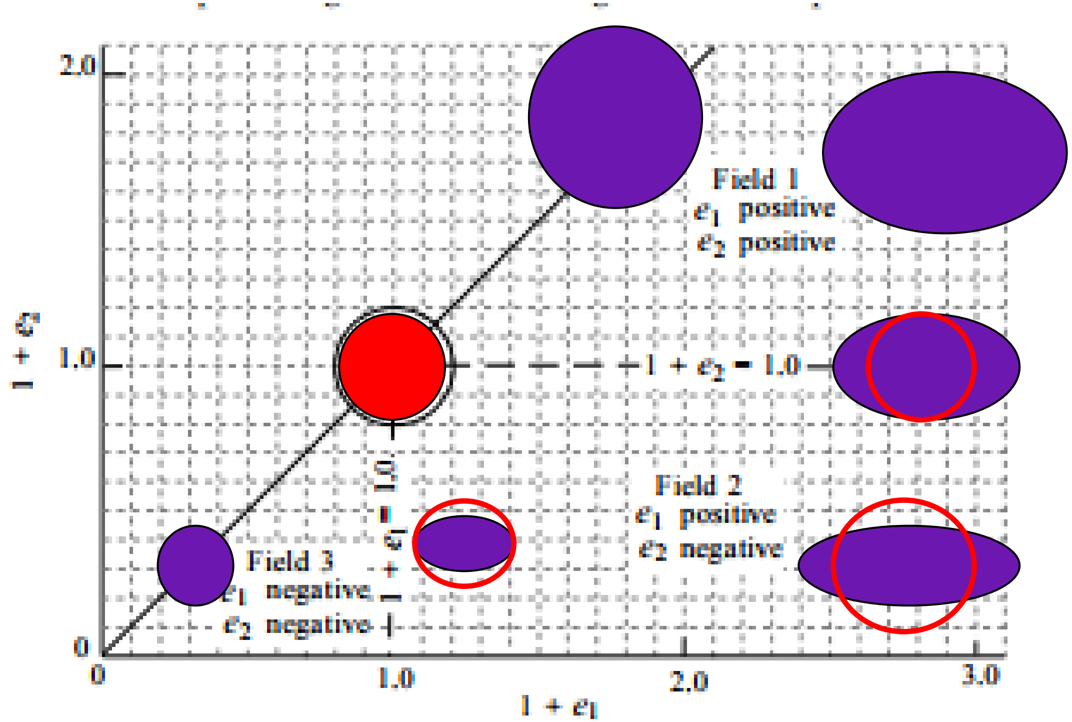
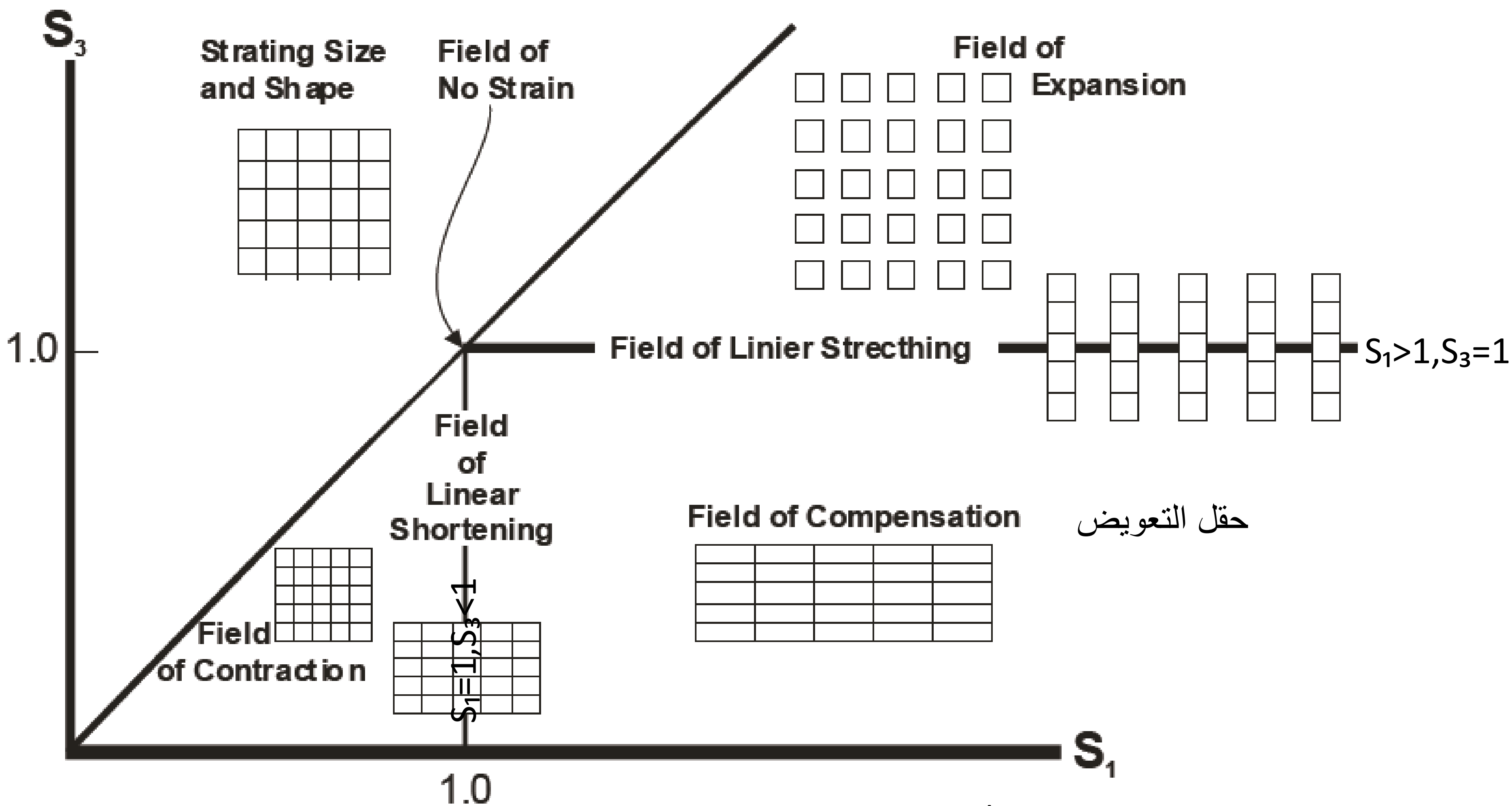



Fig. 14.6 Graph on which 1 + e2 is plotted against 1 + e1, showing three fields

# Strain Field Diagram



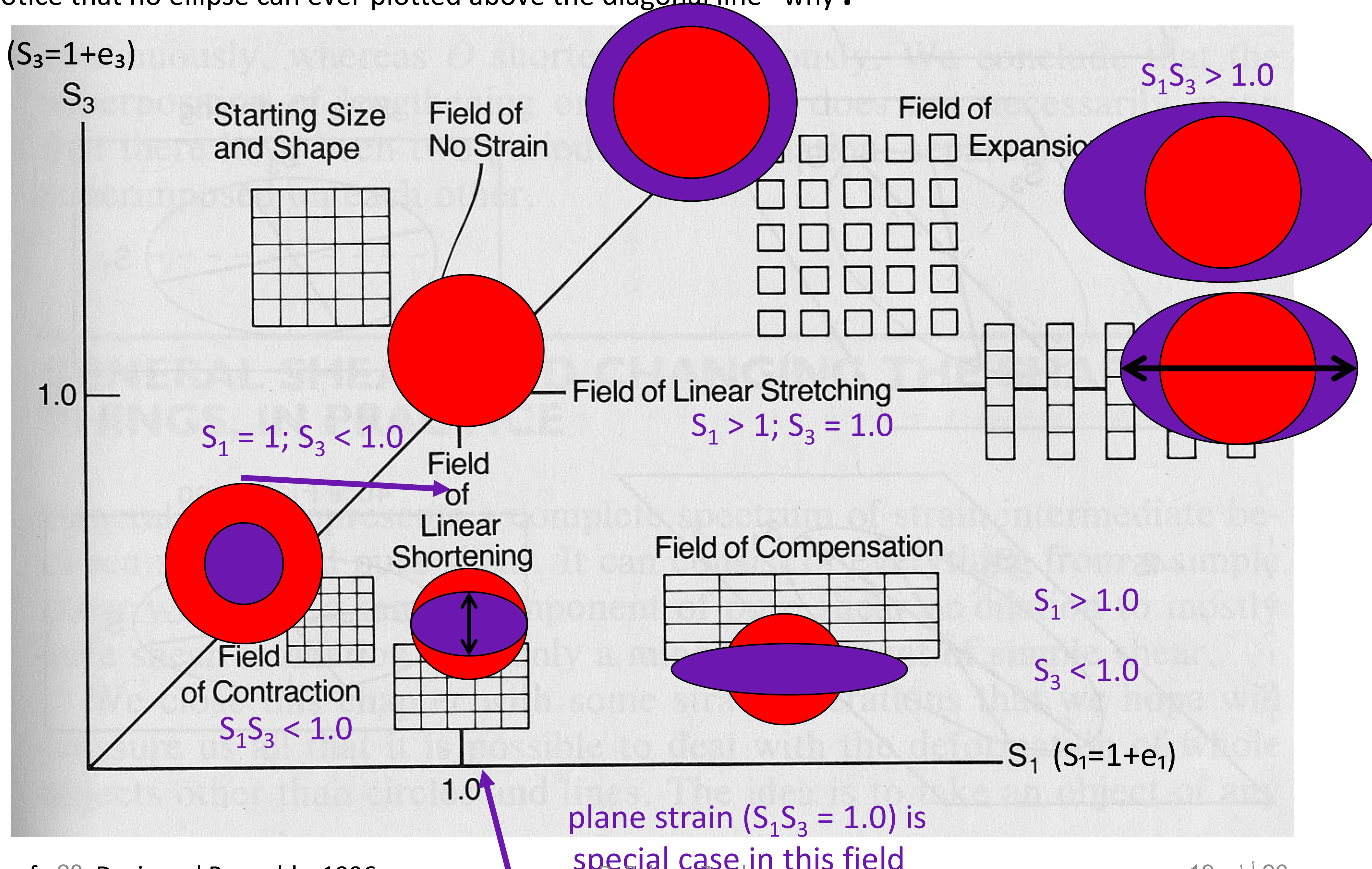

$S_1S_3=1$ no area change

$S_1S_3<1$= area decrease

$S_1S_3>1$ = area increase

# Strain Field Diagram

Notice that no ellipse can ever plotted above the diagonal line why ?




from: Davis and Reynolds, 1996

## التراكيب الجيولوجية التي ممكن ان تتطور ضمن حقول اهليج الانقعال. the geological structure that may be developed in the three fields of strain ellipse.

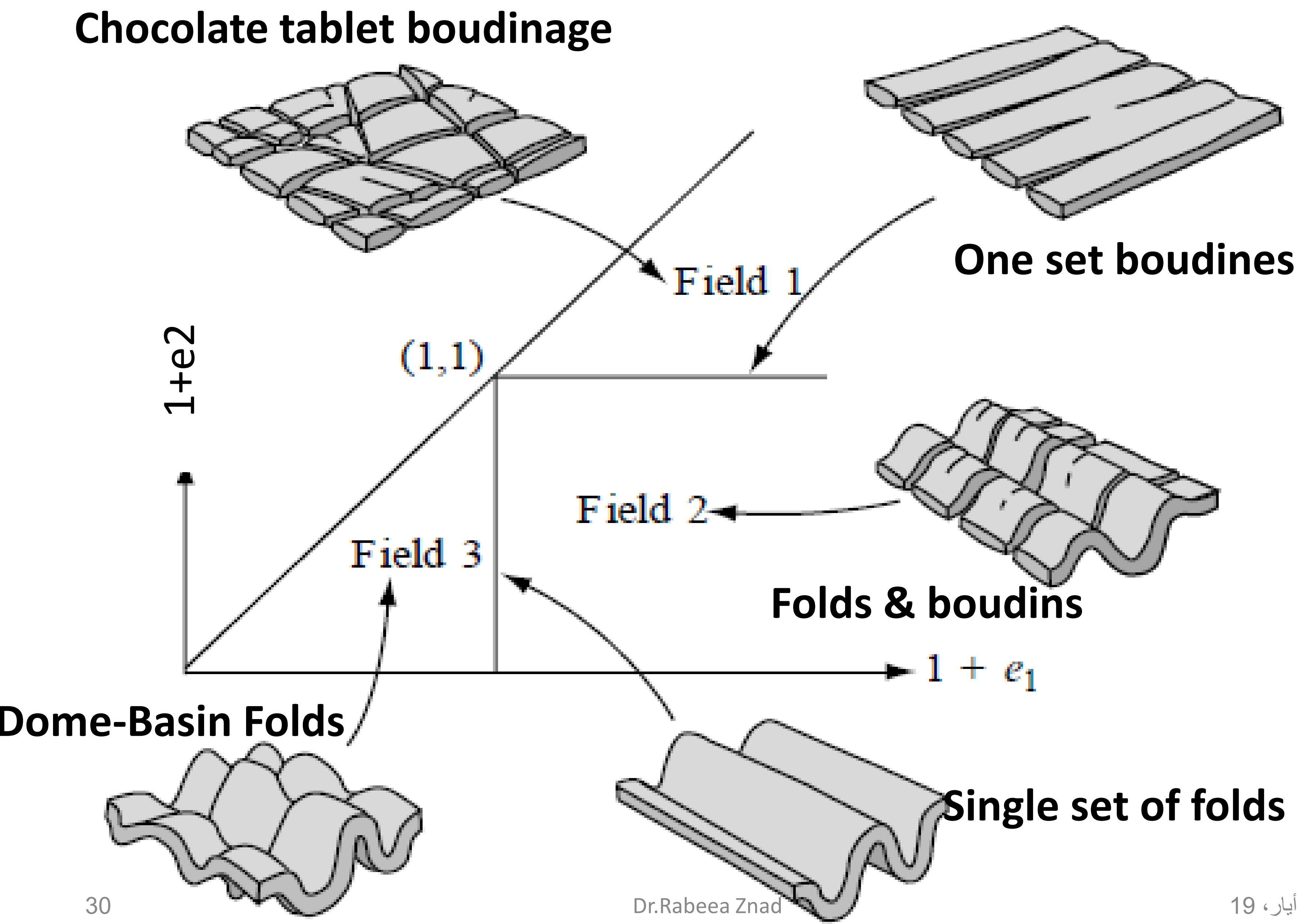
اذا حدث الانفعال في طبقة صلبة competent (مثل الحجر الجيري او اشرطة من الجيرت chert band) محاطة بطبقات اكثر لدانة more ductile (مثل الصخور الطينية او المارل ) , سوف تتطور مختلف التراكيب الثانوية في التتابع الطبقي وحسب موقع حقل اهليج الانفعال ضمن المخطط السابق .

في الحقل الاول.(حقل التوسع للطبقة الصخرية الغير متساوي في جميع الاتجاهات ) سيتطور تركيب من نوع boudinage حيث ان التخصرات necks لاتمتلك توجية موحد اذ ان معظمها تكون شبة عمودية على محور الانفعال الاطول, نتيجة لذلك فان التقاطعات لهذة البودن boudinage تكون بمختلف الاتجاهات ضمن الطبقة الصلبة مما تؤدي الى تطور تركيب يشبة قطع الجكليت chocolate tablet structure وذلك عندما يكون الانفعال كبير حيث تنفصل القطع وتعزل عن بعضها .

اذا وقع اهليج الانفعال بين الحقل الاول والثاني حيث$S_3$=1 $S_1$>1,ستتطور مجموعة واحدة من البودن ويكون طولها عمودي على محور الاستطالة $S_1$

في الحقل الثاني. فان اهليج الانفعال يمتلك محور استطالة في اتجاه واحد ومحور تقصير واحد . وهذا ينتج عنه طيات باتجاه واحد وبودن او كسور متعامدة مع محور الطية.
بين الحقل الثاني والثالث حيث $S_1$=1 لذلك لاتوجد استطالة وانما تقصير باتجاه واحد$S_3$<1(ستتطور طية ).

في الحقل الثالث. كل الانفعال هو تقصير في كل الاتجاهات سيتطور تركيب معقد من القباب والاحواض (طبقة البيض)



Chocolate tablet boudinage
One set boudines
Field 1
(1,1)
1+e2
Field 2
Field 3
Folds & boudins
1 + $e_1$
Dome-Basin Folds
Single set of folds

# Strain in three dimension (Flinn diagram and K – factor) مخطط فلن ومعامل K

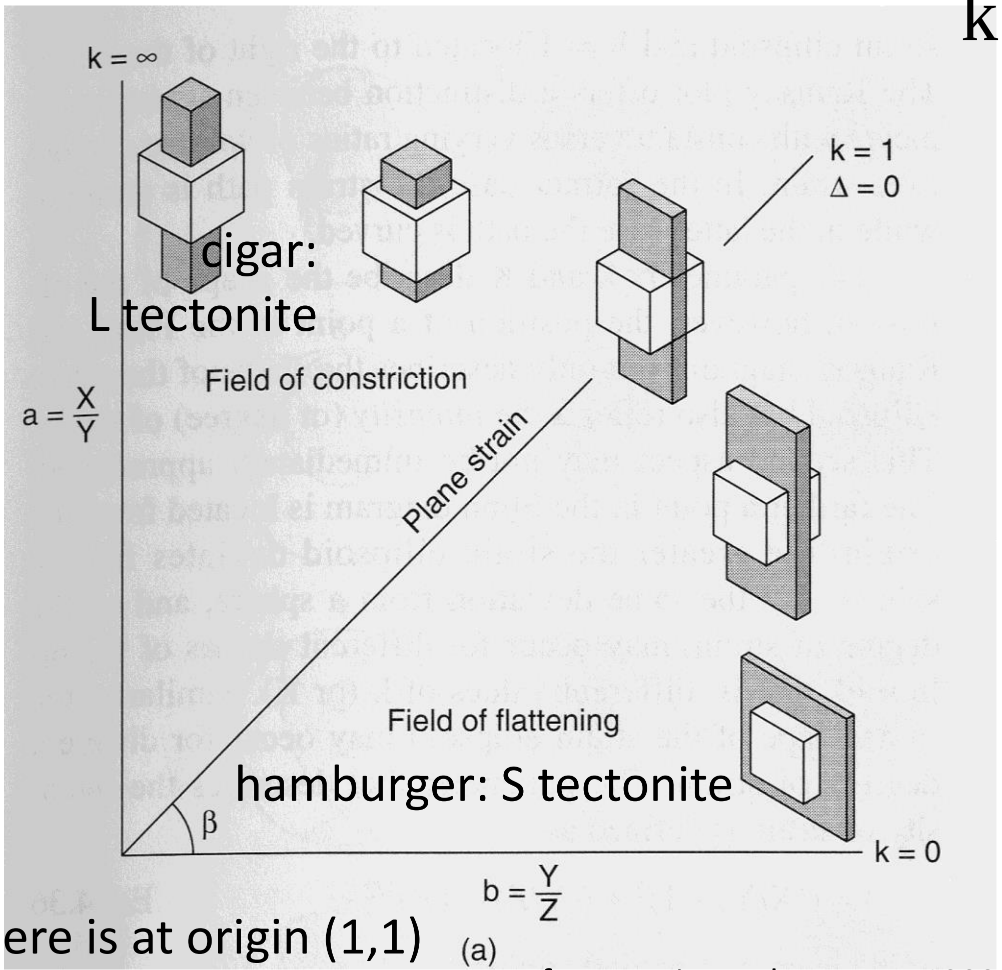
The Flinn diagram is a graphical representation used to plot finite strain ellipsoids. The Flinn diagram describes two main types of strain ellipsoids, cigar and pancake. The cigar type ellipsoid is when the main strain is in the S1 direction. The resultant strain ellipsoid is long and looks like a cigar standing upright. The pancake strain ellipsoid looks like a pancake with the S3 strain direction being extended.

يستخدم مخطط فلن لمعرفة ان كان الانفعال المؤثر في النماذج المدروسة قد حصل على شكل تفلطح Flatting حيث يكون اهلج الانفعال في هذه الحالة مفلطح Oblate . او ان كان الانفعال المؤثر في النماذج ا لمدروسة قد حصل بسكل تخصر Constriction ويكون شكل اهليج الانفعال مغزلي Prolate . ويعتمد تحديد شكل اهليج الانفعال على معرفة مقدار المعامل K له وموقعة من الخط الذي يمثل K=1

إن من الصعب قياس القيمة المطلقة لمحاور الاستطالة (محاور الجهد) بشكل مباشر من الصخور المتشوهه Deformed rocks، ولكن أحياناً من الممكن قياس التشوه الحاصل بين زوايا معروفة الأصل لخطوط متقاطعة، ومنها ممكن إيجاد نسبة محاور الاستطالة. فإذا كان ذلك ممكناً، فإننا نستطيع تمثيل شكل الجهد البيضوي Strain ellipsoid على مخطط ذو بعدين Two dimensional graph وذلك بإسقاط a= x\y=(1+e1) /(1+e2) على المحور الصادي Ordinate و(1+e3)/(1+e2)=b=y/z على المحور السيني abscissa (وكما في الشكل المرفق).

استخدم زنك Zingg هذه الطريقة لتمثيل أشكال الحصى، وطورها فلن (1962) Flinn ليستخدمها في تمثيل الأجسام البيضوية Strain ellipsoids.

examine Flinn diagram more closely…




sphere is at origin (1,1)

a=b=1

from: Twiss and Moores, 1992

k is shape parameter:

(a-1)/(b-1)

K is describes slope of line that passes through origin (angle β)

oblate ellipsoids المفلطحة as k approaches to 0

المستطالةprolate ellipsoids as k approaches infinity

k=1 is special case of plane strain (Y=1)

…line separates field of constriction from field of flattening