# **Operations on Sets**

The basic set operations are:

**1. Union of Sets:** Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by A∪B.

1. A∪B = {x: x ∈ A or x ∈ B}

**Example:** Let A = {1, 2, 3},       B= {3, 4, 5, 6}
A∪B = {1, 2, 3, 4, 5, 6}.



**2. Intersection of Sets:** Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by A ∩ B.

1. A ∩ B = {x: x ∈ A and x ∈ B}

**Example:** Let A = {11, 12, 13},       B = {13, 14, 15}
A ∩ B = {13}.



**3. Difference of Sets:** The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by A - B.

1. A - B = {x: x ∈ A and x ∉ B}

**Example:** Let A = {1, 2, 3, 4} and B = {3, 4, 5, 6} then A - B = {3, 4} and B - A = {5, 6}



**4. Complement of a Set:** The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by Ac.

Ac = U - A = {x: x ∈ U and x ∉ A} = {x: x ∉ A}

**Example:** Let U is the set of all natural numbers.
A = {1, 2, 3}
Ac = {all natural numbers except 1, 2, and 3}.



**5. Symmetric Difference of Sets:** The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by A ⨁ B i.e.

# **Algebra of Sets**

Sets under the operations of union, intersection, and complement satisfy various laws (identities) which are listed in Table 1.

**Table: Law of Algebra of Sets**

|  |  |  |
| --- | --- | --- |
| **Idempotent Laws** | (a) A ∪ A = A | (b) A ∩ A = A |
| **Associative Laws** | (a) (A ∪ B) ∪ C = A ∪ (B ∪ C) | (b) (A ∩ B) ∩ C = A ∩ (B ∩ C) |
| **Commutative Laws** | (a) A ∪ B = B ∪ A | (b) A ∩ B = B ∩ A |
| **Distributive Laws** | (a) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) | (b) A ∩ (B ∪ C) =(A ∩ B) ∪ (A ∩ C) |
| **De Morgan's Laws** | (a) (A ∪B)c=Ac∩ Bc | (b) (A ∩B)c=Ac∪ Bc |
| **Identity Laws** | (a) A ∪ ∅ = A(b) A ∪ U = U | (c) A ∩ U =A(d) A ∩ ∅ = ∅ |
| **Complement Laws** | (a) A ∪ Ac= U(b) A ∩ Ac= ∅ | (c) Uc= ∅(d) ∅c = U |
| **Involution Law** | (a) (Ac)c = A |  |

Table 1 shows the law of algebra of sets.

### **Example 1: Prove Idempotent Laws:**

1. (a) A ∪ A = A

**Solution:**

Since, B ⊂ A ∪ B, therefore A ⊂ A ∪ A

Let x ∈ A ∪ A ⇒ x ∈ A or x ∈ A ⇒ x ∈ A

∴ A ∪ A ⊂ A

As A ∪ A ⊂ A and A ⊂ A ∪ A ⇒ A =A ∪ A. Hence Proved.

1. (b) A ∩ A = A

**Solution:**

Since, A ∩ B ⊂ B, therefore A ∩ A ⊂ A

Let x ∈ A ⇒ x ∈ A and x ∈ A

⇒ x ∈ A ∩ A ∴ A ⊂ A ∩ A

As A ∩ A ⊂ A and A ⊂ A ∩ A ⇒ A = A ∩ A. Hence Proved.

### **Example 2: Prove Associative Laws:**

1. (a) (A ∪ B) ∪ C = A ∪ (B ∪ C)

**Solution:**

Let some x ∈ (A'∪ B) ∪ C

 ⇒ (x ∈ A or x ∈ B) or x ∈ C

 ⇒ x ∈ A or x ∈ B or x ∈ C

 ⇒ x ∈ A or (x ∈ B or x ∈ C)

 ⇒ x ∈ A or x ∈ B ∪ C

 ⇒ x ∈ A ∪ (B ∪ C).

Similarly, if some x ∈ A ∪ (B ∪ C), then x ∈ (A ∪ B) ∪ C.

Thus, any x ∈ A ∪ (B ∪ C) ⇔ x ∈ (A ∪ B) ∪ C. Hence Proved.

1. (b) (A ∩ B) ∩ C = A ∩ (B ∩ C)

**Solution:**

Let some x ∈ A ∩ (B ∩ C) ⇒ x ∈ A and x ∈ B ∩ C

 ⇒ x ∈ A and (x ∈ B and x ∈ C) ⇒ x ∈ A and x ∈ B and x ∈ C

 ⇒ (x ∈ A and x ∈ B) and x ∈ C) ⇒ x ∈ A ∩ B and x ∈ C

 ⇒ x ∈ (A ∩ B) ∩ C.

Similarly, if some x ∈ A ∩ (B ∩ C), then x ∈ (A ∩ B) ∩ C

Thus, any x ∈ (A ∩ B) ∩ C ⇔ x ∈ A ∩ (B ∩ C). Hence Proved.

### **Example3: Prove Commutative Laws**

1. (a)  A ∪ B = B ∪ A

**Solution:**

To Prove

 A ∪ B = B ∪ A

 A ∪ B = {x: x ∈ A or x ∈ B}

 = {x: x ∈ B or x ∈ A} (∵ Order is not preserved in case of sets)

 A ∪ B = B ∪ A. Hence Proved.

1. (b) A ∩ B = B ∩ A

**Solution:**

To Prove

 A ∩ B = B ∩ A

 A ∩ B = {x: x ∈ A and x ∈ B}

 = {x: x ∈ B and x ∈ A} (∵ Order is not preserved in case of sets)

 A ∩ B = B ∩ A. Hence Proved.

### **Example 4: Prove Distributive Laws**

1. (a) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

**Solution:**

To Prove

 Let x ∈ A ∪ (B ∩ C) ⇒ x ∈ A or x ∈ B ∩ C

 ⇒ (x ∈ A or x ∈ A) or (x ∈ B and x ∈ C)

 ⇒ (x ∈ A or x ∈ B) and (x ∈ A or x ∈ C)

 ⇒ x ∈ A ∪ B and x ∈ A ∪ C

 ⇒ x ∈ (A ∪ B) ∩ (A ∪ C)

Therefore, A ∪ (B ∩ C) ⊂ (A ∪ B) ∩ (A ∪ C)............(i)

Again, Let y ∈ (A ∪ B) ∩ (A ∪ C) ⇒ y ∈ A ∪ B and y ∈ A ∪ C

 ⇒ (y ∈ A or y ∈ B) and (y ∈ A or y ∈ C)

 ⇒ (y ∈ A and y ∈ A) or (y ∈ B and y ∈ C)

 ⇒ y ∈ A or y ∈ B ∩ C

 ⇒ y ∈ A ∪ (B ∩ C)

Therefore, (A ∪ B) ∩ (A ∪ C) ⊂ A ∪ (B ∩ C)............(ii)

Combining (i) and (ii), we get A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). Hence Proved

1. (b) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

**Solution:**

To Prove

 Let x ∈ A ∩ (B ∪ C) ⇒ x ∈ A and x ∈ B ∪ C

 ⇒ (x ∈ A and x ∈ A) and (x ∈ B or x ∈ C)

 ⇒ (x ∈ A and x ∈ B) or (x ∈ A and x ∈ C)

 ⇒ x ∈ A ∩ B or x ∈ A ∩ C

 ⇒ x ∈ (A ∩ B) ∪ (A ∪ C)

Therefore, A ∩ (B ∪ C) ⊂ (A ∩ B) ∪ (A ∪ C)............ (i)

Again, Let y ∈ (A ∩ B) ∪ (A ∪ C) ⇒ y ∈ A ∩ B or y ∈ A ∩ C

 ⇒ (y ∈ A and y ∈ B) or (y ∈ A and y ∈ C)

 ⇒ (y ∈ A or y ∈ A) and (y ∈ B or y ∈ C)

 ⇒ y ∈ A and y ∈ B ∪ C

 ⇒ y ∈ A ∩ (B ∪ C)

Therefore, (A ∩ B) ∪ (A ∪ C) ⊂ A ∩ (B ∪ C)............ (ii)

Combining (i) and (ii), we get A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∪ C). Hence Proved

### **Example 5: Prove De Morgan's Laws**

(a) (A ∪B)c=Ac∩ Bc

**Solution:**

To Prove (A ∪B)c=Ac∩ Bc

Let x ∈ (A ∪B)c ⇒ x ∉ A ∪ B (∵ a ∈ A ⇔ a ∉ Ac)

 ⇒ x ∉ A and x ∉ B

 ⇒ x ∉ Ac and x ∉ Bc

 ⇒ x ∉ Ac∩ Bc

Therefore, (A ∪B)c ⊂ Ac∩ Bc............. (i)

Again, let x ∈ Ac∩ Bc ⇒ x ∈ Ac and x ∈ Bc

 ⇒ x ∉ A and x ∉ B

 ⇒ x ∉ A ∪ B

 ⇒ x ∈ (A ∪B)c

Therefore, Ac∩ Bc ⊂ (A ∪B)c............. (ii)

Combining (i) and (ii), we get Ac∩ Bc =(A ∪B)c. Hence Proved.

(b) (A ∩B)c = Ac∪ Bc

**Solution:**

Let x ∈ (A ∩B)c ⇒ x ∉ A ∩ B (∵ a ∈ A ⇔ a ∉ Ac)

 ⇒ x ∉ A or x ∉ B

 ⇒ x ∈ Ac and x ∈ Bc

 ⇒ x ∈ Ac∪ Bc

∴ (A ∩B)c⊂ (A ∪B)c.................. (i)

Again, Let x ∈ Ac∪ Bc ⇒ x ∈ Ac or x ∈ Bc

 ⇒ x ∉ A or x ∉ B

 ⇒ x ∉ A ∩ B

 ⇒ x ∈ (A ∩B)c

∴ Ac∪ Bc⊂ (A ∩B)c.................... (ii)

Combining (i) and (ii), we get(A ∩B)c=Ac∪ Bc. Hence Proved.

### **Example 6: Prove Identity Laws.**

1. (a) A ∪ ∅ = A

**Solution:**

To Prove A ∪ ∅ = A

 Let x ∈ A ∪ ∅ ⇒ x ∈ A or x ∈ ∅

 ⇒ x ∈ A (∵x ∈ ∅, as ∅ is the null set )

 Therefore, x ∈ A ∪ ∅ ⇒ x ∈ A

 Hence, A ∪ ∅ ⊂ A.

We know that A ⊂ A ∪ B for any set B.

 But for B = ∅, we have A ⊂ A ∪ ∅

From above, A ⊂ A ∪ ∅ , A ∪ ∅ ⊂ A ⇒ A = A ∪ ∅. Hence Proved.

1. (b) A ∩ ∅ = ∅

**Solution:**

To Prove A ∩ ∅ = ∅

If x ∈ A, then x ∉ ∅ (∵∅ is a null set)

Therefore, x ∈ A, x ∉ ∅ ⇒ A ∩ ∅ = ∅. Hence Proved.

1. (c) A ∪ U = U

**Solution:**

To Prove A ∪ U = U

Every set is a subset of a universal set.

 ∴ A ∪ U ⊆ U

 Also, U ⊆ A ∪ U

Therefore, A ∪ U = U. Hence Proved.

1. (d) A ∩ U = A

**Solution:**

To Prove A ∩ U = A

We know A ∩ U ⊂ A................. (i)

So we have to show that A ⊂ A ∩ U

Let x ∈ A ⇒ x ∈ A and x ∈ U (∵ A ⊂ U so x ∈ A ⇒ x ∈ U )

 ∴ x ∈ A ⇒ x ∈ A ∩ U

 ∴ A ⊂ A ∩ U................. (ii)

From (i) and (ii), we get A ∩ U = A. Hence Proved.

### **Example7: Prove Complement Laws**

(a) A ∪ Ac= U

**Solution:**

To Prove A ∪ Ac= U

 Every set is a subset of U

 ∴ A ∪ Ac ⊂ U.................. (i)

We have to show that U ⊆ A ∪ Ac

 Let x ∈ U ⇒ x ∈ A or x ∉ A

 ⇒ x ∈ A or x ∈ Ac ⇒ x ∈ A ∪ Ac

 ∴ U ⊆ A ∪ Ac................... (ii)

From (i) and (ii), we get A ∪ Ac= U. Hence Proved.

(b) A ∩ Ac=∅

**Solution:**

As ∅ is the subset of every set

 ∴ ∅ ⊆ A ∩ Ac..................... (i)

We have to show that A ∩ Ac ⊆ ∅

Let x ∈ A ∩ Ac ⇒ x ∈ A and x ∈ Ac

 ⇒ x ∈ A and x ∉ A

 ⇒ x ∈ ∅

 ∴ A ∩ Ac ⊂∅..................... (ii)

From (i) and (ii), we get A∩ Ac=∅. Hence Proved.

(c) Uc= ∅

**Solution:**

Let x ∈ Uc ⇔ x ∉ U ⇔ x ∈ ∅

 ∴ Uc= ∅. Hence Proved. (As U is the Universal Set).

(d) ∅c = U

**Solution:**

Let x ∈ ∅c ⇔ x ∉ ∅ ⇔ x ∈ U (As ∅ is an empty set)

 ∴ ∅c = U. Hence Proved.

### **Example8: Prove Involution Law**

(a) (Ac )c A.

**Solution:**

Let x ∈ (Ac )c ⇔ x ∉ Ac⇔ x ∈ a

 ∴ (Ac )c =A. Hence Proved.

## **Duality:**

The dual E∗ of E is the equation obtained by replacing every occurrence of ∪, ∩, U and ∅ in E by ∩, ∪, ∅, and U, respectively. For example, the dual of

1. (U ∩ A) ∪ (B ∩ A) = A is (∅ ∪ A) ∩ (B ∪ A) = A

It is noted as the principle of duality, that if any equation E is an identity, then its dual E∗ is also an identity.

## **Principle of Extension:**

According to the Principle of Extension two sets, A and B are the same if and only if they have the same members. We denote equal sets by A=B.

1. If A= {1, 3, 5} and B= {3, 1, 5}, then A=B i.e., A and B are equal sets.
2. If A= {1, 4, 7} and B= {5, 4, 8}, then A≠ B i.e.., A and B are unequal sets.

## **Cartesian product of two sets:**

The Cartesian Product of two sets P and Q in that order is the set of all ordered pairs whose first member belongs to the set P and second member belong to set Q and is denoted by P x Q, i.e.,

1. P x Q = {(x, y): x ∈ P, y ∈ Q}.

**Example:** Let P = {a, b, c} and Q = {k, l, m, n}. Determine the Cartesian product of P and Q.

**Solution:** The Cartesian product of P and Q is

