Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	First lecture	Bakr Noori Alhasan/Lecturer
	The major contents: The major contents: 1- The aim of this class. 2- Course contents 3- Ch.1 Introduction		
	The detailed contents: The detailed contents: 1- The aim of this class 2- Course contents 3- Text book 4- References 4- Ch.1 Introduction 5- History of Dynamics 6- Dynamics 7- Kinematics 8- Kinetics 9- Applications of Dynamics 10- Basic Concepts 11- A particle		

	12- A rigid body **13- Vector and Scalar** **14- 1/3 Newton's Laws** **15- Units** **16- Standard value of 'g'**

University of Mosul/College of Engineering
Department of Mechanics
ENGINEERING MECHANICS 'DYNAMICS'
2018 - 2019

The Aim of This Class

The object of this class is to develop the students' abilities in understanding and solving dynamic problems related particles and rigid bodies.

Course Contents

Kinematics of Particles
Rectilinear Motion
Plane curvilinear motion
Relative motion (Translating axes)
Constrained motion of connected particles
Kinetics of Particles
Force , Mass, and Acceleration
Work and Energy
Impulse and Momentum
Special Applications; Impact
Plane Kinematics of Rigid Bodies
Absolute motion
Relative velocity
Instantaneous center of zero velocity
Relative acceleration
Plane Kinetics of Rigid Bodies
Force, Mass, and Acceleration
Work and Energy
Impulse and Momentum
Appendix B: Mass Moment of Inertia

Text Book: Engineering Mechanics "Dynamics"
J.L. Meriam and L.D. Kraige 5th ed.

References:

1. Engineering Mechanics 'Dynamics"
R. C. Hibbeler
2. Engineering Mechanics Dynamics
Andrew Pyel and Jan Kiwsalaas

Engineering Mechanics: The study of how rigid bodies react to forces acting on them. It can be divided into two parts:

Statics: The study of bodies in equilibrium $\sum \vec{F} = 0$
Dynamics: The study of motion of the bodies under the action of forces.

CHAPTER ONE
Introduction to Dynamics

History of Dynamics

The beginning of a rational understanding of dynamics is credited to Galileo (1564-1642) who made observations on free fall, motion on an inclined plane, and motion of the pendulum.

Newton (1642-1727), guided by Galileo's work, formulated the laws of motion and the law of universal gravitation. His famous work was published in the first edition of *Principia*. Other scientists such as Euler, D'Alembert, Lagrange made important contributions to mechanics.

Dynamics: Is that branch of mechanics which deals with the motion of bodies under the action of forces. Dynamics is divided in to two parts: kinematics and kinetics

Kinematics: is the study of motion without reference to the forces which cause motion.

Kinetics: is the study of action of forces on bodies and their resulting motion.

Applications of Dynamics

1. Analysis and design of moving structures.
2. Fixed structures subject to shock loads
3. Robotic devices.
4. Machining of turbines and pumps.
5. Automatic control system
6. Rockets.
7. Missiles and spacecrafts.
8. Ground and air transportation vehicles. And others.

Basic Concepts

Space: Is the geometric region occupied by bodies

Time: Is a measure of the succession of events (considered absolute in Newtonian Mechanics)

Mass : Is the quantitative measure of inertia or resistance to change in motion of the body.

Force: Is the vector action of one body on another.

A particle: is a body of negligible dimensions.
The body may be treated as a particle when its dimensions are irrelevant to the description of its motion or the action of forces on it.

A rigid body: is a body whose changes in shape are negligible compared with the overall dimensions of the body or with the changes in position of the body as a whole.

Vector and Scalar ……..

1/3 Newton's Laws

Law 1. A particle remains at rest or continuous to move with uniform velocity (in a straight line with a constant speed) if there is no unbalance force acting on it.

Law 2. The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force. $\vec{F} = m\vec{a}$

Law 3. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

Units

The units and symbols of the four fundamental quantities of mechanics for the International System of metric units SI And U.S. customary system are summarized in the following table:

Quantity	Symbol	SI units	Unit Symbol	U.S. unit	Customary unit symbol
Mass	M	kilogram	kg	Slug	-
Length	L	Meter	m	foot	ft
Time	T	Second	s	second	s
Force	F	Newton	N	pound	lb

The SI system is termed an absolute system because the standard for the base unit kilogram (a platinum - iridium cylinder kept at the International Bureau of standards near Paris, France) is independent of the gravitational attraction of the earth. On the other hand, the U.S. customary system is termed a gravitational system because the standard for the base unit pound (the weight of a standard mass located at sea level and at latitude of 45°) requires the presence of the gravitational field of the earth.

In SI units, by definition, one Newton is that force which will give a one- kilogram mass an acceleration of one meter per second squared.
In U.S. customary system a 32.1740 pound mass (1 slug) will have an acceleration of one foot per second squared when acted on by a force of one pound.

Standard value of 'g'

The standard value which has been adopted internationally for the gravitational acceleration relative to the rotating earth at sea level and at a latitude of 45° is 9.806 65 m/s^2 or 32.1740 ft/sec^2. The values 9.81 m/sec^2 in SI units and 32.2 ft/sec^2 in US customary units are used for the sea level value of g.

Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	Second lecture	Bakr Noori Alhasan/Lecturer
	The major contents: Chapter Two/ Kinematics of Particle Particle motion Choice of coordinates 2/2 Rectilinear Motion		
	The detailed contents: Particle motion Choice of coordinates 2/2 Rectilinear Motion Velocity and acceleration Relation between s, v, and a Graphical Interpretation Slope Area under the curves Analytical Integration		

CHAPTER TWO
KINEMATICS of PARTICLE

Particle motion:

Ways of describing the motion: Depends on the experience and how the data are given.
Figure 2.1 shows the ways covered in this chapter.

Constrained motion: If the particle is confined to specified path
Unconstrained motion: There are no physical guide

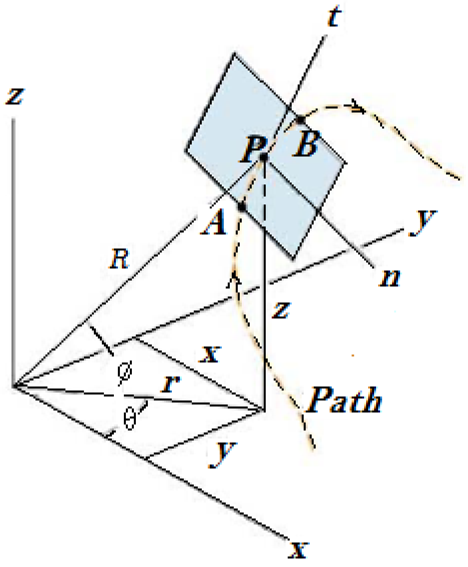


Figure 2/1

Choice of coordinates

The position of particle P at any time t can be described by:

1. *Rectangular coordinates* x, y, z
2. *Cylindrical coordinates* r, θ, z
3. *Spherical coordinates* $R, \theta, \emptyset$

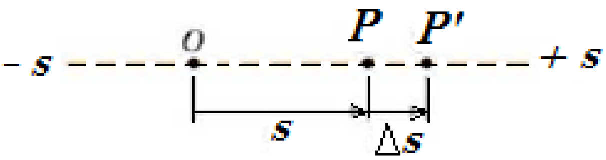
And path variables n-t where t is tangent and n normal to the path.

The motion of particle can be described by using:
Absolute motion analysis: coordinates measured from fixed reference axis.
Relative-motion analysis: coordinates measured from moving reference axis.

Plane motion: all movements can be represented as occurring in a single plane.

Three dimensional motion: motion in space.

2/2 Rectilinear Motion



O reference point
s distance (position coordinate of particle at time t)
$s + \Delta s$ the position coordinate of particle at time $t + \Delta t$
Δs displacement: the change in position coordinate
Δs is positive if it is in the positive s direction and negative if it is in the negative s.

Velocity and acceleration

$$\text{Average speed} \quad v_{av} = \frac{\Delta s}{\Delta t}$$

$$\text{Instantaneous speed} \quad \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\text{Instantaneous speed} \quad v = \frac{ds}{dt} = \dot{s} \quad \ldots\ldots\ldots\ldots \; 2/1$$

Velocity: Is the time rate of change of the position coordinate s.
the velocity is positive if the displacement is positive.

$$\text{Average acceleration} \quad a_{av} = \frac{\Delta v}{\Delta t}$$

$$\text{Instantaneous acceleration} \quad \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$\text{Instantaneous acceleration} \quad a = \frac{dv}{dt} = \dot{v} = \ddot{s} \quad \ldots\ldots\ldots\ldots \; 2/2$$

a is positive when velocity is increasing
a is negative when velocity is decreasing and called deceleration

2/3 The velocity of a particle which moves along the s-axis is given by $v = 2 + 5t^{3/2}$, where t is in seconds and v is in meters per second. Evaluate the displacement s, velocity v, and acceleration a when $t = 4$ s. The particle is at the origin $s = 0$ when $t = 0$.
Ans. $s = 72$ m, $v = 42$ m/s, $a = 15$ m/s^2

Relation between s, v, and a

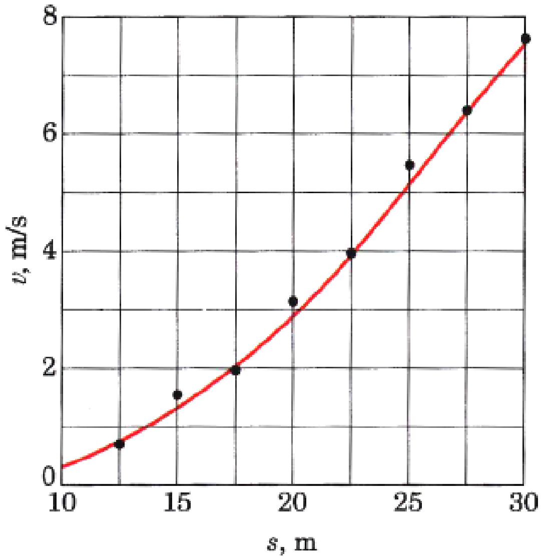
From 2/1 and 2/2

$$v\,dv = a\,ds \quad \ldots\ldots\ldots \; 2/3$$

2/17

Experimental data for the motion of a particle along a straight line yield measured values of the velocity v for various position coordinates s. A smooth curve is drawn through the points as shown in the graph. Determine the acceleration of the particle when $s = 20$ m.

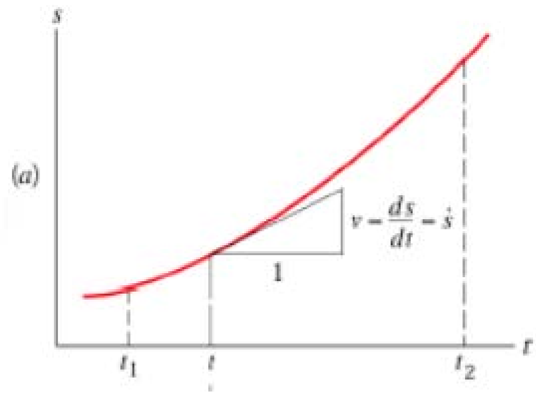
Ans. $a = 1.2\ \text{m/s}^2$



Graphical Interpretation

Slope

$$slope\ of\ s-t\ curve = velocity$$



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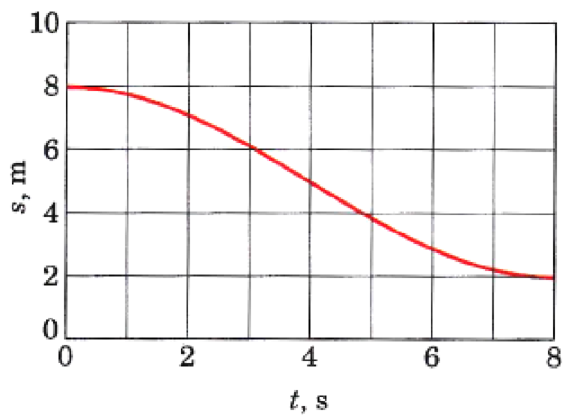
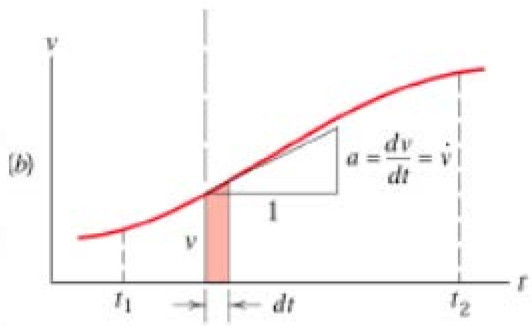
The graph shows the displacement-time history for the rectilinear motion of a particle during an 8-second interval. Determine the average velocity v_{av} during the interval and, to within reasonable limits of accuracy, find the instantaneous velocity v when $t = 4$ s.



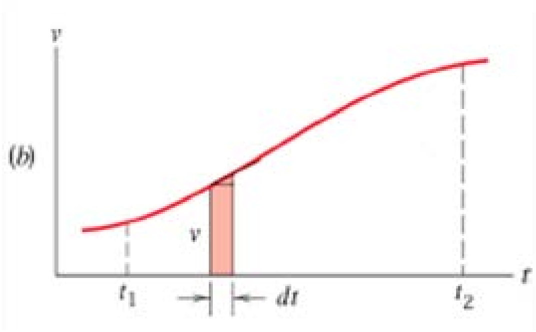
$Slope\ of\ v-t\ curve =$ acceleration.



Area under the curves

Area under v-t curve = net displacement

$$\int_{t_1}^{t_2} v dt = \int_{s_1}^{s_2} ds = s_2 - s_1$$

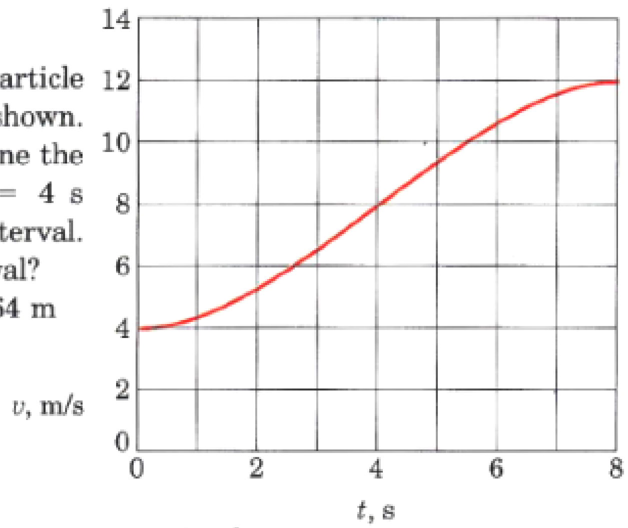


Area under $a - t$ curve = changes in

2/21

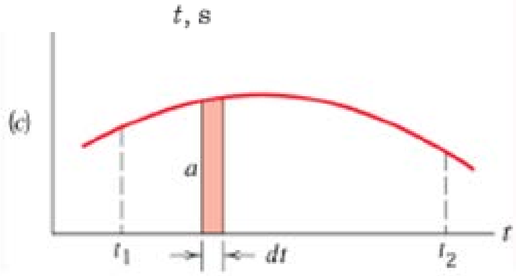
During an 8-second interval the velocity of a particle moving in a straight line varies with time as shown. Within reasonable limits of accuracy, determine the amount Δa by which the acceleration at $t = 4$ s exceeds the average acceleration during the interval. What is the displacement Δs during the interval?

Ans. $\Delta a = 0.50 \text{ m/s}^2$, $\Delta s = 64$ m



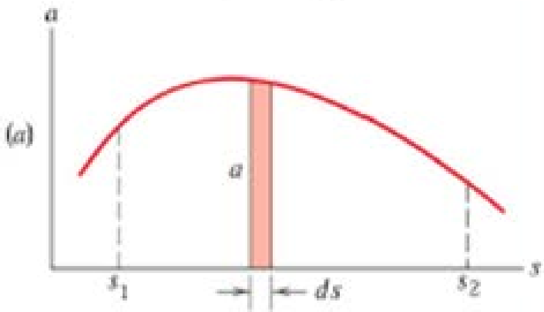
velocity

$$\int_{t_1}^{t_2} a dt = \int_{v_1}^{v_2} dv = v_2 - v_1$$



Area under $a - s$ curve

$$\int_{s_1}^{s_2} a ds = \int_{v_1}^{v_2} v dv = \frac{1}{2}(v_2^2 - v_1^2)$$



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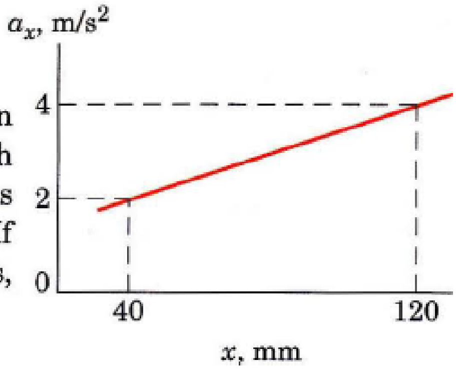
A particle moves along the positive x-axis with an acceleration a_x in meters per second squared which increases linearly with x expressed in millimeters, as shown on the graph for an interval of its motion. If the velocity of the particle at $x = 40$ mm is 0.4 m/s, determine the velocity at $x = 120$ mm.

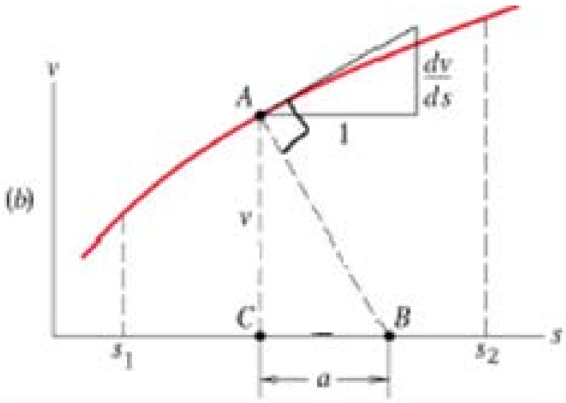


The acceleration a can be found from v-s curve with the aid of similar triangles

$$\frac{dv}{ds} = \frac{\overline{CB}}{v}$$

$$vdv = ads$$

$$\overline{CB} = a$$



Analytical Integration

(a) Constant acceleration
from equation 2/2 $dv = a\,dt$

$$\int_{v_o}^{v} dv = a\int_{0}^{t} dt \qquad or \quad v = v_o + a\,t$$

or $vdv = ads$

$$\int_{v_o}^{v} v\,dv = a\int_{s_o}^{s} ds \quad or \quad v^2 = v_o^2 + 2a(s - s_o)$$

or $ds = vdt$

$$\int_{s_o}^{s} ds = \int_{0}^{t} (v_o + at)dt \quad or \quad s = s_o + v_o t + \frac{1}{2}a\,t^2$$

2/8 A rocket is fired vertically up from rest. If it is designed to maintain a constant upward acceleration of 1.5g, calculate the time t required for it to reach an altitude of 30 km and its velocity at that position.

b. Acceleration given as a function of time, $a = \text{f(t)}$

$$dv = a\,dt$$

$$\int_{v_o}^{v} dv = \int_{0}^{t} f(t)dt \qquad or \quad v = v_o + \int_{0}^{t} f(t)\,dt$$

Note s can be found with the aid of $\int ds = \int vdt$

Note also if $a = f(t)$

$$\ddot{s} = f(t)$$

$$s = \iint f(t)dt$$

2/7 The acceleration of a particle is given by $a = 4t - 30$, where a is in meters per second squared and t is in seconds. Determine the velocity and displacement as functions of time. The initial displacement at $t = 0$ is $s_0 = -5$ m, and the initial velocity is $v_0 = 3$ m/s.

Ans. $v = 3 - 30t + 2t^2$ m/s

$s = -5 + 3t - 15t^2 + \frac{2}{3}t^3$ m

(c) Acceleration given as a function of velocity $a = f(v)$

From eq. 2/2

$$a = \frac{dv}{dt}$$

$$f(v) = \frac{dv}{dt}$$

$$\int_{0}^{t} dt = \int_{v_o}^{v} \frac{dv}{f(v)} \qquad or \;\; t = f(v) \quad \rightarrow \quad v = f(t)$$

$$ds = v\,dt$$

$$\int_{s_o}^{s} ds = \int_{0}^{t} f(t)dt$$

Another approach

$$a = f(v)$$

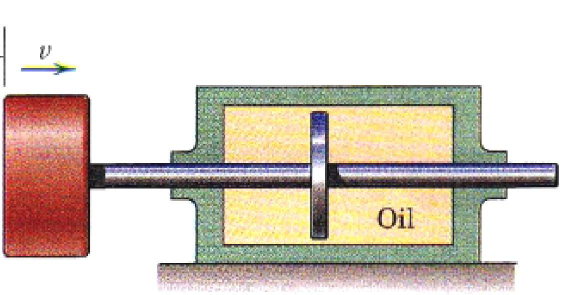
$$vdv = a\,ds$$

$$vdv = f(v)\,ds$$

$$\int_{v_o}^{v} \frac{vdv}{f(v)} = \int_{s_o}^{s} ds \quad or \quad s = s_o + \int_{v_o}^{v} \frac{vdv}{f(v)}$$

2/40

The horizontal motion of the plunger and shaft is arrested by the resistance of the attached disk which moves through the oil bath. If the velocity of the plunger is v_0 in the position A where $x = 0$ and $t = 0$, and if the deceleration is proportional to v so that $a = -kv$, derive expressions for the velocity v and position coordinate x in terms of the time t. Also express v in terms of x.



(d) Acceleration given as a function of displacement, $a = f(s)$

$$vdv = ads$$

$$vdv = f(s)ds \quad or \quad v^2 = v_o^2 + 2\int_{s_o}^{s} f(s)ds$$

$$or\ v = g(s)$$

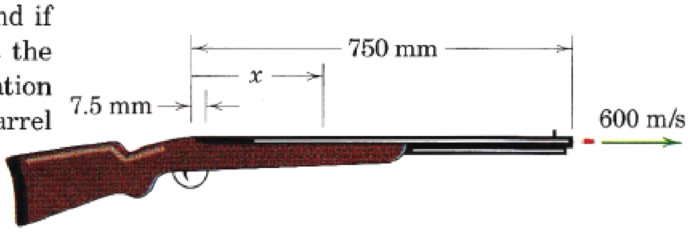
$$v = \frac{ds}{dt}$$

$$\int_0^t dt = \int_{s_o}^{s} \frac{ds}{g(s)} \quad or \quad t = \int_{s_o}^{s} \frac{ds}{g(s)}$$

arrange to give $s = f(t)$

2/48

To a close approximation the pressure behind a rifle bullet varies inversely with the position x of the bullet along the barrel. Thus the acceleration of the bullet may be written as $a = k/x$ where k is a constant. If the bullet starts from rest at $x = 7.5$ mm and if the muzzle velocity of the bullet is 600 m/s at the end of the 750-mm barrel, compute the acceleration of the bullet as it passes the midpoint of the barrel at $x = 375$ mm.



a, b, c and d depend on the form of function. If the integration is difficult the graphical, numerical or computer method are used.

Problems 2/3 2/4 2/5 2/6 2/14 2/17 2/18 2/20 2/22 2/26 2/28 2/31 2/35 2/32 2/45

Sample problem 2/1

The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine (a) the time required for the particle to reach a velocity of 72 m/s from its initial condition at t = 0 , (b) the acceleration of the particle when v=30 m/s, and (c) the net displacement of the particle during the interval from t=1 s to t=4 s.

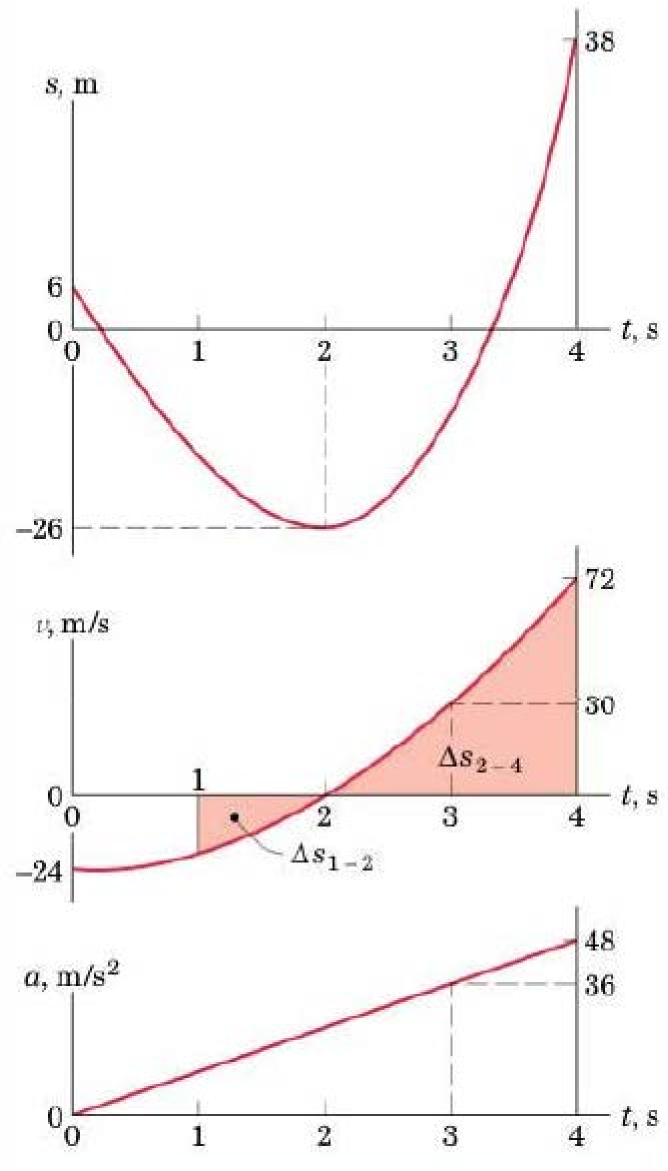
Solution

$$(a) \qquad s = 2t^3 - 24t + 6$$
$$[v = \dot{s}] \qquad v = 6t^2 - 24$$
$$[a = \dot{v}] \qquad a = 12t$$

$$when\ \ v = 72\,m/s \qquad 72 = 6t^2 - 24$$
$$t = \pm 4\,s$$
the negative root is of no physical interest
$$so \qquad t = 4\ \ s \quad Ans.$$

$$(b) \qquad v = 6t^2 - 24$$
$$30 = 6t^2 - 24$$
$$t = \pm 3$$
$t = 3\,s$ *substituting* $t =$ $3\,s$ *into the expression for a gives*
$$a = 12(3) = 36\ \ m/s \quad Ans.$$

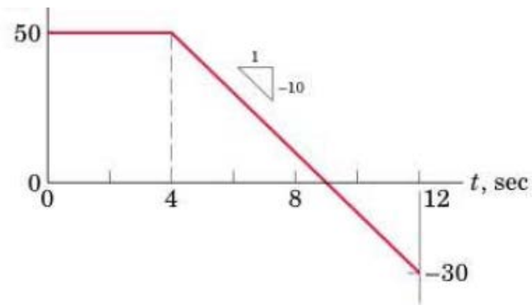
$$(c) \qquad \Delta s = s_4 - s_1$$
$$\Delta s = [2(4^3) - 24(4) + 6] - [2(1^3) - 24(1) + 6]$$
$$= 54\,m \qquad Ans.$$



Sample problem 2/2

A particle moves along the x-axis with an initial velocity $v_x = 50\,m/s$ at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10\,m/s^2$. Calculate the velocity and the x-coordinate of the particle for the conditions of $t = 8\ s$ and $t = 12\,s$ and find the maximum positive x-coordinate reached by the particle.

Solution



After $t = 4\ s$ the velocity is

$[\int dv = \int a dt]$ $\int_{50}^{v_x} dv_x = -10\int_4^t dt$ $v_x = 90 - 10t\ \ m/s$

$At\ t = 8\,s$ $v_x = 90 - 10(8) = 10\,m/s$

$At\ t = 12\,s$ $v_x = 90 - 10(12) = -30\ m/s$ $Ans.$

To find the x-coordinate

$[\int ds = \int v dt]$ $x = 50(4) + \int_4^t (90 - 10t)dt = -5t^2 + 90t - 80\ \ m$

$At\ t = 8\,s$ $x = -5(8^2) + 90(8) - 80 = 320\,m$

$At\ t = 12\,s$ $x = -5(12^2) + 90(12) - 80 = 280\,m$ $Ans.$

The x-coordinate for $t = 12\,s$ is less than that for $t = 8\,s$ since the direction of motion is changed at $t = 9\,s$

$x_{max} = -5(9^2) + 90(9) - 80 = 325\,m$ $Ans.$

2/4 The velocity of a particle along the s-axis is given by $v = 5s^{3/2}$, where s is in millimeters and v is in millimeters per second. Determine the acceleration when s is 2 millimeters.

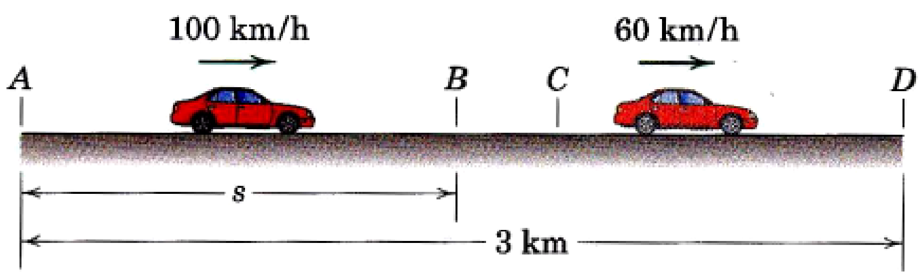
2/5 The position of a particle in millimeters is given by $s = 27 - 12t + t^2$, where t is in seconds. Plot the s-t and v-t relationships for the first 9 seconds. Determine the net displacement Δs during that interval and the total distance D traveled. By inspection of the s-t relationship, what conclusion can you reach regarding the acceleration?

Ans. $\Delta s = -27$ mm, $D = 45$ mm, $a =$ constant

2/6 A particle moves along a straight line with a velocity in millimeters per second given by $v = 400 - 16t^2$ where t is in seconds. Calculate the net displacement Δs and total distance D traveled during the first 6 seconds of motion.

2/14 A projectile is fired vertically with an initial velocity of 200 m/s. Calculate the maximum altitude h reached by the projectile and the time t after firing for it to return to the ground. Neglect air resistance and take the gravitational acceleration to be constant at 9.81 m/s^2.

2/20 In traveling a distance of 3 km between points A and D, a car is driven at 100 km/h from A to B for t seconds and 60 km/h from C to D also for t seconds. If the brakes are applied for 4 seconds between B and C to give the car a uniform deceleration, calculate t and the distance s between A and B.



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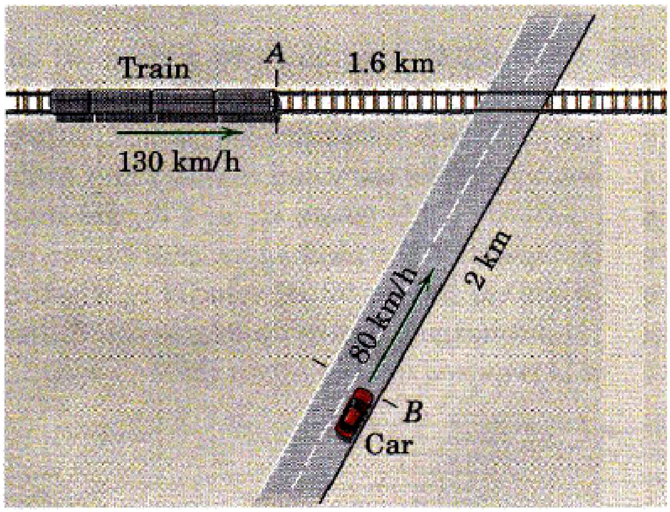
A train which is traveling at 130 km/h applies its brakes as it reaches point A and slows down with a constant deceleration. Its decreased velocity is observed to be 96 km/h as it passes a point 0.8 km beyond A. A car moving at 80 km/h passes point B at the same instant that the train reaches point A. In an unwise effort to beat the train to the crossing, the driver "steps on the gas." Calculate the constant acceleration a that the car must have in order to beat the train to the crossing by 4 s and find the velocity v of the car as it reaches the crossing.

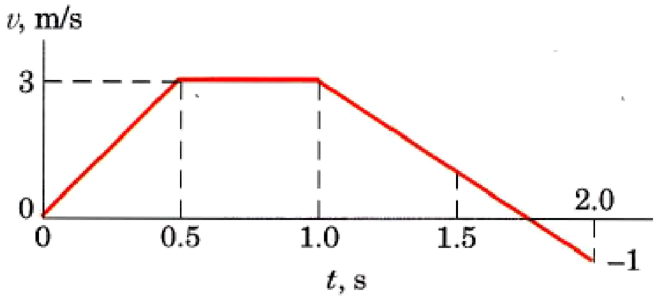


2/28 A single-stage rocket is launched vertically from rest, and its thrust is programmed to give the rocket a constant upward acceleration of 6 m/s^2. If the fuel is exhausted 20 s after launch, calculate the maximum velocity v_m and the subsequent maximum altitude h reached by the rocket.

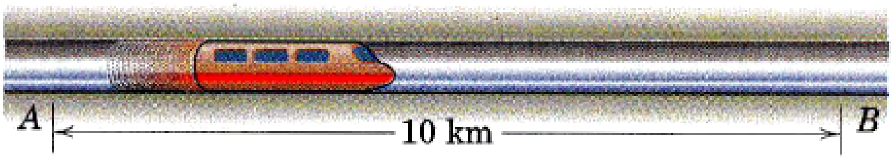
2/31

A particle starts from rest at $x = -2$ m and moves along the x-axis with the velocity history shown. Plot the corresponding acceleration and the displacement histories for the 2 seconds. Find the time t when the particle crosses the origin.

Ans. $t = 0.917$ s



2/32 A vacuum-propelled capsule for a high-speed tube transportation system of the future is being designed for operation between two stations A and B, which are 10 km apart. If the acceleration and deceleration are to have a limiting magnitude of $0.6g$ and if velocities are to be limited to 400 km/h, determine the minimum time t for the capsule to make the 10-km trip.



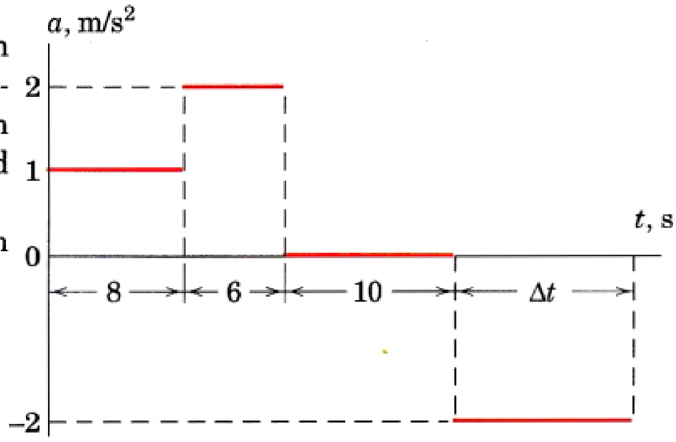
2/35 A car starts from rest with an acceleration of 6 m/s^2 which decreases linearly with time to zero in 10 seconds, after which the car continues at a constant speed. Determine the time t required for the car to travel 400 m from the start.

Ans. t = 16.67 s

2/45

A subway train travels between two of its station stops with the acceleration schedule shown. Determine the time interval Δt during which the train brakes to a stop with a deceleration of 2 m/s^2 and find the distance s between stations.

Ans. Δt = 10 s, s = 416 m



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

	Lecture sequences:	3d lecture	Bakr Noori Alhasan/Lecturer
Lecture Contents	*The major contents* 2/3 Plane Curvilinear Motion Velocity Acceleration 2/4 Rectangular Coordinates (x-y) Projectile Motion		
	The detailed contents 2/3 Plane Curvilinear Motion Velocity The magnitude of the derivative and the derivative of the magnitude Characteristics of the derivative of vectors Acceleration Visualization of motion 2/4 Rectangular Coordinates (x-y) Vector representation Projectile Motion Sample problem 2/5 Sample problem 2/6		

2/3 Plane Curvilinear Motion

If the plane of motion is x-y plane, coordinates: z, ϕ, are zeros and R → r

The vast majority of the motions of particles are plane motion.

We will use vector analysis that is time derivative of vector.

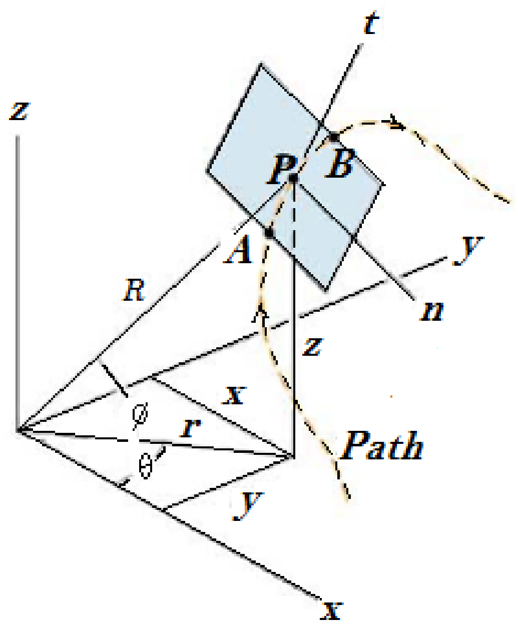
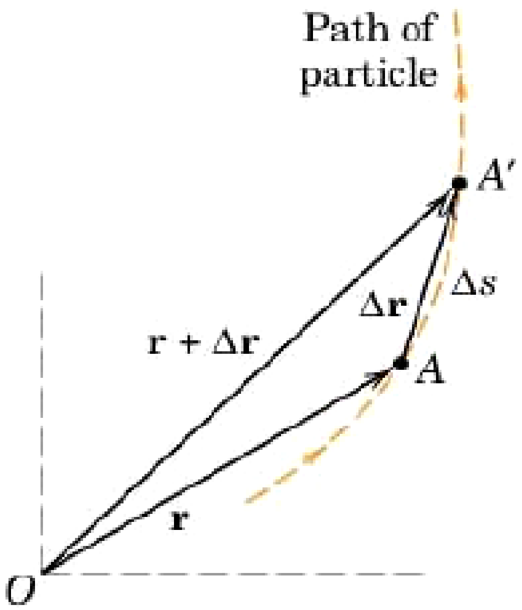


Figure 2/1



$\vec{r}$: *the position vector of the particle at position A at time t.*

$\vec{r} + \Delta\vec{r}$: *the position vector of particle at position À at time* $t + \Delta t$

$\Delta\vec{r}$: *the displacement of particle during time* Δt

(*is independent on the choice of origion*)

Δs : *the distance along the path*)

Velocity

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta r}}{\Delta t} \quad (\text{whose direction is that of } \Delta\vec{r} \text{ and magnitude is that of } \Delta\vec{r}/\Delta t)$$

$$\frac{\Delta s}{\Delta t} \quad \text{is the average speed between } A \text{ and } \grave{A} \ (\text{scalar})$$

$$|\vec{v}_{av}| \approx \text{average speed when } \Delta t \text{ decreasing and } A \text{ becomes close to } \grave{A}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} \quad \ldots\ldots\ldots \ 2/4$$

where $\vec{v}$ *is the instantaneous velocity of the particle always tangent to the path.*

$$The\ direction\ of\ \Delta\vec{r}\ becomes\ tangent\ to\ the\ path\ as\ \Delta t\ \rightarrow 0$$

$$v = |\vec{v}| = \frac{ds}{dt} = \dot{s}\ \ is\ the\ speed\ or\ the\ magnitude\ of\ \vec{v}$$

The magnitude of the derivative and the derivative of the magnitude

$\left|\frac{d\vec{r}}{dt}\right| = |\dot{\vec{r}}| = \dot{s} = |\vec{v}| = v\ \ ex: the\ magnitude\ of\ velocity\ that\ is\ the\ speed$

$\frac{d|\vec{r}|}{dt} = \frac{dr}{dt} = \dot{r}\quad the\ rate\ at\ which\ \vec{r}\ is\ changing$

Characteristics of the derivative of vectors:

$\Delta\vec{v}\ depends\ on\ the\ magnitude\ of\ \vec{v}\ (length)\ and\ the\ direction\ of\ \vec{v}\ \ as\ follows:$
$where\ \ \vec{v} + \Delta\vec{v} = \vec{\acute{v}}$

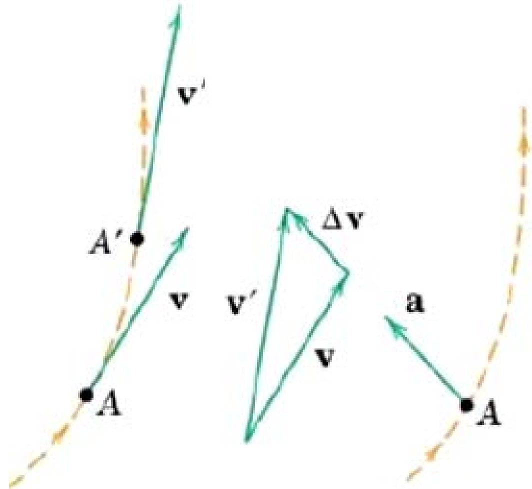
Acceleration

$\vec{a}_{av} = \frac{\overrightarrow{\Delta v}}{\Delta t}$ $(it\ is\ a\ vector\ whose\ direction\ is\ that\ of\ \Delta\vec{v}\ and\ magnitude\ of\ \Delta\vec{v}\ divided\ by\ \Delta t)$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}\ \ \ldots\ldots\ 2/5$$

$As\ \Delta t\ \rightarrow 0, the\ direction\ of\ \Delta\vec{v}\ \rightarrow d\vec{v}\ and\ thus\ of\ \vec{a}$



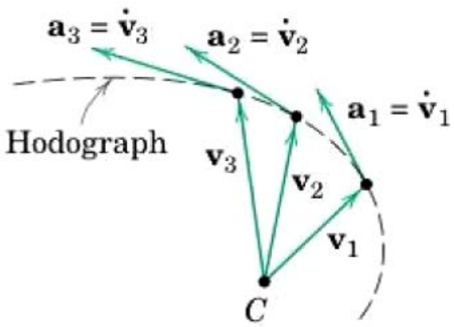
$\vec{a}\ \ includes\ the\ effects\ of\ both; the\ change\ in\ magnitude\ and\ direction\ of\ \vec{v}$

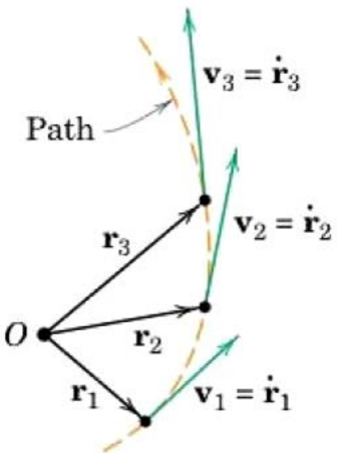
Visualization of motion

a further approach to the visualization of acceleration is explained as follows

If these velocities are plotted from some arbitrary point C, a curve called hodograph is formed.

The derivatives of these velocity vectors will be the acceleration vectors $\vec{a} = \dot{\vec{v}}$ which are tangents to the hodograph.

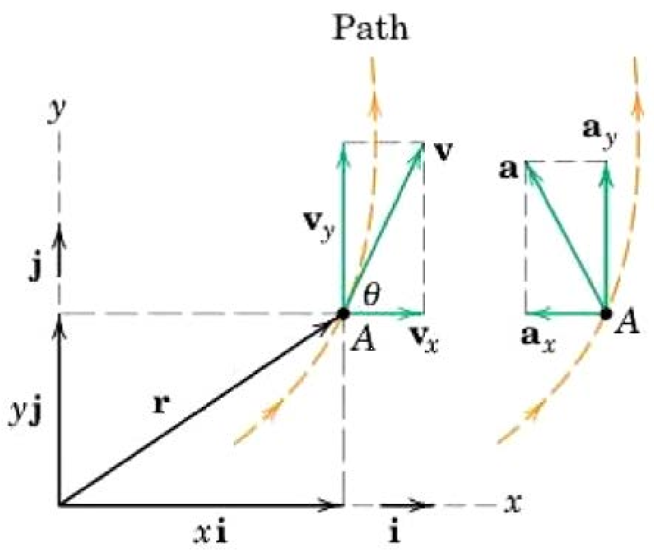




Three different coordinate systems are used:
1. Rectangular coordinates.
2. Normal and Tangential coordinates.
3. Polar coordinates.

2 / 4 Rectangular Coordinates (x-y)

Vector representation



The coordinates are x, y

with the aid of unit vectors, $\vec{\imath}\,,\vec{\jmath}$

$$\vec{r} = x\,\vec{\imath} + \,y\,\vec{\jmath}$$

$$\vec{v} = \dot{\vec{r}} = \dot{x}\,\vec{\imath} + \dot{y}\vec{\jmath}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\vec{\imath} + \ddot{y}\vec{\jmath} \quad \dots\dots\dots \; 2/6$$

The time derivatives of the unit vectors are zero because their magnitudes and directions remain constant.

The scalar values of the components of $\vec{v}$ and $\vec{a}$ are:

$$v_x = \dot{x} \quad v_y = \dot{y} \quad a_x = \ddot{x} \quad a_y = \ddot{y}$$

$$v = \sqrt{v_x^2 + v_y^2} \qquad a = \sqrt{a_x^2 + a_y^2} \qquad \tan\theta = \frac{v_y}{v_x} = \frac{d_y}{d_x}$$

$$\theta \;\; is\; measured\; counterclockwise\; from\; x - axis\; to\; \vec{v}$$

$$If\; x = f_1(t) \quad and \quad y = f_2(t)$$

$$for\; any\; value\; of\; t\;\; we\; can\; find\;\; \vec{r}$$

$$Similarly\; for\; \dot{x}\; and\; \dot{y}\;\; to\; obtain\; \vec{v}\; and\; for\; \ddot{x}\;\; and\; \ddot{y}\;\; to\; obtain\; \vec{a}$$

$$or \quad a_x = g_1(t) \quad and \quad a_y = g_2(t)$$

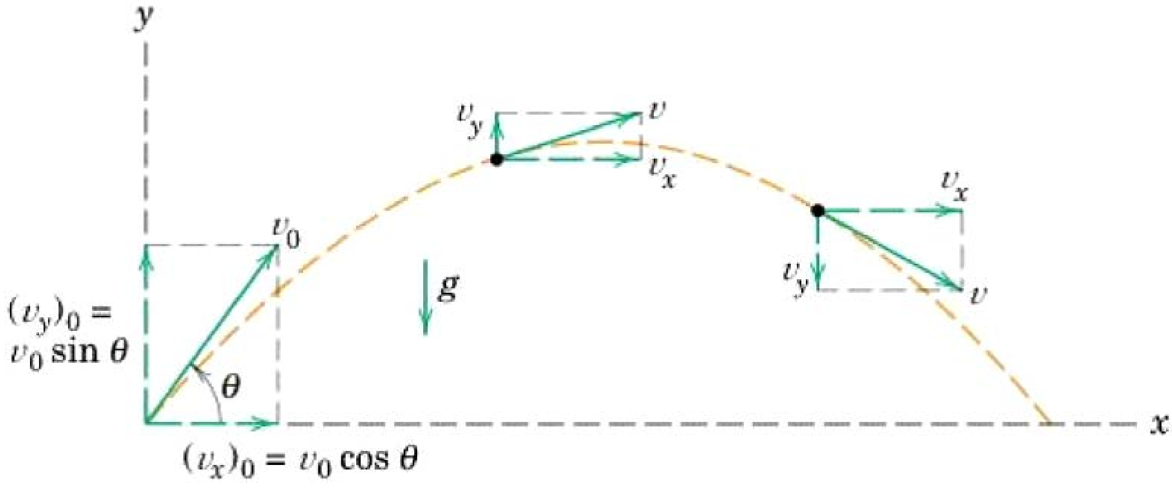
$$with\; integration\; and\; parametric\; equation,$$

$$y = f(x)\;\; be\; found\; that\; is\; the\; equation\; of\; the\; path.$$

Projectile Motion

Assumptions

1. Neglect aerodynamic drag. (function of x^2).
2. Neglect curvature and rotation of the earth.
3. The gravity is assumed constant.



$$a_x = 0$$

$$a_y = -g$$

$$v_x = v_{x_o}$$

$$v_y = v_{y_o} - gt$$

$$x = x_o + v_{x_o} t \qquad or \quad = v_{x_o} t$$

$$y = y_o + v_{y_o} t - \frac{1}{2} gt^2$$

$$v_y^2 = v_{y_o}^2 - 2g(y - y_o)$$

$$With\ elimination\ t \ \rightarrow y = f(x) \rightarrow parabolic.$$

$$If\ the\ drag\ is\ introduced, the\ path\ will\ not\ be\ parabolic$$

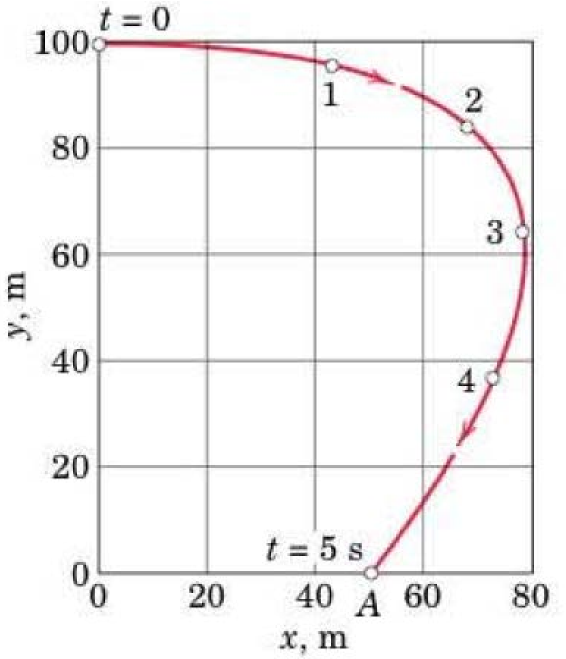
$At\ high\ velocities\ and\ high\ altitude, the\ shape\ of\ projectile, varying\ of\ g, varying\ of\ density, and\ rotation\ of\ the\ earth\ should\ be\ taken\ in\ to\ account.$

Problems

2/62 2/64 2/66 2/71 2/72 2/77 2/87 2/92

Sample problem 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.



Given:

$$v_x = 50 - 16t$$

$$y = 100 - 4t^2$$

$$x = 0\ (t = 0)$$

$$path = ?$$

$$\begin{matrix}\vec{v}=? \\ \vec{a}=?\end{matrix}\{y = 0\}$$

Solution

$$\int_0^x dx = \int_0^t v_x\, dt$$

$$x = \int_0^t 50 - 16t\ \ dt$$

$$x = 50t - 8t^2 \quad m$$

$$a_x = -16\, m/s^2$$

$$y = 100 - 4t^2 \quad m$$

$$v_y = -8\,t \quad m/s$$

$$a_y = -8 \quad m/s^2$$

$$at \quad y = 0 = 100 - 4t^2$$

$$t = 5\ \ sec$$

$$v_x = 50 - 16(5) = -30\ \ m/s$$

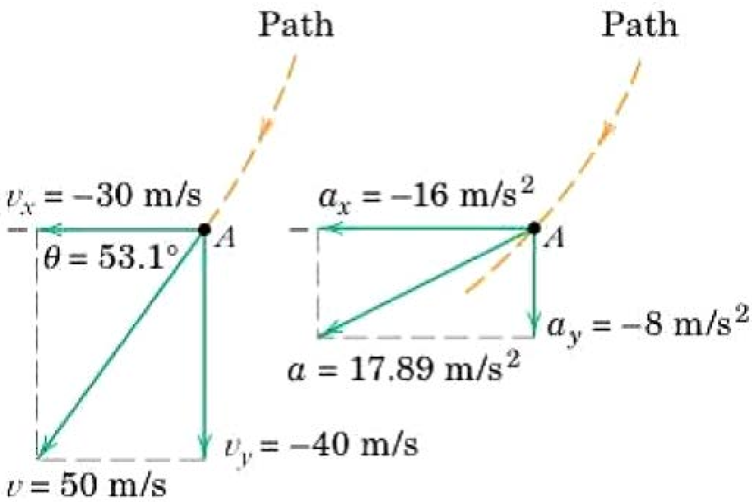
$$v_y = -8(5) = -40\ \ m/s$$

$$a_x = -16\ \ m/s^2$$

$$a_y = -8\ \ m/s^2$$

$$\vec{v} = -30\,\vec{\imath} - 40\vec{\jmath}\ \ m/s$$

$$\vec{a} = -16\,\vec{\imath} - 8\vec{\jmath}\ \ m/s^2$$



Sample problem 2/6

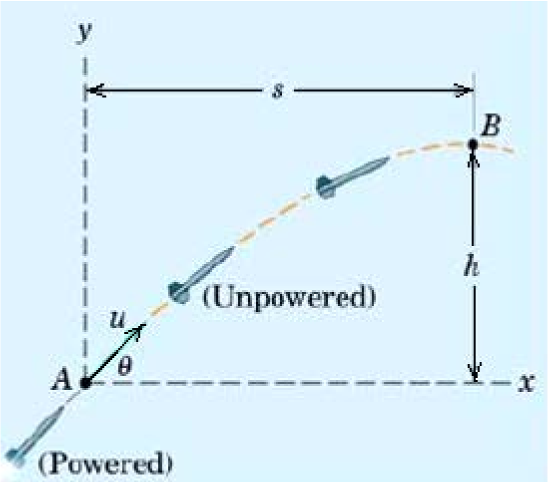
A rocket has expended all its fuel when it reaches position A, where it has a velocity of ***u*** at an angle θ with respect to the horizontal. It then begins unpowered flight and attains a maximum added height h at position B after a traveling a horizontal distance s from A. Determine the expression for h and s, the time t of flight from A to B, and the equation of the path. For the interval concerned, assume a flat earth with a constant gravitational acceleration g and neglect any atmospheric resistance.

Given

$v_o = u$

$max.height = h$

$horizontal\ distance = s$



Required

$$h = ?$$

$$s = ?$$

$$t = ?$$

$$eq.of\ path = ?$$

Solution

$$a_x = 0$$

$$a_y = -g$$

$$\int_0^x d_x = \int_0^t v_x\,dt \rightarrow x = \int_0^t u\cos\theta\,dt \rightarrow x = u\,t\cos\theta$$

$$\int_{usin\theta}^{v_y} dv_y = \int_0^t a_y\,dt \rightarrow v_y - u\sin\theta = -gt \rightarrow v_y = u\sin\theta - gt \rightarrow \boldsymbol{t = \frac{u\sin\theta}{g}}$$

$$\int_0^y d_y = \int_0^t v_y\,dt \rightarrow y = \int_0^t (usin\theta - gt)dt \rightarrow y = utsin\theta - 1/2gt^2$$

$$h = y = u\left(\frac{usin\theta}{g}\right)sin\theta - \frac{1}{2}g\left(\frac{usin\theta}{g}\right)^2$$

$$\boldsymbol{h = \frac{u^2 sin^2\theta}{2g}}$$

$$x = u\frac{usin\theta}{g}\boldsymbol{cos}\,\theta \rightarrow \boldsymbol{s = \frac{u^2 sin2\theta}{2g}}$$

Eliminating t from x and y expression:

$$\boldsymbol{y = x\,tan\theta - \frac{gx^2}{2u^2}\,sec^2\theta} \ldots.(parabola)$$

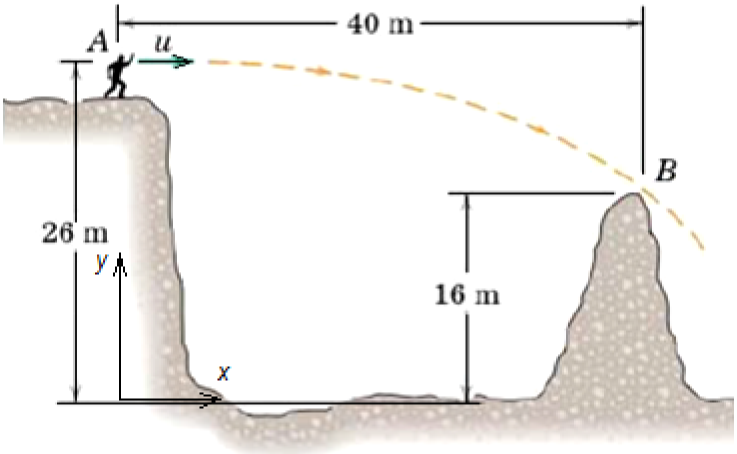
Problems

2/71 With what minimum horizontal velocity u can a boy throw a rock at A and have it just clear the obstruction at B?

Required

$$u = ?$$

Solution



$$y = y_o + v_{o_y}\, t + \frac{1}{2} a\, t^2$$

$$16 = 26 + 0 - \frac{1}{2}\, 9.81\, t^2$$

$$t = 1.43 \ \ s$$

$$x = v_x\, t$$

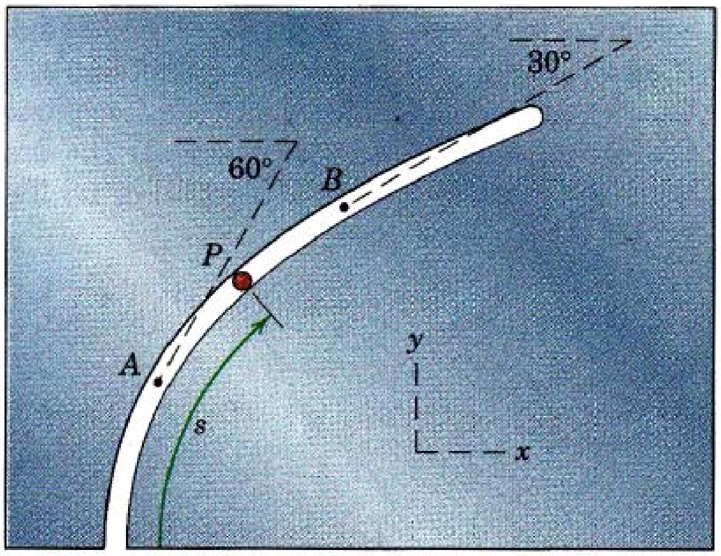
$$40 = u\; 1.43$$

$$u = 28 \ \ m/s \quad Ans.$$

2/62 A particle which moves with curvilinear motion has coordinates in millimeters which vary with the time t in seconds according to $x = 2t^2 - 4t$ and $y = 3t^2 - \frac{1}{3}t^3$. Determine the magnitudes of the velocity **v** and acceleration **a** and the angles which these vectors make with the x-axis when $t = 2$ s.

2/64

The particle P moves along the curved slot, a portion of which is shown. Its distance in meters measured along the slot is given by $s = t^2/4$, where t is in seconds. The particle is at A when $t = 2.00$ s and at B when $t = 2.20$ s. Determine the magnitude a_{av} of the average acceleration of P between A and B. Also express the acceleration as a vector $\mathbf{a}_{av}$ using unit vectors **i** and **j**.

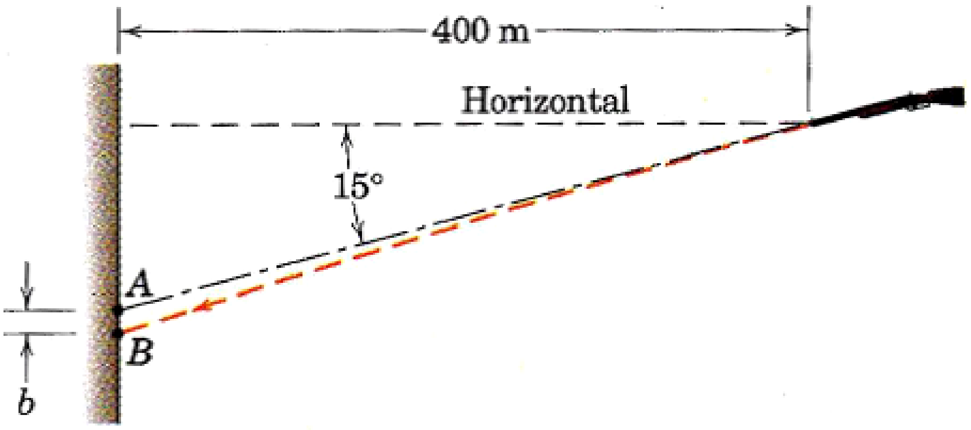


2/66 The position vector of a point which moves in the x-y plane is given by

$$\mathbf{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\mathbf{i} + \frac{t^4}{12}\mathbf{j}$$

where $\mathbf{r}$ is in meters and t is in seconds. Determine the angle between the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ when (*a*) $t = 2$ s and (*b*) $t = 3$ s.

2/72 The rifle is aimed at point A and fired. Calculate the distance b below A to the point B where the bullet strikes. The muzzle velocity is 800 m/s.



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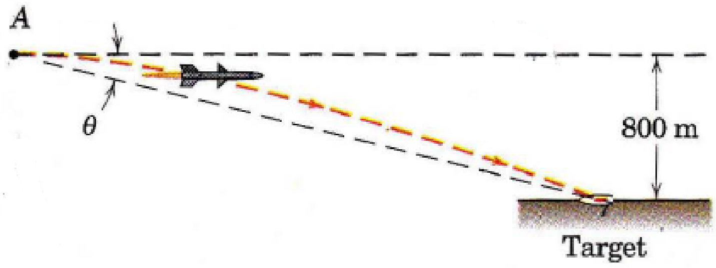
A rocket is released at point A from a jet aircraft flying horizontally at 1000 km/h at an altitude of 800 m. If the rocket thrust remains horizontal and gives the rocket a horizontal acceleration of $0.5g$, determine the angle θ from the horizontal to the line of sight to the target.

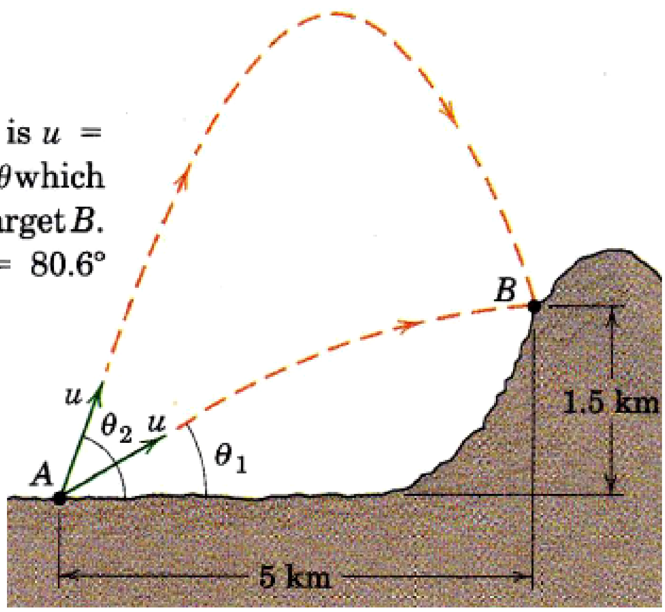
Ans. $\theta = 11.46°$



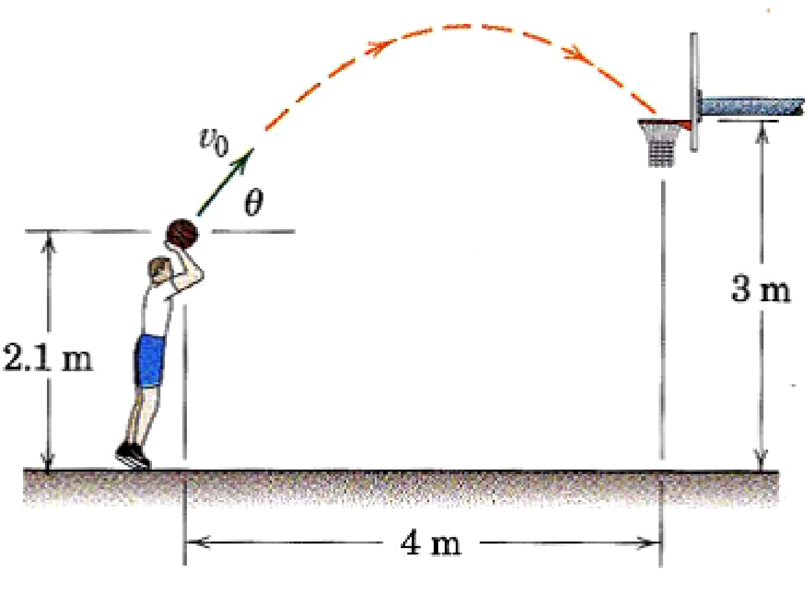
2/87

The muzzle velocity of a long-range rifle at A is $u = 400$ m/s. Determine the two angles of elevation θ which will permit the projectile to hit the mountain target B.

Ans. $\theta_1 = 26.1°$, $\theta_2 = 80.6°$



2/92 The basketball player likes to release his foul shots at an angle $\theta = 50°$ to the horizontal as shown. What initial speed v_0 will cause the ball to pass through the center of the rim?



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

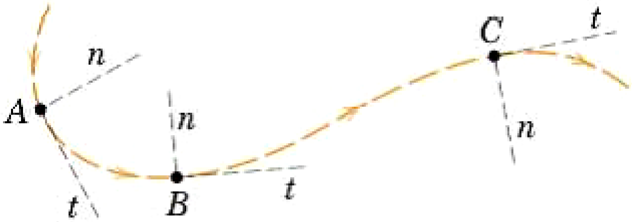
Class: 2nd Year

Lecture Contents	**Lecture sequences:**	**4d lecture**	**Bakr Noori Alhasan/Lecturer**
	The major contents **Normal and Tangential Coordinates (n-t)** **Circular Motion**		
	The detailed contents **2/5 Normal and Tangential Coordinates (n-t)** **Velocity** **Acceleration** **Geometric Interpretation** **Circular Motion** **Sample Problem 2/7** **Sample Problem 2/8**		

2/5 Normal and Tangential Coordinates (n-t)

It is one of the very natural description of the curvilinear motion along the tangent t and normal n to the path of the particle.

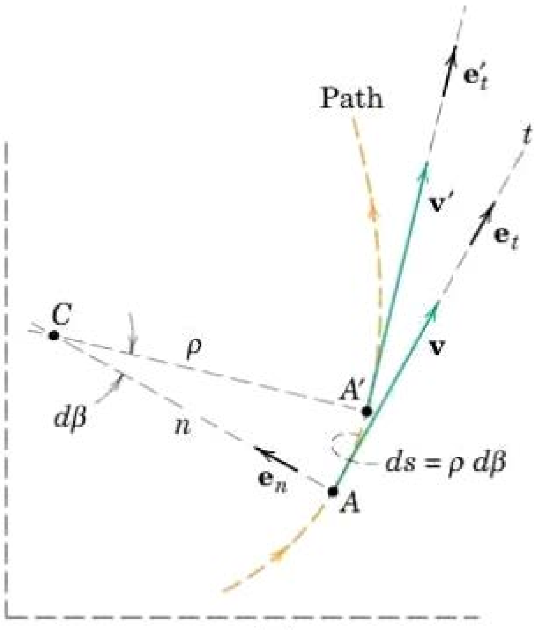
When the particle advances from A to B to C, the positive n is always taken toward the center of the curvature of the path.



Velocity

We introduce:

$\overrightarrow{e_t}$ *a unit vector in the t − direction.*
$\overrightarrow{e_n}$ *a unit vector in the n − direction*



(a)

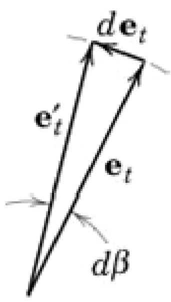
During dt, the particle moves a differential distance ds along the curve from A to Á

ρ the radius of curvature of the path.

$ds = \rho\, d\beta$
ρ *is unchanged (higher order)*
$v = \frac{ds}{dt} = \frac{\rho d\beta}{dt}$

$$\vec{v} = v\,\vec{e}_t = \rho\,\dot{\beta}\,\vec{e}_t \quad \text{.......... 2/7}$$

$\vec{a} = \frac{d\vec{v}}{dt}$ is a vector reflects both the change in magnitude and the change in direction of $\vec{v}$



(b)

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(v\vec{e}_t)}{dt} = v\dot{\vec{e}}_t + \dot{v}\vec{e}_t \quad \dots\dots\dots\ 2/8$$

$$\dot{\vec{e}}_t \neq 0 \qquad (\textit{the direction changes})$$

In Fig. a $\quad A \rightarrow A` \quad \vec{e}_t \rightarrow \grave{\vec{e}}_t \quad \equiv d\vec{e}_t$

The magnitude of $d\vec{e}_t = |\vec{e}_t| d\beta = d\beta$ and the direction of $d\vec{e}_t$ is given by $\vec{e}_n$

$$d\vec{e}_t = d\beta\, \vec{e}_n$$

$$\frac{d\vec{e}_t}{d\beta} = \vec{e}_n \rightarrow \frac{d\vec{e}_t}{dt} = \frac{d\beta}{dt}\vec{e}_n \rightarrow \dot{\vec{e}}_t = \dot{\beta}\vec{e}_n \quad \dots\dots\dots \; 2/9$$

With the substitution of eq. 2/9 and $\dot{\beta}$ from the relation $(v = \rho\dot{\beta})$ equation 2/8 becomes:

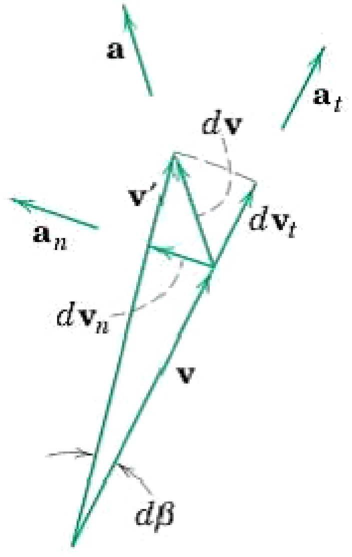
$$\vec{a} = v\dot{\beta}\vec{e}_n + \dot{v}\vec{e}_t$$

$$\boxed{\vec{a} = \frac{v^2}{\rho}\vec{e}_n + \dot{v}\vec{e}_t} \quad \dots\dots\dots \; 2/10$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$

$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{{a_n}^2 + a_t^2}$$



(c)

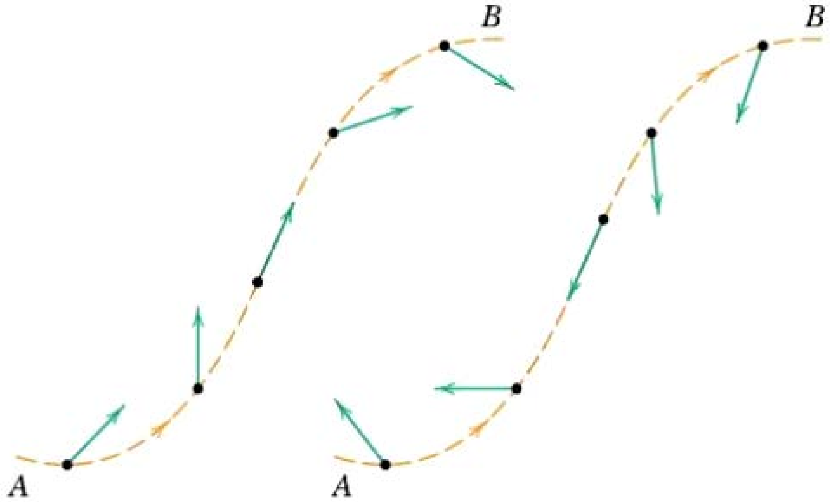
Geometric Interpretation

Referring to Fig. c $\quad |d\vec{v}_n| = v d\beta$

$$a_n = \frac{|dv_n|}{dt} = v\frac{d\beta}{dt} = v\dot{\beta}$$

$|d\vec{v}_t| = dv \quad$ *change in the length of* $\vec{v}$

$$a_t = \frac{dv}{dt} = \dot{v} = \ddot{s} \text{ as before}$$



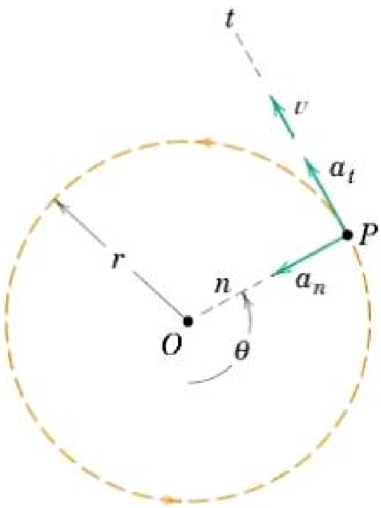
(a) Speed increasing (b) Speed decreasing

Acceleration vectors for particle moving from A to B

$a_n \quad is\ always\ directed\ toward\ the\ center\ of\ the\ curvature$

$a_t \quad is\ in\ the\ positive\ t - coor.\ if\ the\ speed\ is\ increasing.$

Circular Motion



It is a special case of plane curvilinear motion.

$$\rho \rightarrow r$$

$$\beta \rightarrow \theta$$

$$v = r\dot{\theta}$$

$$a_n = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$$

$$a_t = \dot{v} = r\ddot{\theta}$$

.... 2/11

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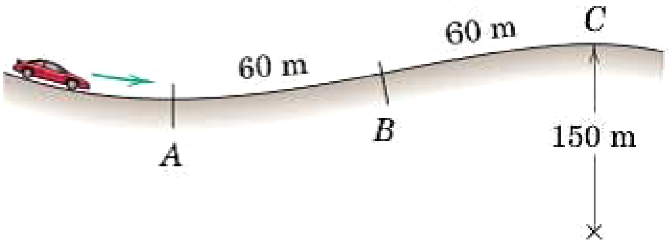
Sample Problem 2/7

To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a uniform deceleration. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, calculate (a) the radius of curvature ρ at A, (b) the acceleration at the inflection point B, and (c) the total acceleration at C.



Solution

The dimensions of the car are small compared with those of the path, so we will treat the car as a particle.

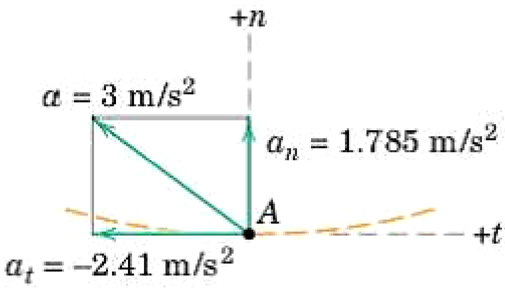
$v_A = \frac{100}{3.6} = 27.8\ m/s$

$v_C = \frac{50}{3.6} = 13.89\ m/s$

${v_C}^2 = {v_A}^2 + 2a_t\Delta s$

$(13.89)^2 = (27.8)^2 + 2a_t(120)$

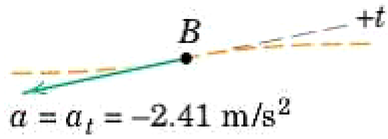
$a_t = -2.41\ m/s^2$



(a) Condition at A

$[a^2 = {a_n}^2 + {a_t}^2]$ ${a_n}^2 = 3^2 - 2.41^2 = 3.19$ $a_n = 1.785\ m/s^2$

$[a_n = \frac{v^2}{\rho}]$ $\rho = \frac{v^2}{a_n} = \frac{27.8^2}{1.785} = 432\ m\ \ Ans.$



(b) Condition at B

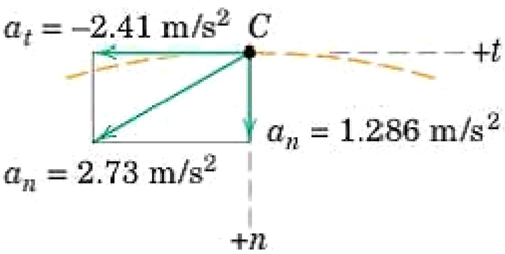
Since ρ is infinite, $a_n = 0$:

$a = a_t = -2.41\ m/s^2\ \ Ans.$

(c) Condition at C

$[a_n = \frac{v^2}{\rho}]$ $a_n = \frac{13.89^2}{150} = 1.286\ m/s^2$

$[a^2 = {a_n}^2 + {a_t}^2]$ $a = \sqrt{1.286^2 + 2.41^2} = 2.73\ m/s^2\ \ Ans.$



With unit vectors, the acceleration may written

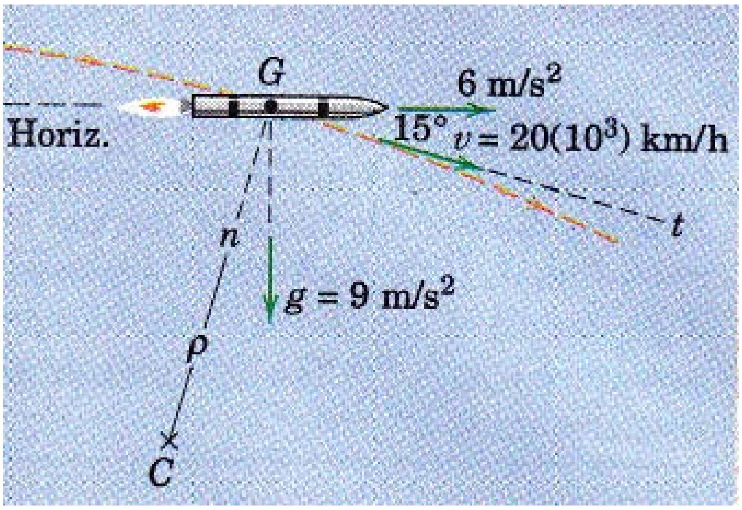
$\vec{a} = 1.286\,\vec{e}_n - 2.41\vec{e}_t\ \ m/s^2$

The acceleration vectors representing the conditions at each of the three points are shown for clarification.

Sample Problem 2/8

A certain rocket maintains a horizontal attitude of its axis during the powered phase of its flight at high altitude. The thrust imparts a horizontal component of acceleration of 6 m/s^2, and the downward acceleration component is the acceleration due to gravity at that altitude, which is g= 9 m/s^2. At the instant represented, the velocity of the mass center G of the rocket along the 15° direction of its trajectory is $20(10)^3$ km/h. For this position determine (a) the radius of curvature of the flight trajectory, (b) the rate at which the speed v is increasing, (c) the angular rate $\dot{\beta}$ of the radial line from G to the center of curvature C, and (d) the vector expression for the total acceleration $\vec{a}$ of the rocket.

Solution



$$a_n = 9\cos 15^o - 6\sin 15^o = 7.14\ m/s^2$$

$$a_t = 9\sin 15^o + 6\cos 15^o = 8.12\ m/s^2$$

a. $\left[a_n = \frac{v^2}{\rho}\right]$ $\quad 7.14 = \frac{\left(20(10)^3\right)^2}{\rho\times 3.6}$

$$\rho = 4.32(10)^6\ m\ \ Ans.$$

b. *It means* a_t $\quad a_t = 8.12\ m/s^2\ \ Ans.$

c. $[v = \rho\dot{\beta}]$ $\quad 20(10)^3 = 4.32(10)^6\dot{\beta}$

$$\dot{\beta} = 12.85(10)^{-4}\ \ rad/\sec\ \ Ans.$$

d. $[\vec{a} = a_t\vec{e}_t + a_n\vec{e}_n]$ $\quad \vec{a} = 8.12\vec{e}_t + 7.14\vec{e}_n\ \ Ans.$

2/97 A particle moves in a circular path of 0.4-m radius. Calculate the magnitude a of the acceleration of the particle (*a*) if its speed is constant at 0.6 m/s and (*b*) if its speed is 0.6 m/s but is increasing at the rate of 1.2 m/s each second.

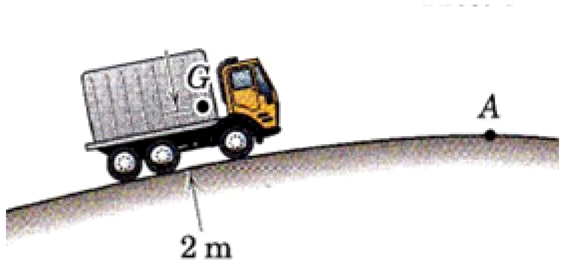
Ans. (*a*) $a = 0.9\ m/s^2$, (*b*) $a = 1.5\ m/s^2$

2/99 A car is traveling around a circular track of 240-m radius. If the magnitude of its total acceleration is 3 m/s^2 at the instant when its speed is 75 km/h, determine the rate at which the car is changing its speed.

Ans. $a_t = +2.39$ m/s^2

2/101

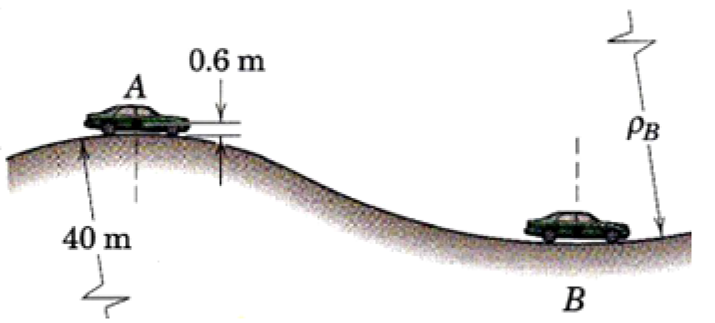
The driver of the truck has an acceleration of 0.4g as the truck passes over the top A of the hump in the road at constant speed. The radius of curvature of the road at the top of the hump is 98 m, and the center of mass G of the driver (considered a particle) is 2 m above the road. Calculate the speed v of the truck.



Ans. $v = 71.3$ km/h

2/105

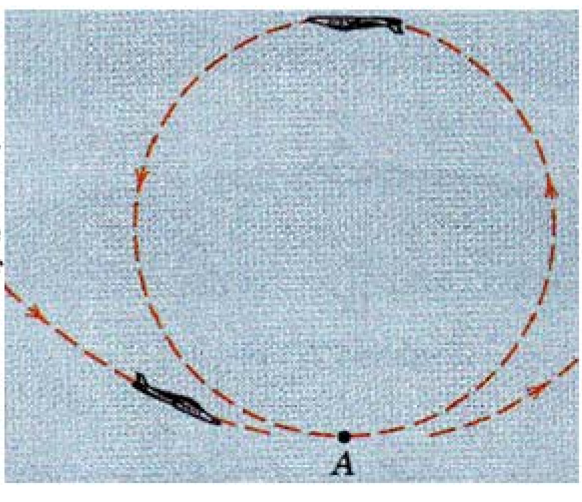
The speed of a car increases uniformly with time from 50 km/h at A to 100 km/h at B during 10 seconds. The radius of curvature of the hump at A is 40 m. If the magnitude of the total acceleration of the mass center of the car is the same at B as at A, compute the radius of curvature ρ_B of the dip in the road at B. The mass center of the car is 0.6 m from the road.



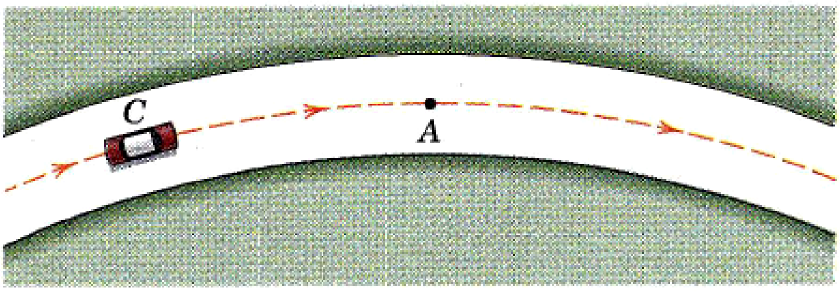
Ans. $\rho_B = 163.0$ m

2/112

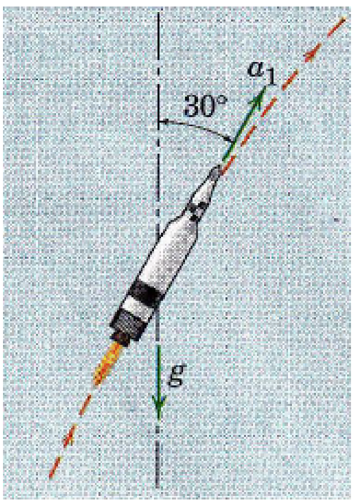
At the bottom A of the vertical inside loop, the magnitude of the total acceleration of the airplane is 3g. If the airspeed is 800 km/h and is increasing at the rate of 20 km/h per second, calculate the radius of curvature ρ of the path at A.



2/114 The car C increases its speed at the constant rate of 1.5 m/s^2 as it rounds the curve shown. If the magnitude of the total acceleration of the car is 2.5 m/s^2 at the point A where the radius of curvature is 200 m, compute the speed v of the car at this point.

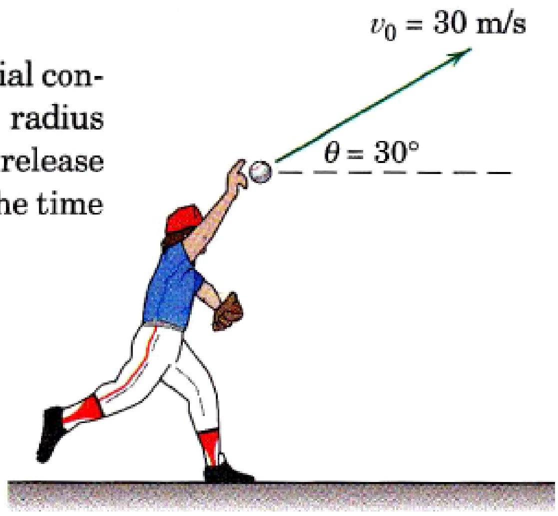


2/116 A rocket traveling above the atmosphere at an altitude of 500 km would have a free-fall acceleration $g = 8.43$ m/s^2 in the absence of forces other than gravitational attraction. Because of thrust, however, the rocket has an additional acceleration component a_1 of 8.80 m/s^2 tangent to its trajectory, which makes an angle of 30° with the vertical at the instant considered. If the velocity v of the rocket is 30 000 km/h at this position, compute the radius of curvature ρ of the trajectory and the rate at which v is changing with time.



2/120

A baseball player releases a ball with the initial conditions shown in the figure. Determine the radius of curvature of the trajectory (a) just after release and (b) at the apex. For each case, compute the time rate of change of the speed.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	5th lecture	Bakr Noori Alhasan/Lecturer
	The major contents **Velocity** **2/6 Polar Coordinates (r-θ)** **Acceleration** **Circular Motion** **Sample Problem 2/9 P71** **Sample Problem 2/10**		
	The detailed contents **2/6 Polar Coordinates (r-θ)** **Time Derivatives of the Unit Vectors** **Velocity** **Acceleration** **Geometric Interpretation** **Circular Motion** **Sample Problem 2/9 P71** **Sample Problem 2/10**		

2/6 Polar Coordinates (r-θ)

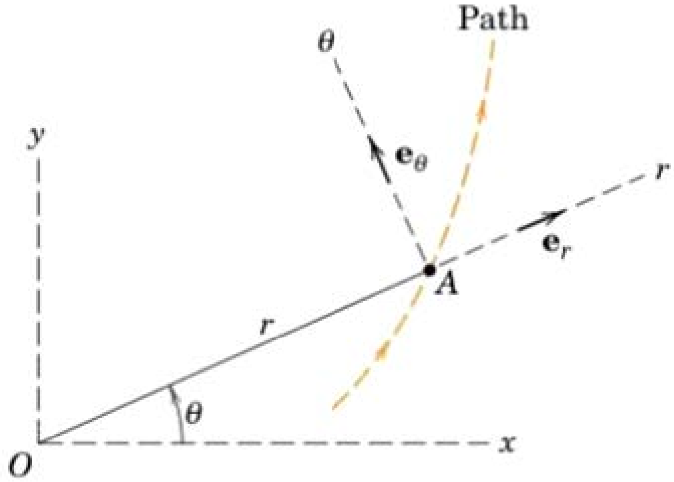
The description particle is located by the radial distance r from a fixed point and by an angular measurement θ to the radial line.

Polar coordinates is used when:
A motion is constrained through the control of a radial distance and angular position. or: An unconstrained motion is observed by measurements of radial distance and angular position.

Let a particle A traveling on a curved path located by the polar coordinates r and θ.

x-axis: an arbitrary fixed line can be used as a reference for the measurement of θ.

$\vec{e}_r$ and $\vec{e}_\theta$ are unit vectors in the positive r and θ-direction respectively.



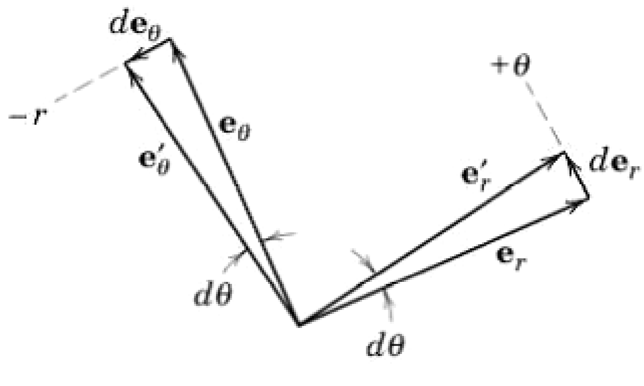
$\vec{r} = r\vec{e}_r \qquad \vec{r}$ *is the position vector of the paeticle A.*

Time Derivatives of the Unit Vectors

During time dt, the coordinates r, θ rotate through dθ, the unit vectors $\vec{e}_r$ and $\vec{e}_\theta$ also rotate through the same angle.

$\vec{e}_r \rightarrow \vec{e}_r'$

$\vec{e}_\theta \rightarrow \vec{e}_\theta'$



(b)

Note that:

$d\vec{e}_r$ *in the* $+\theta$ *direction*

$d\vec{e}_\theta$ *in the* $-r$ *direction*

$$d\vec{e}_r = \vec{e}_\theta\, d\theta \quad \& \quad d\vec{e}_\theta = -\,\vec{e}_r d\theta$$

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \quad \text{and} \quad \frac{d\vec{e}_\theta}{d\theta} = -\,\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\theta}{dt}\vec{e}_\theta \quad \text{and} \quad \frac{d\vec{e}_\theta}{dt} = -\,\frac{d\theta}{dt}\vec{e}_r$$

$$\dot{\vec{e}}_r = \dot{\theta}\vec{e}_\theta \quad \text{and} \quad \dot{\vec{e}}_\theta = -\dot{\theta}\vec{e}_r \quad \ldots\ldots\ldots 2/12$$

Velocity

$$\vec{r} = r\vec{e}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r$$

$$\boxed{\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta}$$

……. 2/13 (scalr and vectors)

$v_r = \dot{r}$ *the rate at wich the vector* $\vec{r}$ *stretches.*

$$v_\theta = r\dot{\theta} \quad \textit{due to the rotation of } r$$
$$v = \sqrt{v_r^2 + v_\theta^2}$$

Acceleration

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dr}\left(\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta\right) \quad recall\ \dot{\vec{e}}_r = \dot{\theta}\vec{e}_\theta \ and\ \dot{\vec{e}}_\theta = -\dot{\theta}\vec{e}_r$$
$$= \ddot{r}\,\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$\boxed{\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta} \quad \text{............. 2/14}$$

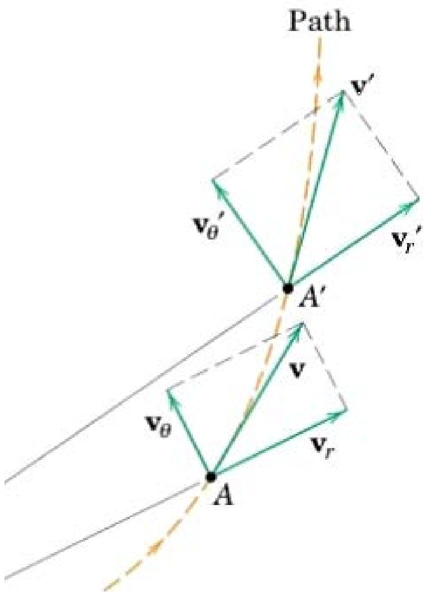
Where: $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$$a = \sqrt{a_r^2 + a_\theta^2}$$

We can write $a_\theta = \frac{1}{r}\frac{d}{dt}(\dot{r}\dot{\theta})$

Geometric Interpretation

Equation 2/14 can be best understood when the geometry of the physical changes can be clearly seen.



The components in r and θ- direction undergoe a change in magnitude and direction:

(a) Magnitude change of $\vec{v}_r$

$$dv_r = d\dot{r} \ \rightarrow\ \ddot{r} \quad \textit{in the positive } r - \textit{direction}$$

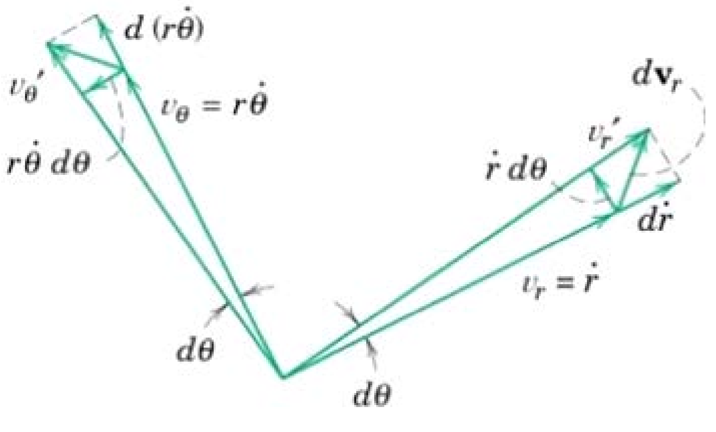
(b) Direction change of $\vec{v}_r$

$v_r\, d\theta = \dot{r} d\theta \ \rightarrow$
$\dot{r}\dot{\theta}$ *in the positive* $\theta -$ *direction*

(c) Magnitude change of $\vec{v}_\theta$

$$d(r\dot{\theta}) = \frac{d(r\dot{\theta})}{dt} \rightarrow r\ddot{\theta} +$$

$\dot{r}\dot{\theta}$ *in the positive* $\theta -$ *direction*



(b)

(d) Direction change of $\vec{v}_\theta$:
$v_\theta d\theta = r\dot{\theta} d\theta \rightarrow r\dot{\theta}\, {d\theta}/{dt} =$
$r\dot{\theta}^2$ *in the negative* $r -$ *direction.*
Collcting terms gives $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

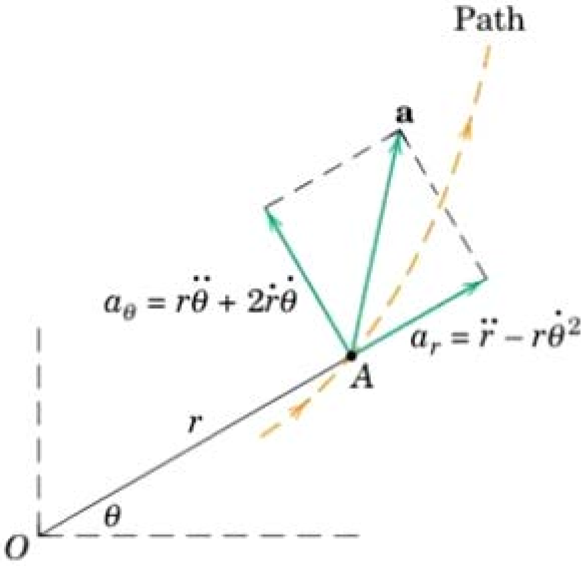
$\ddot{r}$ is the acceleration which the particle would have along the radius in the absence of a change in θ.

$-r\dot{\theta}^2$ is the normal component of acceleration if r were constant (as in circular motion).

$r\ddot{\theta}$ is the tangential acceleration which the particle would have if r were constant, but is only a part of the acceleration due to the change in magnitude of $\vec{v}_\theta$ when r is variable.

$2\dot{r}\dot{\theta}$ is composed of two effects:
comes from that portion of the change in magnitude $d(r\dot{\theta})$ of v_θ due to the change in r.

comes from the change in direction of $\vec{v}_r$.



The Difference between $d\vec{v}_r$ and dv_r or $d\vec{v}_\theta$ *and* dv_θ
To obtain expressions for the derivatives, divide by dt:

$$d\vec{v}_r \rightarrow \frac{d\vec{v}_r}{dt} \quad and \quad dv_r \rightarrow \frac{dv_r}{dt}$$

$\left|\frac{d\vec{v}_r}{dt}\right|$ is the magnitude of the derivative.
$\frac{dv_r}{dt}$ is the derivative of the magnitude and they are different $\left(\frac{d}{dt}|\vec{v}_r|\right)$

Circular Motion

For motion in a circular path, r is constant

$$v_r = \dot{r} = 0 \qquad a_r = -r\dot{\theta}^2$$
$$v_\theta = r\dot{\theta} \qquad a_\theta = r\ddot{\theta}$$

They are the same of that obtained for n and t component , θ and t coincide, positive r is in the negative n-direction and $a_r = -a_n$.

a_r *and* a_θ may be found directly by differentiation of the coordinates relation.

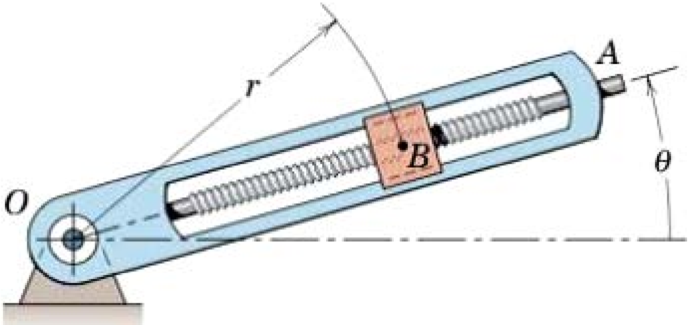
$$x = r\cos\theta \qquad y = r\sin\theta$$
$$\dot{x} = \qquad \dot{y} =$$
$$\ddot{x} = a_x = \qquad \ddot{y} = a_y =$$

a_x *and* a_y are resolved in r and θ direction which when combined , will yield the expressions of eq. 2/14.

Problems 2/131 2/135 2/138 2/141 2/142 2/144 2/152 2/166

Sample Problem 2/9 P71

Rotation of the radially slotted arm is governed by $\theta = 0.2t + 0.02\,t^3$, where θ is in radians and t is in seconds. Simultaneously, the power screw in the arm engages the slider B and controls its distance from O according to $r = 0.2 + 0.04\,t^2$, where r is in meters and t is in seconds. Calculate the magnitude of the velocity and acceleration of the slider for the instant when $t = 3\,sec$.



Given

$\theta = 0.2t + 0.02\,t^3$

$r = 0.2 + 0.04\,t^2$

Required

$v = ?$

$a = ?$

Solution

$r = 0.2 + 0.04\,t^2$ $\qquad r_3 = 0.56\,m$

$\theta = 0.2t + 0.02\,t^3$ $\qquad \theta_3 = 1.14\;rad$

$\dot{r} = 0.08t$ $\qquad \dot{r}_3 = 0.24\;m/s$

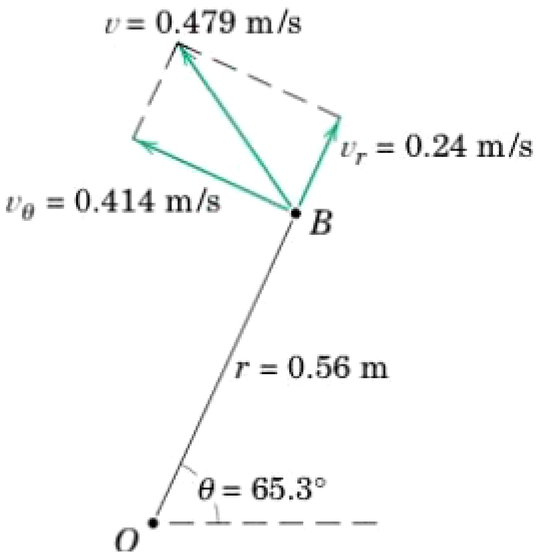
$\dot{\theta} = 0.2 + 0.06\,t^2$ $\qquad \dot{\theta}_3 = 0.74\;rad/s$

$\ddot{r} = 0.08$ $\qquad \ddot{r}_3 = 0.08\;m/s^2$

$\ddot{\theta} = 0.012t$ $\qquad \ddot{\theta}_3 = 0.36\;rad/s^2$

$v_r = \dot{r}$ $\qquad v_r = 0.24\;m/s$

$v_\theta = r\dot{\theta}$ $\qquad v_\theta = 0.56(0.74) = 0.414\;m/s$



$$v = \sqrt{v_r^2 + v_\theta^2}$$

$v = \sqrt{0.24^2 + 0.414^2} \;=\; 0.479\;m/s\;\;$ ***Ans.***

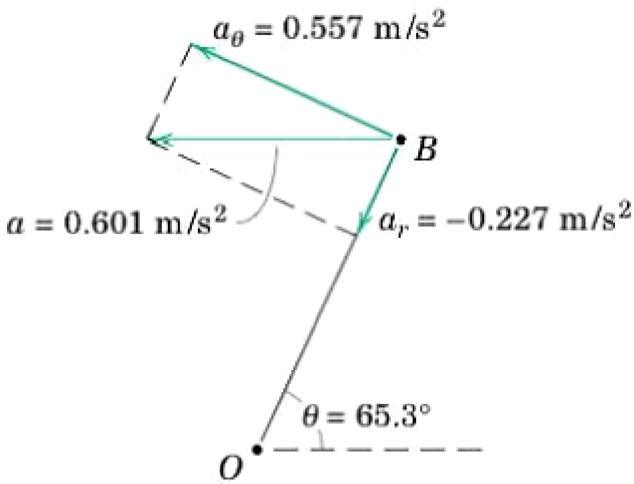
$a_r \;=\; \ddot{r} - r\dot{\theta}^2$

$a_r = 0.08 - 0.56(0.74)^2 = -0.227\;m/s^2$

$a_\theta \;=\; r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_\theta \;=\; 0.56(0.36) + 2(0.24)(0.74) = 0.557\;m/s^2$

$a = \sqrt{a_r^2 + a_\theta^2}$

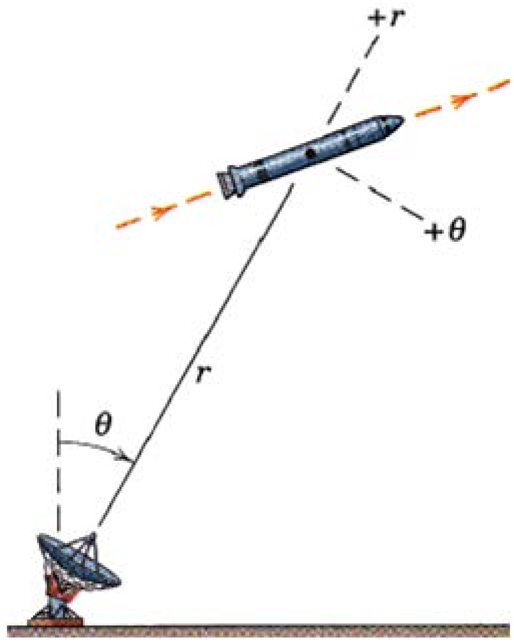


$a = \sqrt{-0.227^2 + 0.577^2}$

$a = 0.601\;m/s^2\;\;$ ***Ans.***

Sample Problem 2/10

A tracking radar lies in the vertical plane of the path of a rocket which is coasting in unpowered flight above the atmosphere. For the instant when $\theta = 30^{\circ}$, the tracking data give $r = 8(10)^4\ m$, $\dot{r} = 1200\ m/s$, and $\dot{\theta} = 0.8\ °/s$. The acceleration of the rocket is due only to gravitational attraction and for its particular altitude is $9.20\ m/s^2$ vertically down. For the conditions determine the velocity v of the rocket and the values of $\ddot{r}$ and $\ddot{\theta}$.



Given $\theta = 30^{\circ}$ $\quad \dot{\theta} = 0.8\ deg/s$

$r = 8(10)^4\ m$ $\quad g = 9.20\ m/s^2$

$\dot{r} = 1200\ m/s$

Required

$v = ?$

$\ddot{r} = ?$

$\ddot{\theta} = ?$

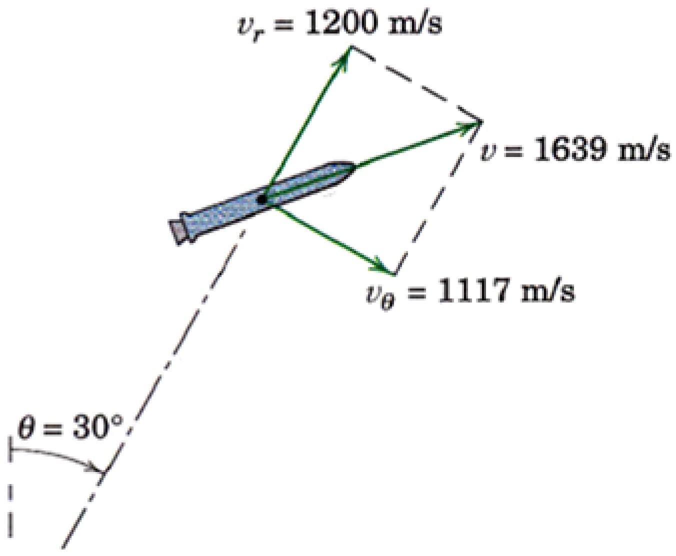
Solution

$v_r = \dot{r}$ $\qquad v_r = 1200\ m/s$

$v_\theta = r\dot{\theta}$ $\qquad 8(10)^4(0.8)\,\pi/180 = 1117\ \text{m/s}$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$v = \sqrt{1200^2 + 1117^2} = 1639\ m/s \quad \textbf{Ans.}$$



The total acceleration of the rocket is g

$a_r = -9.20\cos 30 = -7.97\ m/s^2$

$a_\theta = 9.20\sin 30 = 4.60\ m/s^2$

$a_r = \ddot{r} - r\dot{\theta}^2$

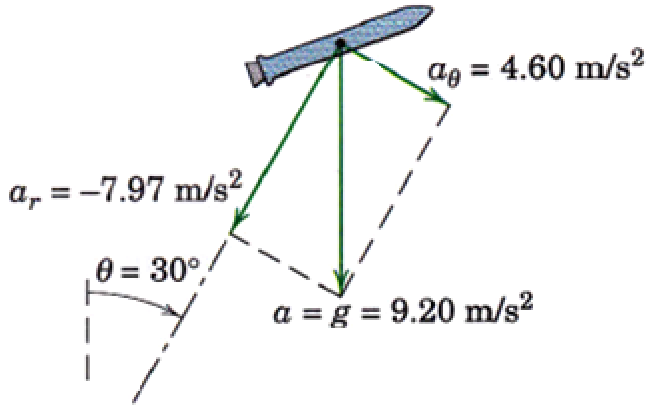
$-7.97 = \ddot{r} - 8(10)^4(0.8\frac{\pi}{180})^2$

$\ddot{r} = 7.63\ m/s^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$4.6 = 8(10)^4\ddot{\theta} + 2(1200)$

$\ddot{\theta} = -3.61(10)^{-4}\ rad/s^2$ **Ans.**



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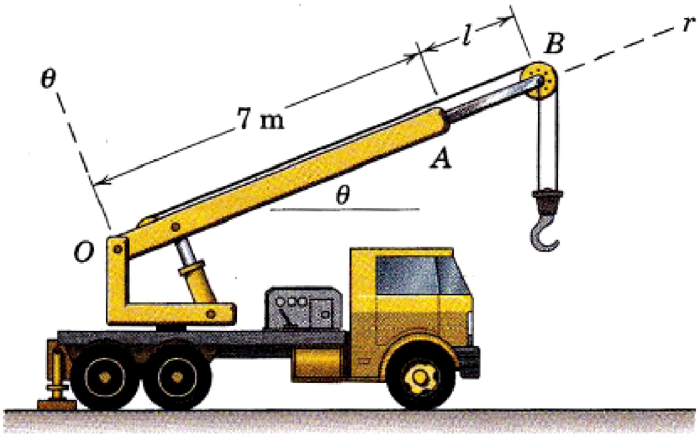
The boom *OAB* pivots about point *O*, while section *AB* simultaneously extends from within section *OA*. Determine the velocity and acceleration of the center *B* of the pulley for the following conditions: $\theta = 20°$, $\dot{\theta} = 5$ deg/s, $\ddot{\theta} = 2$ deg/s^2, $l = 2$ m, $\dot{l} = 0.5$ m/s, $\ddot{l} = -1.2$ m/s^2. The quantities $\dot{l}$ and $\ddot{l}$ are the first and second time derivatives, respectively, of the length l of section *AB*.

Ans. $\mathbf{v} = 0.5\mathbf{e}_r + 0.785\mathbf{e}_\theta$ m/s

$\mathbf{a} = -1.269\mathbf{e}_r + 0.401\mathbf{e}_\theta$ m/s^2



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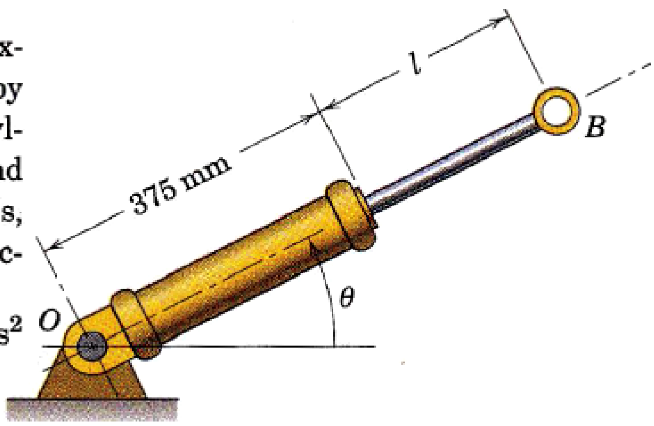
As the hydraulic cylinder rotates around *O*, the exposed length *l* of the piston rod *P* is controlled by the action of oil pressure in the cylinder. If the cylinder rotates at the constant rate $\dot{\theta} = 60$ deg/s and *l* is decreasing at the constant rate of 150 mm/s, calculate the magnitudes of the velocity **v** and acceleration **a** of end *B* when $l = 125$ mm.

Ans. $v = 545$ mm/s, $a = 632$ mm/s^2



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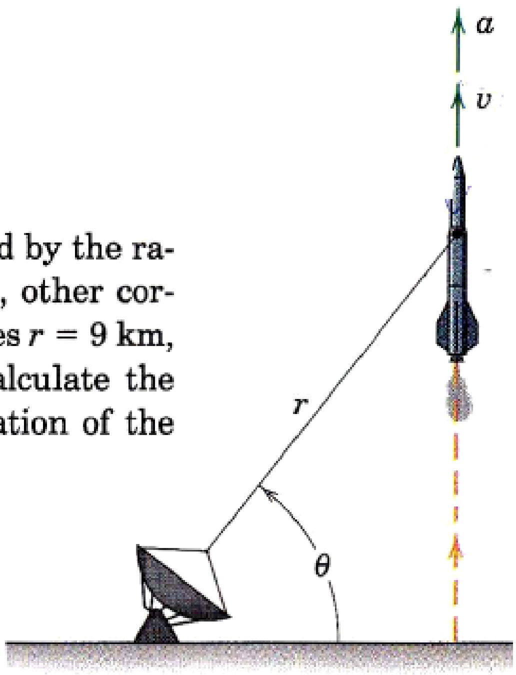
The rocket is fired vertically and tracked by the radar station shown. When θ reaches 60°, other corresponding measurements give the values $r = 9$ km, $\ddot{r} = 21$ m/s^2, and $\dot{\theta} = 0.02$ rad/s. Calculate the magnitudes of the velocity and acceleration of the rocket at this position.



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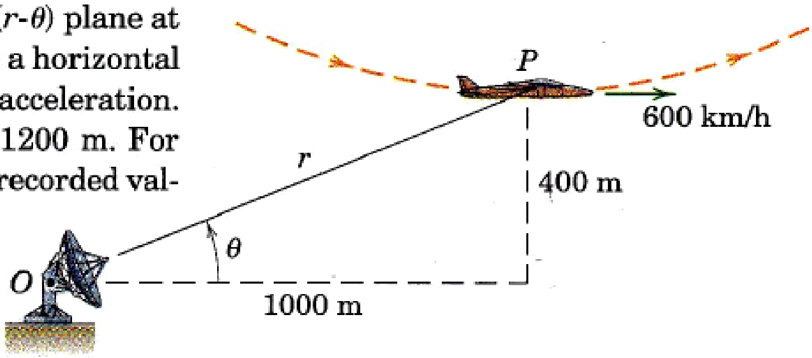
At the bottom of a loop in the vertical (r-θ) plane at an altitude of 400 m, the airplane P has a horizontal velocity of 600 km/h and no horizontal acceleration. The radius of curvature of the loop is 1200 m. For the radar tracking at O, determine the recorded values of $\ddot{r}$ and $\ddot{\theta}$ for this instant.



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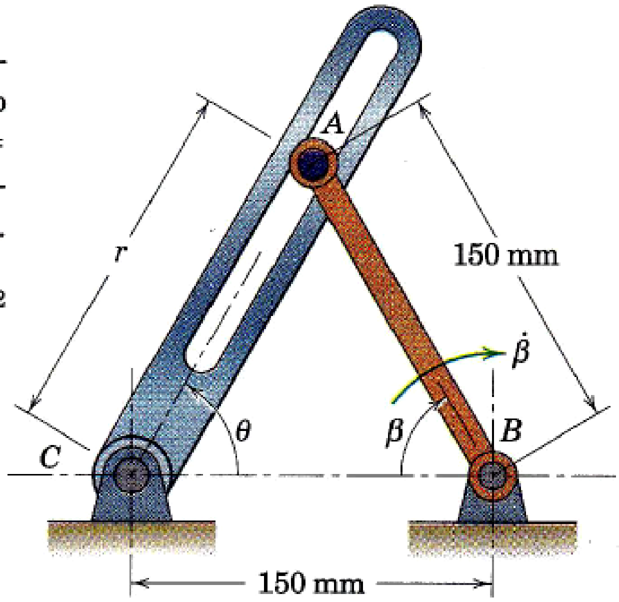
Link AB rotates through a limited range of the angle β, and its end A causes the slotted link AC to rotate also. For the instant represented where $\beta = 60°$ and $\dot{\beta} = 0.6$ rad/s constant, determine the corresponding values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$. Make use of Eqs. 2/13 and 2/14.

Ans. $\dot{r} = 77.9$ mm/s, $\ddot{r} = -13.5$ mm/s^2
$\dot{\theta} = -0.3$ rad/s, $\ddot{\theta} = 0$



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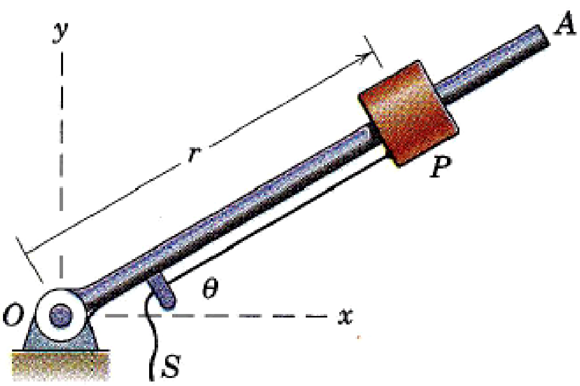
The slider P can be moved inward by means of the string S as the bar OA rotates about the pivot O. The angular position of the bar is given by $\theta = 0.4 + 0.12t + 0.06t^3$, where θ is in radians and t is in seconds. The position of the slider is given by $r = 0.8 - 0.1t - 0.05t^2$, where r is in meters and t is in seconds. Determine and sketch the velocity and acceleration of the slider at time $t = 2$ s. Find the angles α and β which **v** and **a** make with the positive x-axis.



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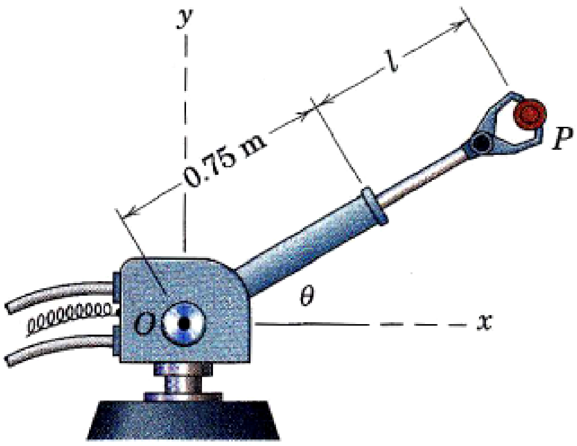
The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30°$, $\dot{\theta} = 10$ deg/s = constant, $l = 0.5$ m, $\dot{l} = 0.2$ m/s, and $\ddot{l} = -0.3$ m/s^2. Compute the magnitudes of the velocity **v** and acceleration **a** of the gripped part P. In addition, express **v** and **a** in terms of the unit vectors **i** and **j**.

Ans. $v = 0.296$ m/s, $a = 0.345$ m/s^2
$\mathbf{v} = 0.064\mathbf{i} + 0.289\mathbf{j}$ m/s
$\mathbf{a} = -0.328\mathbf{i} - 0.1086\mathbf{j}$ m/s^2



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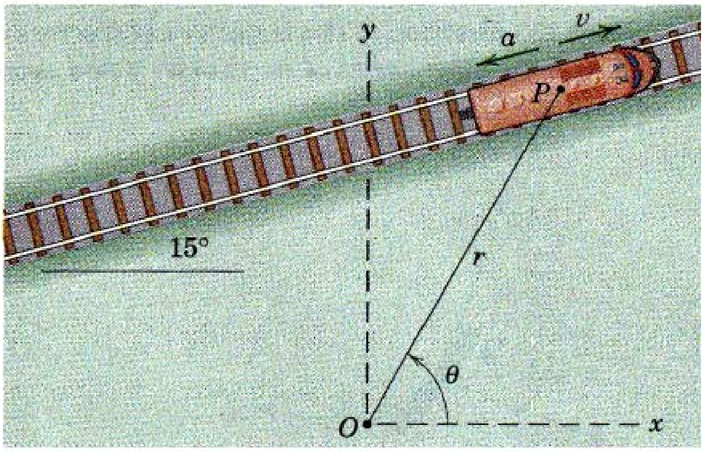
A locomotive is traveling on the straight and level track with a speed $v = 90$ km/h and a deceleration $a = 0.5$ m/s^2 as shown. Relative to the fixed observer at O, determine the quantities $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ at the instant when $\theta = 60°$ and $r = 400$ m.



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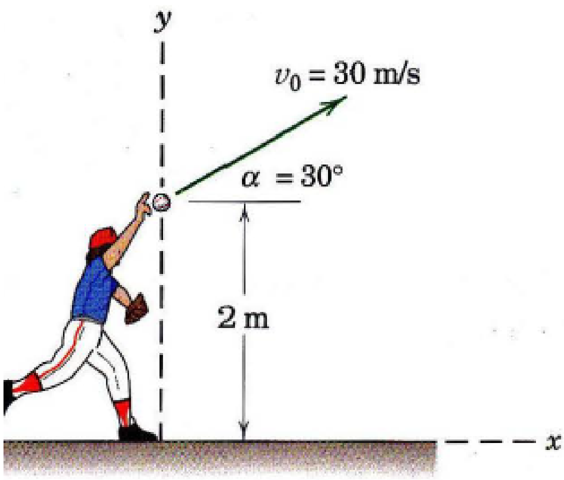
The baseball player of Prob. 2/120 is repeated here with additional information supplied. At time $t = 0$, the ball is thrown with an initial speed of 30 m/s at an angle of 30° to the horizontal. Determine the quantities r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, and $\ddot{\theta}$, all relative to the x-y coordinate system shown, at time $t = 0.5$ s.

Ans. $r = 15.40$ m, $\dot{r} = 27.3$ m/s
$\ddot{r} = -3.35$ m/s^2, $\theta = 32.5°$
$\dot{\theta} = -0.353$ rad/s, $\ddot{\theta} = 0.717$ rad/s^2



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

	Lecture sequences:	6th lecture	Bakr Noori Alhasan/Lecturer
Lecture Contents	*The major contents* 2/8 Relative Motion (Translating Axes) Velocity Acceleration Additional consideration Sample Problem 2/12 Sample Problem 2/13		
	The detailed contents 2/8 Relative Motion (Translating Axes) Choice of Coordinate System Vector Representation Velocity Acceleration Additional consideration Sample Problem 2/12 Sample Problem 2/13		

2/8 Relative Motion (Translating Axes)

In the previous articles, we have described particle motion using coordinates referred to fixed reference axes. $s, v, and\ a$ are absolute.

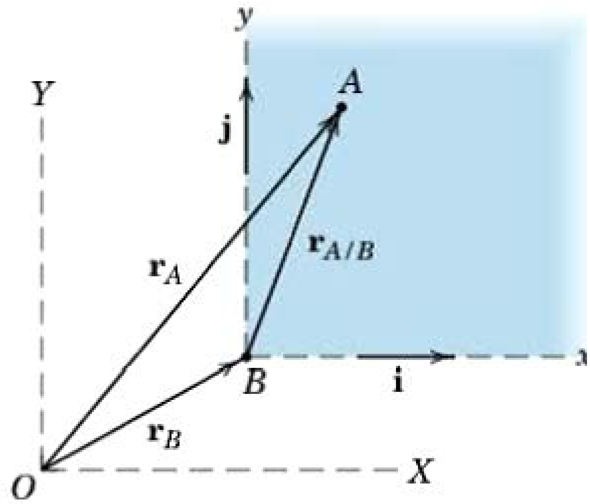
Choice of Coordinate System

The motion of moving coordinate system is specified with respect to a fixed coordinate system.

This article restricted to the relative motion for plane motion. We will take the moving reference systems which translate but do not rotate. Motion measured in rotating systems will be discussed in Ch.5.

The choice of the fixed axis depends on the following:
- In Newtonian mechanics, the fixed system is the primary inertial system which is assumed to have no motion in the space).
- For engineering purposes, the fixed system is any system whose absolute motion is negligible for the problem.
- For earth bound engineering problems, the fixed system is a set of axes attached to the earth. (motion of earth is neglected).
- For the motion of satellites around the earth, a non-rotating coordinate system is chosen with its origin on the axis of rotation of the earth.
- For interplanetary travel, a non-rotating coordinate system fixed to the sun would be used.

Vector Representation



Consider two particles A and B which may have separate curvilinear motion in a plane or parallel planes.
Attach the origin of a set of a translating axis x-y to particle B.
Observe the motion of A from our moving position on B.
The position vector of A as measurement relative to the frame x-y: $\vec{r}_{A/B} = x\vec{i} + y\vec{j}$
x and y are the coordinates of particle A measured in the x-y frame.

The absolute position of B is $\vec{r}_B$ measured from the origin of the fixed axis X-Y.

The absolute position of A is determined by the vector:

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

Velocity

$$\vec{v}_A = \dot{\vec{r}}_A = \dot{\vec{r}}_B + \dot{\vec{r}}_{A/B} \quad or \quad \vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

Acceleration

$$\ddot{\vec{r}}_A = \ddot{\vec{r}}_B + \ddot{\vec{r}}_{A/B} \quad or \quad \vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$\vec{v}_{A/B}$ is the velocity which we observe A to have from our position at B attached to the moving axis x-y. or: the velocity of A with respect to B.

$\vec{a}_{A/B}$ is the acceleration which we observe A to have from our non-rotating position on B. or: the acceleration of A with respect to B

$$\vec{a}_{A/B} = \ddot{\vec{r}}_{A/B} = \dot{\vec{v}}_{A/B} = \ddot{x}\vec{\imath} + \ddot{y}\vec{\jmath}$$

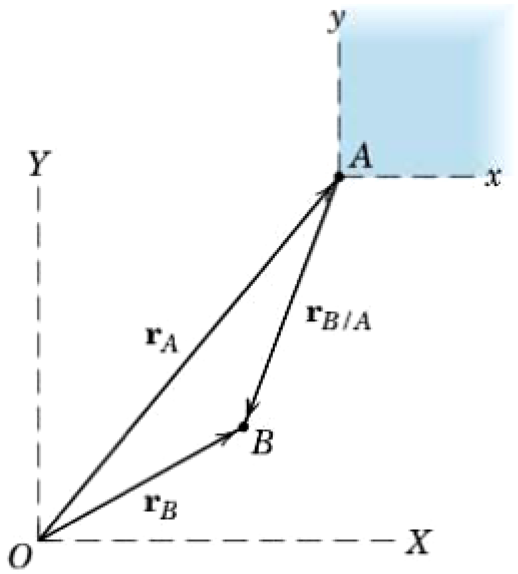
or: Is the acceleration measurement which an observer attached to the moving coordinate system x-y would make.

$\vec{\imath}$ *and* $\vec{\jmath}$ have zero derivatives because their magnitudes as well as their directions remain unchanged. (Later when we discuss rotating reference axes, we must account for the derivatives of the unit vectors when they change direction).

The relative terms may be rectangular, normal and tangential, or polar provided that the fixed system of the previous articles becomes the moving system in the present article.

Additional consideration

The attachment of point B to the reference moving coordinate is arbitrary. Point A could be used as well for the attachment of the moving system.



$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{r}_{B/A} = -\vec{r}_{A/B}$, $\vec{v}_{B/A} = -\vec{v}_{A/B}$ *and* $\vec{a}_{B/A} = -\vec{a}_{A/B}$

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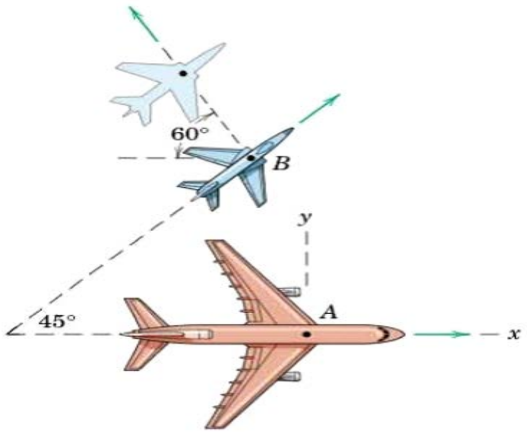
Sample Problem 2/12

Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60° angle as shown. Determine the true velocity of B.



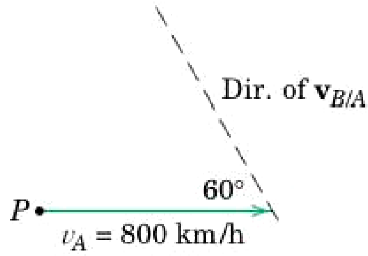
solution

The moving reference axes x-y are attached to A. We write therefore,

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

We may solve the vector equation in any one of three ways.

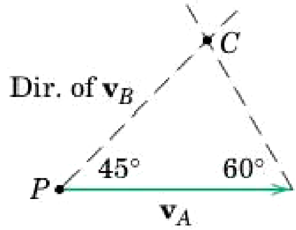
(1) Graphical



We start the vector sum at some point P by drawing $\vec{v}_A$ to a covenient scale and then construct a line through the tip of $\vec{v}_A$ with known direction of $\vec{v}_{B/A}$. The known direction of $\vec{v}_B$ is then drawn through P, and the intersection C yields the unique solution enabling us to complete the vector triangle and scale off the unknown magnitudes, which are found to be

$$v_{B/a} = 586\ km/h \quad and \quad v_B = 717\ km/h \quad Ans$$

(2) Trigonometric.



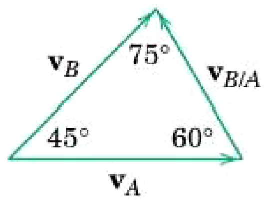
A sketch of the vector triangle is made, which gives

$$\frac{v_B}{\sin 60^o} = \frac{v_A}{\sin 75^o} \quad v_B = \frac{\sin 60^o}{\sin 75^o} = 717\, km/h \quad Ans.$$

(3) Vector Algebra

$$\vec{v}_A = 800\,\vec{\imath}\ \ \text{km/h} \qquad \vec{v}_B = (v_B \cos 45^o)\vec{\imath} + (v_B \sin 45^o)\vec{\jmath}$$

$$\vec{v}_{B/A} = \left(v_{B/A} \cos 60^o\right)(-\vec{\imath}) + (v_{B/A} \sin 60^o)\vec{\jmath}$$



Substituting these relations into the relative velocity equation and solving separately for the $\vec{\imath}$ *and* $\vec{\jmath}$ *terms give*

$$(\vec{\imath}\ \ terms) \quad v_B cos45^o = 800 - v_{B/A} cos60^o$$

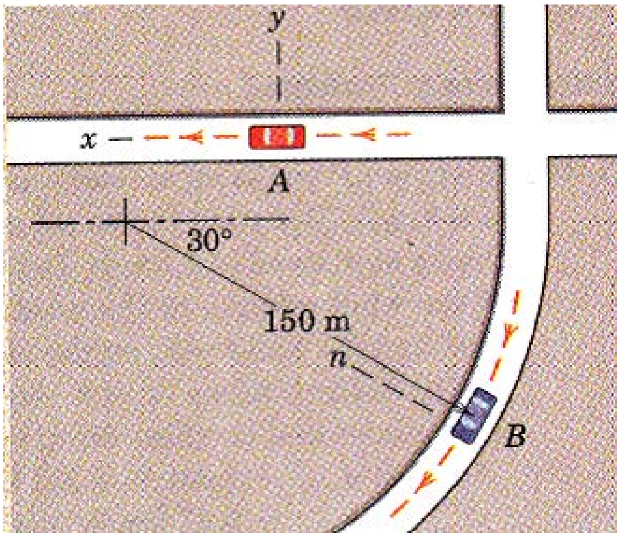
$$(\vec{\jmath}\ \ terms) \quad v_B cos45^o = v_{B/A} sin60^o$$

Solving simultaneously yields the unknown velocity magnitudes

$$v_{B/A} = 586\ km/h \quad v_B = 717\ km/h\ \ Ans.$$

Sample Problem 2/13

Car A is accelerating in the direction of its motion at the rate of 1.2 m/s^2 . Car B is rouding a curve of 150 m radius at a constant speed of 54 km/h. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of 72 km/h for the positions represented.



Solution

We choose nonrotating rference axes attached to car A since the motion of A is desired.

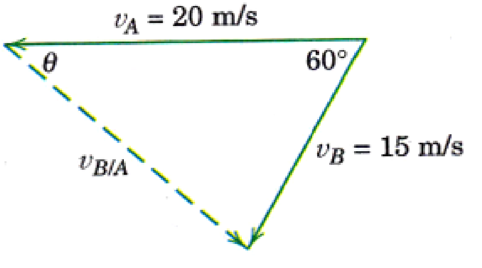
Velocity

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$v_A = 72/3.6 = 20\ m/s$

$v_B = 54/3.6 = 15\, m/s$

The triangle of velocity is drawn and with the aid of law of cosines and law of sines gives



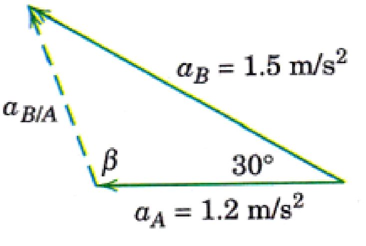
$$v_{B/A} = 18.03\ m/s \quad \theta = 46.1^o \quad Ans.$$

Acceleration

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$[a_n = \frac{v^2}{\rho}]$ $\qquad a_B = 15^2/150 = 1.5\ m/s^2$

The triangle of acceleration vectors is drawn in the sequence required by the equation as illustraited.



$$(a_{B/A})_x = 1.5\cos 30^o - 1.2 = 0.0990\ m/s^2$$

$$(a_{B/A})_y = 1.5\sin 30^o = 0.750\ m/s^2$$

$$a_{B/A} = \sqrt{0.0990^2 + 0.75^2} = 0.757\ m/s^2 \quad Ans.$$

By the law of sines

$$\frac{1.5}{\sin\beta} = \frac{0.757}{\sin 30^o} \quad \beta = sin^{-1}\left(\frac{1.5}{0.757}0.5\right) = 97.5^o \quad Ans$$

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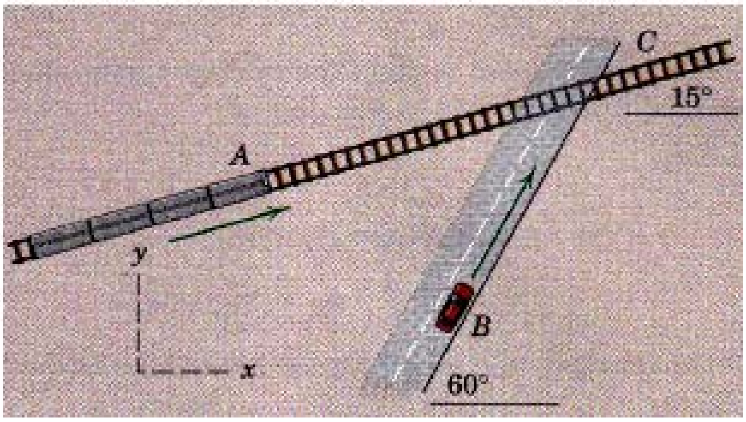
Train A travels with a constant speed v_A = 12(km/h along the straight and level track. The drive of car B, anticipating the railway grade crossing C decreases the car speed of 90 km/h at the rate o 3 m/s^2. Determine the velocity and acceleration o the train relative to the car.

Ans. $\mathbf{v}_{A/B} = 70.9\mathbf{i} - 46.9\mathbf{j}$ km/h
$\mathbf{a}_{A/B} = 1.5\mathbf{i} + 2.60\mathbf{j}$ m/s^2



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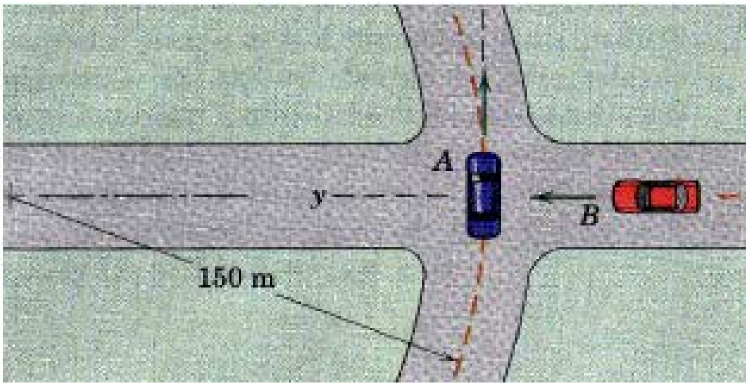
Car A rounds a curve of 150-m radius at a constant speed of 54 km/h. At the instant represented, car B is moving at 81 km/h but is slowing down at the rate of 3 m/s^2. Determine the velocity and acceleration of car A as observed from car B.

Ans. $\mathbf{v}_{A/B} = 15\mathbf{i} - 22.5\mathbf{j}$ m/s
$\mathbf{a}_{A/B} = 4.5\mathbf{j}$ m/s^2



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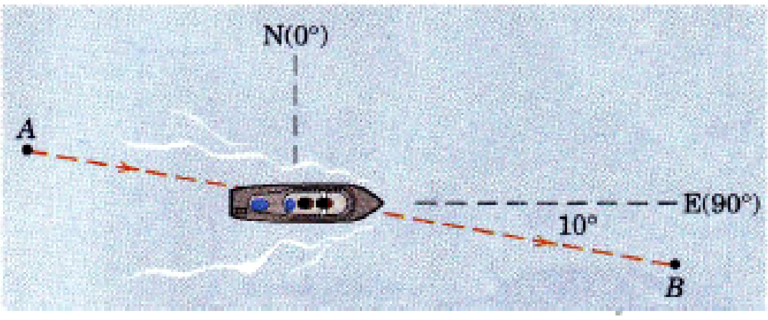
A small ship capable of making a speed of 6 knots through still water maintains a heading due east while being set to the south by an ocean current. The actual course of the boat is from A to B, a distance of 10 nautical miles that requires exactly 2 hours. Determine the speed v_w of the current and its direction measured clockwise from the north.



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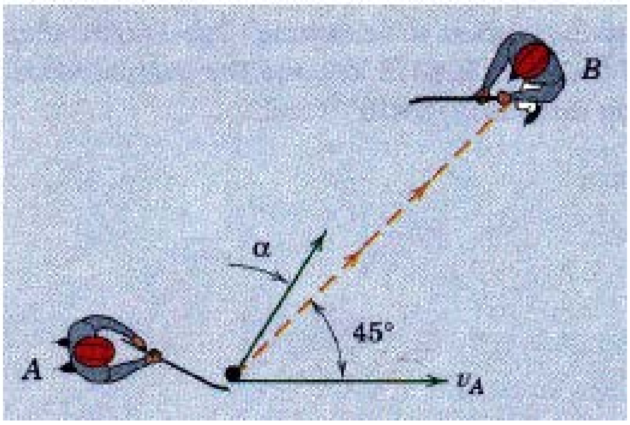
Hockey player A carries the puck on his stick and moves in the direction shown with a speed v_A = 4 m/s. In passing the puck to his stationary teammate B, by what shot angle α should the direction of his shot trail the line of sight if he launches the puck with a speed of 7 m/s relative to himself?

Ans. $\alpha = 23.8°$



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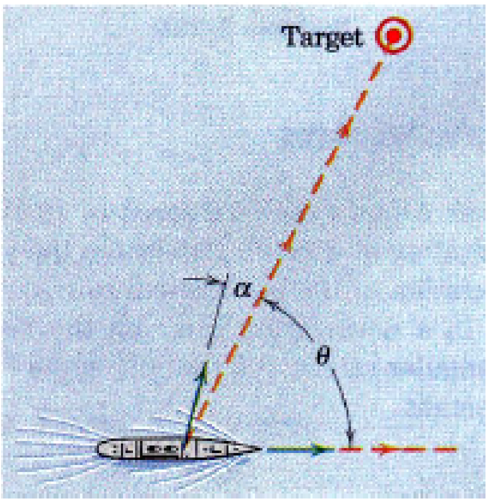
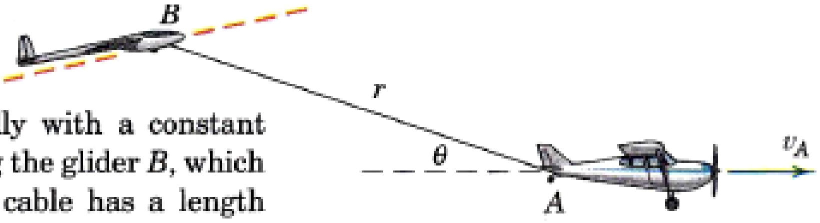
The destroyer moves at 30 knots (1 knot = 1.852 km/h) and fires a rocket at an angle which trails the line of sight to the fixed target by the angle α. The launching velocity is 75 m/s relative to the ship and has an angle of elevation of 30° above the horizontal. If the missile continues to move in the same vertical plane as that determined by its absolute velocity at launching, determine α for $\theta = 60°$.





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Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitude. If the tow cable has a length $r = 60$ m and θ is increasing at the constant rate of 5 degrees per second, determine the magnitudes of the velocity **v** and acceleration **a** of the glider for the instant when $\theta = 15°$.

Ans. $v_B = 206$ km/h, $a_B = 0.457$ m/s^2

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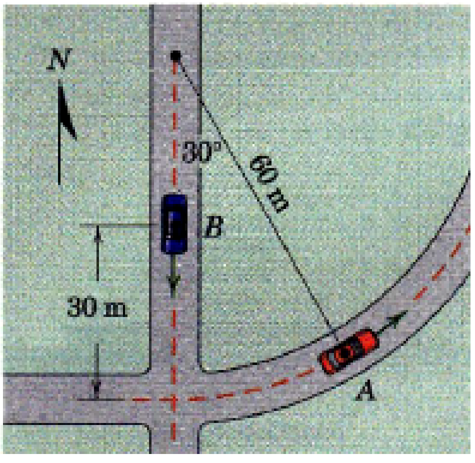
Car A negotiates a curve of 60-m radius at a constant speed of 50 km/h. When A passes the position shown, car B is 30 m from the intersection and is accelerating south toward the intersection at the rate of 1.5 m/s^2. Determine the acceleration which A appears to have when observed by an occupant of B at this instant.

Ans. $a_{A/B} = 4.58$ m/s^2, $\beta = 20.6°$ west of north



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

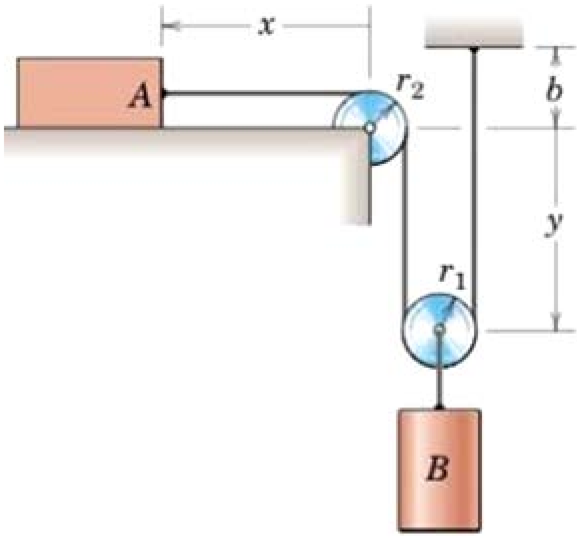
Lecture Contents	Lecture sequences:	7th lecture	Bakr Noori Alhasan/Lecturer
	The major contents 2/9 Constrained Motion of Connected Particles One Degree of Freedom Two Degrees of Freedom		
	The detailed contents 2/9 Constrained Motion of Connected Particles One Degree of Freedom Two Degrees of Freedom Sample Problem 2/14 Sample Problem 2/15		

2 / 9 Constrained Motion of Connected Particles

The motions of particles are interrelated because of the constraints imposed by interconnecting members.

One Degree of Freedom

Consider a system of two connected particles A and B.



The motion of B = The motion of the center of its pulley

Establish position coordinates y and x from a convenient fixed datum.

L: the total length of the cable.

$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$

$0 = \dot{x} + 2\dot{y}$

$L, r_1, r_2,\ and\ b\ are\ constants$

$0 = v_A + 2v_B$

$0 = \ddot{x} + 2\ddot{y}$

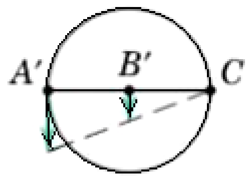
$0 = a_A + 2a_B$

Of these constrained equations:

v_A has a sign opposite to v_B and similarly for the accelerations.

v_A is positive to the left and v_B is positive down.

The figure shown is enlarged view of the horizontal diameter $\acute{A}\acute{B}\acute{C}$ of the lower pulley at an instant of time.



The motion magnitude of $\acute{A} = A$

The motion magnitude of $\acute{B} = B$

During an infinitesimal motion of $\acute{A}$, the movement of $\acute{B} = {}^1/_2\ of\ \acute{A}$

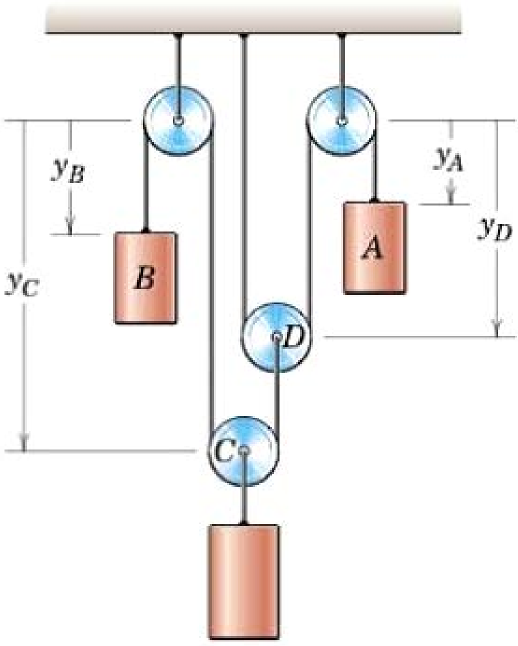
(because C momentarily has no motion)

Differentiate: $v_B = \frac{1}{2} v_A \quad and \quad a_B = \frac{1}{2} a_A$

The system is <u>one degree of freedom</u> since only one variable either x or y is needed to specify the positions of all parts of the system.

Two Degrees of Freedom

The figure shows the system with two degrees of freedom:
The positions of the lower cylinder and pulley C depend on the separate coordinates y_A *and* y_B.



$$L_A = y_A + 2y_D + const.$$

$$L_B = y_B + y_C + y_C - y_D + const.$$

$$0 = \dot{y}_A + 2\dot{y}_D \quad and \quad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$
$$0 = \ddot{y}_A + 2\ddot{y}_D \quad and \quad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

Eliminating the terms in $\dot{y}_D$ *and* $\ddot{y}_D$ *gives*

$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \ or \ v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C \ or \ a_A + 2a_B + 4a_C = 0$$

Note : It is impossible for the signs of all three terms to be positive simultaneously. For example if both v_A and v_B have downward velocity (positive), then v_C will have an upward (negative) velocity.

By Inspection

1. With y_B held fixed:

for an increment $dy_A \rightarrow dy_D = \frac{dy_A}{2}$

$dy_C = \frac{dy_A}{4}$ ……… (1)

2. With y_A held fixed:

for an increment $dy_B \rightarrow dy_C = \frac{dy_B}{2}$ ………… (2)

A combination of the two movements gives an upward movement:

$-dy_C = \frac{dy_B}{2} + \frac{dy_A}{4}$

So that $-v_C = \frac{v_B}{2} + \frac{v_A}{4}$

A second type of constraint: The direction of the connecting member changes with the motion (Sample problem 2/16).

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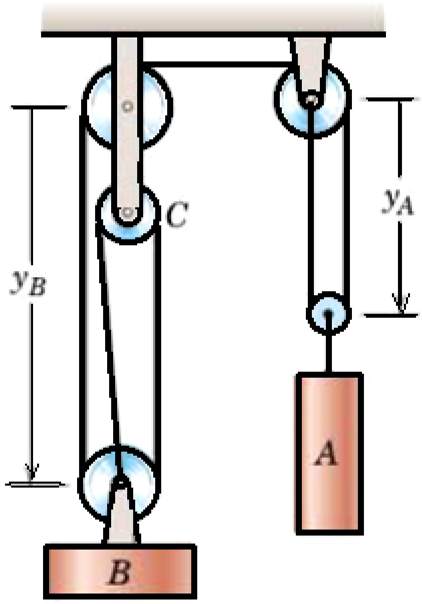
Sample Problem 2/14

In the pulley configuration shown, cylinder A has a downward velocity of 0.3 m/s. Determine the velocity of B. Solve in two ways.



Solution (1)

$L = 3y_B + 2y_A + constants$

Differentiation with time gives

$$0 = 3\dot{y}_B + 2\dot{y}_A$$

Substitution

$$v_A = \dot{y}_A = 0.3\ m/s \quad and \quad v_B = \dot{y}_B \ gives$$
$$0 = 3(v_B) + 2(0.3) \quad or \quad v_B = -0.2\ m/s \quad \textbf{Ans.}$$

Solution (2)

During a differential movement ds_A of the center of pulley A (downward)[1]

$2ds_A = 3ds_B$

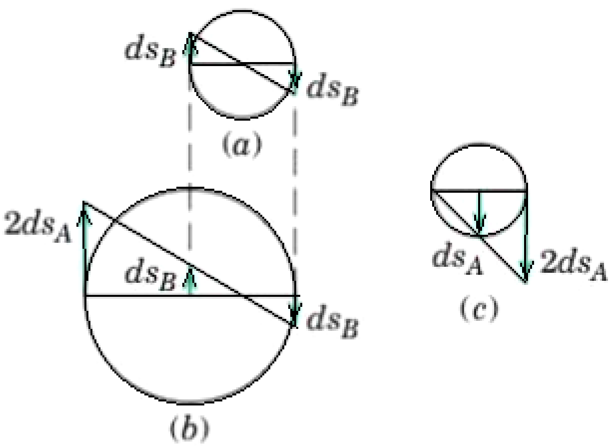
$ds_B = \frac{2}{3}ds_A$

$|v_B| = \frac{2}{3}v_A$

$v_B = \frac{2}{3}(0.3)$

$v_B = 0.2\ m/s \quad \textbf{Ans.}$

upward (According to Figure)



1. According to v_A in question.

Sample Problem 2/15

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.

Solution

x: The position of the tractor

y: The position of the bale

$$L = l + 2(h - y)$$

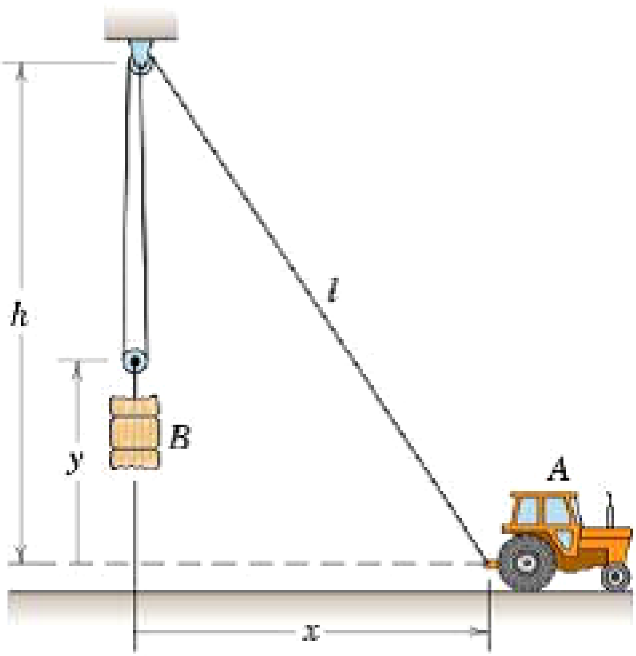
$$L = \sqrt{x^2 + h^2} + 2(h - y)$$

$$0 = \frac{x\dot{x}}{\sqrt{x^2+h^2}} - 2\dot{y}$$

Substituting $v_A = \dot{x}$ & $v_B = \dot{y}$

$$\dot{y} = \frac{1}{2}\frac{xv_A}{\sqrt{x^2+h^2}}$$

$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{x^2+h^2}} \qquad \textbf{Ans.}$$



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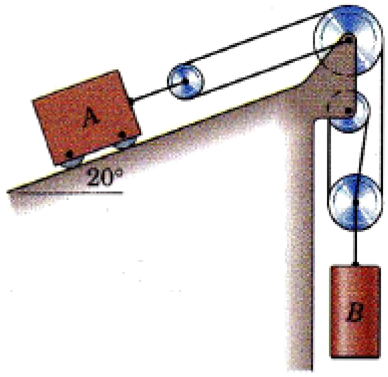
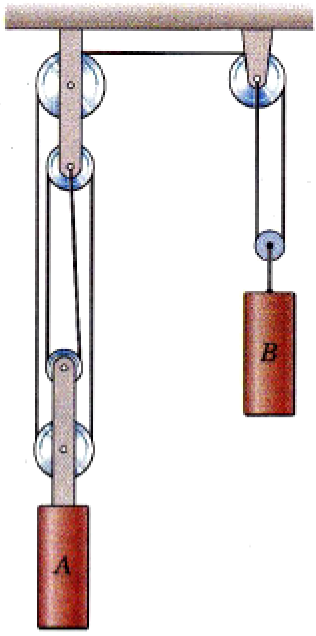
Cylinder B has a downward velocity of 0.6 m/s and an upward acceleration of 0.15 m/s^2. Calculate the velocity and acceleration of block A.

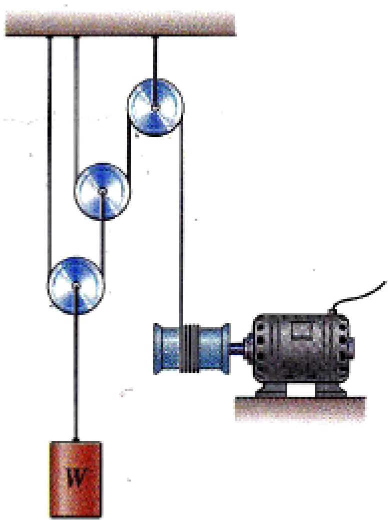


2/210 Cylinder B has a downward velocity in meters per second given by $v_B = t^2/2 + t^3/6$, where t is in seconds. Calculate the acceleration of A when $t = 2$ s.



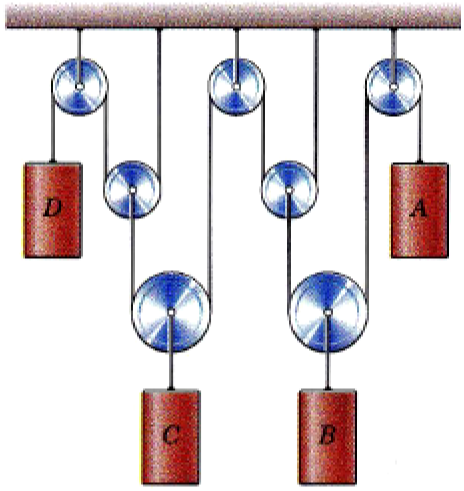
2/211 Determine the vertical rise h of the load W during 5 seconds if the hoisting drum wraps cable around it at the constant rate of 320 mm/s.

Ans. $h = 400$ mm

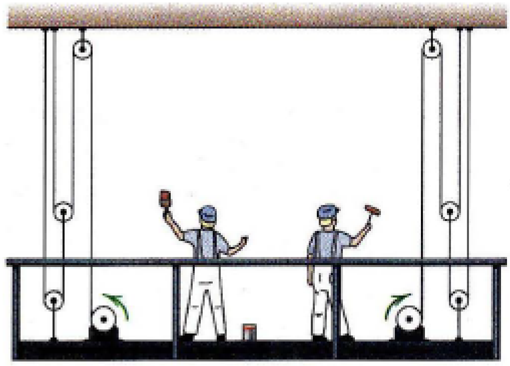


2/213 Determine the relationship which governs the velocities of the four cylinders. Express all velocities as positive down. How many degrees of freedom are there?

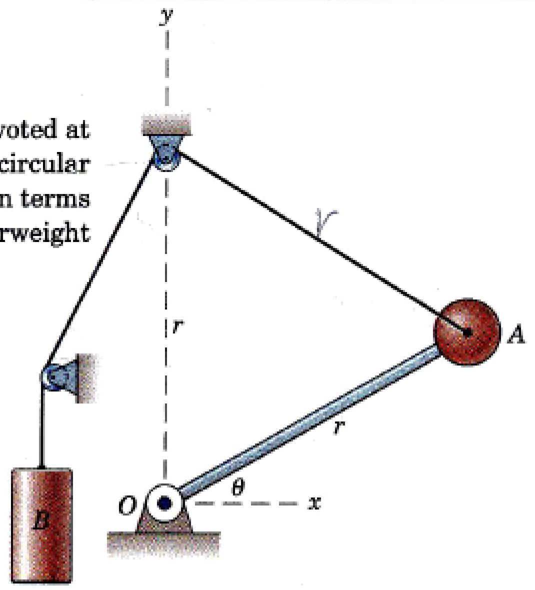
Ans. $4v_A + 8v_B + 4v_C + v_D = 0$
3 degrees of freedom



2/216 The power winches on the industrial scaffold enable it to be raised or lowered. For rotation in the sense indicated, the scaffold is being raised. If each drum has a diameter of 200 mm and turns at the rate of 40 rev/min, determine the upward velocity v of the scaffold.

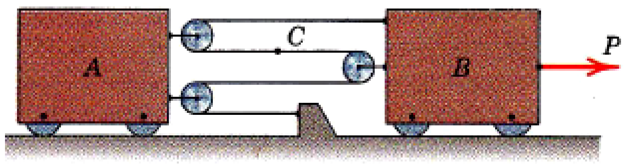


2/220 The particle A is mounted on a light rod pivoted at O and therefore is constrained to move in a circular arc of radius r. Determine the velocity of A in terms of the downward velocity v_B of the counterweight for any angle θ.



►2/222 Under the action of force P, the constant acceleration of block B is 3 m/s² to the right. At the instant when the velocity of B is 2 m/s to the right, determine the velocity of B relative to A, the acceleration of B relative to A, and the absolute velocity of point C of the cable.

Ans. $v_{B/A} = 0.5$ m/s, $a_{B/A} = 0.75$ m/s²
$v_C = 1$ m/s, all to the right



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

	Lecture sequences:	8th lecture	Bakr Noori Alhasan/Lecturer
Lecture Contents	*The major contents* CHAPTER THREE/ KINETICS OF PARTICLES Introduction Section A. Force , Mass, and Acceleration 3/2 Newton's Second Law 3/3 Equation of Motion and Solution of Problems 3/4 Rectilinear Motion 3/5 Curvilinear Motion		
	The detailed contents CHAPTER THREE/ KINETICS OF PARTICLES Introduction Section A. Force , Mass, and Acceleration 3/2 Newton's Second Law Inertial System System of Units Force and Mass Units 3/3 Equation of Motion and Solution of Problems Two Types of Dynamics Problems Constrained and Unconstrained Motion		

	Free Body Diagram **Steps of solutions** **3/4 Rectilinear Motion** **3/5 Curvilinear Motion** **Sample problem 3/1** **Sample problem 3/2** **Sample problem 3/3** **Sample problem 3/4** **Sample Problem 3/6** **Sample Problem 3/8**

CHAPTER THREE
KINETICS OF PARTICLES

Introduction

According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces . Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion.

Kinetics of particles can be solved by three general approaches:

a) Direct application of Newton's second law (Force- mass - acceleration method).
b) Work and energy principles.
c) Impulse and momentum methods.

Section A. Force , Mass, and Acceleration

3/2 Newton's Second Law

$$\vec{F} = m\,\vec{a} \;\ldots\ldots\ldots\; (3/1)$$

The verification of this equation is entirely experimental. The experiment involves subjecting a mass particle to the action of a single force $\vec{F}_1$, and the acceleration $\vec{a}_1$ is measured in the primary inertial system. By repeating the procedure with different forces, The followings were noticed:

$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \ldots \frac{F}{a} = C$ C is constant.

1. C is the inertial of the particle (invariable) which is the resistance to rate of change of velocity. Mass, m is a quantitative measure of inertia.
$C = km$, k is a constant accounting for the units used.
2. The acceleration is always in the direction of the applied force.

$$\vec{F} = km\vec{a} \;\ldots\ldots\ldots\; 3/2$$

Inertial System

The results of the ideal experiment (eq. 3/2) are obtained with respect to fixed primary inertial system. The results are also valid for measurements made with respect to any non-rotating reference systems which translates with a constant velocity.

Eq. 3/2 may be applied with negligible error to experiments made on the surface of the earth.

System of Units

$$\vec{F} = m\,\vec{a}$$

A system of units for which k is unity is known as a kinetic system.
In SI units, the units of force derived from the basic units of mass times acceleration (known as an absolute system).
In US customary units, the units of mass (slugs) are derived from the units of force (pounds force, lb) divided by acceleration (ft/sec^2). This system is known as a gravitational system.
$g = 9.80665\ m/s^2$ (Accepted measured value relative to the earth at sea level and at a latitude of 45°) or $g = 32.174\ ft/s^2$

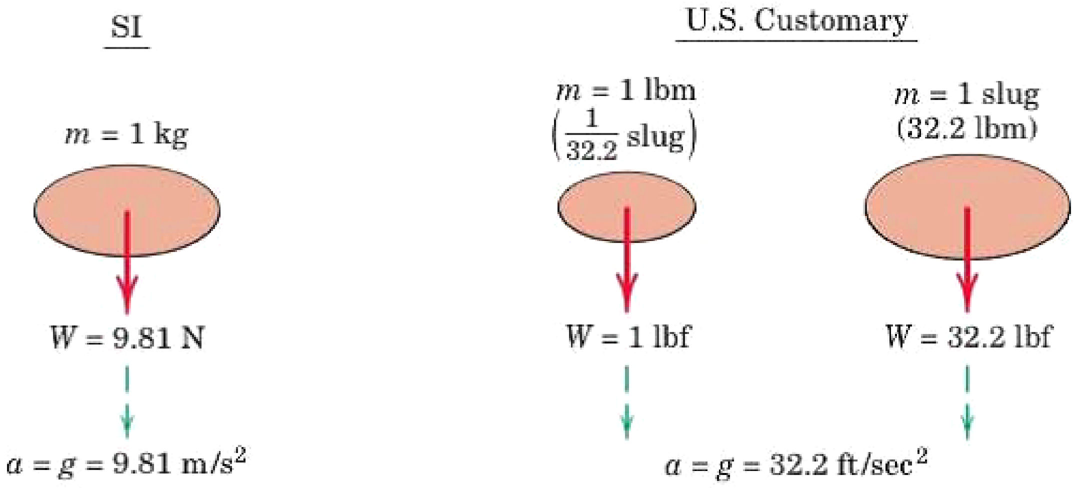
g = 9.81 m/s^2	or	g = 32.2 ft/s^2

Force and Mass Units

Consider the free -fall experiment as shown in the figure below:

In SI units, for a mass $m = 1$ kg, the weight is $W = 9.81\ N$ and the corresponding *downward* acceleration a is $g = 9.81\ \mathrm{m/s^2}$

In US customary units, for a mass $m = 1\ lbm$ (1/32 slug), the weight is $W = 1\ lbf$ and the acceleration is $g = 32.2\ \mathrm{ft/s^2}$



3/3 Equation of Motion and Solution of Problems

$$\sum \vec{F} = m\,\vec{a} \quad \dots\dots\dots \quad 3/3$$

Equation 3/3 is usually called the equation of motion, $\sum \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots\dots\dots$, $\vec{F_1}, \vec{F_2}, \vec{F_3}, \dots$ are concurrent forces and they are instantaneous values corresponding to the instantaneous accelerations. To solve problems, eq. 3/3 is usually expressed in scalar components form with the use of one of the following coordinates, $x-y$, $n-t$, $r-\theta$, … … …

Two Types of Dynamics Problems

1. The acceleration of the particle is specified or determined directly from known kinematic conditions. Then the corresponding forces are determined by direct substitution into Eq. 3/3 .
2. The forces are specified and the resulting motion must be determined.
If the forces are constant $\rightarrow$ a is constant and is easily found from Eq. 3/3
When the forces are functions of time , position ,or velocity Eq. 3/3 becomes a differential equation and must be integrated.

Constrained and Unconstrained Motion

There are two physically distinct types of motion specified by Eq. 3/3 .
1. Unconstrained motion : The particle is free of mechanical guides. Air planes , rockets are examples of unconstrained motion.
2. Constrained motion :
a) Partially ; Ice hockey puck (constrained to move in the horizontal plane).
b) Totally ; Train moving along its track, a collar sliding along a fixed shaft.

Free Body Diagram

All forces acting on the particle must be accounted for when force- mass acceleration equations of motion are applied. The only reliable way is by drawing the FBD . By which every force known or unknown is represented and accounted for.

Steps of solutions:

1) Recognizing the known and unknown.
2) Drawing the FBD.
3) Indicate the coordinate axes and their positive directions.
4) Plan of attack. (including choice of particular direction).
5) Equations of motion.

3/4 Rectilinear Motion

In this article, we will analyze the motions of bodies which can be treated as particles. This is possible as long as we are interested only in the motion of the mass center of the body.

The forces should be concurrent through the mass center (the action of non-concurrent forces will be accounted for *the kinetics of rigid bod*ies).

$$\sum F_x = m\,a_x$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

If x-direction ,for example, is chosen as the direction of the rectilinear motion of a particle.

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

If we are not free to choose a coordinate direction along the direction of the motion.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} \qquad \left|\sum \vec{F}\right| = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2 + (\sum F_z)^2}$$

Problems **3/1 3/2 3/3 3/4 3/7 3/12 3/15 3/17 3/20 3/323**

3/5 Curvilinear Motion

Equation 3/3 is rewriten in three ways :

- ❖ Rectangular coordinates.

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \longrightarrow \boxed{3/6}$$

Where $a_x = \ddot{x}$ and $a_y = \ddot{y}$

- ❖ Normal and tangential coordinates.

$$\sum F_n = ma_n \qquad \sum F_t = ma_t \longrightarrow \boxed{3/7}$$

Where:

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta}$$
$$a_t = \dot{v}\,, \qquad v = \rho\dot{\beta}$$

- ❖ Polar coordinates.

$$\sum F_r = ma_r \qquad \sum F_\theta = ma_\theta \longrightarrow \boxed{3/8}$$

Where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Problems **3/50 3/51 3/54 3/60 3/64 3/69 3/73 3/75 3/81 3/86**

Sample problem 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in Newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

Given :

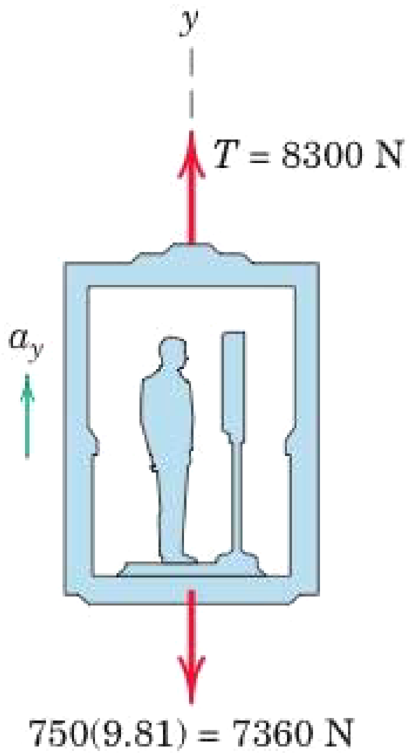
$m_m = 75\ kg$ $\quad\quad t = 1\ to\ 3\ sec.$

$T = 8300\ N\ (during\ 3\ sec.)$ $\quad\quad m_t = 750\ kg$

Required

$R = ?$

$v_3 = ?$



Solution

From the FBD of the total

$[\sum F_y = ma_y]$

$8300 - 750(9.81) = 750\ a_y$

$a_y = 1.257\ m/s^2$

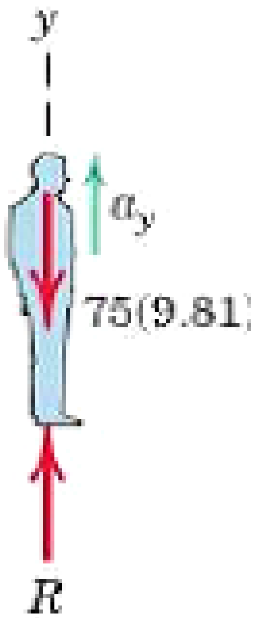
From the FBD of the man alone

$R - 75(9.81) = 75(1.257)$

$R = 830\ N$

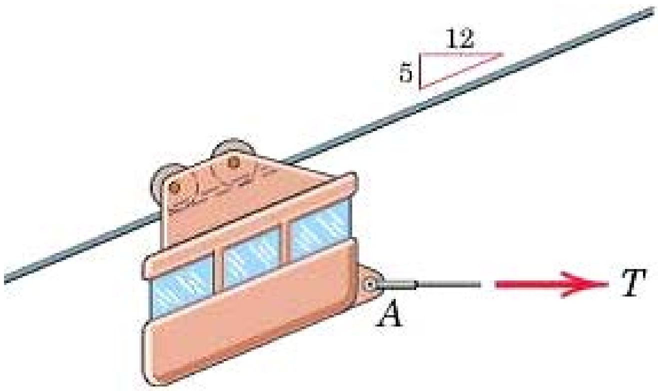
$v = v_0 + a\ t$

$v_3 = 0 + 1.257(3) = 3.77\ m/s \quad Ans.$



- As tension force (T) is constant , a becomes constant
- a_y is the acceleration of man as well as of the total
- If the scale is calibrated to 830 / 9.81= 84.6 kg ≠ 75 kg , because of the acceleartion.

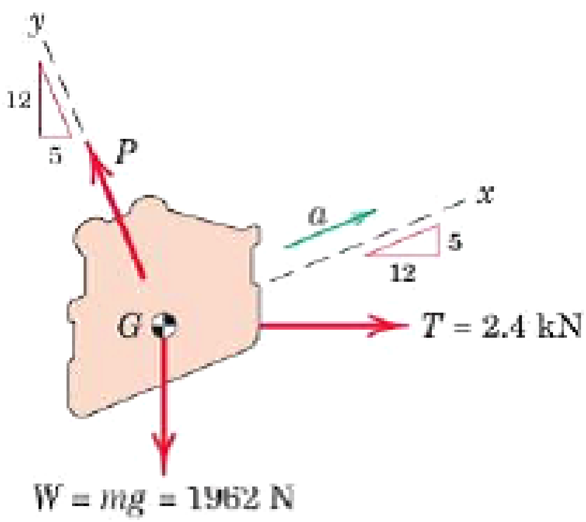
Sample problem 3/2
A small inspection car with a mass of 200-kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension T=2.4 kN. Also find the total force P exerted by the supporting cable on the wheels.



Given :
$m = 200\ kg$
$T = 2400\ N$
Required
$a = ?\quad ,\quad P = ?$

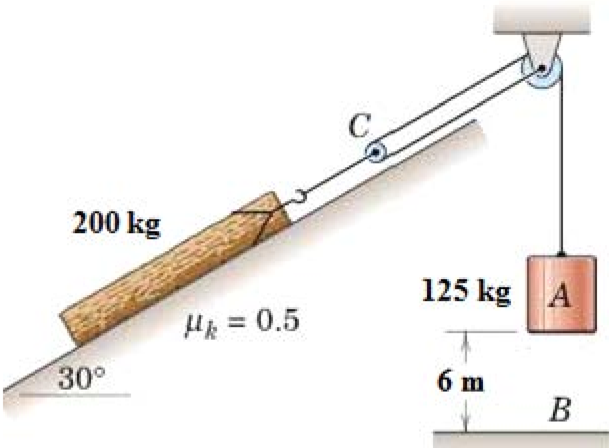
Solution
From the FBD of the total
$[\sum F_y = 0]$
$P - 200\,(9.81)\left(\frac{12}{13}\right) - 2400\left(\frac{5}{13}\right) = 0$
$P = 2.73\ kN \quad Ans.$

$[\sum F_x = ma_x]$
$2400\left(\frac{12}{13}\right) - 200(9.81)\left(\frac{5}{13}\right) = 200\ a$
$a = 7.3\ \ m/s^2 \quad Ans.$



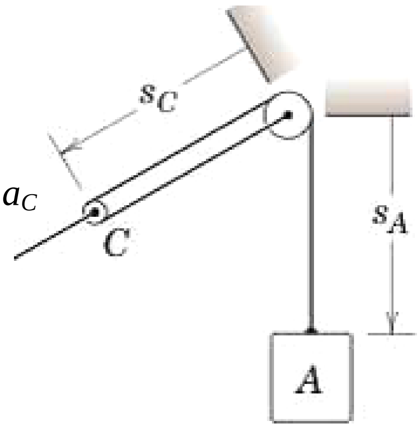
- All parts have been taken as one units and one acceleartion.
- There is no acceleration , a_y so $\sum F_y = 0$
- Try to solve by choosing x and y as horizontal and vertical.

Sample problem 3/3
The 125-kg concrete block A is released from rest in the position shown and pulls the 200-kg log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.



Given :
$m_A = 125$ kg $v_0 = 0$, $m_{log} = 200$ kg
$\theta = 30°$, $\mu_k = 0.5$
Required
$v_B = ?$

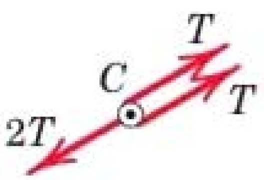
Solution
$L = 2\,S_c + S_A + C$
$0 = 2\,\dot{S}_C + \dot{S}_A$
$0 = 2\,a_C + a_A$ …. (1)



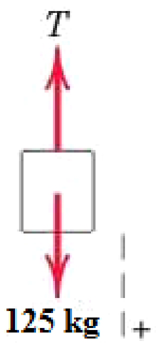
For the FBD of the pully C T (log) = 2T (block)
For the FBD of the log
$[\sum F_y = 0]$ N – 200 (9.81) cos 30° = 0
N = 1699 N
$[\sum F_x = ma_x]$ 0.5 (1699) + 200 (9.81) sin 30° - 2T = 200 a_C

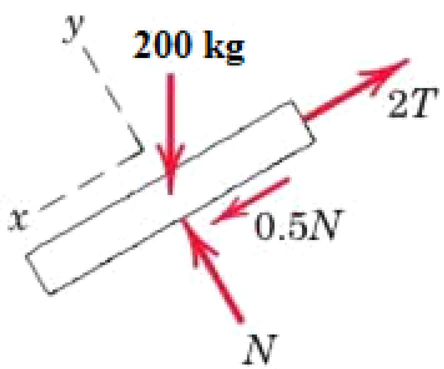
For the block $[\sum F = m_a \downarrow^{+}]$
125 (9.81) – T = 125 a_A ……. (3)

Solving the three equations (1), (2) and (3):
$a_A = 1.777$ $a_C = -0.888$ T = 1004 N



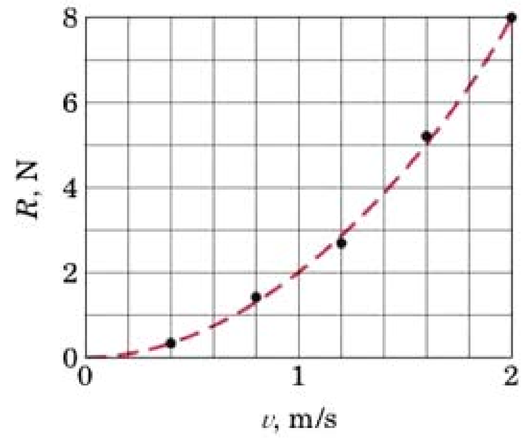
$v^2 = v_0^2 + 2\,a\,\Delta S$
$v_A = \sqrt{2\,(1.777)(6)}$
$v_A = 4.62\ m/s\ \ Ans.$

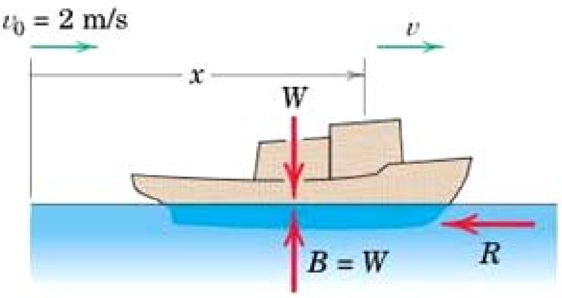




Sample problem 3/4
The design model for a new ship has a mass of 10-kg and is tested in an experimental towing tank to determine its resistance to motion through the water at various speeds. The test results are plotted on the accompanying graph, and the resistance R may be closely approximated by the dashed parabolic curve shown. If the model is released when it has a speed of 2 m/s, determine the time t required for it to reduce its speed to 1 m/s and the corresponding travel distance x.

Given : $m = 10\ kg,\ v_o = 2\ m/s\ , v = 1\ m/s$
Required : $\boldsymbol{t} = ?\quad \boldsymbol{x} = ?$





Solution
Approximate $R = k\,v^2$
$at\ v = 2\ and\ R = 8$
$8 = k(2)^2\ \ lead\ to\ k = 2$
$R = 2v^2$

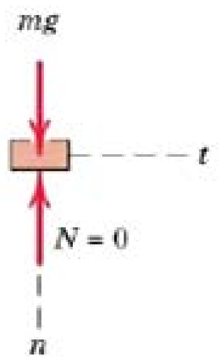
$\sum F_x = ma_x$
$-R = 10\ a_x$
$-2v^2 = 10\ a_x \quad a_x = -0.2\ v^2$
$a = \frac{dv}{dt}$
$\frac{dv}{dt} = \frac{-1}{5}\ v^2$

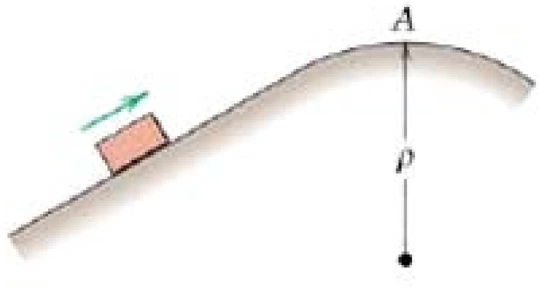
$\int_0^t dt = -5\ \int_2^v \frac{dv}{v^2} \longrightarrow t = 5\left(\frac{1}{v} - \frac{1}{2}\right)$
$substitute\ v = 1\ leads\ to\ t = 2.5\ \text{sec}\ \ Ans.$
$v = \frac{dx}{dt}$
$\int_0^x dx = \int_0^{2.5} \frac{10}{5+2t}\, dt$
$x = \frac{10}{2}\ln(5 + 2t)$
$x = 3.47\ m\ \ Ans.$

Suggestion : express x in terms of v
$v\,dv = a\,dx \longrightarrow v\,dv = \frac{-1}{5} v^2\,dx$
$\int_{v_0}^{v} -5\,\frac{dv}{v} = \int_0^x dx \longrightarrow x = -5\ln v|_{v_0}^{v}$
$x = -5\,(\ln v - \ln v_0) \longrightarrow x = 5\ln\frac{v_0}{v}$

Sample Problem 3/6

Determine the maximum speed v which the sliding block may have as it passes point A without lossing contact with the surface.





Solution

Loss of contact means $N = 0$

From the FBD of the block $[\sum F_n = m\,a_n]$

$$mg - 0 = m\,\frac{v^2}{\rho}$$

$$v = \sqrt{g\rho} \qquad Ans.$$

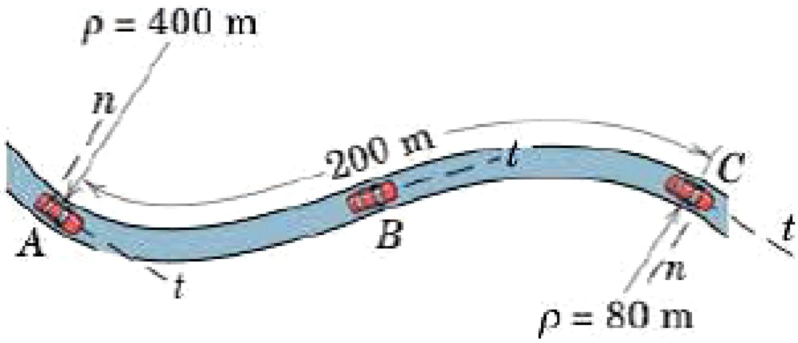
If $\quad v < \sqrt{g\rho}\,,\ the\ force\ N\ exists$

$In\ order\ for\ the\ block\ \ v > \sqrt{g\rho}\,,$

$some\ type\ of\ constraint\ would\ have\ to\ be\ introduced\ to\ provide\ additional\ \ downward\ force.$

Sample Problem 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at A to a speed of 50 km/h as it passes at C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. Determine the total horizontal force exerted by the road on the tires at positions A, B, and C. Point B is the inflection point where the curvature changes direction.



Solution

$v_c^2 = v_A^2 + 2a_t \Delta s$

$(50/3.6)^2 = (100/3.6)^2 + 2\, a_t(200)$

$a_t = -1.447\ m/s^2$

$[a_n = \frac{v^2}{\rho}]$

$At\ \ A, \quad a_n = \frac{(100/3.6)^2}{400} = 1.929\ m/s^2$

$At\ \ B, \quad a_n = 0$

$At\ \ C, \quad a_n = \frac{(50/3.6)^2}{80} = 2.41\ m/s^2$

$[\sum F_t = m\, a_t]$

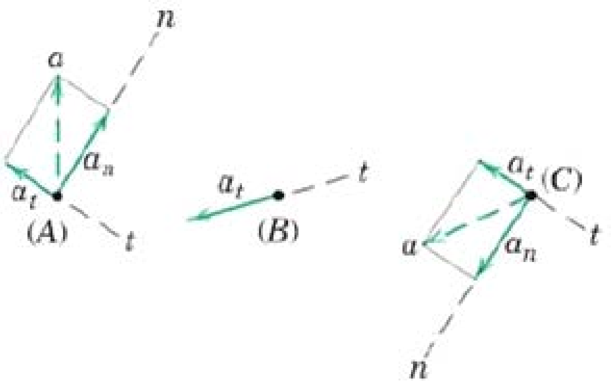
$F_t = 1500(1.447) = 2170\ N$

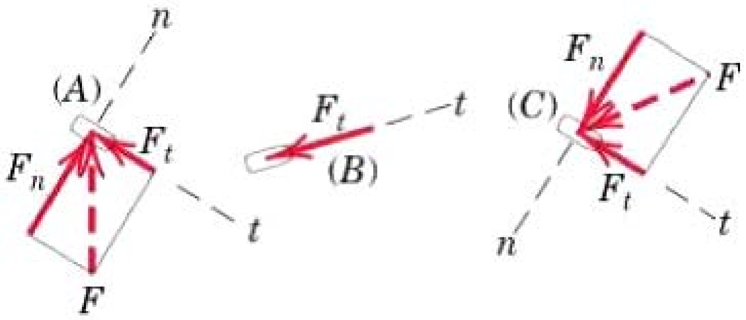
$\sum F_n = m\, a_n$

$At\ A, \quad F_n = 1500(1.929) = 2890\ N$

$At\ B, \quad F_n = 0$

$At\ C, \quad F_n = 1500(2.41) = 3620\ N$





$At\ A, \quad F = \sqrt{F_n^2 + F_t^2} = \sqrt{(2890)^2 + (2170)^2} = 3620\ N\ \ Ans.$

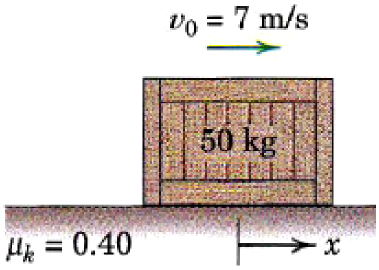
$At\ B, \quad F = F_t = 2170\ N\ \ Ans.$

$At\ C, \quad F = \sqrt{F_n^2 + F_t^2} = \sqrt{(3620)^2 + (2170)^2} = 4220\ N\ \ Ans.$

3/1

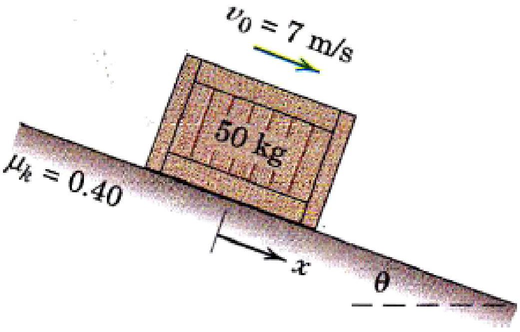
The 50-kg crate is projected along the floor with an initial speed of 7 m/s at $x = 0$. The coefficient of kinetic friction is 0.40. Calculate the time required for the crate to come to rest and the corresponding distance x traveled.

Ans. $t = 1.784$ s, $x = 6.24$ m



3/2

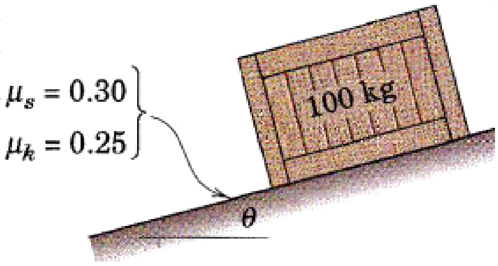
The 50-kg crate of Prob. 3/1 is now projected down an incline as shown with an initial speed of 7 m/s. Investigate the time t required for the crate to come to rest and the corresponding distance x traveled if (*a*) $\theta = 15°$ and (*b*) $\theta = 30°$.



3/3

The 100-kg crate is carefully placed with zero velocity on the incline. Describe what happens if (*a*) $\theta = 15°$ and (*b*) $\theta = 20°$.

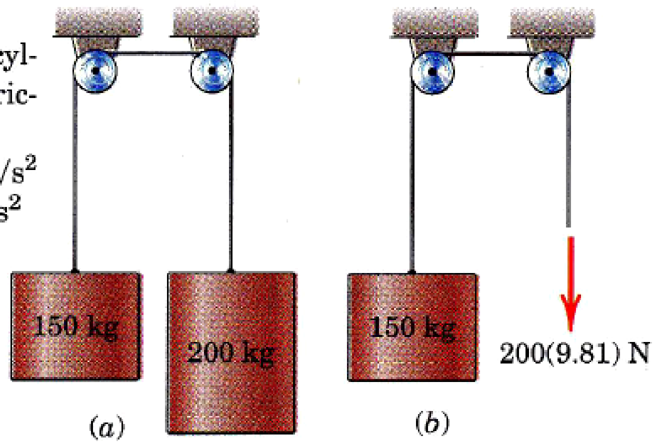
Ans. (*a*) $a = 0$; no motion
(*b*) $a = 1.051$ m/s^2 down incline



3/7

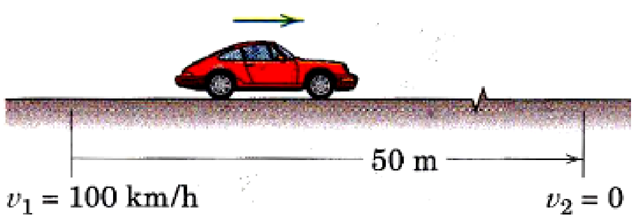
Calculate the vertical acceleration a of the 150-kg cylinder for each of the two cases illustrated. Neglect friction and the mass of the pulleys.

Ans. (*a*) $a = 1.401$ m/s^2
(*b*) $a = 3.27$ m/s^2



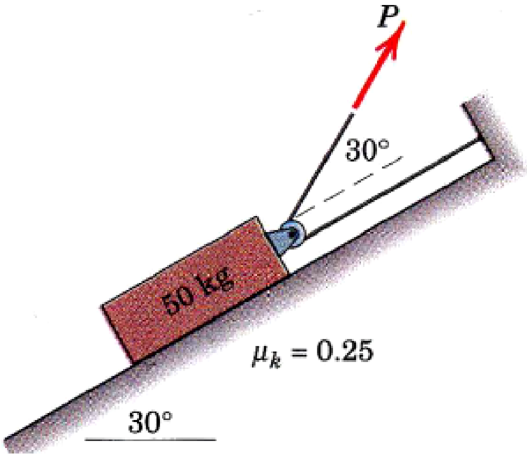
3/4

During a brake test, the rear-engine car is stopped from an initial speed of 100 km/h in a distance of 50 m. If it is known that all four wheels contribute equally to the braking force, determine the braking force F at each wheel. Assume a constant deceleration for the 1500-kg car.



3/12

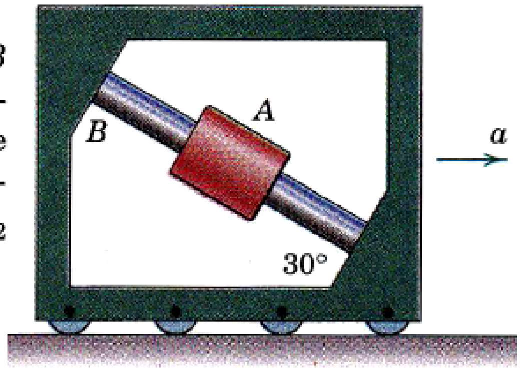
Determine the tension P in the cable which will give the 50-kg block a steady acceleration of 2 m/s^2 up the incline.



3/15

The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the horizontal acceleration a of the frame necessary to maintain the collar in a fixed position on the shaft.

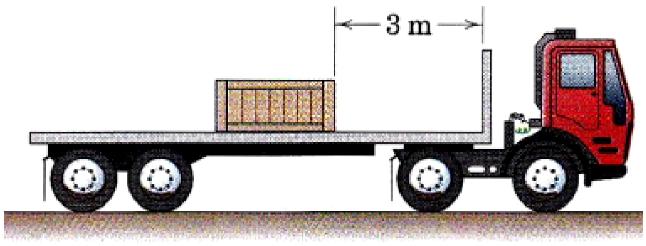
Ans. $a = 5.66 \text{ m/s}^2$



3/17

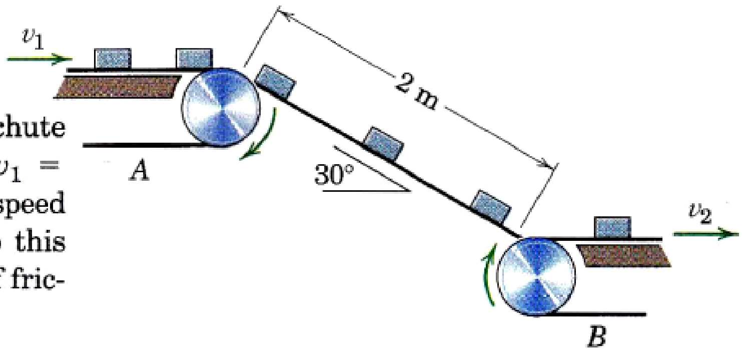
The coefficient of static friction between the flat bed of the truck and the crate it carries is 0.30. Determine the minimum stopping distance s which the truck can have from a speed of 70 km/h with constant deceleration if the crate is not to slip forward.

Ans. $s = 64.3$ m



3/20

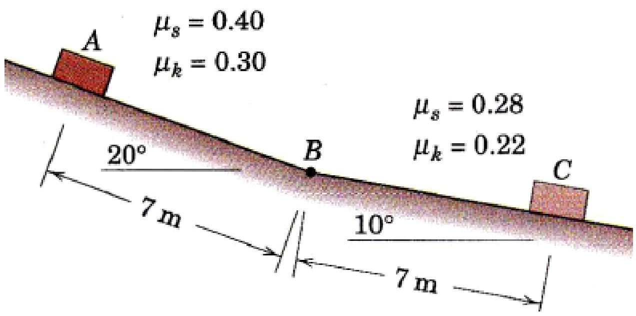
Small objects are delivered to the 2-m inclined chute by a conveyor belt A which moves at a speed $v_1 = 0.4$ m/s. If the conveyor belt B has a speed $v_2 = 0.9$ m/s and the objects are delivered to this belt with no slipping, calculate the coefficient of friction μ_k between the objects and the chute.



3/323

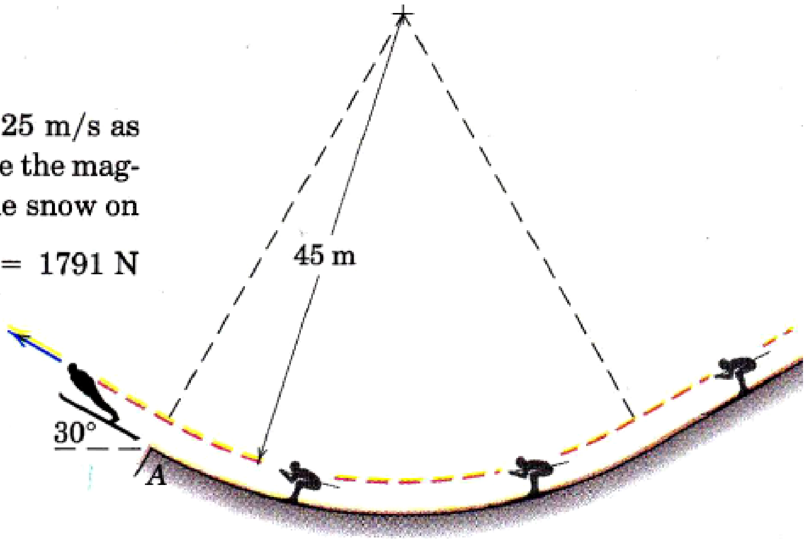
The crate is at rest at point A when it is nudged down the incline. If the coefficient of kinetic friction between the crate and the incline is 0.30 from A to B and 0.22 from B to C, determine its speeds at points B and C.

Ans. $v_B = 2.87$ m/s, $v_C = 1.533$ m/s



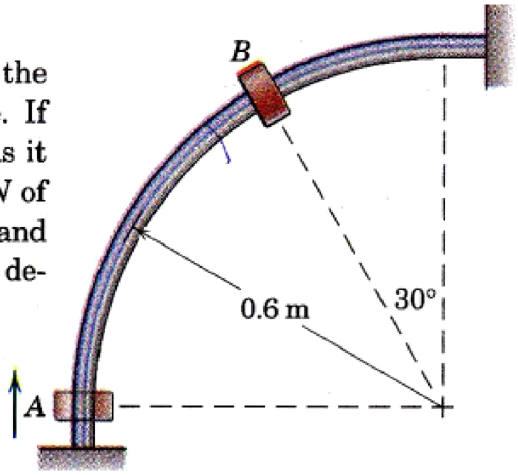
3/51

If the 80-kg ski-jumper attains a speed of 25 m/s as he approaches the takeoff position, calculate the magnitude N of the normal force exerted by the snow on his skis just before he reaches A. *Ans.* $N = 1791$ N



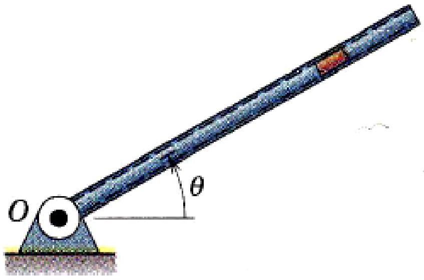
3/52

A 0.8-kg slider is propelled upward at A along the fixed curved bar which lies in a vertical plane. If the slider is observed to have a speed of 4 m/s as it passes position B, determine (a) the magnitude N of the force exerted by the fixed rod on the slider and (b) the rate at which the speed of the slider is decreasing. Assume that friction is negligible.



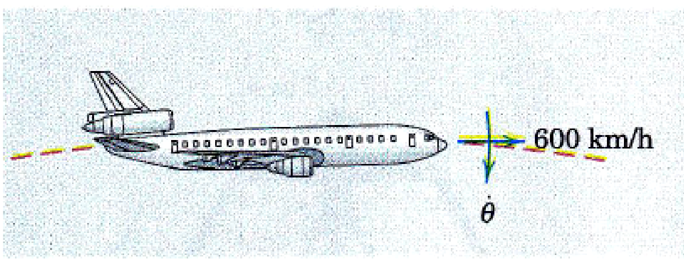
3/54

The hollow tube is pivoted about a horizontal axis through point O and is made to rotate in the vertical plane with a constant counterclockwise angular velocity $\dot{\theta} = 3$ rad/s. If a 0.1-kg particle is sliding in the tube toward O with a velocity of 1.2 m/s relative to the tube when the position $\theta = 30°$ is passed, calculate the magnitude N of the normal force exerted by the wall of the tube on the particle at this instant.



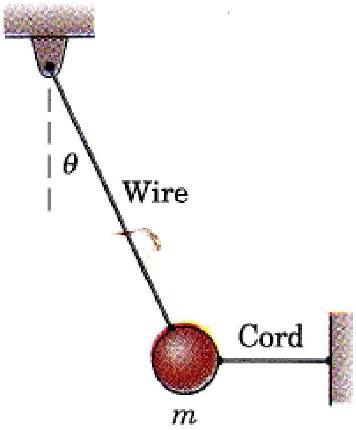
3/62

In order to simulate a condition of apparent "weightlessness" experienced by astronauts in an orbiting spacecraft, a jet transport can change its direction at the top of its flight path by dropping its flight-path direction at a prescribed rate $\dot{\theta}$ for a short interval of time. Specify $\dot{\theta}$ if the aircraft has a speed $v = 600$ km/h.



3/64

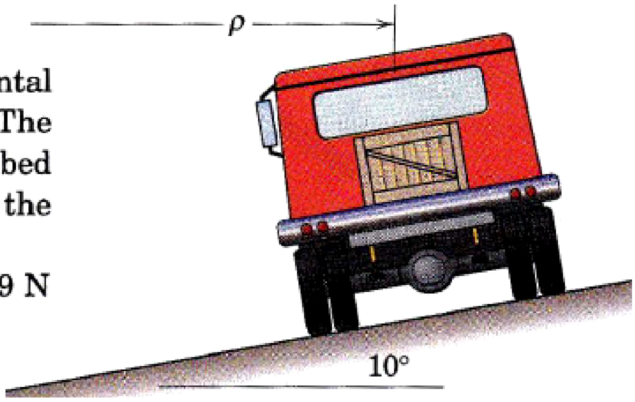
The small ball of mass m and its supporting wire become a simple pendulum when the horizontal cord is severed. Determine the ratio k of the tension T in the supporting wire immediately after the cord is cut to that in the wire before the cord is cut.



3/69

A flatbed truck going 100 km/h rounds a horizontal curve of 300-m radius inwardly banked at 10°. The coefficient of static friction between the truck bed and the 200-kg crate it carries is 0.70. Calculate the friction force F acting on the crate.

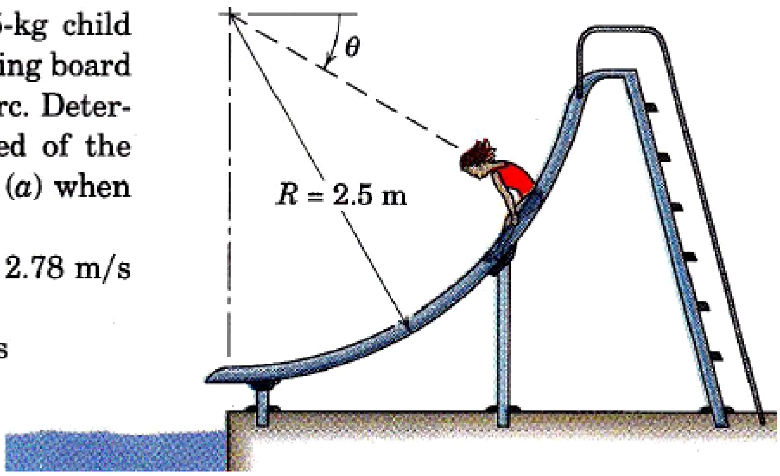
Ans. $F = 165.9$ N



3/75

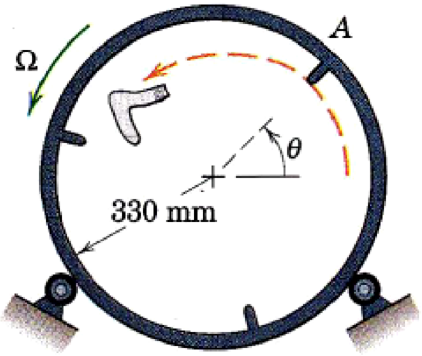
Beginning from rest when $\theta = 20°$, a 35-kg child slides with negligible friction down the sliding board which is in the shape of a 2.5-m circular arc. Determine the tangential acceleration and speed of the child, and the normal force exerted on her (*a*) when $\theta = 30°$ and (*b*) when $\theta = 90°$.

Ans. (*a*) $a_t = 8.50$ m/s^2, $v = 2.78$ m/s
$N = 280$ N
(*b*) $a_t = 0$, $v = 5.68$ m/s
$N = 795$ N



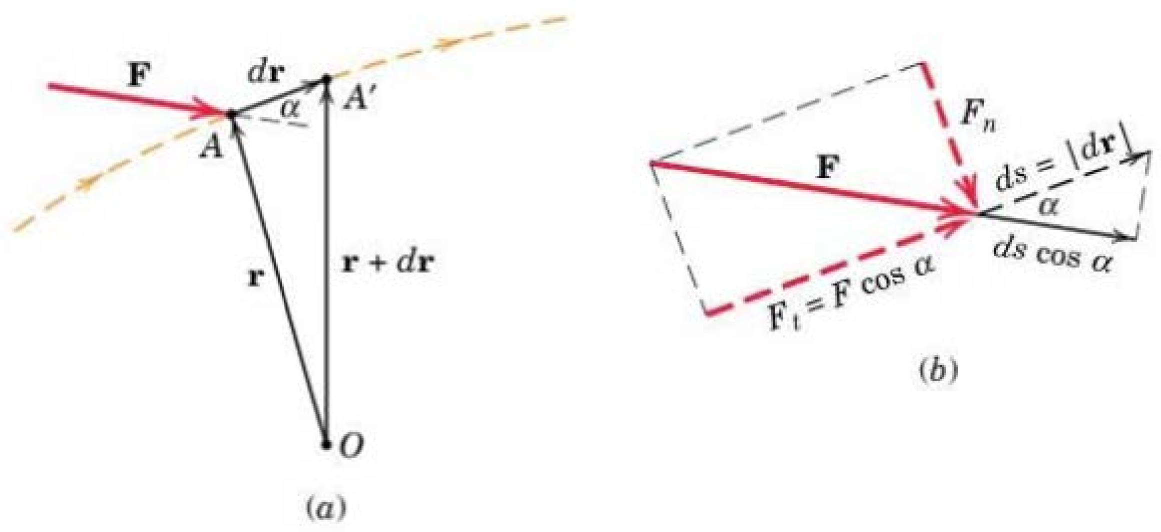
3/82

The rotating drum of a clothes dryer is shown in the figure. Determine the angular velocity Ω of the drum which results in loss of contact between the clothes and the drum at $\theta = 50°$. Assume that the small vanes prevent slipping until loss of contact.



Section B Work and Energy
Work and Kinetic Energy

Definition of work:



$\vec{r}$: is the position vector measured from some convenient origin locates the particle at point A.

$\overrightarrow{dr}$ **:** is the deferential displacement with an infinitesimal movement A to A' .

$dU = \vec{F}.d\vec{r}$ The magnitude of the dot product is $dU = Fds\cos\alpha$. This expression may be interpreted:

- As the displacement multiplied by the force component $F_t ds$ in the direction of displacement as represented in the dashed line in the figure $(or\ F\cos\alpha\ \ ds)$.
- As the force multiplied by the displacement component $ds\cos\alpha$ in the direction of the force $(F\ ds\cos\alpha)$ as represented in the Full lines in the figure.

The component F_n normal to the displacement does not work.

Work is positive if the working component F_t is in the direction of displacement and negative if it is in the opposite direction of the displacement.

Active forces: Are forces which do work like F_t.
Reactive forces: Are forces which do not work like F_n and the constrained forces.

Units of Work

SI units	US Customary units
Joule = N.m	ft-lb (lb- ft is the unit of torque)

Joule is used to avoid ambiguity with moment or torque.

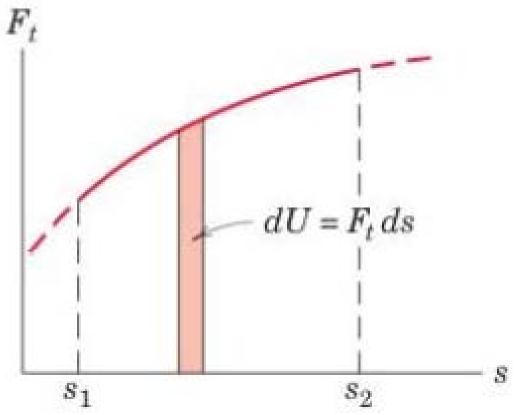
The Discrimination between Work and Moment:

Work	***Moment***
Scalar	Vector
Dot product	Cross product
The product of a force and a distance both are measured along the same line.	The product of a force and distance are measured at right angle to the force.

Calculation of Work

During a finite movement of point of application of a force, the work is:

$$U = \int_1^2 \vec{F} \,.\, d\vec{r}$$
$$= \int_1^2 (Fx\,dx + Fy\,dy + Fz\,dz)$$
$$or \quad U = \int_1^2 F_t\,ds$$



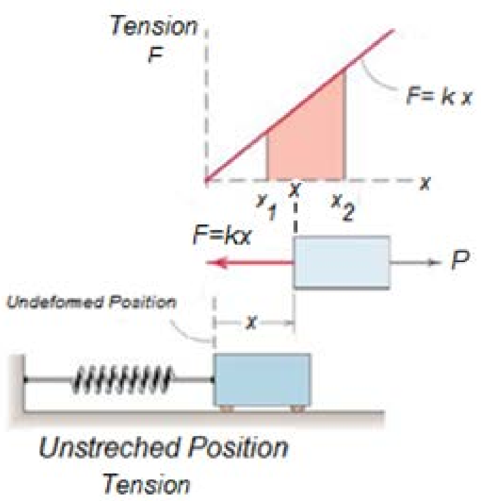
Examples of Work

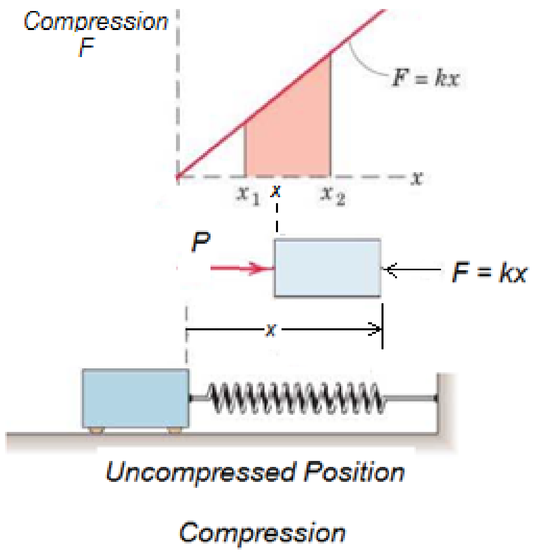
1. Work Associated with a Constant External Force.

$$U = \int_1^2 \vec{F} \,.\, d\vec{r}$$

2. Work Associated With a Spring Force

We consider here the linear spring of stiffness k where the force required to stretch or compress the spring is proportional to its deformation x.



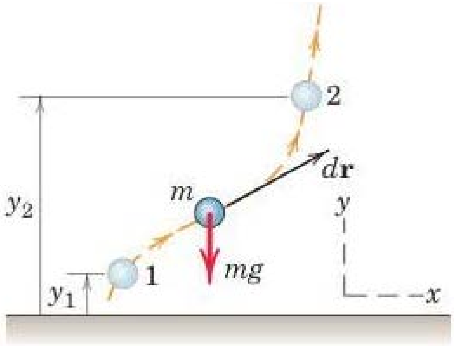


For both cases, tension or compression, the work done on the body by the spring is negative and is given by:

$$U_{1-2} = -\int_{x_1}^{x_2} F\,dx = -\int_{x_1}^{x_2} kx\,dx = \frac{-1}{2}k(x_2^2 - x_1^2)$$

For both cases, when the spring is being released, the force exerted on the body by the spring is in the same sense as the displacement, and therefore, the work is positive.

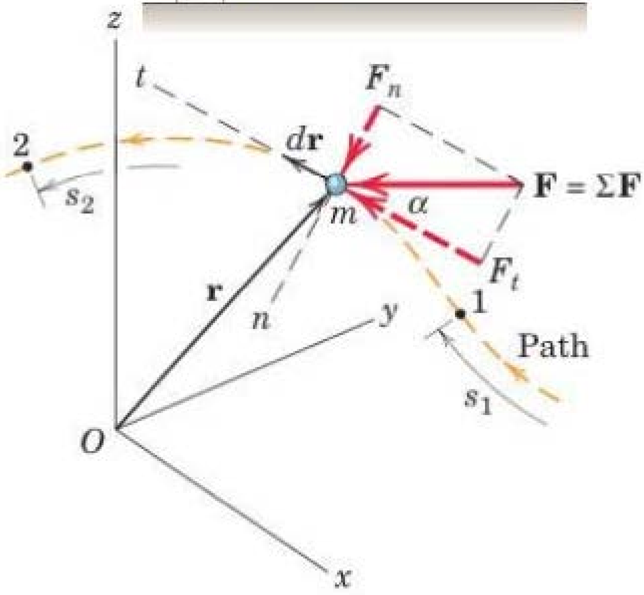
3. Work Associated With Weight



$$U = \int_1^2 \vec{F}\,.d\vec{r} \int_1^2 -mg\,\vec{j}.(dx\,\vec{i} + dy\,\vec{j})$$

$$U = -mg(y_2 - y_1)$$

Work and Curvilinear Motion



m: mass of the particle.

$\vec{F}$: a force acting on the particle $(= \sum \vec{F})$

$\vec{r}$: The position vector of the particle.

$d\vec{r}$: The change in position vector during time dt.

The work done during a finite movement of the particle from point 1 to point 2 is:

$$U_{1-2} = \int_1^2 \vec{F}\,.d\vec{r}$$

$U_{1-2} = \int_1^2 m\vec{a}\,.d\vec{r} \qquad (a_t ds = vdv)$

$U_{1-2} = \int_{v_1}^{v_2} mvdv = \,^1/_2\, m({v_2}^2 - {v_1}^2) \ldots\ldots\ldots\ 3/9$

Principle of Work and Kinetic Energy

The kinetic energy T of the particle is defined as:

$$T = \frac{1}{2}\,m\,v^2 \qquad Joule\ or\ J\ \ (or\ ft - lb)$$

So $\quad U_{1-2} = T_2 - T_1 = \Delta T \ldots\ldots\ldots\ 3/11 \qquad Work - Energy\ equation$

Alternatively, the work-energy relation may be expressed as the initial kinetic energy T_1 plus the work done U_{1-2} equals the final kinetic energy T_2, or:

$T_1 + U_{1-2} = T_2$,In this form, the terms correspond the natural sequence of events

Efficiency

The ratio of the work done by a machine to the work done on the machine during the same time interval is called the mechanical efficiency e_m. Provided that no depletion or accumulation of energy (uniform operation).

In mechanical devices → kinetic friction forces → negative work → heat energy → dissipated to the surrounding.

$e_m = \frac{P_{output}}{P_{input}}$ at any instant of time 3/13

$$e = e_m e_e e_t$$

e : Overall efficiency

e_m: Mechanical efficiency.

e_e : Electrical efficiency.

e_t: Thermal efficiency.

Advantages of Work- Energy Method

1. No necessity of computing the acceleration.
2. Involves only forces do work.
3. Enables us to analyze a system of particles joined without dismembering the system. (interconnected member)

Power

The total work or energy output is not a measure of the capacity of a machine since a motor can deliver a large amount of energy if given a sufficient time. The powerful machine is required to deliver a large amount of energy in a short period of time.

Hence the capacity of machine is measured by the time rate at which it can do work or deliver energy.

Power = the time rate of doing work.

$$P = \frac{dU}{dt} = \vec{F}.\frac{d\vec{r}}{dt} \rightarrow \boldsymbol{P} = \vec{\boldsymbol{F}}\,\vec{\boldsymbol{v}} \;\ldots\ldots\ldots\; 3/12$$

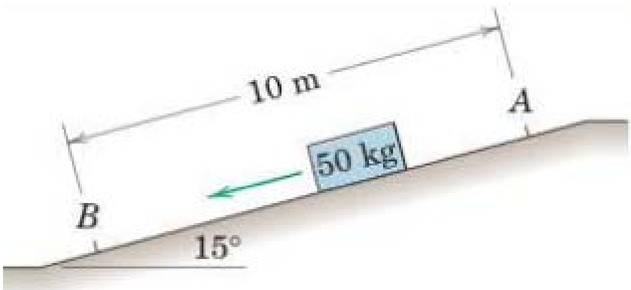
The units of power is W (watt) = N.m/s in SI, or hp (Horsepower) in US customary units.

$$1\ hp = 746\ W = 0.746\ kW$$

Problems 3/103 3/106 3/113 3/117 3/121 3/134

Sample Problem 3/11

Calculate the velocity v of the 50-kg crate when it reaches the bottom of the chute at B if it is given an initial velocity of 4 m/s down the chute at A. The coefficient of kinetic friction is 0.3.



Solution

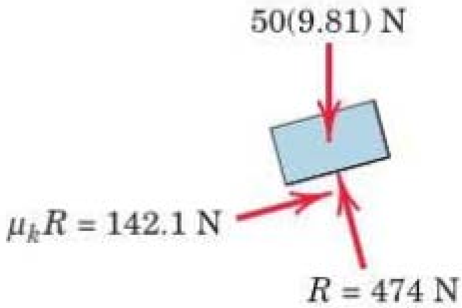
$[\sum Fy = 0]$

$R - 50(9.81)\, cos\, 15^o$

$R = 474\, N$

$F = \mu_k\, R = 0.3(474)$

$F = 142.1\ \ N$



$U = F\ \times s$

$U = 50(9.81)\, sin\, 15\ \times 10 - 142.1 \times 10$

$U = -151.9\ J$

(The work done by the weight component is positive, where as that done by the friction force is negative)

$U_{1-2} = \Delta T$

$-151.9 = \frac{1}{2}\, 50\, ({v_B}^2 - 4^2)$

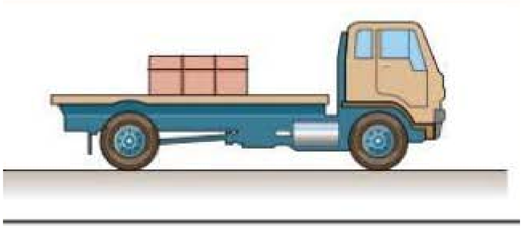
$v_B = 3.15\ \ m/s \quad \boldsymbol{Ans.}$

The kinetic energy is decreased since the work done is negative.

Sample Problem 3/12

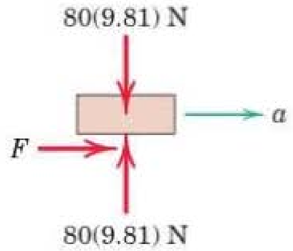
The flatbed truck, which carries an 80-kg crate, starts from rest and attains a speed of 72 km/h in a distaance of 75 m on a level road with constant acceleration. Calculate the work done by the friction force acting on the crate during this interval if the static and kinetic coefficients of friction btween the crate and the truck bed are (a) 0.30 and 0.28, respectively, or (b) 0.25 and 0.2, respectively.

Solution



$$[v^2 = {v_o}^2 + 2\,a(\Delta s)]$$

$$a = \frac{v^2}{2s} = \frac{(72/3.6)^2}{2(75)} = 2.67\ m/s^2$$



Case (a)
This acceleration requires a friction force on the block of:

$$[F = ma] \quad F = 80(2.67) = 213\ N < \mu_s N = 0.3(80)(9.81) = 235\ N$$

Therefore the crate does not slip and:

$$[U = Fs] \quad U_{1-2} = 213(75) = 16000\ J\ or\ 16\ kJ\ \ \boldsymbol{Ans}.$$

Case (b)
$$\mu_s N = 0.25(80)(9.81) = 196.2\ N < 213$$

Therefore, we conclude that the crate slips

$$F = \mu_k N = 0.2(80)(9.81) = 157\ N$$
$$[F = ma] \quad a = \frac{F}{m} = \frac{157}{80} = 1.962\ m/s^2$$

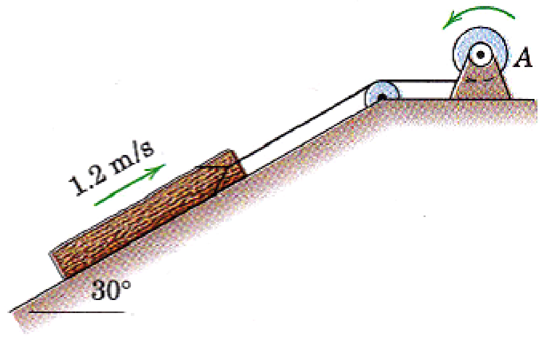
The distance traveled by the crate and the truck are in proportion to thier acceleration. Thus

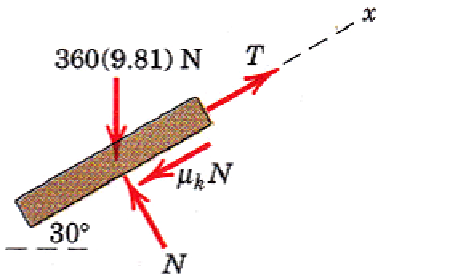
$$(1.962/2.67)75 = 55.2\,m$$

$$[U = Fs] \quad U_{1-2} = 157(55.2) = 8660\ J\ or\ 8.66\ kJ\ \ \boldsymbol{Ans}.$$

Sample problem 3/14

The power winch A hoists the 360-kg log up the 30° incline at a constant speed of 1.2 m/s. If the power output of the winch is 4 kW, Compute the coefficient of kinetic friction μk between the log and the incline. If the power is suddenly decreased to 6 kW, what is the corresponding instantaneous a of the log?





Solution

From the FBD diagram of the log,

$$N = 369(9.81)\cos 30^O = 3060\ N,$$

$$[\sum F_x = 0]$$

$$T - 3060\ \mu_k - 360(9.81)\ sin\ 30^o = 0$$
$$T = 3060\ \mu_k + 1766$$

$$[P = T\ v] \quad T = 4000/1.2 = 3330\ N$$

Substituting T gives

$$3330 = 3060\ \mu_k + 1766 \quad \mu_k = 0.513\ \ Ans.$$

When the power is increased, the tension momentarily becomes

$$[P = T\ v] \quad T = 6000/1.2 = 5000\ N$$

$$[\sum F_x = ma_x]$$

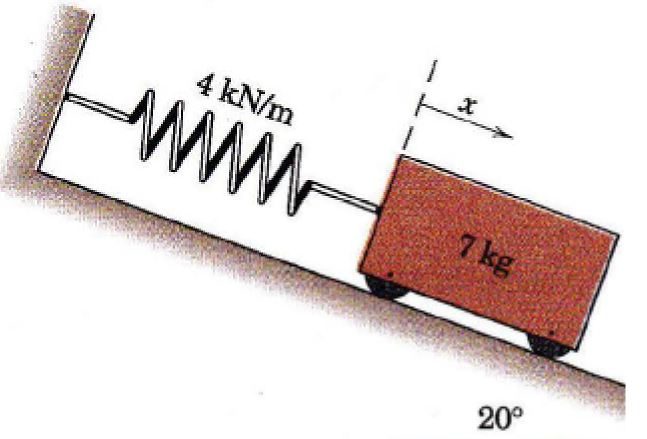
$$5000 - 3060(0.513) - 360(9.81) sin\ 30^o = 360\ a$$

$$a = 4.63\ \ m/s^2 \quad Ans.$$

3/103

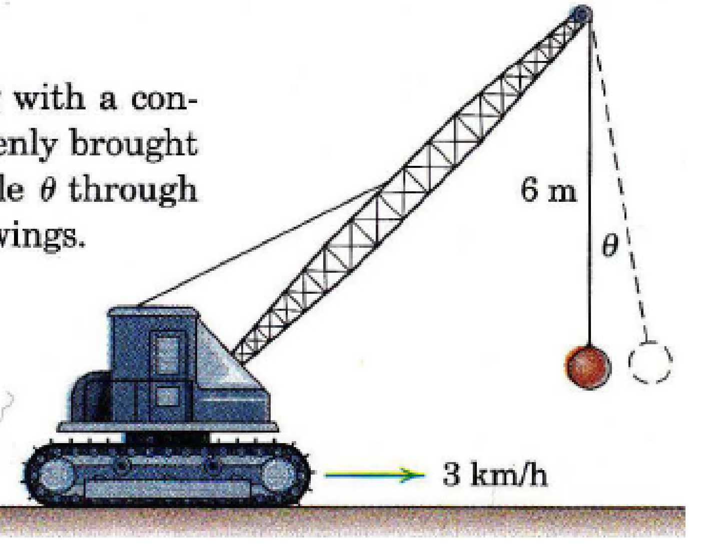
The spring is unstretched when $x = 0$. If the body moves from the initial position $x_1 = 100$ mm to the final position $x_2 = 200$ mm, (*a*) determine the work done by the spring on the body and (*b*) determine the work done on the body by its weight.

Ans. (*a*) $U_{1\text{-}2} = -60$ J, (*b*) $U_{1\text{-}2} = 2.35$ J



3/106

The crawler wrecking crane is moving with a constant speed of 3 km/h when it is suddenly brought to a stop. Compute the maximum angle θ through which the cable of the wrecking ball swings.



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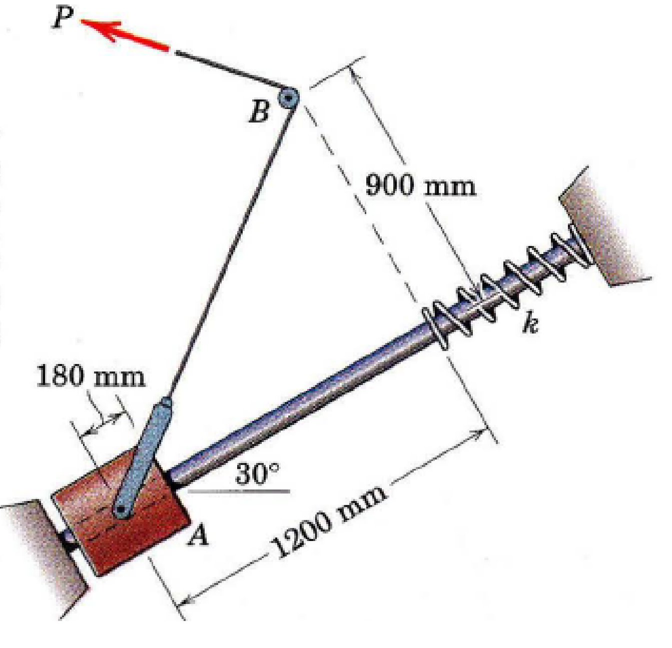
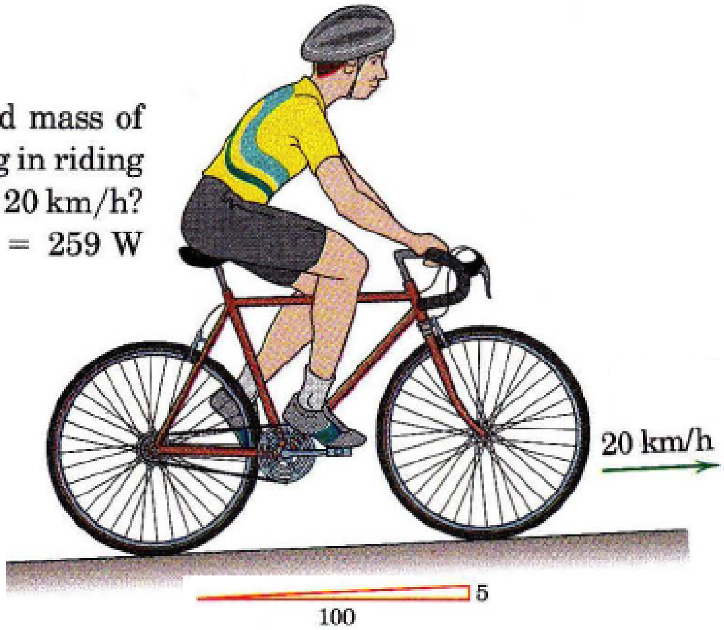
The 15-kg collar A is released from rest in the position shown and slides with negligible friction up the fixed rod inclined 30° from the horizontal under the action of a constant force $P = 200$ N applied to the cable. Calculate the required stiffness k of the spring so that its maximum deflection equals 180 mm. The position of the small pulley at B is fixed.

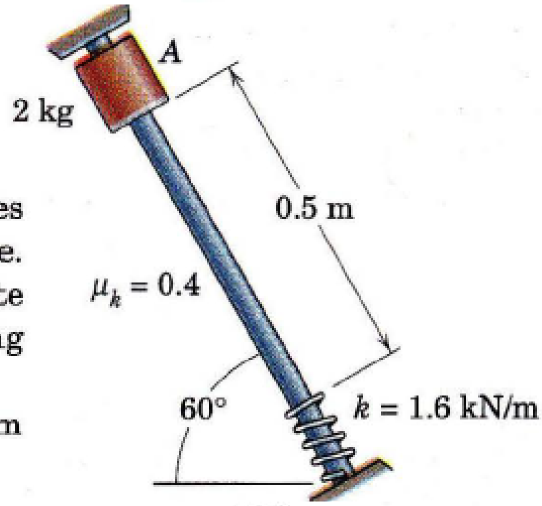


3/113

The man and his bicycle have a combined mass of 95 kg. What power P is the man developing in riding up a 5-percent grade at a constant speed of 20 km/h?

Ans. $P = 259$ W





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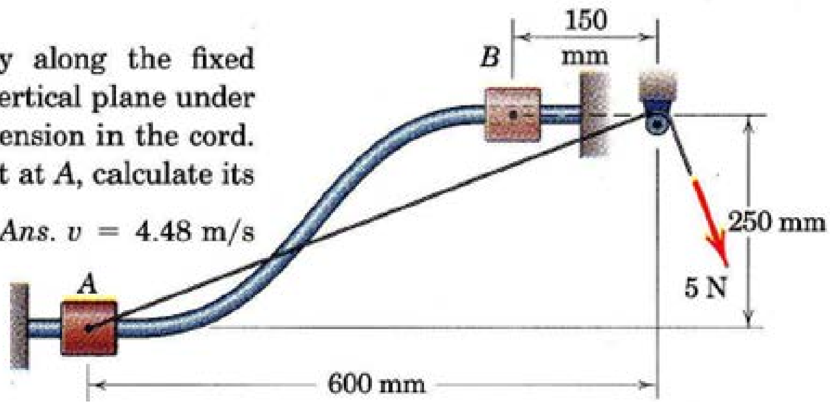
The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (a) the velocity v of the collar as it strikes the spring and (b) the maximum deflection x of the spring.

Ans. (a) $v = 2.56$ m/s, (b) $x = 98.9$ mm

3/121

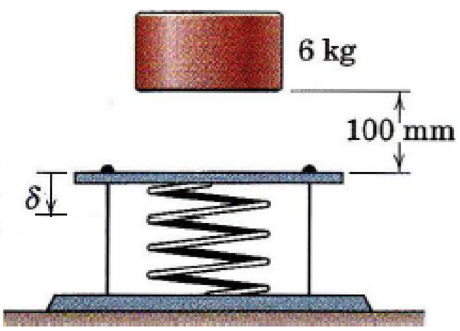
The 0.2-kg slider moves freely along the fixed curved rod from A to B in the vertical plane under the action of the constant 5-N tension in the cord. If the slider is released from rest at A, calculate its velocity v as it reaches B.

Ans. $v = 4.48$ m/s



3/134

The 6-kg cylinder is released from rest in the position shown and falls on the spring, which has been initially precompressed 50 mm by the light strap and restraining wires. If the stiffness of the spring is 4 kN/m, compute the additional deflection δ of the spring produced by the falling cylinder before it rebounds.



Lectures of the Department of Mechanical Engineering

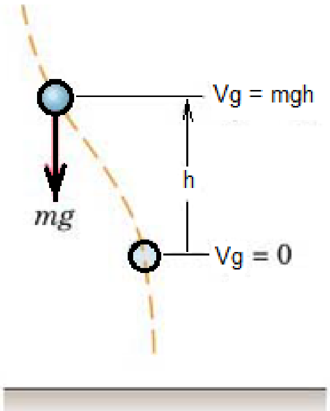
Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	10th lecture	Bakr Noori Alhasan/Lecturer
	The major contents Potential Energy Gravitational Potential Energy Elastic Potential Energy Work Energy Equation		
	The detailed contents Potential Energy Gravitational Potential Energy Elastic Potential Energy Work Energy Equation Work Mechanical Energy Sample Problem 3/16 Sample problem 3/17		

Potential Energy

Gravitational Potential Energy:



The gravitational potential energy V_g of the particle is defined as the work mgh done against the gravitational field to elevate the particle a distance h above a reference plane (called a datum), where V_g is taken to be zero. This work is called potential energy because it may be converted into energy if the particle is allowed to do work on a supporting body while it returns to its lower original plane.

Going from one level at $h = h_1$ to a higher level at $h = h_2$, the change in potential energy, $\Delta V_g = mg(h_2 - h_1) = mg\,\Delta h$. The corresponding work done by the gravitational force on the particle is $-mg\,\Delta h$. Thus, The work done by the gravitational force is the negative of the change in potential energy.

Elastic Potential Energy

The work which is done on the spring to deform it is stored in the spring and is called its elastic potential energy V_e. This energy is recoverable to work done by the spring on the body during the release of the deformation of the spring.

The elastic potential energy is defined as the work done on the spring to deform it an amount x.

$V_e = \int_0^x kx\,dx \quad \rightarrow \quad V_e = \frac{1}{2}kx^2$ ……… 3/16

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2)$ is the change in potential energy.

ΔV_e is positive when the deformation of the spring increases from $x_1 \rightarrow x_2$

ΔV_e is negative when the deformation decreases.

The work done on the spring = - The work done on the body (because the force exerted on the spring by the body is equal and opposite to the force exerted on the body by the spring)

We may replace the work Udone by the spring on the body by $-\Delta V_e$

Work Energy Equation

$$U = \Delta T$$

$$U'_{1-2} + (-\Delta V_g) + (-\Delta V_e) = \Delta T$$

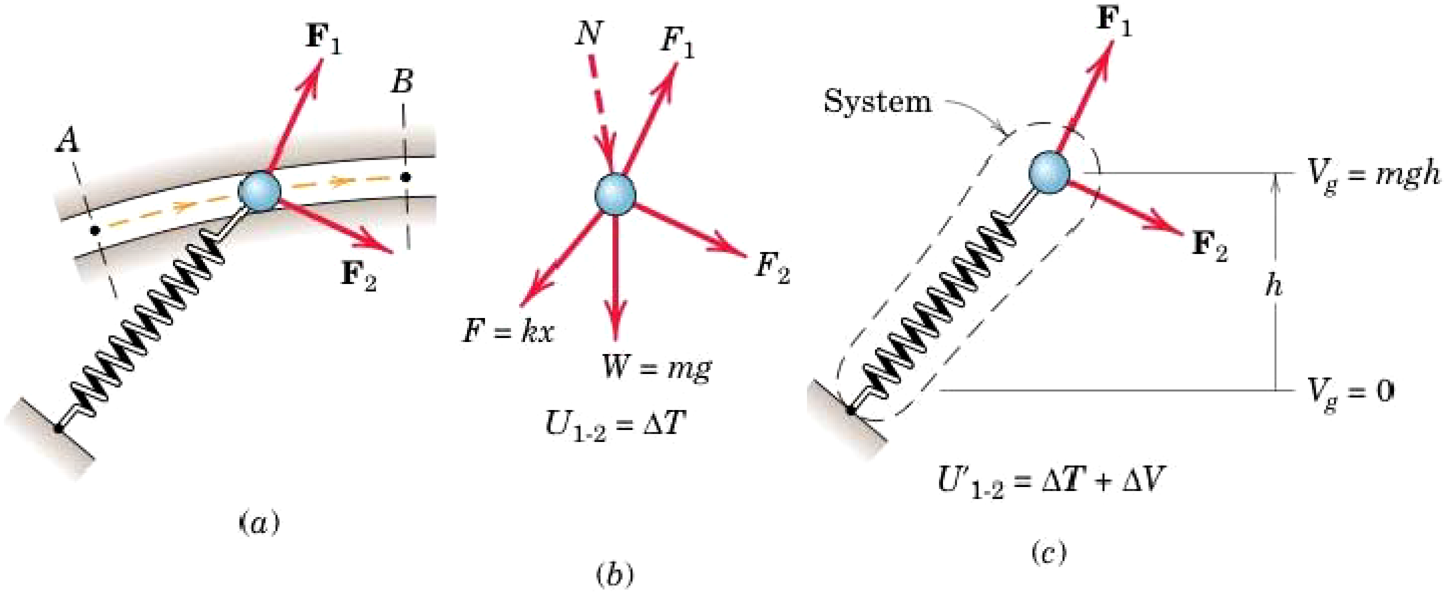
$$U'_{1-2} = \Delta V_g + \Delta V_e + \Delta T \ldots \ldots \ldots 3/17$$

U'_{1-2}: Is the work of all external forces other than gravitational and spring forces.

The advantage: The work of both gravity and spring forces is accounted for by focusing attention on the end point positions of the particles and on the end point lengths of the spring. The path between these end positions are not of concern.

$$T_1 + Vg_1 + Ve_1 + U'_{1-2} = T_2 + Vg_2 + Ve_2 \ldots \ldots\ 3/17a$$

The clarification of the difference between the use of Eqs. 3/11 and 3/17:



Work Mechanical Energy

$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$
$$U'_{1-2} = \Delta\left(T + V_g + V_e\right) = \Delta E \ldots \ldots .3/17b$$

Where $E = T + Vg + Ve$ is the total mechanical energy.
The net work done on the system = The change in the total mechanical energy

when $U' = 0$

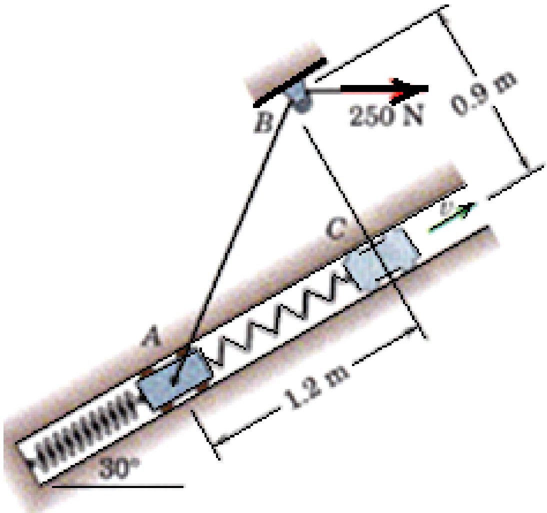
$$\Delta E = 0$$

or E is constant Eq. 3/17b expresses the law of conservation of dynamical energy.

Problems 3/143 3/145 3/150 3/151 3/145 3/155 3/168 3/171

Sample Problem 3/16

The 10-kg slider A moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position A, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v of the slider as it passes point C.



Solution

The slider, the cord, and the spring will be analyzed as a one system.

$$[U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e]$$

$$U'_{1-2} = 250(\overline{AB} - \overline{BC})$$
$$= 250(1.5 - 0.9) = 150\,J$$

$$\Delta T = \frac{1}{2}m({v_2}^2 - {v_1}^2) = 5\,{v_C}^2\,J$$

$$\Delta V_g = mg\Delta h = 10(9.81) \times 1.2\sin 30^o = 58.9\,J$$

The displacement component of the mass center is positive.

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

$$x_2 = 1.2 + 0.6 = 1.8\,m \quad and \quad x_1 = 0.6\,m$$

$$\Delta V_e = \frac{1}{2}(60)(1.8^2 - 0.6^2) = 86.4\,J$$

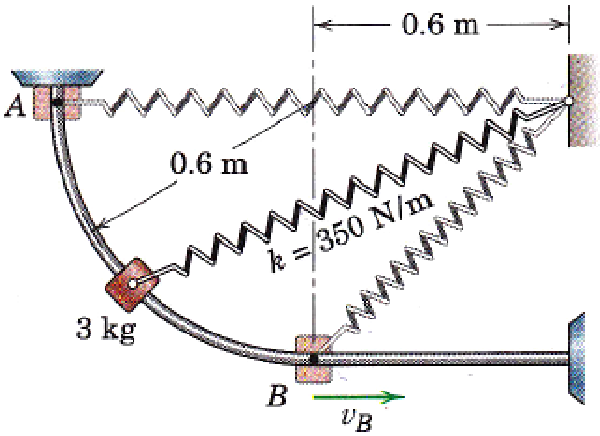
Substitution into the alternative work-energy equation

$$150 = 5{v_C}^2 + 58.9 + 86.4$$

$$v_C = 0.974\,m/s \quad Ans.$$

Sample problem 3/17

The 3-kg slider is released from rest at point A and slides with negligible friction in a vertical plane along the circlular rod. The attached spring has a stiffness of 350 N/m and has an unstretched length of 0.6 m. Determine the velocity of the slider as it passes position B.



Solution

The alternative work-energy equation:

$$T_1 + Vg_1 + Ve_1 + U'_{1-2} = T_2 + Vg_2 + Ve_2$$

$$T_1 = 0 \quad starts\ from\ rest$$

$$V_{g_1} = 0 \quad datum$$

$$x_1 = 0.12 - 0.6 = 0.6\, m$$

$$V_{e_1} = \frac{1}{2}kx_1{}^2 = \frac{1}{2}(350)\,(0.6)^2 = 630\, J$$

$$U'_{1-2} = 0 \quad no\ external\ forces$$

$$T_2 = \frac{1}{2}(0.3)v_B{}^2 = 1.5\, v_B{}^2\, J$$

$$V_{g_2} = mgh_2 = 3(9.81)(-0.6) = -17.66\, J \qquad (h_2 = -0.6\, m)$$

$$V_{e_2} = \frac{1}{2}kx_2{}^2 = \frac{1}{2}350(\sqrt{0.72} - 0.6)^2 = 577.5\, J$$

$$0 + 0 + 630 + 0 = 1.5v_B{}^2 - 17.66 + 577.5$$

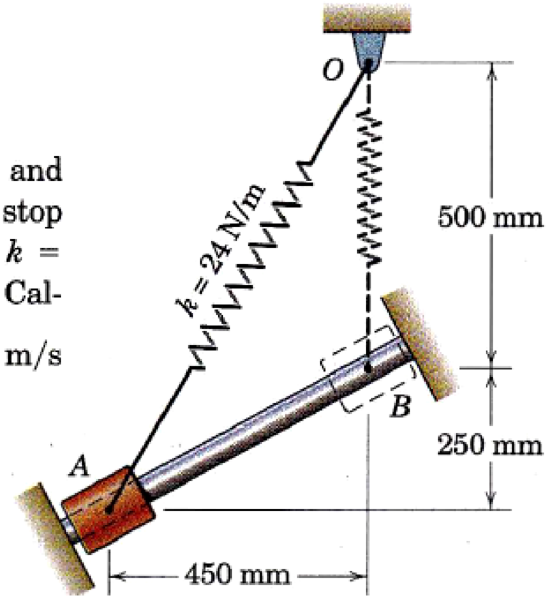
$$v_B = 6.82\ m/s \quad Ans.$$

Note: v_B depends only on the end conditions of the motion and does not require knowledge of the shape of the path.

3/143

The 0.9-kg collar is released from rest at A and slides freely up the inclined rod, striking the stop at B with a velocity v. The spring of stiffness k = 24 N/m has an unstretched length of 375 mm. Calculate v.

Ans. v = 1.156 m/s



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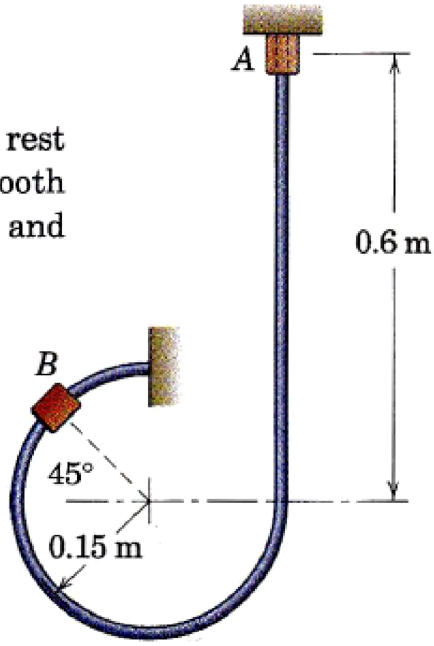
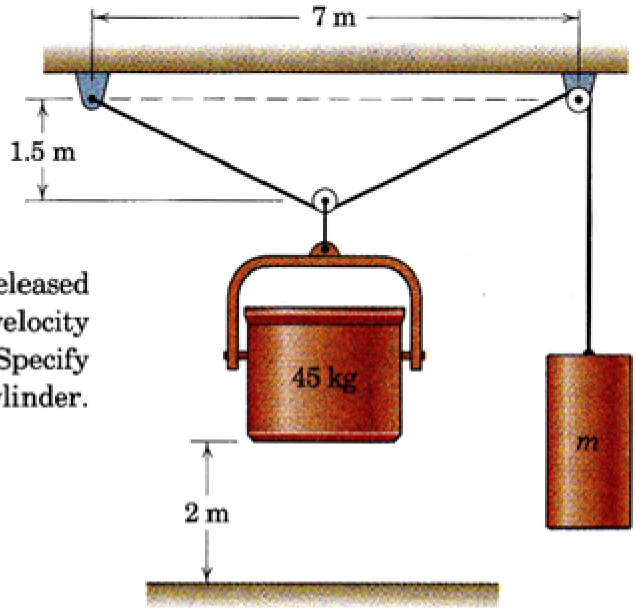
A bead with a mass of 0.25 kg is released from rest at A and slides down and around the fixed smooth wire. Determine the force N between the wire and the bead as it passes point B.



3/150

It is desired that the 45-kg container, when released from rest in the position shown, have no velocity after dropping 2 m to the platform below. Specify the proper mass m of the counterbalancing cylinder.



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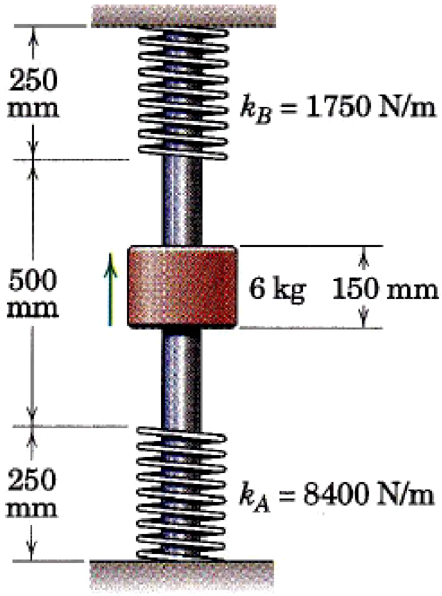
The springs are undeformed in the position shown. If the 6-kg collar is released from rest in the position where the lower spring is compressed 125 mm, determine the maximum compression x_B of the upper spring.



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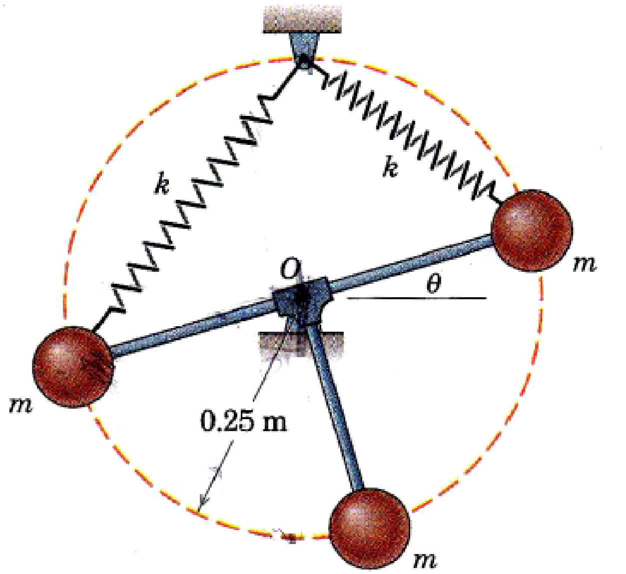
The two springs, each of stiffness $k = 1.2$ kN/m, are of equal length and undeformed when $\theta = 0$. If the mechanism is released from rest in the position $\theta = 20°$, determine its angular velocity $\dot{\theta}$ when $\theta = 0$. The mass m of each sphere is 3 kg. Treat the spheres as particles and neglect the masses of the light rods and springs.

Ans. $\dot{\theta} = 4.22$ rad/s



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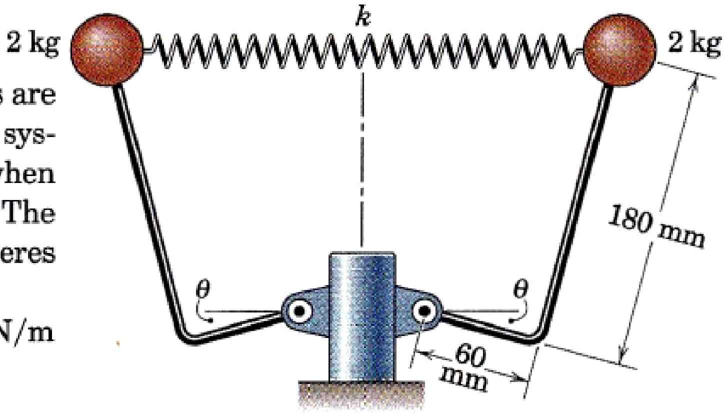
The two right-angle rods with attached spheres are released from rest in the position $\theta = 0$. If the system is observed to momentarily come to rest when $\theta = 45°$, determine the spring constant k. The spring is unstretched when $\theta = 0$. Treat the spheres as particles and neglect friction.

Ans. $k = 155.1$ N/m



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

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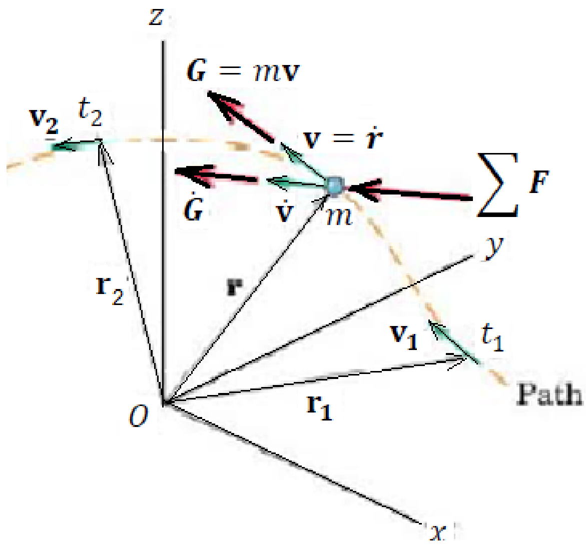
Section C Impulse and Momentum

In this article, the equation of motion will be integrated with respect to time rather than displacement, leading to the equations of impulse and momentum. These equations facilitate the solution of many problems in which the applied forces act during extremely short periods of time (as in impact problems) or over specified intervals of time.

3/9 Linear Impulse and Linear Momentum

Refer to the basic equation of motion for the particle:

$$\sum \vec{F} = m\vec{a}$$
$$\sum \vec{F} = m\dot{\vec{v}}$$
$$\sum \vec{F} = \frac{d}{dt}(m\vec{v})$$
$$\sum \vec{F} = \dot{\vec{G}} \quad \ldots\ldots \quad 3/21$$



Where the product of the mass and velocity is defined as the linear momentum $\vec{G} = m\vec{v}$

Assuming m is not changing with time.

Equation 3/21 states that the resultant of all forces acting on a particle equals its time rate of change of linear momentum.

$$\vec{G} = m\vec{v} \qquad N.s \qquad or \qquad lb - sec$$

The scalar components of equation 3/21:

$$\sum F_x = \dot{G}_x$$
$$\sum F_y = \dot{G}_y$$
$$\sum F_z = \dot{G}_z$$

The Linear Impulse- Momentum Principle

The Newton's second Law in terms of momentum:

$$\sum \vec{F} = \dot{\vec{G}}$$
$$\sum \vec{F} = \frac{d\vec{G}}{dt}$$

$\int_{t_1}^{t_2} \sum \vec{F}\, dt = \int_{\vec{G_1}}^{\vec{G_2}} d\vec{G}$

$\int_{t_1}^{t_2} \sum \vec{F}\, dt = \Delta\vec{G} = \vec{G}_2 - \vec{G}_1 \ldots\ldots\ldots\ 3/23$

Where:

$\overrightarrow{G_1}$ The linear momentum at time $t_1 = m\vec{v}_1$

$\overrightarrow{G_2}$ The linear momentum at time $t_2 = m\vec{v}_2$

Equation 3/23 states that the total linear impulse on m equals the corresponding change in linear momentum of m.

Alternatively we may write Eq. 3/23 as

$$\vec{G}_1 + \int_{t_1}^{t_2} \sum \vec{F} dt = \vec{G}_2 \ \ldots\ldots\ 3/23\ a$$

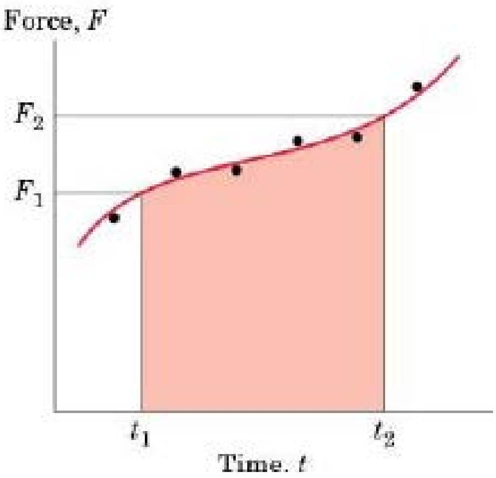
In scalar components of Eq. 3/23:

$$\int_{t_1}^{t_2} \sum F_x\, dt = (mv_x)_2 - (mv_x)_1$$
$$\int_{t_1}^{t_2} \sum F_y\, dt = \left(mv_y\right)_2 - (mv_y)_1$$
$$\int_{t_1}^{t_2} \sum F_z\, dt = (mv_z)_2 - (mv_z)_1$$

All forces must be included. This is achieved by drawing the FBD.

When a force acting on a particle in a manner determined by experimental measurements or by other approximate means, a graphical or numerical integration must be performed.

$$\int_{t_1}^{t_2} F\, dt = Shaded\ area$$



Conservation of Linear Momentum

If the resultant force on a particle is zero during an interval of time:

$$\sum \vec{F} = \dot{\vec{G}}$$
$$\dot{\vec{G}} = 0 \ \ \rightarrow \vec{G}\ \ is\ constant.$$
$$\Delta\vec{G} = \vec{G}_2 - \vec{G}_1 = 0 \quad or \quad \vec{G}_2 = \vec{G}_1 \ \ldots\ldots\ \ 3/24$$

Eq. 3/24 expresses the principle of conservation of linear momentum.

The linear momentum may be conserved in one coordinate direction but not necessarily in the other directions, hence the FBD is important.

Interacted two particles A and B:

The linear impulse on particle A = - The linear impulse on particle B

$$\Delta\vec{G}_A = -\Delta\vec{G}_B \quad \rightarrow \Delta\left(\vec{G}_A + \vec{G}_B\right) = 0$$

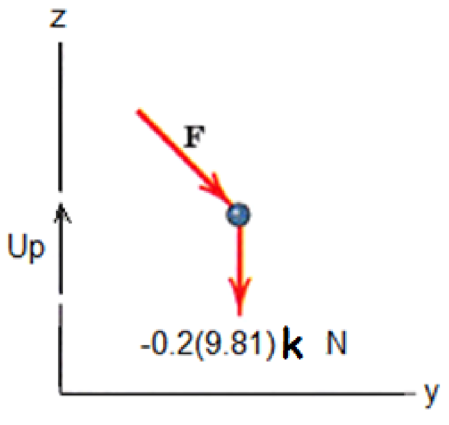
$$\vec{G}_A + \vec{G}_B = constant \;\; or\; \vec{G}\; is\; constant.$$

Problems 3/180 3/185 3/186 3/191 3/192 3/196 3/199

Sample Problem 3/18

A 0.2-kg particle moves in the vertical y-z plane (z up, y horizontal) under the action of its weight and a force $\vec{F}$ which varies with time. The linear momentum of the particle in newton-seconds is given by the expression $\vec{G} = \frac{3}{2}(t^2+3)\vec{j} - \frac{2}{3}(t^3-4)\vec{k}$, where t is the time in seconds. Determine the force $\vec{F}$ and its magnitude for the instant when $t = 2\,s$.

Solution



The force- momentum equation is

$$[\sum \vec{F} = \dot{\vec{G}}]$$

$$\vec{F} - 0.2(9.81)\vec{k} = 3t\,\vec{j} - 2t^2\vec{k}$$

$$\vec{F} = 0.2(9.81)\vec{k} + 3t\,\vec{j} - 2t^2\vec{k}$$

$$at\;\; t = 2\;\; s$$

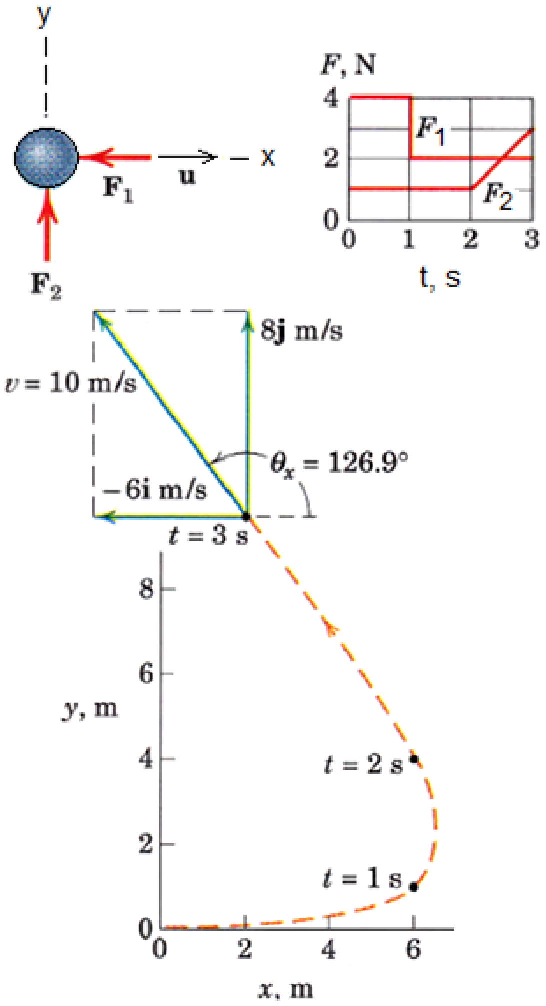
$$\vec{F} = 0.2(9.81)\vec{k} + 3(2)\,\vec{j} - 2(2)^2\vec{k}$$

$$\vec{F} = 6\,\vec{j} - 6.04\,\vec{k} \quad Ans.$$

$$F = \sqrt{6^2 + 6.04^2} = 8.51\;N\;\; Ans.$$

Sample Problem 3/19

A particle with a mass of 0.5 kg has a velocity $u = 10\ m/s$ in the $x - direction$ at time $t = 0$. Forces $\vec{F}_1$ and $\vec{F}_2$ act on the particle, and their magnitudes change with time according to the graphical schedule shown. Determine the velocity $\vec{v}$ of the particle at the end of the $3\ sec$.



Solution

The impulse momentum equation in components form:

$[\int \sum F_x\, dt = m\Delta v_x]$

$-[4 \times 1 + 2 \times 2] = 0.5\,(v_x - 10)$
$v_x = -6\, m/s$

$[\int \sum F_y\, dt = m\Delta v_y]$
$1 \times 2 + 2 \times 1 = 0.5(v_y - 0)$
$v_y = 8\, m/s$

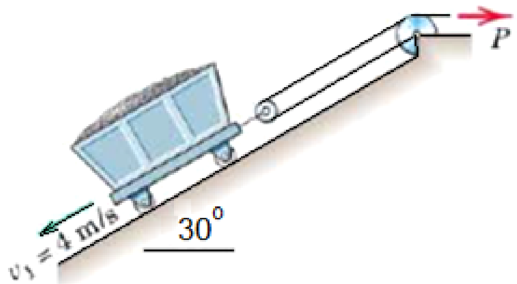
Thus, $\vec{v} = -6\vec{i} + 8\vec{j} \quad m/s \qquad Ans.$

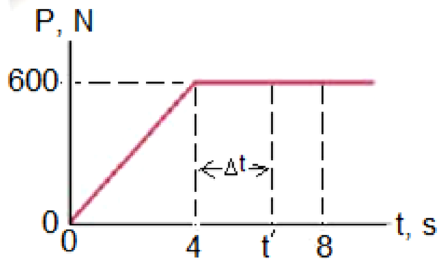
$v = \sqrt{6^2 + 8^2} = 10\ m/s$

$\theta_x = tan^{-1} \frac{8}{-6} = 126.9^o\ Ans.$

Sample Problem 3/20

The loaded $150\ kg$ skip is rolling down the incline at $4\ m/s$ when a force P is applied to the cable as shown at time $t = 0$. The force P is increased uniformly with the time until it reaches $600\ N$ at $t = 4s$, after which time it remains constant at this value. Calculate (a) the time t' at which the skip reverses its direction and (b) the velocity v of the skip at $t = 8\ sec.$. Treat the skip as a particle.

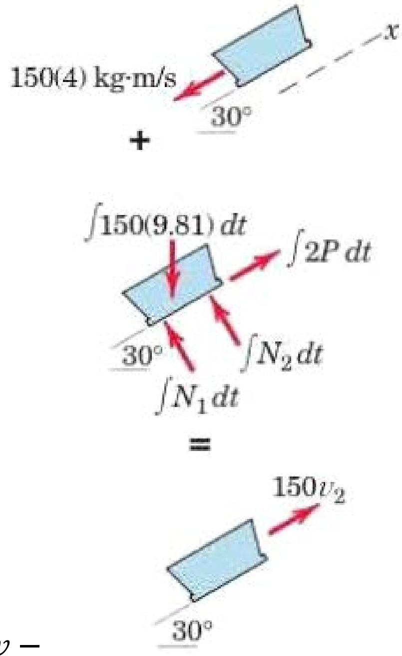




Solution

a) The variation of P with time is plotted.

The FBD of the skip is drawn.



Assume the condition at which the direction of the skip is reversed occurs at $t = 4 + \Delta t$

$[\int \sum F_x \, dt = m\Delta v_x]$

$2\left[\frac{1}{2} \times 4 \times 600 + 600\Delta t\right] - 150(9.81)\sin 30^o\,(t + \Delta t) = 150(0 - (-4))$

$\Delta t = 2.46\, s \quad t = 4 + 2.46 = 6.46 \; s \;\; Ans.$

b)$[\int \sum F_x \, dt = m\Delta v_x]$

$2\left[\frac{1}{2} \times 4 \times 600 + 600(4)\right] - 150(9.81)\sin 30^o\,(8) = 150(v - (-4))$

$$v = 4.76 \; m/s \;\; Ans$$

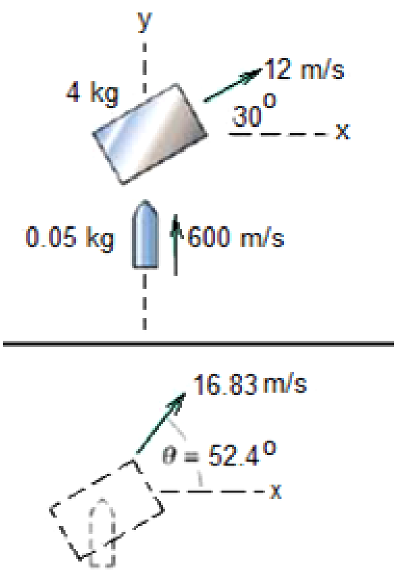
The same result is obtained by analyzing the interval from t' to $t = 8\, s$. The FBD keeps us from making the errors.

Sample Problem 3/21

The 50 gm bullet traveling at 600 m/s strikes the 4-kg block centrally and is embedded within it. If the block slides on a smooth horizontal plane with a velocity of 12 m/s in the direction shown prior to impact, determine the velocity v_2 of the block and embedded bullet immediately after impact.

Solution

Since the force of impact is internal and there are no other external forces acting on the system, it follows that the linear momentum of the system is conserved.



$\vec{G}_1 = \vec{G}_2$

$0.05(600\vec{j}) + 4(12)(\cos 30^o\,\vec{i} + \sin 30^o\,\vec{j}) = (4 + 0.05)\vec{v}$

$\vec{v} = 10.26\,\vec{i} + 13.33\vec{j} \;\; m/s \;\; Ans.$

$v = \sqrt{10.26^2 + 13.33^2} = 16.83 \;\; m/s$

$\tan\theta = \frac{v_y}{v_x} = \frac{13.33}{10.26} = 1.299$

$\theta = 52.4^o \;\; Ans.$

The same result is obtained by working with components form.

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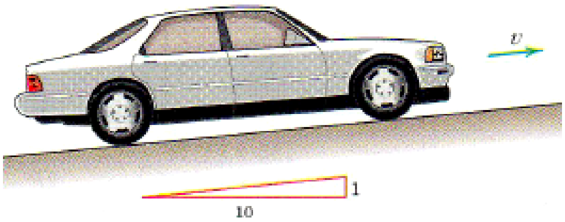
The 1500-kg car has a velocity of 30 km/h up the 10-percent grade when the driver applies more power for 8 s to bring the car up to a speed of 60 km/h. Calculate the time average F of the total force tangent to the road exerted on the tires during the 8 s. Treat the car as a particle and neglect air resistance.

Ans. $F = 3.03$ kN



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A jet-propelled airplane with a mass of 10 Mg is flying horizontally at a constant speed of 1000 km/h under the action of the engine thrust T and the equal and opposite air resistance R. The pilot ignites two rocket-assist units, each of which develops a forward thrust T_0 of 8 kN for 9 s. If the velocity of the airplane in its horizontal flight is 1050 km/h at the end of the 9 s, calculate the time-average increase ΔR in air resistance. The mass of the rocket fuel used is negligible compared with that of the airplane.

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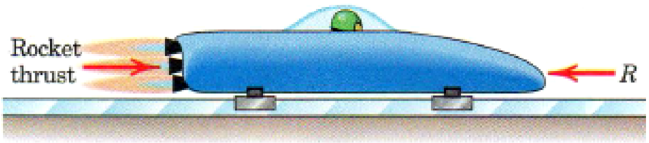
A 3-Mg experimental rocket sled is propelled by six rocket motors each with an impulse rating of 100 kN·s. The rockets are fired at $\frac{1}{4}$-s intervals, and the duration of each rocket firing is 1.5 s. If the velocity of the sled 3 s from the start is 150 m/s, determine the time average R of the total aerodynamic and mechanical resistance to motion. Neglect the loss of mass due to exhausted fuel compared with the mass of the sled.



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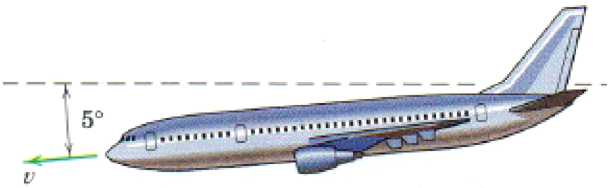
The pilot of a 40-Mg airplane which is originally flying horizontally at a speed of 650 km/h cuts off all engine power and enters a 5° glide path as shown. After 120 s the airspeed is 600 km/h. Calculate the time-average drag force D (air resistance to motion along the flight path).

Ans. $D = 38.8$ kN



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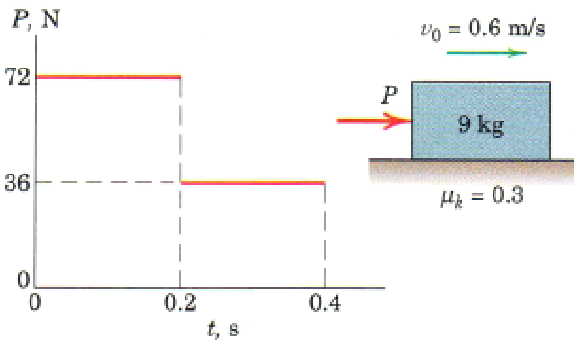
The 9-kg block is moving to the right with a velocity of 0.6 m/s on a horizontal surface when a force P is applied to it at time $t = 0$. Calculate the velocity v of the block when $t = 0.4$ s. The kinetic coefficient of friction is $\mu_k = 0.3$.

Ans. $v = 1.823$ m/s



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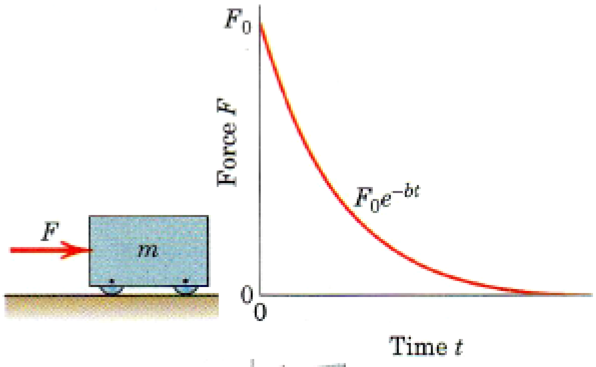
The cart of mass m is subjected to the exponentially decreasing force F, which represents a shock or blast loading. If the cart is stationary at time $t = 0$, determine its velocity v and displacement s as functions of time. What is the value of v for large values of t?



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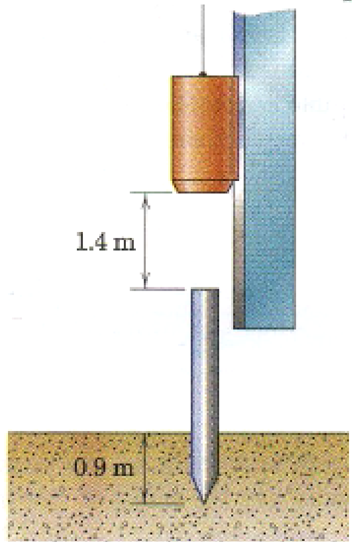
The 450-kg ram of a pile driver falls 1.4 m from rest and strikes the top of a 240-kg pile embedded 0.9 m in the ground. Upon impact the ram is seen to move with the pile with no noticeable rebound. Determine the velocity v of the pile and ram immediately after impact. Can you justify using the principle of conservation of momentum even though the weights act during the impact?

Ans. $v = 3.42$ m/s



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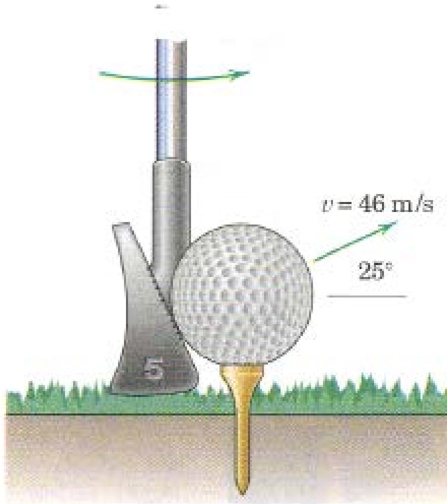
The 45.9-g golf ball is struck by the five-iron and acquires the velocity shown in a time period of 0.001 s. Determine the magnitude R of the average force exerted by the club on the ball. What acceleration magnitude a does this force cause, and what is the distance d over which the launch velocity is achieved, assuming constant acceleration?



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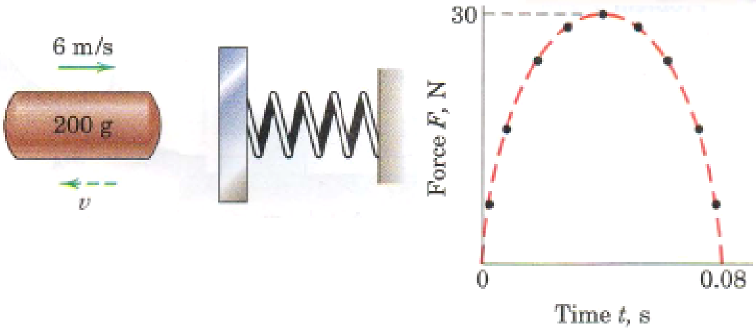
Careful measurements made during the impact of the 200-g metal cylinder with the spring-loaded plate reveal a semielliptical relation between the contact force F and the time t of impact as shown. Determine the rebound velocity v of the cylinder if it strikes the plate with a velocity of 6 m/s.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

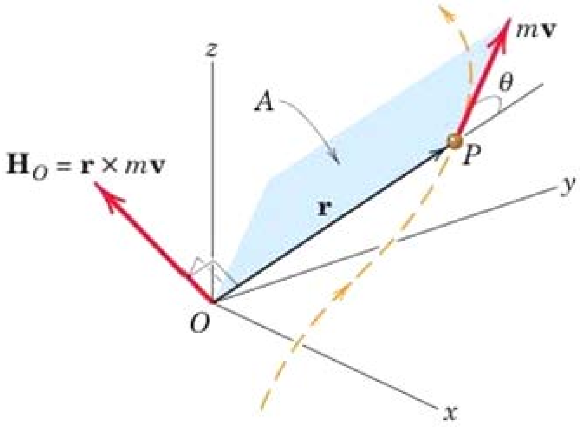
Class: 2nd Year

Lecture Contents	Lecture sequences:	12th lecture	Bakr Noori Alhasan/Lecturer
	The major contents Angular Impulse and Angular Momentum Rate of Change of Angular Momentum The Angular Impulse - Momentum Principle		
	Angular Impulse and Angular Momentum Rate of Change of Angular Momentum The Angular Impulse - Momentum Principle Plane Motion Applications Conservation of Angular Momentum Interacted Particles Sample Problem 3/22 Sample Problem 3/23		

Angular Impulse and Angular Momentum

Angular momentum is the moment of linear momentum.

$$\vec{H}_O = \vec{r} \times m\vec{v} \dots\dots \; 3/25$$



$\vec{H}_O$ is the angular momentum of P about O and is given by the cross-product.

The sense of $\vec{H}_O$ is defined by the right-hand rule for cross products.

Obtaining scalar angular momentum from Eq. 3/25 has two interpretations:

$either \;\; mvr\sin\theta \quad or \quad mv\sin\theta\, r$

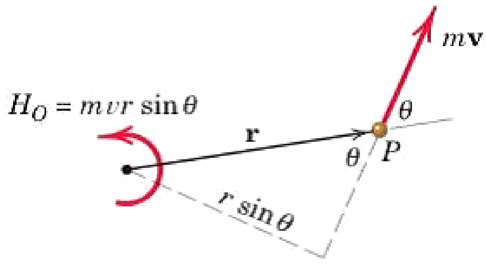
The scalar components of angular momentum may be obtained from the expression:

$$\vec{H}_o = m\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} \dots\dots\dots \; 3/26$$

$$= m(v_z y - v_y z)\vec{i} + m(v_x z - v_z x)\vec{j} + m(v_y x - v_x y)\vec{k}$$

$$H_{o_x} = m(v_z y - v_y z), \qquad H_{o_y} = m(v_x z - v_z x),$$

$$H_{o_z} = (v_y x - v_x y)$$



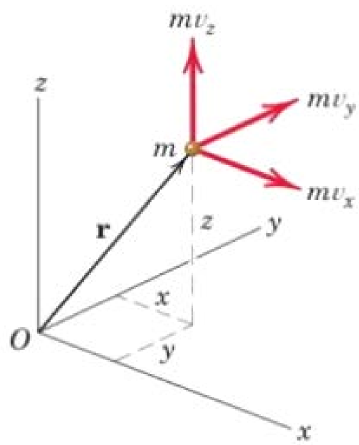
View in plane *A*

Units

SI Units	US Customary units
kg.m/s m	[lb/ft/s^2][ft/s.ft]
N.m.s	ft-lb-sec

Rate of Change of Angular Momentum

Let $\sum\vec{F}$ represents the resultant of all forces acting on the particle P, the moment $\vec{M}_o$ is the vector cross product



$\sum\vec{M}_o = \vec{r} \times \sum\vec{F}$

$= \vec{r} \times m\vec{a}$

$\sum\vec{M}_o = \vec{r} \times \dot{\vec{v}}$

$recall \; \vec{H}_o = \vec{r} \times m\vec{v}$

$\quad \dot{\vec{H}}_o = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$

The cross product of parallel vectors is identically zero

$\vec{H}_o = \vec{r} \times m\vec{v}$

$$\boxed{\sum \vec{M}_o = \dot{\vec{H}}_o} \quad \text{............... } 3/27$$

Equation 3/27 states that the moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O.

The scalar components:

$\sum M_{o_x} = \dot{H}_{o_x}$
$\sum M_{o_y} = \dot{H}_{o_y}$
$\sum M_{o_z} = \dot{H}_{o_z}$

The Angular Impulse - Momentum Principle

$\sum \vec{M}_o = \dot{\vec{H}}_o$
$\sum \vec{M}_o = \frac{d\vec{H}_o}{dt}$
$\sum \vec{M}_o \, dt = d\vec{H}_o$

To obtain the effect of the moment over a finite period of time :

$$\int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \int_{H_{o1}}^{H_{o2}} d\,\vec{H}_o$$

$$\boxed{\int_{t_1}^{t_2} \sum \vec{M}_o \, dt = \Delta\vec{H}_o = (\vec{H}_o)_2 - (\vec{H}_o)_1} \quad \text{... ... } 3/29$$

Where $(\vec{H}_o)_1 = \vec{r}_1 \times m\vec{v}_1 \qquad (\vec{H}_o)_2 = \vec{r}_2 \times m\vec{v}_2$

Equation 3/29 states that the total angular impulse on m about the fixed point O equals the corresponding change in angular momentum of m about O.

Alternatively:

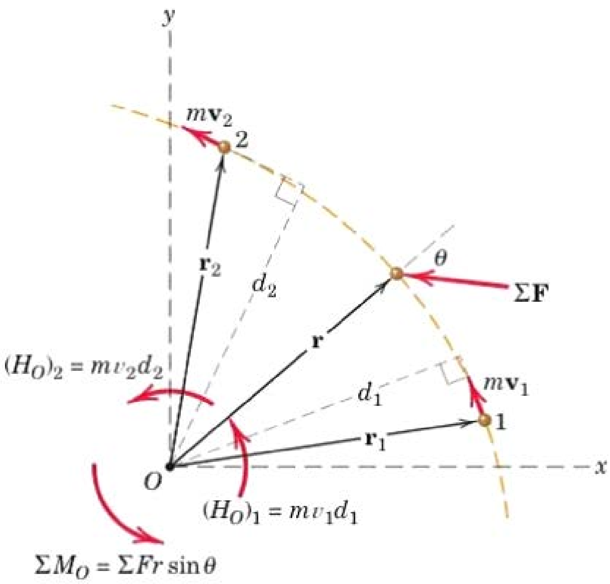
$(\vec{H}_o)_1 + \int_{t_1}^{t_2} \sum \vec{M}_o \, dt = (\vec{H}_o)_2$ 3/29

a

In components form:

$\int_{t_1}^{t_2} \sum M_{o_x} \, dt = (H_{o_x})_2 - (H_{o_x})_1$

$\int_{t_1}^{t_2} \sum M_{o_y} \, dt = (H_{o_y})_2 - (H_{o_y})_1$



Plane Motion Applications

Most of the applications of interest to us can be analyzed as plane-motion problems where moments are taken about a single axis normal to the plane of motion. In this case, the angular momentum may change magnitude and sense, but the direction of the vector remains unaltered.

In x-y plane

$H_{o_1} = |\vec{r}_1 \times m\vec{v}_1| = mv_1 d_1 \quad CCW$
$H_{o_2} = |\vec{r}_2 \times m\vec{v}_2| = mv_2 d_2 \quad CCW$

The scalar form of equation 3/29:

$$\int_{t_1}^{t_2} \sum M_o \, dt = H_{o_2} - H_{o_1}$$

$$\int_{t_1}^{t_2} \sum F \, r \sin\theta \, dt = mv_2 d_2 - mv_1 d_1$$

Conservation of Angular Momentum

$$\sum \vec{M}_o = \dot{\vec{H}}_o$$

When $\sum \vec{M}_o = 0$ *during an interval of time*

$$\dot{\vec{H}}_o = 0 \rightarrow H_o \text{ is constant}$$

In this case, the angular momentum of the particle is said to be conserved.
Angular momentum may be conserved about one axis but not about another axis.

Interacted Particles

Consider the motion of two particles a and b which interact during an interval of time.
F and -F are the interactive forces and they are only unbalanced forces acting on the particles during the interval.
The moments of them about any point O are equal and opposite.

$$(\int_{t_1}^{t_2} \sum \vec{M} \, dt = \Delta\vec{H})_a + (\int_{t_1}^{t_2} \sum \vec{M} \, dt = \Delta\vec{H})_b \rightarrow \Delta\vec{H}_a + \Delta\vec{H}_b = 0$$

Where all angular momenta are referred to point O.

$$\vec{H}_a = \vec{H}_b = constant$$

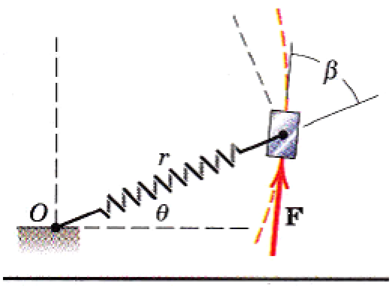
Thus the total angular momentum for the system of the two particles remains constant during the interval and.

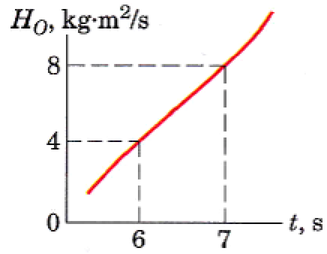
$$\Delta\vec{H}_o = 0 \quad or \quad (\vec{H}_o)_1 = (\vec{H}_o)_2$$ 3/30

Problems 3/222 3/227 3/229 3/230 3/232 3/235 3/236 3/237

Sample Problem 3/22

The small 2-kg block slides on a smooth horizontal surface under the action of the force in the spring and a force $\vec{F}$. The angular momentum of the block about O varies with time as shown in the graph. When t = 6.5 s, it is known that r = 150 mm and $\beta = 60^o$. Determine F for this instant.





Solution

$\sum M_o = Fr \sin \beta$

The spring force passes through O and the force due gravity is normal to the plane of motion.

From the graph $\dot{H}_o$ at $t = 6.5\ s$ is nearly $(8-4)/(7-6)$

$$\dot{H}_o = 4\ kg.m^2/s^2$$

$$\left[\sum M_o = \dot{H}_o\right]$$

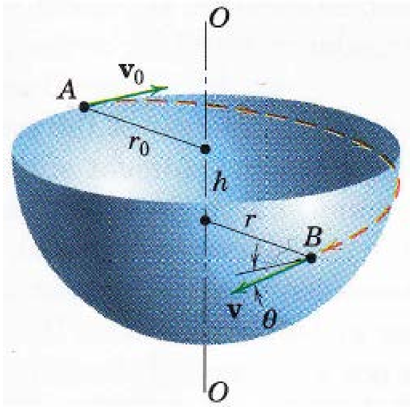
$$F(0.150) \sin 60^o = 4$$

$$F = 30.8\ N \quad Ans.$$

We do not need vector notation here since we have plane motion.

Sample Problem 3/23

A small mass particle is given an initial velocity v_o tangent to the horizontal rim of a smooth hemispherical bowel at a radius r_o from the vertical centerline, as shown at point A. As the particle slides past point B, a distance h below A and a distance r from the vertical centerline, its velocity v makes an angle θ with the horizontal tangent to the bowel through B. Determine θ.



Solution

The weight of the particle and the normal reaction force by the bowel do not moment about the axis O-O

$$(H_o)_1 = (H_o)_2$$

$$mv_o r_o = mvr\cos\theta$$

$$E_1 = E_2$$

$$T_1 + V_{g_1} = T_2 + V_{g_2}$$

$$\frac{1}{2}m\,{v_o}^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{{v_o}^2 + 2gh}$$

Eliminating v and substituting $r^2 = {r_o}^2 - h^2$ give

$$v_o r_o \sqrt{{v_o}^2 + 2gh}\,\sqrt{{r_o}^2 - h^2}\cos\theta$$

$$\theta = cos^{-1}\frac{1}{\sqrt{1 + \frac{2gh}{{v_o}^2}}\;\sqrt{1 - \frac{h^2}{{r_o}^2}}} \quad Ans.$$

3/223

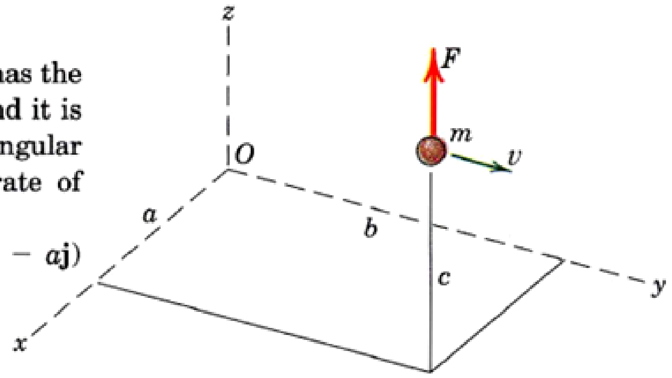
At a certain instant, the linear momentum of a particle is given by $\mathbf{G} = -3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ kg·m/s and its position vector is $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ m. Determine the magnitude H_O of the angular momentum of the particle about the origin of coordinates.

Ans. $H_O = 8.49\ \text{kg}\cdot\text{m}^2/\text{s}$

3/225

At a certain instant, the particle of mass m has the position and velocity shown in the figure, and it is acted upon by the force $\mathbf{F}$. Determine its angular momentum about point O and the time rate of change of this angular momentum.

Ans. $\mathbf{H}_O = mv(-c\mathbf{i} + a\mathbf{k})$, $\dot{\mathbf{H}}_O = F(b\mathbf{i} - a\mathbf{j})$



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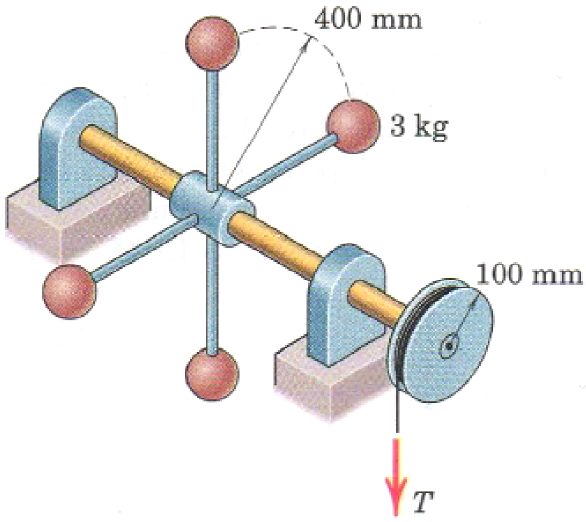
The assembly starts from rest and reaches an angular speed of 150 rev/min under the action of a 20-N force T applied to the string for t seconds. Determine t. Neglect friction and all masses except those of the four 3-kg spheres, which may be treated as particles.

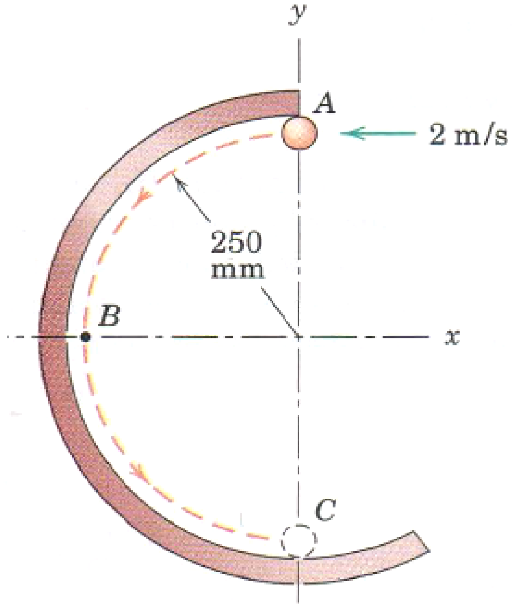
Ans. $t = 15.08$ s



3/229

A small 110-g particle is projected with a horizontal velocity of 2 m/s into the top A of the smooth circular guide fixed in the vertical plane. Calculate the time rate of change $\dot{\mathbf{H}}_B$ of angular momentum about point B when the particle passes the bottom of the guide at C.

Ans. $\dot{\mathbf{H}}_B = 1.519\mathbf{k}$ N·m



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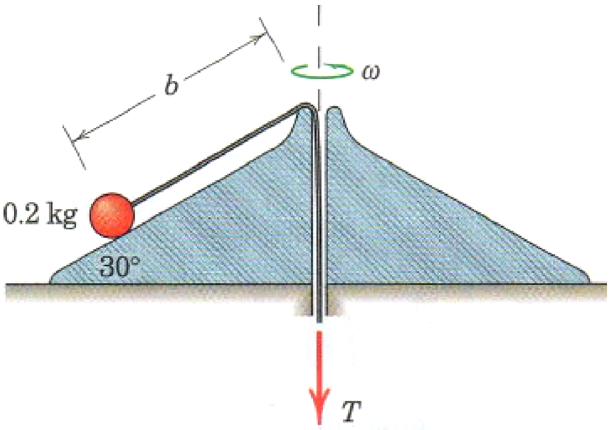
The 0.2-kg ball and its supporting cord are revolving about the vertical axis on the fixed smooth conical surface with an angular velocity of 4 rad/s. The ball is held in the position b = 300 mm by the tension T in the cord. If the distance b is reduced to the constant value of 200 mm by increasing the tension T in the cord, compute the new angular velocity ω and the work $U'_{1\text{-}2}$ done on the system by T.

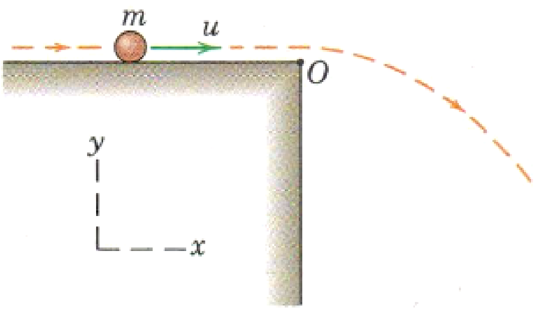
Ans. ω = 9 rad/s, $U'_{1\text{-}2}$ = 0.233 J



3/237

The particle of mass m is launched from point O with a horizontal velocity $\mathbf{u}$ at time t = 0. Determine its angular momentum $\mathbf{H}_O$ relative to point O as a function of time.

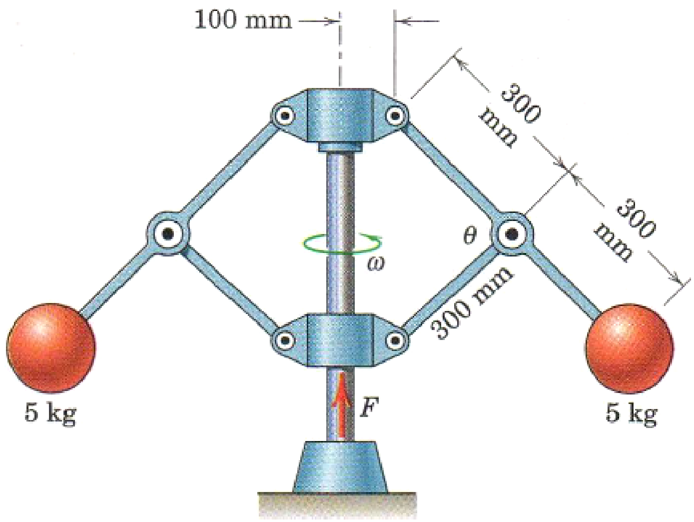
Ans. $\mathbf{H}_O = -\frac{1}{2}mgut^2\mathbf{k}$



►3/243

The assembly of two 5-kg spheres is rotating freely about the vertical axis at 40 rev/min with θ = 90°. If the force F which maintains the given position is increased to raise the base collar and reduce θ to 60°, determine the new angular velocity ω. Also determine the work U done by F in changing the configuration of the system. Assume that the mass of the arms and collars is negligible.

Ans. ω = 3.00 rad/s, U = 5.34 J



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	13th lecture	Bakr Noori Alhasan/Lecturer
	The major contents Section D Special Applications Impact Direct Central Impact Oblique Central Impact		
	Section D Special Applications Impact Direct Central Impact Coefficient of Restitution e Energy Loss During Impact Oblique Central Impact Sample Problem 3/24 Sample Problem 3/25 Sample Problem 3/26		

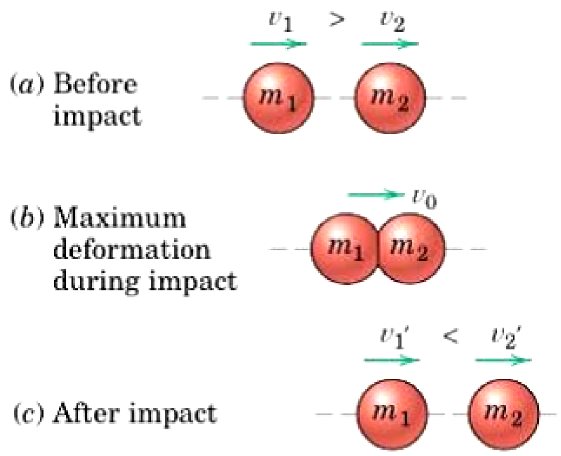
Section D Special Applications
Impact

It refers to the collision between two bodies and is characterized by the generation of relatively large contact forces which act over a very short interval of time. It involves material deformation and recovery and generation of heat and sound.

Direct Central Impact

Consider a collinear motion of two spheres of masses m_1 and m_2 and travelling with velocities v_1 and v_2 respectively. The linear momentum is constant because the contact forces are equal and opposite.

$$G = G'$$
$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \dots\dots \; 3/31$$

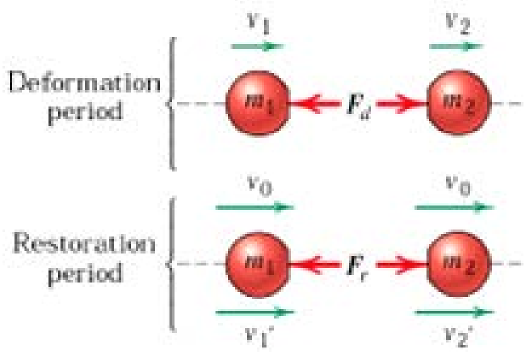


Assumptions:
All forces other than internal forces are small and neglected.
No appreciate change in the position of centers of the bodies during impact.

Coefficient of Restitution e

It is the capacity of the contacting bodies to recover from the impact.

$$e = \frac{The\ magnitude\ \ of\ the\ restoration\ \ impulse}{The\ magnitude\ \ of\ the\ deformation\ \ impulse}$$



For particle 1

$$e = \frac{\int_{t_o}^{t} F_r\, dt}{\int_0^{t_o} F_d\, dt} = \frac{m_1[-v_1' - (-v_o)]}{m_1(-v_o - (-v_1))} = \frac{v_o - v_1'}{v_1 - v_o} \quad \dots\dots \; 1$$

For particle 2

$$e = \frac{m_2(v_2' - v_o)}{m_2(v_o - v_2)} = \frac{v_2' - v_o}{v_o - v_2} \quad \dots\dots \; 2$$

Eliminating v_o

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{|Relative\ \ velocity\ \ of\ seperation\ |}{|Relative\ \ velocity\ \ of\ approac\ h|} \quad \dots\dots \; 3/32$$

F_r is the contact force during restoration.
F_d is the contact force during deformation.
t_o time of deformation.
t the total time of contact.

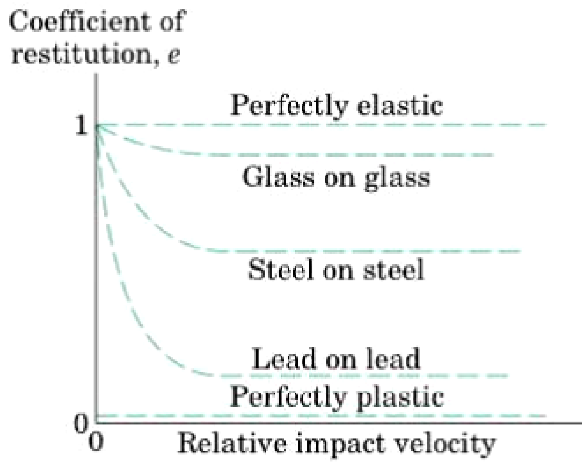
Energy Loss During Impact

Impact is always accompanied by the energy loss.

$$Energy\ Loss = Kinetic\ energy\ just\ before\ impact - Kinetic\ energy\ just\ after\ impact$$

Ways of Energy Loss

- Generation of heat during the inelastic deformation.
- Generation and dissipation of elastic stress waves.
- Generation of sound energy.



$e = 1$ The capacity of the particle to recover equals their tendency to deform. (elastic impact → no energy loss)

$e = 0$ The particle cling together after collision (energy loss is max)

All impact conditions lie somewhere between these two extremes.

Oblique Central Impact

Is the case where the initial and final velocities are not parallel.

t: is the coordination along the tangent to the contacting surfaces.

$\vec{v}_1$ *and* $\vec{v}_2$ are the initial velocities (vectors).

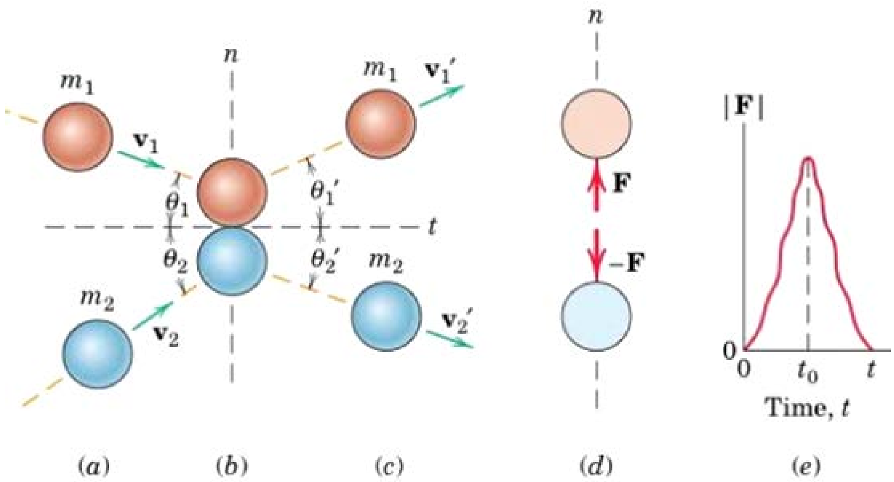
The scalar components:

$(v_1)_n = -v_1 \sin\theta_1$

$(v_1)_t = v_1 \cos\theta_1$

$(v_2)_n = v_2 \sin\theta_2$

$(v_2)_t = v_2 \cos\theta_2$



Rebound conditions are four:

Unknown: $(v_1')_t$, $(v_1')_n$, $(v_2')_t$, *and* $(v_2')_n$,

four equation are needed:

1. Conservation in n-direction:

$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1')_n + m_2(v_2')_n$$

2,3 Conservation of particle 1 in t-direction.

$$m_1(v_1)_t = m_1(v_1')_t$$

and conservation of particle 2 in t-direction.

$$m_2(v_2)_t = m_2(v_2')_t$$

4. The coefficient of restitution in n-direction.

$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$$

$$\theta_1' \ \ and \ \ \theta_2' \ \ can\, be\, found\, easily.$$

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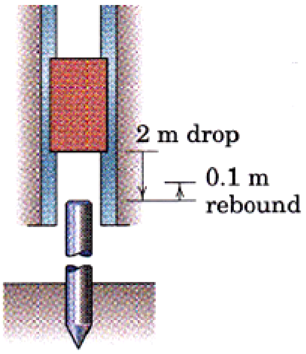
Sample Problem 3/24

The ram of a pile driver has a mass of 800 kg and is released from rest 2 m above the top of the 2400-kg pile. If the ram rebounds to a height of 0.1 m after impact with the pile, calculate (a) the velocity v_p' of the pile immediately after impact, (b) the coefficient of restitution e , and (c) the percentage loss of energy due to the impact.



Solution

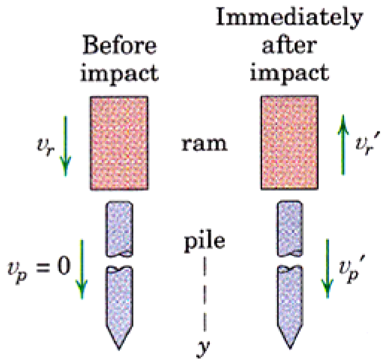
Conservation of energy during free fall gives:

$$T_1 + V_{g1} = T_2 + V_{g2}$$

$$0 + mgh_1 = \frac{1}{2}m{v_r}^2 + 0$$

$$v_r = \sqrt{2(9.81)(2)} = 6.26\ m/s$$

$$v_r' = \sqrt{2(9.81)(0.1)} = 1.401\ m/s$$



Conservation of momentum for the system of the ram and pile gives:

$$G_1 = G_2$$

$$800(6.26) + 0 = 800(-1.4) + 2400\, v_p'$$

$$v_p' = 2.55\ m/s \quad Ans.$$

(b) The coefficient of restitution yields

$$e = \frac{|Relative\ velocity\ of\ seperation|}{|Relative\ velocity\ of\ approach|}$$

$$= \frac{2.55 + 1.401}{6.26 + 0} = 0.631 \ \ Ans.$$

(c) $T = V_g = mgh = 800(9.81)(2) = 15\,700\, J$

$$T' = \frac{1}{2}(800)1.401^2 + \frac{1}{2}(2400)2.55^2 = 8620\, J$$

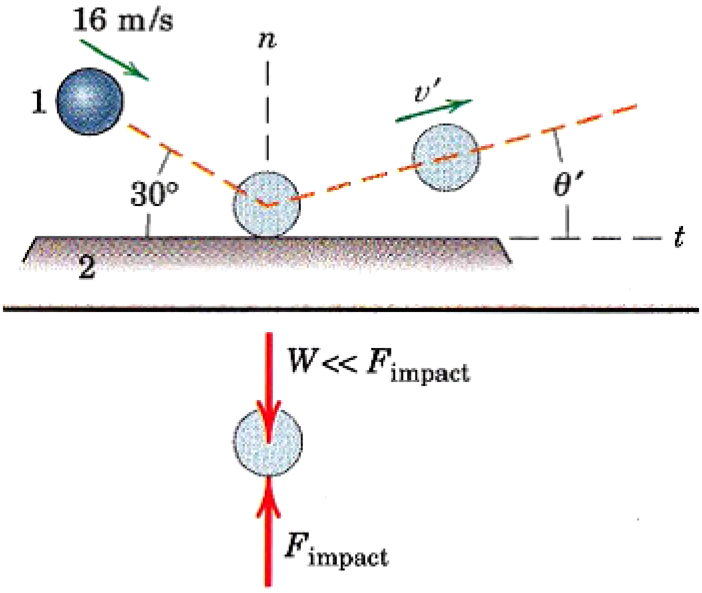
The percentage loss of energy, therefore,

$$\frac{15\,700 - 8620}{15\,700}(100) = 45.1\% \ \ Ans.$$

Sample Problem 3/25

A ball is projected onto the heavy plate with a velocity of $16\ m/s$ at the 30^o angle shown. If the effective coefficient of restitution is 0.5, compute the rebound velocity v' and the angle θ'.

Solution



The mass of the heavy plate may be considered infinite and its corresponding velocities are zero both before and after impact.

$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$$

$$0.5 = \frac{0 - (v_1')_n}{-16 \sin 30^o - 0}$$

$$(v_1')_n = 4\ m/s$$

$$m_1 (v_1)_t = m_1 (v_1')_t$$

$$(v_1)_t = (v_1')_t$$

$$16\ cos\ 30^o = 13.86\ m/s$$

Smooth surfaces make no force acting on the ball in t- direction and the momentum is unchanged in that direction.

$$v_1' = \sqrt{((v_1')_n)^2 + ((v_1')_t)^2}$$

$$= \sqrt{13.86^2 + 4^2} = 14.42\ m/s$$

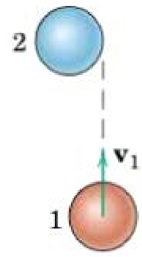
$$\theta' = tan^{-1} \frac{(v_1')_n}{(v_1')_t}$$
$$= tan^{-1} \frac{4}{13.86} = 16.10^o\ Ans.$$

For infinite mass the conservation momentum for the system in n-direction is not applied.

The impulse of the weight W is neglected as its very small compared with the impact force.

Sample Problem 3/26

Spherical particle1 has a velocity $v_1 = 6\ \mathrm{m}/s$ in the direction shown and collides with spherical particle 2 of equal mass and diameter and initially at rest. If the coefficient of restitution for these conditions is e = 0.6, determine the resulting motion of each particle following impact. Also calculate the percentage loss of energy due to the impact.



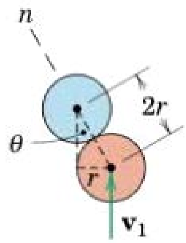
Solution

$v_{1_n} = v_1\ cos\, 30^0 = 6\, cos\, 30^0 = 5.2\ m/s$

$v_{1_t} = v_1\ sin\, 30^0 = 6\, sin\, 30^0 = 3\ m/s$

$v_{2_t} = 0\ \ and \qquad v_{2_n} = 0$

Momentum conservation for the two-particle system in the n-direction gives



$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1^{'})_n + m_2(v_2^{'})_n$$

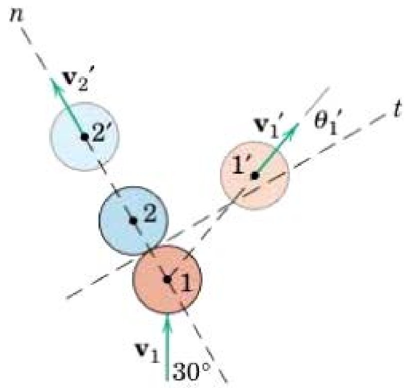
$$5.20 + 0 = (v_1^{'})_n + (v_2^{'})_n \ldots\ldots\ldots\ldots (a)$$

The coefficient-of-restitution relationship

$$e = \frac{(v_2^{'})_n - (v_1^{'})_n}{(v_1)_n - (v_2)_n} \qquad 0.6 = \frac{(v_2^{'})_n - (v_1^{'})_n}{5.20 - 0} \ldots\ldots\ldots (b)$$

Simultaneous solution of (a) and (b) yields

$(v_1^{'})_n = 1.039\ m/s\ \ and\ \ (v_2^{'})_n = 4.16\ m/s$



With assumed smooth surfaces, there is no forces in the t-direction. Thus for particles 1 and 2, we have

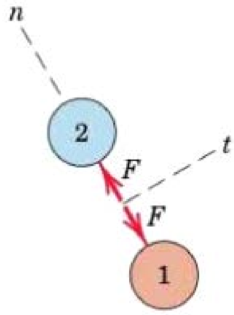
$m_1(v_1)_t = m_1(v_1^{'})_t \qquad (v_1^{'})_t = (v_1)_t = 3\ m/s$

$m_2(v_2)_t = m_2(v_2^{'})_t \qquad (v_2^{'})_t = (v_2)_t = 0$

The final speeds of the particles are

$$v_1^{'} = \sqrt{{(v_1^{'})_n}^2 + {(v_1^{'})_t}^2} = \sqrt{(1.039)^2 + 3^2} = 3.17\ m/s\ \ Ans.$$

$$v_2^{'} = \sqrt{{(v_2^{'})_n}^2 + {(v_2^{'})_t}^2} = \sqrt{(4.16)^2 + 0} = 4.16\, m/s\ \ Ans.$$



The angle θ which $\overrightarrow{v_1^{'}}$ makes with the t-direction is

$$\theta^{'} = tan^{-1}\frac{(v_1^{'})_n}{(v_1^{'})_t} = tan^{-1}\frac{1.039}{3} = 19.11^0 \quad Ans.$$

The kinetic energies just before and just after impact, with $m = m_1 = m_2$, are

$$T = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m(6)^2 + 0 = 18m$$

$$T^{'} = \frac{1}{2}m_1{v_1}^{'2} + \frac{1}{2}m_2{v_2}^{'2} = \frac{1}{2}m(3.17)^2 + \frac{1}{2}m(4.16)^2 = 13.68m$$

The percentage loss of energy is then

$$\frac{|\Delta E|}{E}(100) = \frac{T - T^{'}}{T}(100) = \frac{18m - 13{,}68m}{18m}(100) = 24.0\%\ \ Ans.$$

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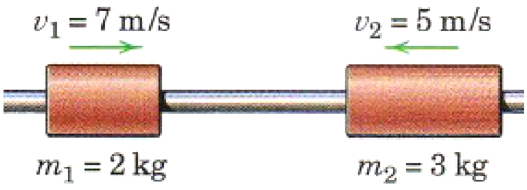
Compute the final velocities v_1' and v_2' after collision of the two cylinders which slide on the smooth horizontal shaft. The coefficient of restitution is $e = 0.6$.

Ans. $v_1' = 4.52$ m/s to the left
$v_2' = 2.68$ m/s to the right



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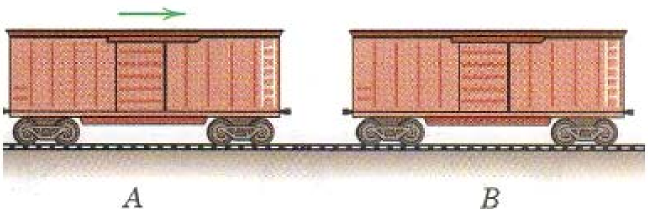
Freight car A of mass m_A is rolling to the right when it collides with freight car B of mass m_B initially at rest. If the two cars are coupled together at impact, show that the fractional loss of energy equals $m_B/(m_A + m_B)$.



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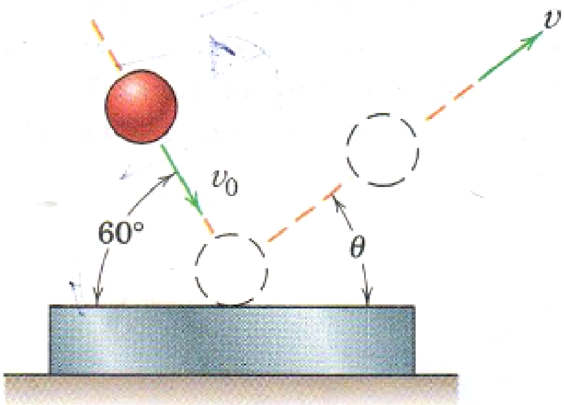
The steel ball strikes the heavy steel plate with a velocity $v_0 = 24$ m/s at an angle of 60° with the horizontal. If the coefficient of restitution is $e = 0.8$, compute the velocity v and its direction θ with which the ball rebounds from the plate.



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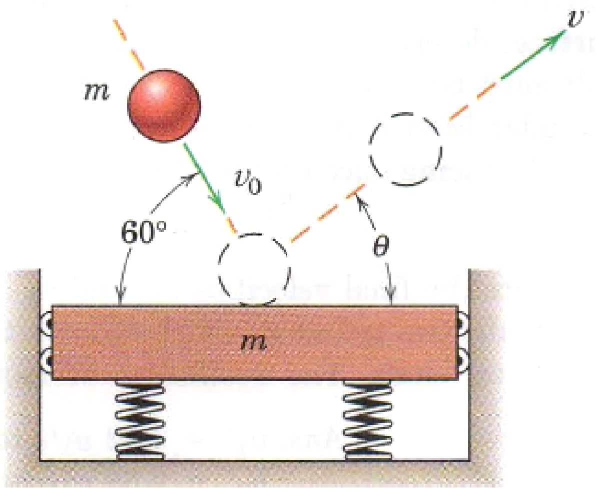
The previous problem is modified in that the plate struck by the ball now has a mass equal to that of the ball and is supported as shown. Compute the final velocities of both masses immediately after impact if the plate is initially stationary and all other conditions are the same as stated in the previous problem.

Ans. Ball, $v_1' = 12.20$ m/s, $\theta = -9.83°$
Plate, $v_2' = 18.71$ m/s down



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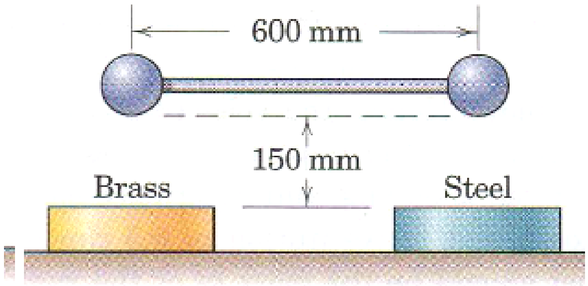
Two steel balls of the same diameter are connected by a rigid bar of negligible mass as shown and are dropped in the horizontal position from a height of 150 mm above the heavy steel and brass base plates. If the coefficient of restitution between the ball and the steel base is 0.6 and that between the other ball and the brass base is 0.4, determine the angular velocity ω of the bar immediately after impact. Assume that the two impacts are simultaneous.

Ans. $\omega = 0.572$ rad/s CCW



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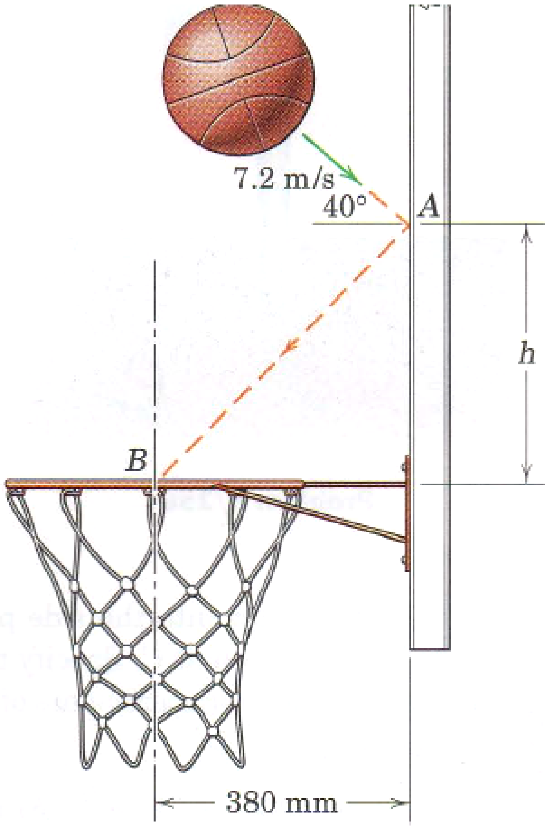
A basketball traveling with the velocity shown in the figure strikes the backboard at A. If the coefficient of restitution for this impact is $e = 0.84$, determine the required distance h above the hoop if the ball is to arrive at the center B of the hoop. Carry out two solutions: (a) an approximate solution obtained by neglecting the effects of gravity from A to B and (b) a solution which accounts for gravity from A to B. Neglect the diameter of the ball compared with h.



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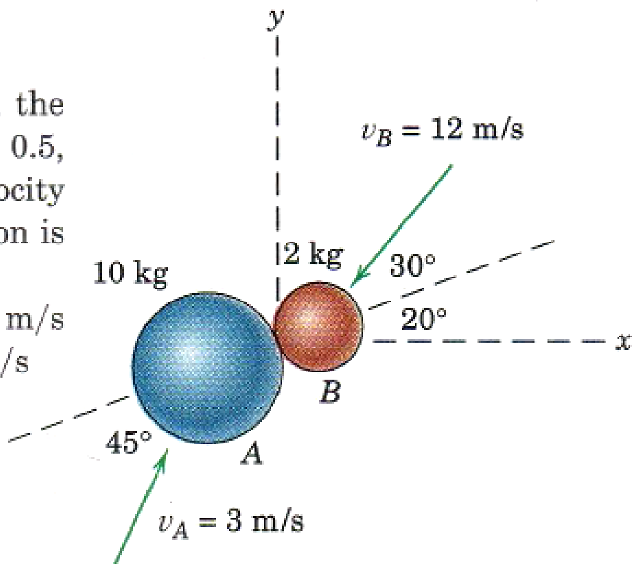
Sphere A collides with sphere B as shown in the figure. If the coefficient of restitution is $e = 0.5$, determine the x- and y-components of the velocity of each sphere immediately after impact. Motion is confined to the x-y plane.

Ans. $(v_A')_x = -1.672$ m/s, $(v_A')_y = 1.649$ m/s
$(v_B')_x = 6.99$ m/s, $(v_B')_y = -3.84$ m/s



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	14th lecture	Bakr Noori Alhasan/Lecturer
	The major contents CHAPTER 5 Plane Kinematics of Rigid Bodies 5.1 Introduction Plane Motion The Categories of Plane Motion of a Rigid Bodies 5/2 Rotation Rotation About a Fixed Axis		
	The detailed contents CHAPTER 5 Plane Kinematics of Rigid Bodies 5.1 Introduction The Reasons of Study the Motion of Rigid Bodies Rigid Body Assumptions Plane Motion The Categories of Plane Motion of a Rigid Bodies 5/2 Rotation Angular Motion Relations Rotation About a Fixed Axis Sample Problem 5/1 Sample Problem 5/2		

CHAPTER 5 Plane Kinematics of Rigid Bodies

5.1 Introduction

In Ch.2, the relationships governing displacement, velocity, and acceleration along straight and curved paths are formulated. In Rigid Body Kinematics, Additional relationships of rotational motion of the body are added.

The Reasons of Study the Motion of Rigid Bodies:

*Generating, transmitting and controlling certain motions by using cams, gears, ... etc.(The motions are calculated to design geometry of the mechanical parts)

*The motion of the rigid body is caused by the forces applied. Ex: studying the motion of the rocket.

Rigid Body Assumptions:

1. The position vector of each particle in the rigid body measured from a reference axis attached to the body remains unchanged (ideal case).

2. The movements associated with the changes in shape are very small compared with the movements of the body as whole. Ex: flutter of an aircraft wing.

The displacements due to the flutter of an aircraft wing do not affect the description of the flight path of the aircraft as whole and thus the rigid body assumption is acceptable. On the other hand, if the problem is one of describing the internal wing stress due to the wing flutter, then the relative motions of portions of the wing cannot be neglected, and the wing may not be considered a rigid body.

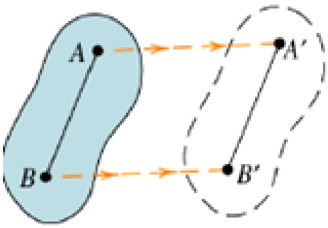
Plane Motion

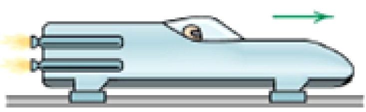
A rigid body executes plane motion when all parts of the body move in parallel planes. The plane of motion is the plane which contains the mass center and is considered as a thin slab.

The Categories of Plane Motion of a Rigid Bodies

1. Translation:

Every line in the body remains parallel to its original position at all times. The motion of the body is the same that of any particle in the body. The relations formulated in Ch.2 are applied.

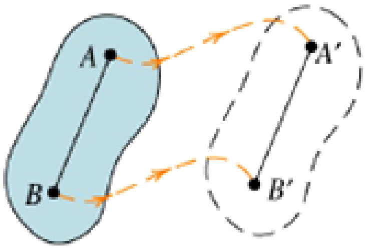


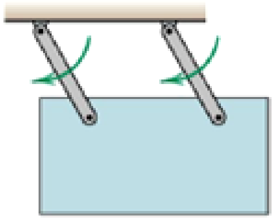


Rocket test sled

a) Rectilinear Translation: All points in the body move in parallel straight lines.

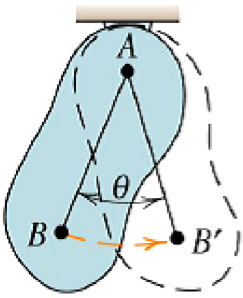
b) Curvilinear Translation: All points move in congruent curves.

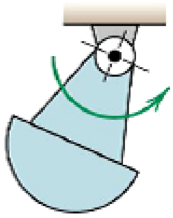




Parallel-link swinging plate

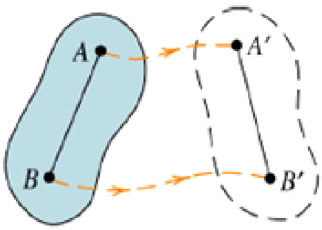
2. Rotation:
All points in the rigid body move in circular paths about a fixed axis. The perpendicular lines to the axis of rotation rotate through the same angle at the same time. (Note the relations of Ch.2 - circular motion)

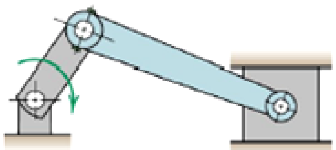




Compound pendulum

3. General Plane Motion:
It is a combination of translation and rotation motion.

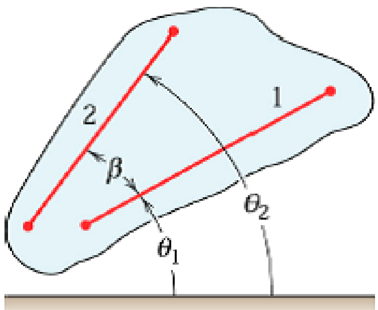




Connecting Rod in a Reciprocating Engine

5/2 Rotation

The rotation of a rigid body is described by its angular motion.
Let θ_1 and θ_2 are the angular positions of any two lines 1 and 2 attached to the body measured from a convenient reference axis.



$$\theta_2 = \theta_1 + \beta$$
$$\dot{\theta}_2 = \dot{\theta}_1 + 0 \;\rightarrow\; \dot{\theta}_2 = \dot{\theta}_1 \;\; (\beta \;\; is\; invariant)$$
$$\ddot{\theta}_2 = \ddot{\theta}_1$$
$$\Delta\theta_1 = \Delta\theta_2 \;\; (during\; a\; finite\; interval)$$

Thus, all lines in a rigid body in its plane of motion have the same angular displacement θ, the same angular velocity $\dot{\theta}$, and the same angular acceleration $\ddot{\theta}$.

The angular motion of a line depends only on its angular position θ and on the time derivative of the displacemet and does not require the precence of a fixed rotation axis, normal to the plane of motion.

Angular Motion Relations

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$
$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad or \quad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$
$$\omega d\omega = \alpha d\theta \quad or \quad \dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta$$

.. 5/1 (Analogy to equations 2/1 2/2 and 2/3)

The positive direction of ω and α is determined by that chosen for θ. All relations in rectilinear motion can be transformed into angular motion.

$$s \rightarrow \theta$$
$$v \rightarrow \omega$$
$$a \rightarrow \alpha$$

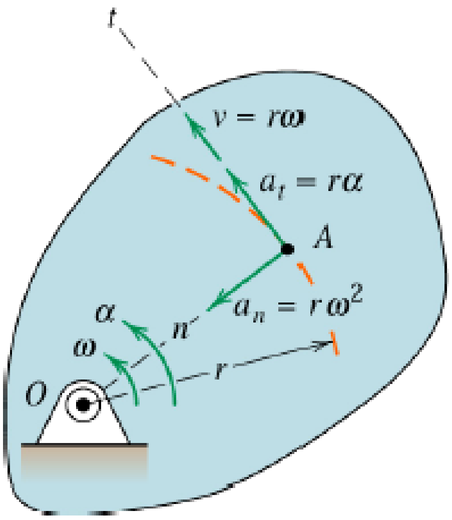
For constant ∝:

$$\omega = \omega_o + \alpha t$$
$$\omega^2 = {\omega_o}^2 + 2\alpha(\theta - \theta_o)$$
$$\theta = \theta_o + \omega_o t + {}^1/_2\, \alpha t^2$$

Rotation About a Fixed Axis

For a rigid body rotates about a fixed axis through O normal to the plane of the figure, all points move in concentric circles about the fixed axis.

Point A moves in a circle of radius r.



The followings are the relationships between the linear motion of the point A and the angular motion of the normal line n (or the body):

$$v = r\omega \qquad (or\ v = r\dot{\theta})$$
$$a_n = r\omega^2 = \frac{v^2}{r} = v\omega \qquad (a_n = r\dot{\theta}^2 \ldots.)$$
$$a_t = r\alpha \qquad\qquad (or \quad a_t = r\ddot{\theta}) \qquad \ldots 5/2$$

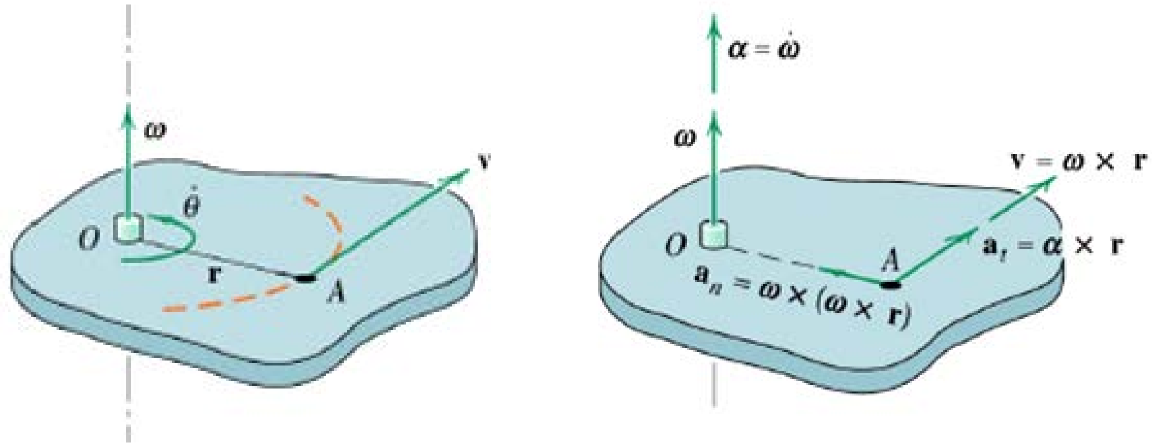
or in vector notation (important in 3D motion)
$\vec{v} = \dot{\vec{r}} = \vec{\omega} \times \vec{r}$ (the order must be retained)

Differentiation:

$$\vec{a} = \dot{\vec{v}} = \vec{\omega} \times \dot{\vec{r}} + \dot{\vec{\omega}} \times \vec{r}$$
$$= \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} \quad or \quad \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

The vectors equivalent to equation 5/2 are:
$\vec{v} = \vec{\omega} \times \vec{r}$
$\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$
$\vec{a}_t = \vec{\alpha} \times \vec{r}$

In 3D motion $\vec{\omega}$ may change direction as well as magnitude.
$\vec{\alpha}$ will no longer be in the same direction as $\vec{\omega}$.



Problems 5/2 5/3 5/10 5/11 5/13 5/15 5/17 5/19

Sample Problem 5/1

A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t\ rad/s^2$, where t is the time in seconds during which the torque is applied. Determine (a) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (b) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Solution

The counterclockwise direction will be taken arbitrarily as positive.

(a) $\omega = \frac{2\pi 1800}{60} = -60\pi\ rad/s$

$[d\omega = \alpha dt]$ $\int_{-60\pi}^{\omega} d\omega = \int_0^t 4t\ dt$ $\omega = -60\pi + 2t^2$

$\omega = \frac{2\pi(-900)}{60} = -30\pi\ rad/s$

$-30\,\pi = -60\pi + 2t^2$ $t^2 = 15\pi$ $t = 6.86\,s$ $Ans.$

(b) The flywheel changes direction when its angular velocity is momentarily zero, thus

$0 = -60\pi + 2t^2$

$t^2 = 30\pi$

$t = 9.71\,s$ $Ans.$

(c) $[d\theta = \omega dt]$ $\int_0^{\theta_1} d\theta = \int_0^{9.71}(-60\pi + 2t^2)\ dt$

$\theta_1 = [-60\,\pi t + \frac{2}{3}t^3]_0^{9.71} = -1220\,rad$

or $N_1 = \frac{1220}{2\pi} = 194.2$ $revolutions\ clockwise.$

For the second interval

$\int_0^{\theta_2} d\theta = \int_{9.71}^{14}(-60\pi + 2t^2)dt$

$\theta_2 = [-60\pi t + \frac{2}{3}t^3]_{9.71}^{14} = 410$ rad

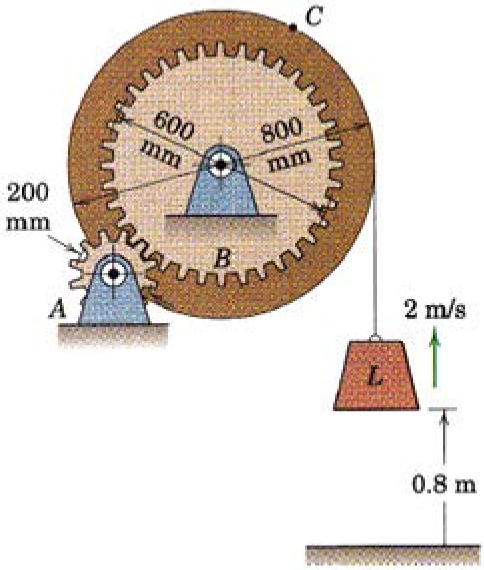
or $N_2 = 410/2\pi = 65.3$ $revolutions\ CCW$

Thus the total number of revolutions turned during the 14 seconds is

$N = N_1 + N_2 = 194.2 + 65.3 = 259$ rev $Ans.$

Sample Problem 5/2

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 2 m/s in a vertical rise of 0.8 m with constant acceleration. As the load passes this position, compute (a) the acceleration of point C on the cable in contact with the drum and (b) the angular velocity and angular acceleration of the pinion A.



Solution

$[v^2 = v_o^2 + 2as]$ $\quad 2^2 = 0 + 2a(0.8)$ $\quad a = 2.5\ m/s^2$

At point C:

$[a_n = \frac{v^2}{r}]$ $\quad a_n = \frac{2^2}{0.4}$ $\quad a_n = 10\ m/s^2$

$a_t = 2.5\ m/s^2$

$[a_c = \sqrt{a_t^2 + a_n^2}]$ $\quad a_c = \sqrt{2.5^2 + 10^2}$ $\quad a_c = 10.31\ m/s^2$ $\quad Ans.$

$[a_t = \alpha r]$ $\quad 2.5 = \alpha(0.4)$ $\quad \alpha = 6.25\ rad/s$

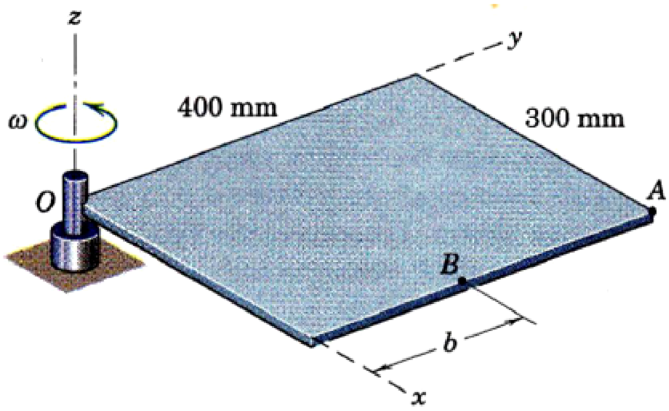
$[v = \omega r]$ $\quad 2 = \omega(0.4)$ $\quad \omega = 5\ rad/s$

$[\omega_B\ r_B = \omega_A\ r_A]$ $\quad 5 \times 0.3 = \omega_A \times 0.1$ $\quad \omega_A = 15\ rad/s$ $\quad Ans.$

$[\alpha_B r_B = \alpha_A r_A]$ $\quad 6.25(0.3) = \alpha_A(0.1)$ $\quad \alpha_A = 18.75\ rad/s^2$ $\quad Ans.$

5/2

The rectangular plate is rotating about its corner axis through *O* with a constant angular velocity $\omega = 10$ rad/s. Determine the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the corner *A* by (*a*) using the scalar relations and (*b*) using the vector relations.



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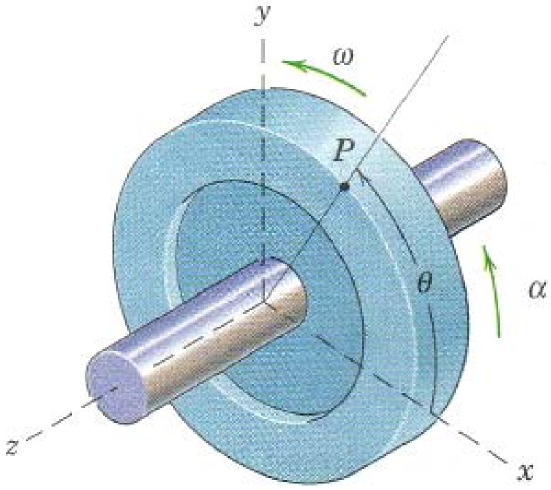
The flywheel has a diameter of 600 mm and rotates with increasing speed about its *z*-axis shaft. When point *P* on the rim crosses the *y*-axis with $\theta = 90°$, it has an acceleration given by $\mathbf{a} = -1.8\mathbf{i} - 4.8\mathbf{j}$ m/s^2. For this instant, determine the angular velocity ω and the angular acceleration α of the flywheel.

Ans. $\alpha = 6$ rad/s^2, $\omega = 4$ rad/s



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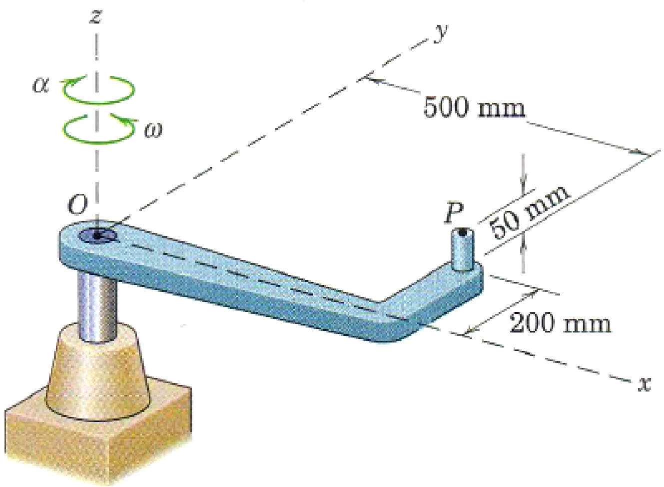
The right-angle bar rotates about the *z*-axis through *O* with an angular acceleration $\alpha = 3$ rad/s^2 in the direction shown. Determine the velocity and acceleration of point *P* when the angular velocity reaches the value $\omega = 2$ rad/s.



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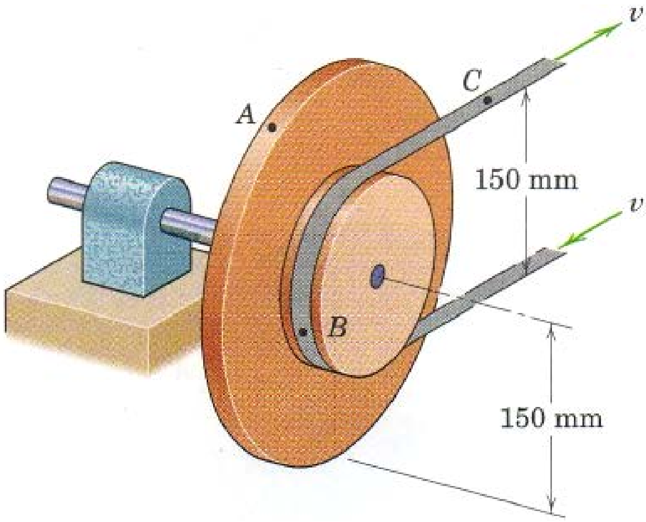
The belt-driven pulley and attached disk are rotating with increasing angular velocity. At a certain instant the speed v of the belt is 1.5 m/s, and the total acceleration of point A is 75 m/s^2. For this instant determine (*a*) the angular acceleration α of the pulley and disk, (*b*) the total acceleration of point B, and (*c*) the acceleration of point C on the belt.

Ans. (*a*) $\alpha = 300$ rad/s^2, (*b*) $a_B = 37.5$ m/s^2
(*c*) $a_C = 22.5$ m/s^2



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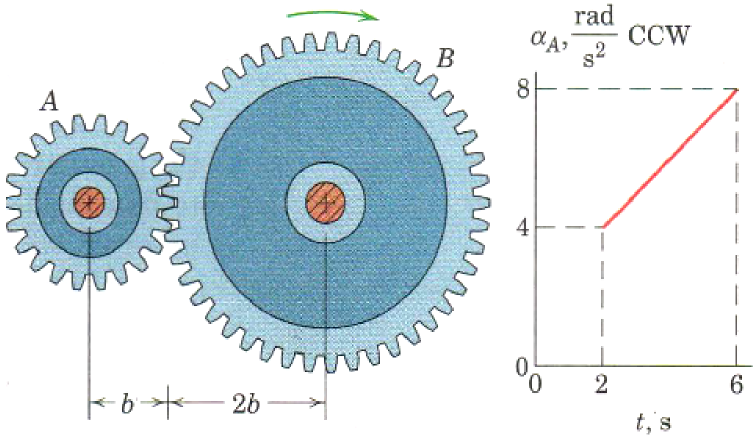
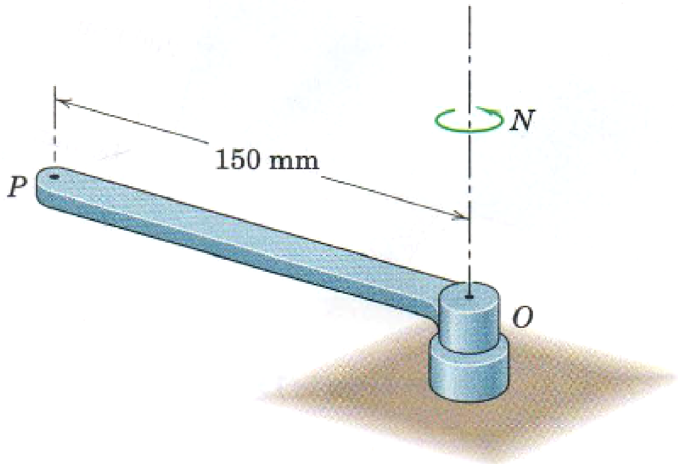
The design characteristics of a gear-reduction unit are under review. Gear B is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear A at time $t = 2$ s to give gear A a counterclockwise acceleration α which varies with time for a duration of 4 seconds as shown. Determine the speed N_B of gear B when $t = 6$ s.

Ans. $N_B = 415$ rev/min



5/20

The rotating arm starts from rest and acquires a rotational speed $N = 600$ rev/min in 2 seconds with constant angular acceleration. Find the time t after starting before the acceleration vector of end P makes an angle of 45° with the arm OP.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

	Lecture sequences:	15th lecture	Bakr Noori Alhasan/Lecturer
Lecture Contents	*The major contents* Chapter 5 5/3 Absolute Motion		
	The detailed contents Chapter 5 5/3 Absolute Motion Sample Problem 5/4 Sample Problem 5/5		

Chapter 5
5/3 Absolute Motion

In this approach, the geometric relation which defines the configuration of the body are constructed and then taking derivatives to obtain velocities and accelerations.

In chapter 2, The absolute motion analysis is applied on the constrained motion of connected particles and successive differentiation of the lengths of the connecting cables is made to determine velocities and accelerations. The geometric relations were quite simple and had no angular quantities.

In rigid body, the defining geometric relations include both linear and angular variables to obtain linear and angular velocities and accelerations.

This approach is used for simple configuration geometries. For more complicated geometry, the relative motion may be preferable. The choice depends on the experience.

The relations 2/1, 2/2, 2/3, 5/1, 5/2, and 5/3 should be mastered.

Problems

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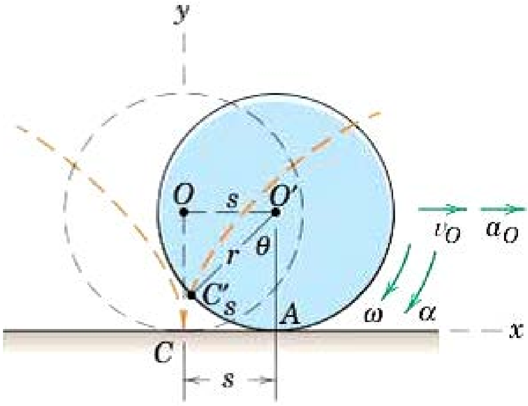
Sample Problem 5/4

A wheel of radius r rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.

Solution.

Rolling to the right from the dashed to the full position without slipping



$$s = r\theta$$
$$v_O = r\omega$$
$$a_O = r \propto \quad Ans.$$

the origin of fixed coordinates is taken at C, the new coordinates of C which is C' become

$$x = s - r\,sin\,\theta = r(\theta - sin\,\theta) \qquad y = r - r\,cos\,\theta = r(1 - cos\,\theta)$$
$$\dot{x} = r\dot{\theta}(1 - cos\,\theta) = v_o(1 - cos\,\theta) \qquad \dot{y} = r\dot{\theta}\sin\theta = v_o\,sin\theta$$
$$\ddot{x} = \dot{v_o}(1 - cos\,\theta) + v_o\,\dot{\theta}\,sin\theta \qquad \ddot{y} = \dot{v}_o\sin\theta + v_o\dot{\theta}cos\theta$$

$$= a_o(1 - cos\,\theta) + r\omega^2\,sin\theta \qquad = a_o\sin\theta + r\omega^2 cos\theta \quad Ans.$$

$At\ \theta = 0$

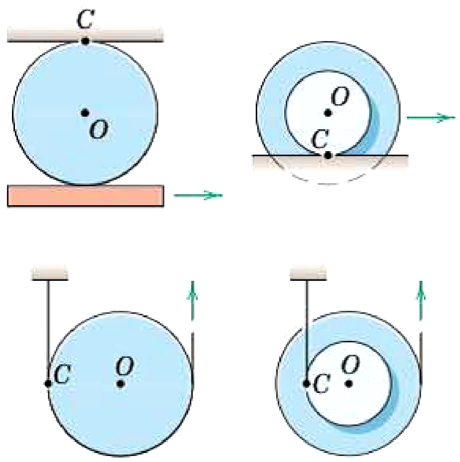
$$\ddot{x} = 0 \qquad and \qquad \ddot{y} = r\omega^2$$

the acceleration at the point of contact depends only on ω and r and directed toward the center of the wheel.

The velocity and acceleration of C at any position θ

$$\vec{v} = \dot{x}\vec{\imath} + \dot{y}\vec{\jmath} \quad and\ \vec{a} = \ddot{x}\vec{\imath} + \ddot{y}\vec{\jmath}$$

The figure shows the various configurations of rolling wheels.



Sample Problem 5/5

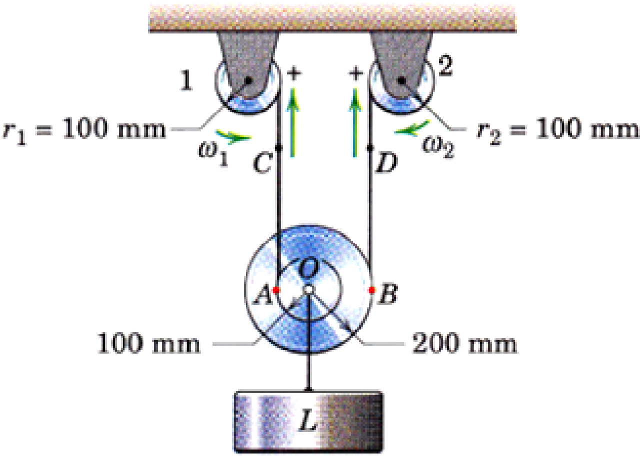
The load L is being hoisted by the pulley and cable arrangement shown. Each cable is wrapped securely around its respective pulley so it does not slip. The two pulleys to which L is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load L and the corresponding angular velocity ω and angular acceleratiom α of the double pulley under the following conditions.

$Case\ (a)\ Pulley\ 1{:}\ \omega_1 = \dot{\omega}_1 = 0\ (pulley\ at\ rest)$

$Pulley\ 2{:}\ \omega_2 = 2\ rad/s.\ \alpha_2 = \dot{\omega}_2 = -3\ rad/s^2.$

$Case\ (b)\ Pulley\ 1{:}\ \omega_1 = 1\ rad/s.\ \alpha_1 = \dot{\omega}_1 = 4\ rad/s^2.$

$Pulley\ 2{:}\ \omega_2 = 2\ rad/s.\ \alpha_2 = \dot{\omega}_2 = -2\ rad/s^2.$



Solution

Case (a)

$ds_B = \overline{AB}\, d\theta \quad v_B = \overline{AB}\omega \quad (a_B)_t = \overline{AB}\alpha$

$ds_O = \overline{AO}\, d\theta \quad v_O = \overline{AO}\omega \quad a_O = \overline{AO}\alpha$

$$v_D = r_2\omega_2 = 0.1(2) = 0.2\ m/s$$

$$a_D = r_2\alpha_2 = 0.1(-3) = -0.3\ m/s^2$$

For the double pulley:

$\omega = v_B/\overline{AB} = v_D/\overline{AB} = 0.2/0.3$

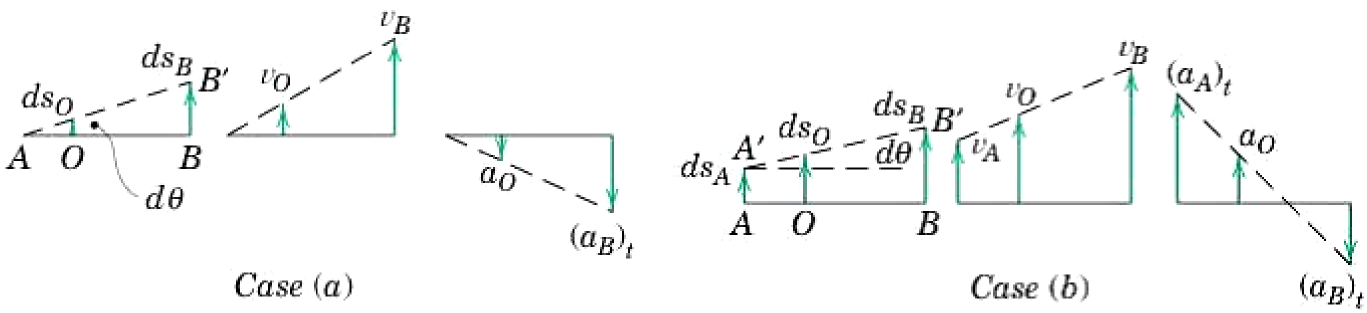
$= 0.667\ rad/s\ CCW\ \ Ans.$

$$\alpha = (a_B)_t/\overline{AB} = a_D/\overline{AB} = -0.3/0.3 = -1\ rad/s^2\ \ CW\ \ Ans.$$

The corresponding motion of O and the load L is

$$v_O = \overline{AO}\omega = 0.1(0.667) = 0.0667\, m/s \quad Ans.$$

$$a_O = \overline{AO}\alpha = 0.1(-1) = -0.1\, m/s^2 \quad Ans.$$



Case B

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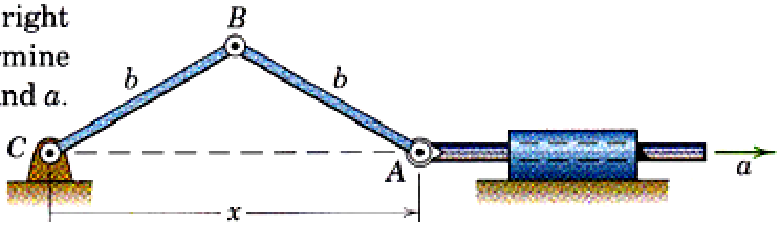
Point A is given a constant acceleration a to the right starting from rest with x essentially zero. Determine the angular velocity ω of link AB in terms of x and a.

$$Ans.\ \omega = \frac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$$



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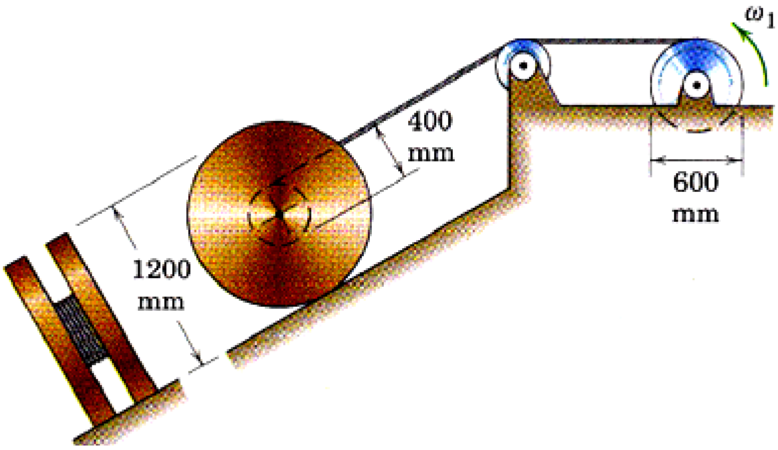
The telephone-cable reel is rolled down the incline by the cable leading from the upper drum and wrapped around the inner hub of the reel. If the upper drum is turned at the constant rate $\omega_1 = 2$ rad/s, calculate the time required for the center of the reel to move 30 m along the incline. No slipping occurs.

Ans. $t = 66.7$ s



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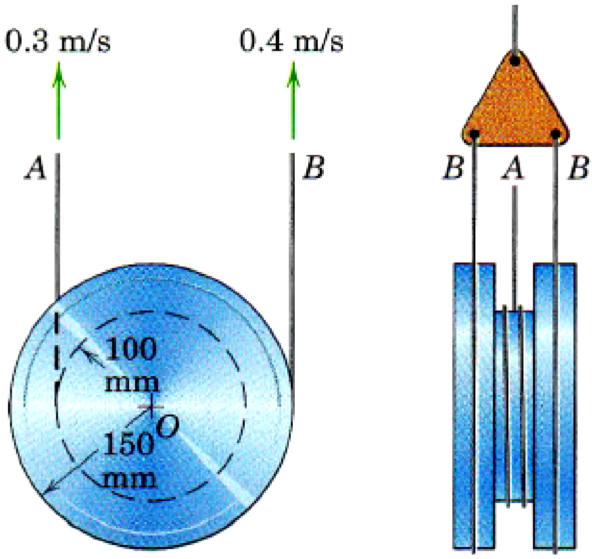
The cables at A and B are wrapped securely around the rims and the hub of the integral pulley as shown. If the cables at A and B are given upward velocities of 0.3 m/s and 0.4 m/s, respectively, calculate the velocity of the center O and the angular velocity of the pulley.



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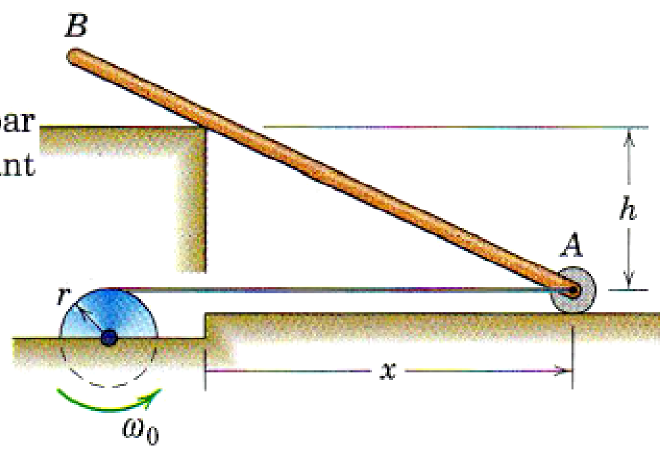
Calculate the angular velocity ω of the slender bar AB as a function of the distance x and the constant angular velocity ω_0 of the drum.

$$Ans.\ \omega = \frac{rh\omega_0}{x^2 + h^2}$$



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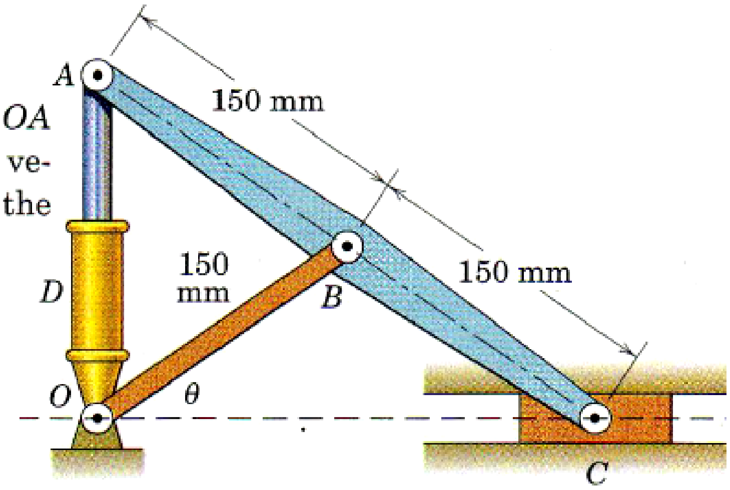
The hydraulic cylinder D is causing the distance OA to increase at the rate of 50 mm/s. Calculate the velocity of the pin at C in its horizontal guide for the instant when $\theta = 50°$.



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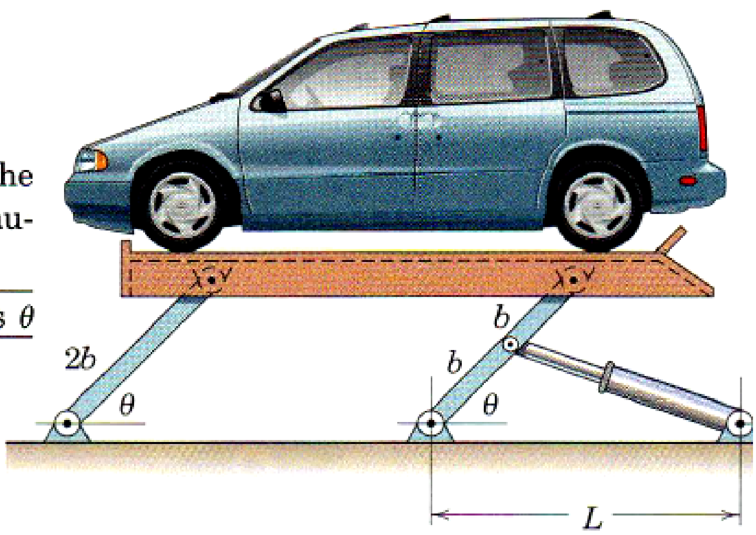
Derive an expression for the upward velocity v of the car hoist in terms of θ. The piston rod of the hydraulic cylinder is extending at the rate $\dot{s}$.

$$Ans.\ v = \frac{2\dot{s}\sqrt{b^2 + L^2 - 2bL\cos\theta}}{L\tan\theta}$$



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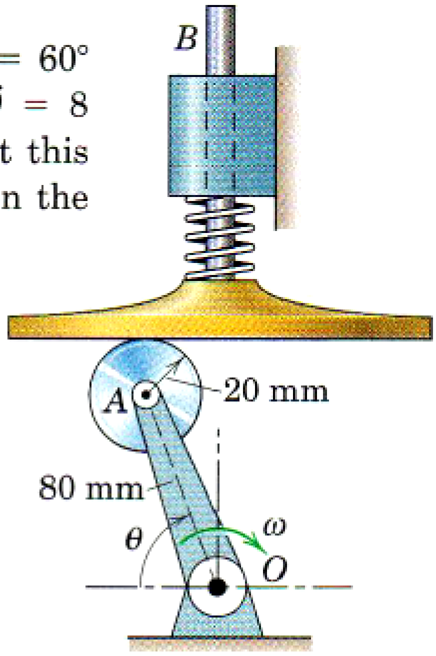
Determine the acceleration of the shaft B for $\theta = 60°$ if the crank OA has an angular acceleration $\ddot{\theta} = 8$ rad/s^2 and an angular velocity $\dot{\theta} = 4$ rad/s at this position. The spring maintains contact between the roller and the surface of the plunger.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

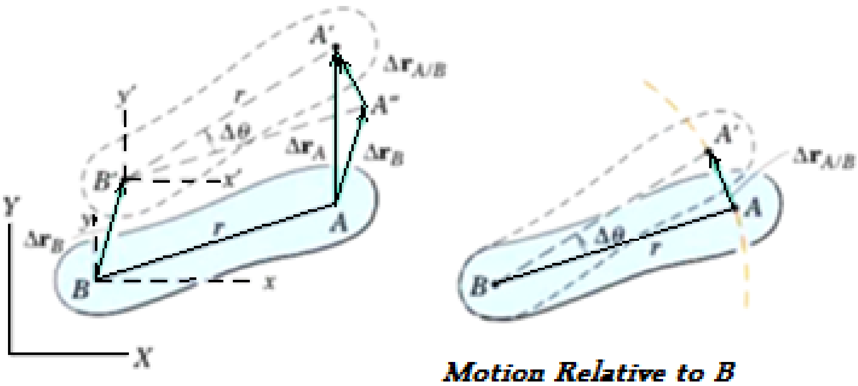
	Lecture sequences:	**16th lecture**	**Bakr Noori Alhasan/Lecturer**
Lecture Contents	***The major contents*** **Chapter 5** **5/4 Relative Velocity** **Relative Velocity Due to Rotation** **Solution of the Relative - Velocity Equation**		
	The detailed contents **Chapter 5** **5/4 Relative Velocity** **Relative Velocity Due to Rotation** **Interpretation of the Relative Velocity Equation** **Solution of the Relative - Velocity Equation** **1. Scalar Geometric** **2. Vector Algebra** **3. Graphical solution** **Sample Problem 5/7** **Sample Problem 5/8** **Sample Problem 5/9**		

Chapter 5
5/4 Relative Velocity

In Art. 2/8, the principles of relative motion was developed for motion relative to translating axis and the relative velocity equation was applied to the motions of two particles.

Relative Velocity Due to Rotation.

Let A and B are two points on the same rigid body. The motion of one point as seen by an observer translating with the other point must be circular since the radial distance between them doesn't change.



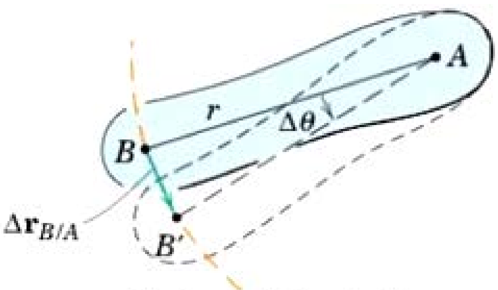
(a) Motion Relative to B (b)

Figure 5/5

Let the body is moving from position AB to A'B' during time Δt. This movement is divided into two parts:

First: Translation to the parallel position A"B' with the displacement $\Delta\vec{r}_B$.
Second: Rotation about B' through $\Delta\theta$ (B' attached to the non-rotating reference axes $x' - y'$ or $\Delta\vec{r}_{A/B}$.

The body appears to undergo fixed axis rotation about B as emphasized in the second figure. (Eqs. 2/11 n-t coordination circular motion) and 5/2 rotation about fixed axis;
$v = r\omega \quad a_n = r\omega^2 \ldots etc.$ describe the relative portion of the motion of point A.



Motion relative to A
(c)

Point A could have been used just as well: B is observed to have circular motion about A and the sense of rotation is CCW $\quad \Delta\vec{r}_{B/A} = -\Delta\vec{r}_{A/B}$

From Fig. 5.5 a $\quad \Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$,

Dividing the expressions by the corresponding Δt and passing to the limit we obtain the relative velocity equation

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} \ldots\ldots 5/4$$

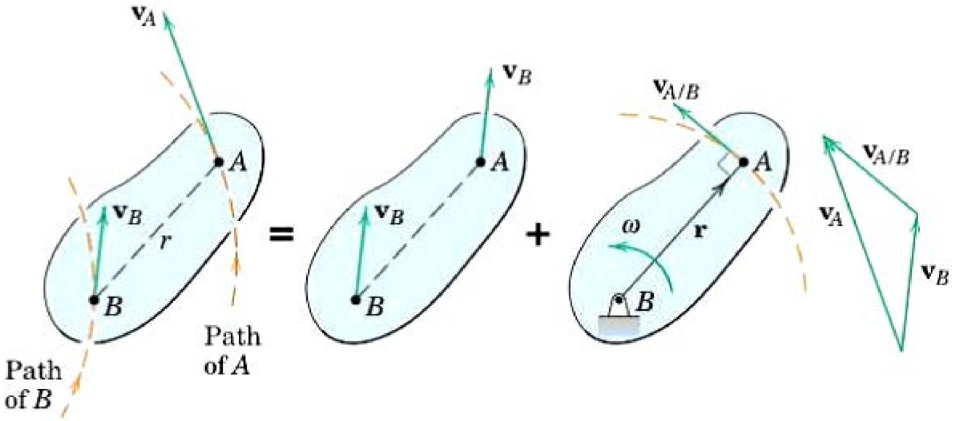
$$\left|\Delta\vec{r}_{A/B}\right| = r\Delta\theta \qquad \lim_{\Delta t \to 0} \frac{r\Delta\theta}{\Delta t} = r\dot{\theta} = r\omega$$

$$v_{A/B} = r\omega$$

$$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$$

ω is the angular velocity vector normal to the plane of motion in a sense determined by the right hand rule.
Equation 5/4 is similar to equation 2/20 except that r between A and B remains constant.

Interpretation of the Relative Velocity Equation



Equation 5/4 can be better understood by visualizing the separate translation and rotation components of the equation.

With B is chosen as the reference point. The velocity of A is the vector sum of the translation portion $\vec{v}_B$ plus the rotational portion $\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$ and $v_{A/B} = r\omega = r\dot{\theta}$

$\dot{\theta}$ is the absolute angular velocity of $\overline{AB}$
The relative linear velocity is always perpendicular to the line joining the two points.

Equation 5/4 may also be used to analyze constrained sliding contact between two links in a mechanism. The two points are on different bodies, so the distance is not fixed. (Sample problem 5/10).

Solution of the Relative - Velocity Equation.

1. Scalar Geometric

Sketch the vector polygon (according to vector equation), then write scalar component equations by projecting the vector along convenient directions. (Avoid simultaneous equations). Apply sine and cosine laws.

2. Vector Algebra

Each term in the relative equation is written in terms of its $\vec{\imath}$ *and* $\vec{\jmath}$ components.
Then the scalar two equations from the later step are obtained.
After that the $\vec{\imath}$ *and* $\vec{\jmath}$ terms may are applied separately.

3. Graphical solution

The known vectors are constructed in their correct positions using a convenient scale. The unknown velocities can be constructed by completing the polygon and satisfying the vector equation. The unknown vectors is directly measured from the drawing.

The choice depends on:

The particular problem.

The accuracy.

Experience.

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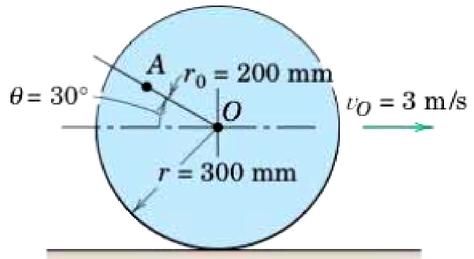
Sample Problem 5/7

The wheel of radius r = 300 mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of center O. Calculate the velocity of point A on the wheel for the instant represented.



Solution 1 (Scalar Geometry)

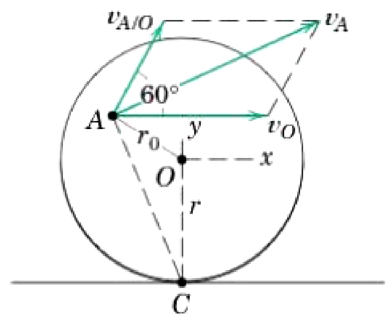
$\vec{v}_A = \vec{v}_O + \vec{v}_{A/O}$

$\omega = \frac{v_o}{r} = \frac{3}{0.3} = 10\ rad/sec$

(ω is the angular velocity of the wheel found from sample problem 5/4 and is the same that of line AO)

$\left[v_{A/O} = r_0\dot{\theta}\right]$ $v_{A/O} = 0.2 \times 10 = 2\ m/s$ (normal to line AO)

$v_A = \sqrt{2^2 + 3^2 + 2 \times 3 \times cos60^o}$ $v_A = 4.36\ m/s\ Ans.$



Alternatively

The velocity of any point on the wheel (including point A) is easily determined by using the contact point C as the reference point,

$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C} = \vec{v}_{A/C}$

$\vec{v}_{A/C} = \overline{AC}\omega = \frac{\overline{AC}}{\overline{OC}} v_o = \frac{0.436}{0.3} \times 3 = 4.36\ m/sec\ Ans. = v_A$

$\overline{AC} = 0.436$ is calculated separately.

Solution 2 (Vector Algebra)

$\vec{v}_A = \vec{v}_O + \vec{v}_{A/O} = \vec{v}_A = \vec{v}_O + \vec{\omega} \times \vec{r_o}$

$\vec{\omega} = -10\vec{k}$ $\vec{r_o} = 0.2(-\vec{i}\cos 30 + \vec{j}\sin 30) = -0.1732\vec{i} + 0.1\vec{j}\ m$ $\vec{v}_O = 3\vec{i}\ m/sec$

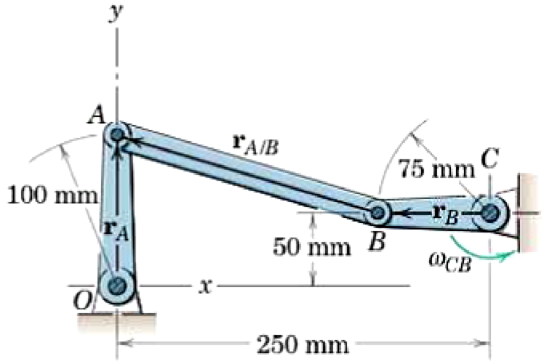
$$\vec{v}_A = 3\vec{i} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -10 \\ -0.1732 & 0.1 & 0 \end{vmatrix} = 3\vec{i} + 1.732\vec{j} + 1\vec{i} = 4\vec{i} + 1.732\vec{j}\ m/s\ Ans.$$

$$v_A = \sqrt{4^2 + 1.732^2} = 4.36\ m/s$$

Sample Problem 5/8

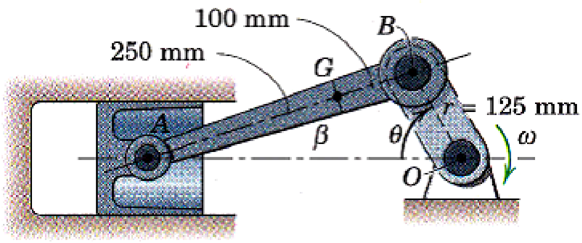
Crank CB oscillates about C through a limited arc, causing crank OA to oscillate about O. When the linkage passes the position shown with CB horizontal and OA vertical, the angular velocity of CB is 2 rad/s counterclockwise. For this instant, determine the angular velocities of OA and AB.

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$



Sample Problem 5/9

The common configuration of a reciprocating engine is that of the slider crank mechanism shown. If the crank OB has a clockwise rotational speed of 1500 rev/min, determine for the position where θ = 60° the velocity of the piston A, the velocity of point G on the connecting rod, and the angular velocity of the connecting rod.



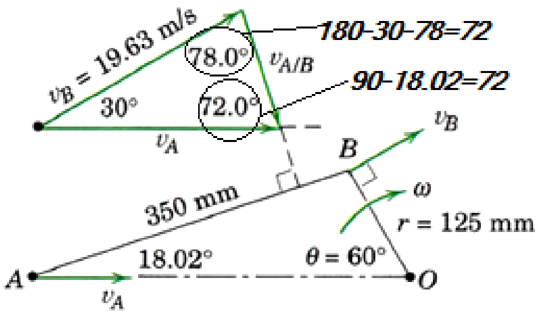
Solution

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$[v = r\omega] \quad v_B = 0.125 \times \frac{2\pi \times 1500}{60} = 19.63 \ m/s$$

$(v_B \textit{ is normal to } \overline{OB}, \ v_A \textit{ is horizontal, and } v_{A/B} \textit{ is normal to } \overline{AB})$

$$\frac{125}{\sin\beta^o} = \frac{350}{\sin 60^0} \qquad \beta = 18.02^o$$

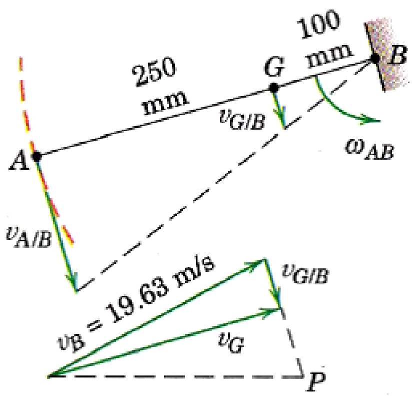


$$\frac{v_A}{\sin 78^o} = \frac{19.63}{\sin 72^0} \quad v_A = 20.2 \ m/sec \quad Ans.$$

$$\frac{v_{A/B}}{\sin 30^o} = \frac{19.63}{\sin 72^o} \qquad v_{A/B} = 10.32 \ m/sec$$

$$\left[\omega = \frac{v}{r}\right] \qquad \omega_{AB} = \frac{v_{A/B}}{AB} = \frac{10.32}{0.350} = 29.5 \ rad/sec \quad Ans.$$

$$\vec{v}_G = \vec{v}_B + \vec{v}_{G/B}$$

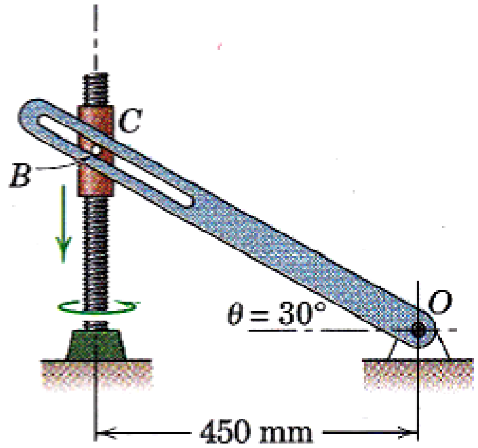


$$v_{G/B} = \overline{GB}\omega_{AB} = \frac{\overline{GB}}{\overline{AB}} v_{A/B} = \frac{100}{350}(10.32) = 2.95 \ m/s$$

$$v_G = 19.24 \ m/sec \quad Ans.$$

Sample Problem 5/10

The power screw turns at a speed which gives the threaded collar C a velocity of 0.25 m/s vertically down. Determine the angular velocity of the slotted arm when $\theta = 30^o$.



Solution

Let A a point on the arm

B a point on the collar and they coincident

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

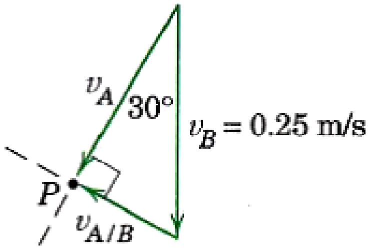
$$v_B \cos 30 = v_A$$

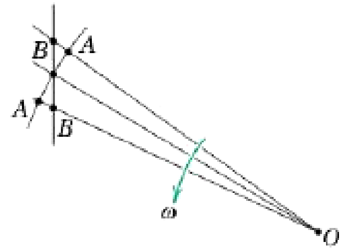
$$v_A = 0.25 \cos 30 = 0.217 \ m/s$$

$$\left[\omega_{OA} = \frac{v}{r}\right]$$

$$\omega_{OA} = \frac{0.217}{0.45/\cos 30^o}$$

$$\omega_{OA} = 0.418 \ rad/s \ \ CCW \quad Ans.$$





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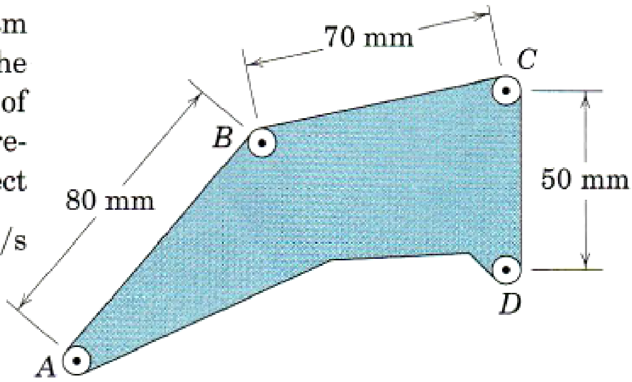
A control element in a special-purpose mechanism undergoes motion in the plane of the figure. If the velocity of B with respect to A has a magnitude of 0.926 m/s at a certain instant, what is the corresponding magnitude of the velocity of C with respect to D?

Ans. $v_{C/D} = 0.579$ m/s



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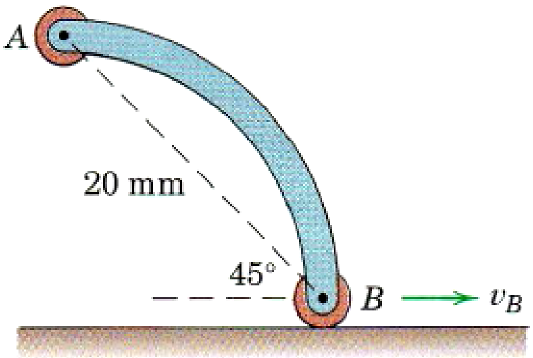
For the instant represented the curved link has a counterclockwise angular velocity of 4 rad/s, and the roller at B has a velocity of 40 mm/s along the constraining surface as shown. Determine the magnitude v_A of the velocity of A.

Ans. $v_A = 58.9$ mm/s



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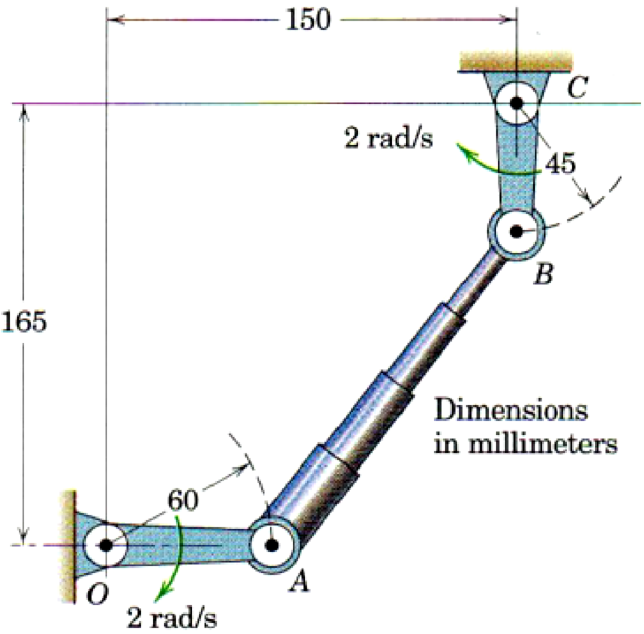
Determine the angular velocity of the telescoping link AB for the position shown where the driving links have the angular velocities indicated.



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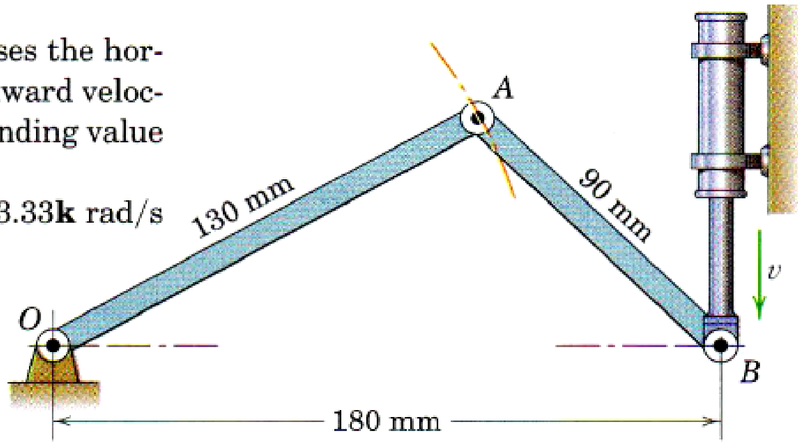
For the instant represented point B crosses the horizontal axis through point O with a downward velocity $v = 0.6$ m/s. Determine the corresponding value of the angular velocity ω_{OA} of link OA.

Ans. $\boldsymbol{\omega}_{OA} = -3.33\mathbf{k}$ rad/s



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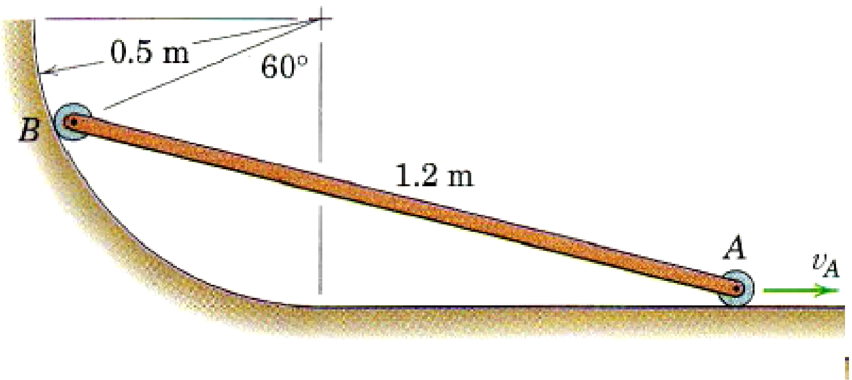
At the instant represented, the velocity of point A of the 1.2-m bar is 3 m/s to the right. Determine the speed v_B of point B and the angular velocity ω of the bar. The diameter of the small end wheels may be neglected.



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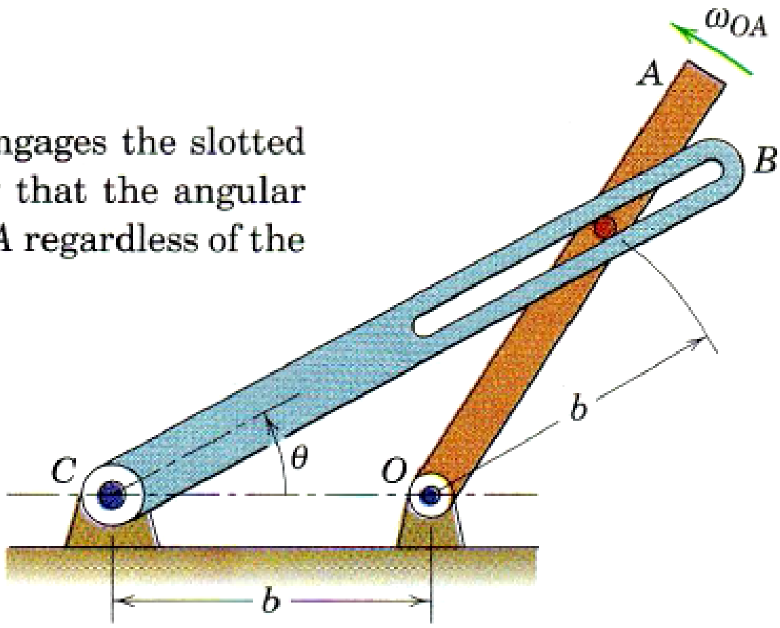
The pin in the rotating arm OA engages the slotted link and causes it to rotate. Show that the angular velocity of CB is one-half that of OA regardless of the angle θ.



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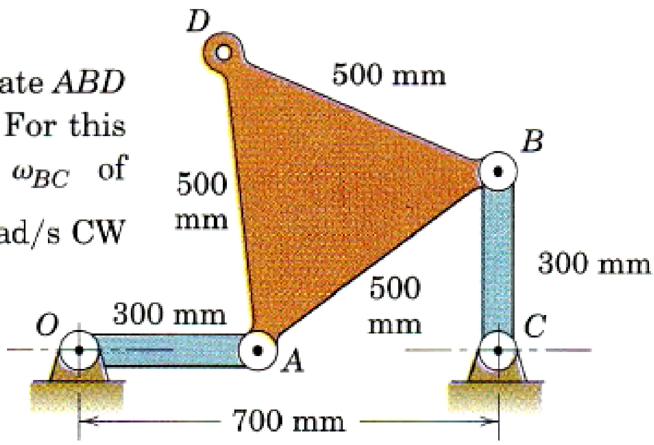
At the instant represented the triangular plate ABD has a clockwise angular velocity of 3 rad/s. For this instant determine the angular velocity ω_{BC} of link BC.

Ans. $\omega_{BC} = 3$ rad/s CW



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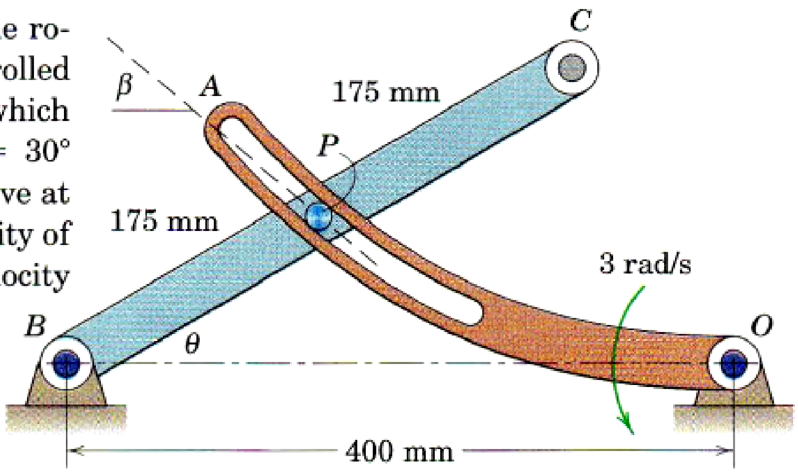
The mechanism is designed to convert from one rotation to another. Rotation of link BC is controlled by the rotation of the curved slotted arm OA, which engages pin P. For the instant represented $\theta = 30°$ and the angle β between the tangent to the curve at P and the horizontal is 40°. If the angular velocity of OA is 3 rad/s for this position, determine the velocity of point C.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

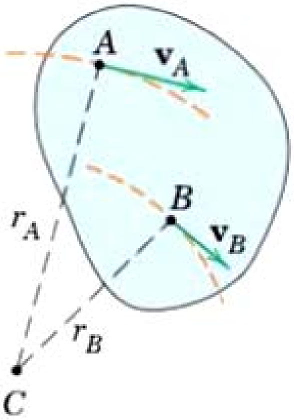
Class: 2nd Year

Lecture Contents	Lecture sequences:	17th lecture	Bakr Noori Alhasan/Lecturer
	The major contents Chapter 5 5/5 Instantaneous Center Zero Velocity p361		
	The detailed contents Chapter 5 5/5 Instantaneous Center Zero Velocity p361 Locating the Instantaneous Center Motion of the Instantaneous Center Sample Problem 5/11 Sample Problem 5/12		

Chapter 5
5/5 Instantaneous Center Zero Velocity p361

In this method , the body may be considered to be in pure rotation about an axis normal to the plane of motion. This axis is called *the instantaneous axis of zero velocity*, and the intersection of this axis with the plane of motion is known as *the instantaneous center of zero velocity*.

Locating the Instantaneous Center



(a)

- Let A and B are two points on the body.
- The directions of the absolute velocities of A and B are known and not parallel.
- Construct a normal to v_A through A. (because the point about which A has absolute circular motion at this instant must lie on the normal)
- Apply the same to B
- The intersection of the two perpendiculars makes the center of rotation.
- C is the point of instantaneous center zero velocity and may lie on or off the body.

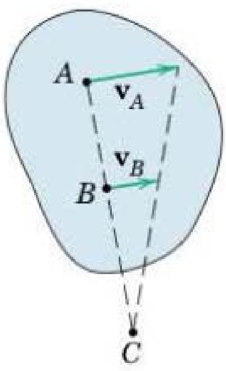
If v_A is known, $\omega = \frac{v_A}{r_A}$ where ω is the angular velocity of the body or any line on the body.

The velocity of any point on the body can be found:

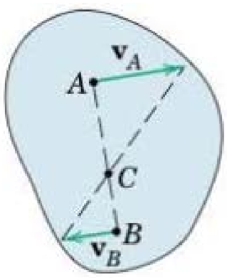
$$v_B = r_B \omega = r_B \frac{v_A}{r_A}$$

Once the instantaneous center is located, the direction of the instantaneous velocity of every point in the body is readily found since it must be perpendicular to the radial line joining the point in question with C.

If the velocities of two points in a body having plane motion are parallel and the line joining the points is perpendicular to the direction of the velocities, the instantaneous center is located by direct proportion. If the parallel velocities become equal in magnitude , the instantaneous center approaches infinity and the body stops rotating and translates only.



(b)



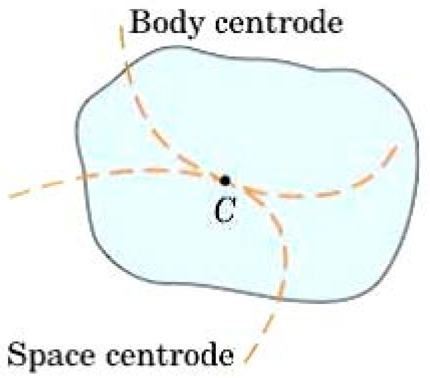
(c)

Motion of the Instantaneous Center

The instantaneous center is changed when the body changes its position.

Body Centrode: Is the locus of the instantaneous centers on the body.
Space Centrode: Is the locus of the instantaneous centers in space.

The two curves are tangent at C. The body-centrode curve rolls on the space centrode curve during the motion of the body.



Although the instantaneous center of zero velocity is momentarily at rest, its acceleration is generally not zero. There is a special topic deals with the instantaneous zero acceleration; its existence, location, and use.

Problems

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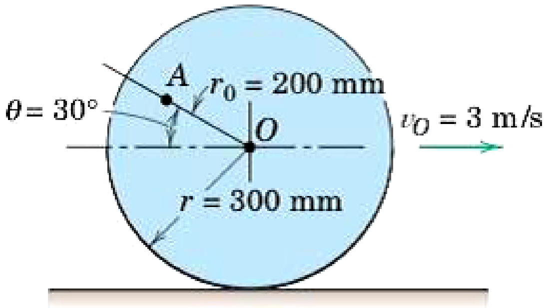
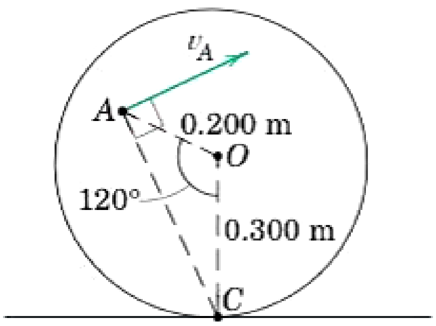
Sample Problem 5/11

The wheel of sample problem 5/7, shown again here, rolls to the right without slipping, With its center O having a velocity $v_O = 3\ m/s$. locate the instantaneous center zero velocity and use it to find the velocity of point A for the position indicated.



Solution

The point of contact with the ground has no velocity is the instantaneous center of zero velocity (no slipping)



$$\left[\omega = \frac{v}{r}\right],\ \omega = v_O/\overline{OC} = 3/0.3 = 10\ rad/s$$

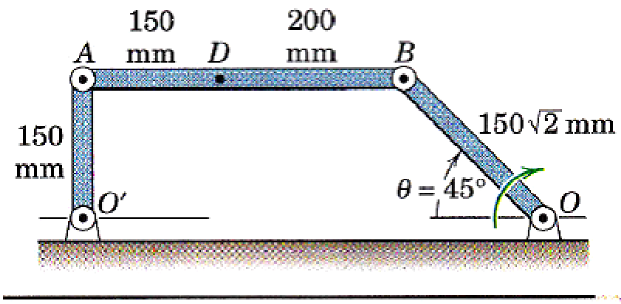
$$\overline{AC} = \sqrt{(0.300)^2 + (0.200)^2 - 2(0.3)(0.2)\cos 120^\circ} = 0.436\, m$$

$$[v = r\omega],\quad v_A = \overline{AC}\,\omega = 4.36\ m/s\quad Ans.$$

The direction of v_A is perpendicular to AC as shown.

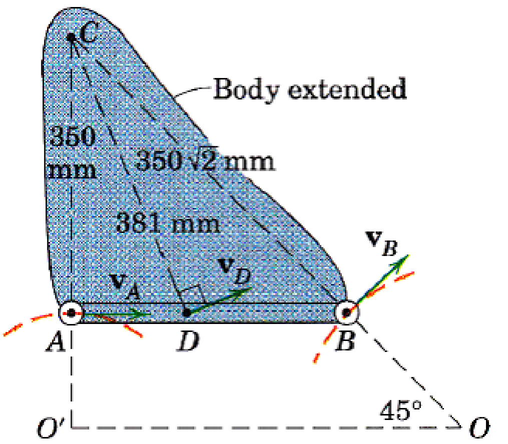
Sample Problem 5/12

Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where θ = 45°. Determine the velocity of A, the velocity of D, and the angular velocity of link AB for the position shown.



Solution

The directions of the velocities v_A and v_B are tangent to their paths. The instantaneous center of zero velocity C is determined by the intersection of the extended perpendicular lines from the two velocities.



$$\left[\omega = \frac{v}{r}\right] \qquad \omega_{AB} = \omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{\overline{OB}\,\omega_{OB}}{\overline{BC}}$$

$$= \frac{150\sqrt{2}\times 10}{350\sqrt{2}} = 4.29\ rad/s\ CCW\ Ans.$$

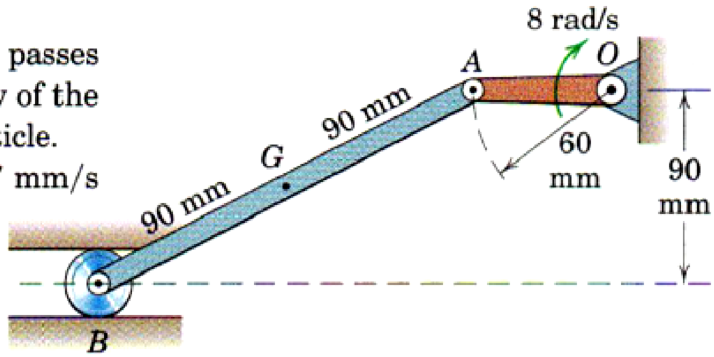
$$[v = r\omega] \qquad v_A = 0.350(4.29)$$
$$= 1.5\ m/s \quad Ans.$$

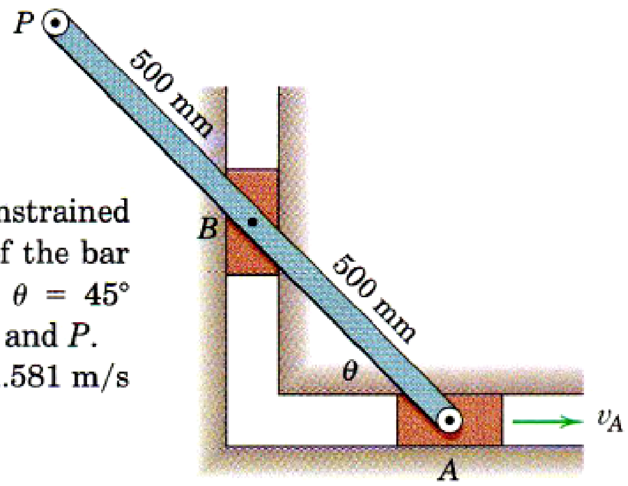
$$v_D = 0.381(4.29)$$
$$= 1.632\ m/s \quad Ans.$$

5/93

For the instant represented, when crank *OA* passes the horizontal position, determine the velocity of the center *G* of link *AB* by the method of this article.

Ans. v_G = 277 mm/s





5/95

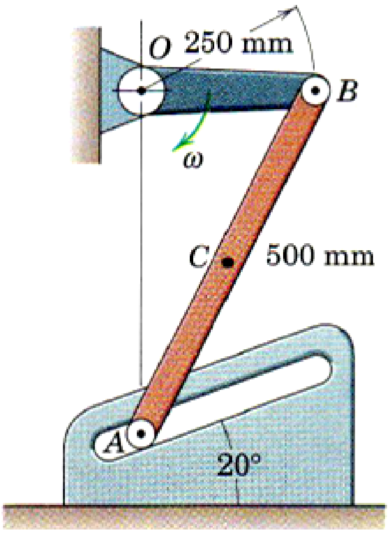
Motion of the bar is controlled by the constrained paths of *A* and *B*. If the angular velocity of the bar is 2 rad/s counterclockwise as the position θ = 45° is passed, determine the speeds of points *A* and *P*.

Ans. v_A = 0.707 m/s, v_P = 1.581 m/s

5/97

The linkage of Prob. 5/76 is repeated here. At the instant represented, crank OB has a clockwise angular velocity $\omega = 0.8$ rad/s and is passing the horizontal position. By the method of this article, determine the corresponding velocity of the guide roller A in the 20° slot and the velocity of point C midway between A and B.

Ans. $v_A = 226$ mm/s, $v_C = 174.7$ mm/s



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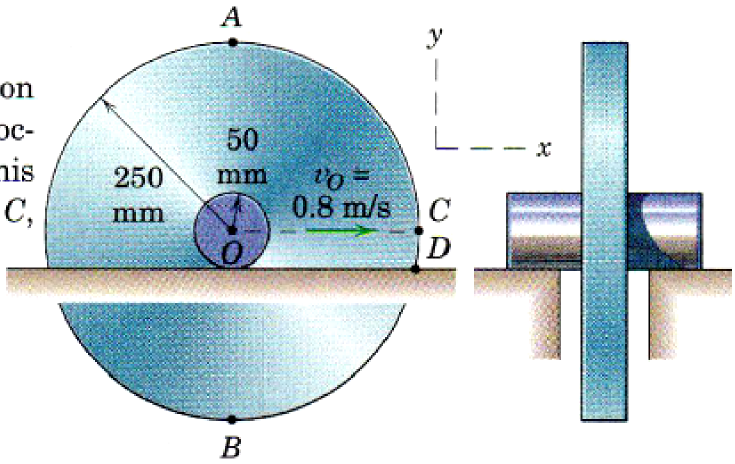
The shaft of the wheel unit rolls without slipping on the fixed horizontal surface, and point O has a velocity of 0.8 m/s to the right. By the method of this article, determine the velocities of points A, B, C, and D.



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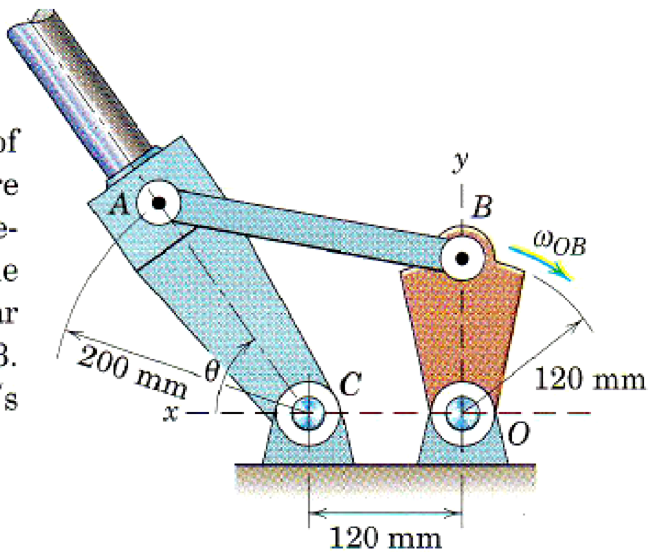
The elements of the mechanism for deployment of a spacecraft magnetometer boom are repeated here from Prob. 5/81. By the method of this article, determine the angular velocity of the boom when the driving link OB crosses the y-axis with an angular velocity $\omega_{OB} = 0.5$ rad/s if at this instant $\tan\theta = 4/3$.

Ans. $\boldsymbol{\omega}_{CA} = 0.429\mathbf{k}$ rad/s



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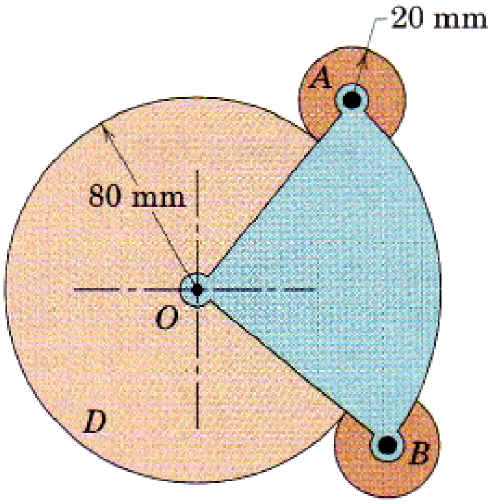
The gear D (teeth not shown) rotates clockwise about O with a constant angular velocity of 4 rad/s. The 90° sector AOB is mounted on an independent shaft at O, and each of the small gears at A and B meshes with gear D. If the sector has a counterclockwise angular velocity of 3 rad/s at the instant represented, determine the corresponding angular velocity ω of each of the small gears.



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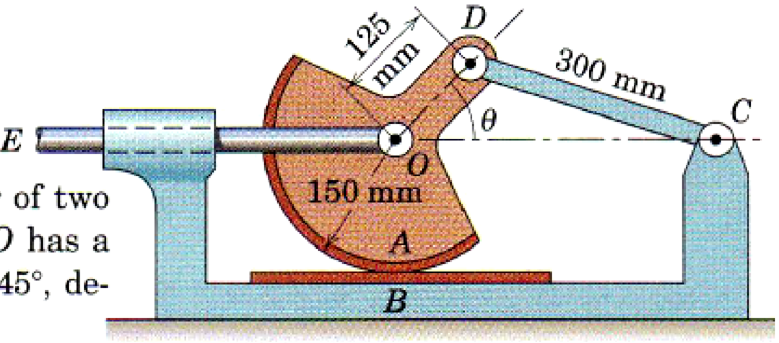
A device which tests the resistance to wear of two materials A and B is shown. If the link EO has a velocity of 1.2 m/s to the right when $\theta = 45°$, determine the rubbing velocity v_A.



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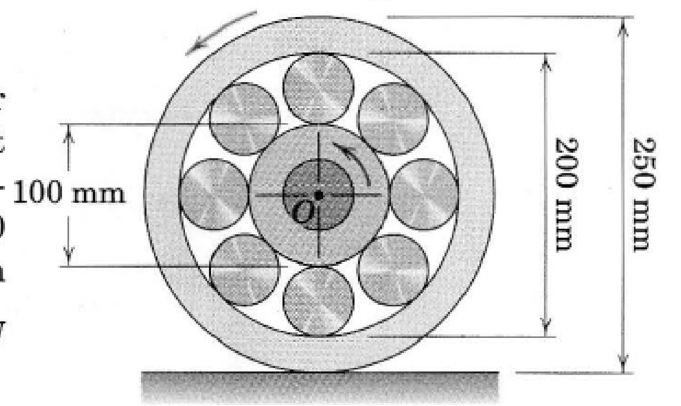
The large roller bearing rolls to the left on its outer race with a velocity of its center O of 0.9 m/s. At the same time the central shaft and inner race rotate counterclockwise with an angular speed of 240 rev/min. Determine the angular velocity ω of each of the rollers.

Ans. ω = 10.73 rad/s CW



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	18th lecture	Bakr Noori Alhasan/Lecturer
	The major contents Chapter 5 5/4 Relative Acceleration Relative Acceleration Due to Rotation Solution of the Relative Acceleration Equation		
	The detailed contents Chapter 5 5/4 Relative Acceleration Relative Acceleration Due to Rotation Interpretation of the Relative Acceleration Equations Solution of the Relative Acceleration Equation Sample Problem 5/13 Sample Problem 5/14		

Chapter 5
5/4 Relative Acceleration

Consider the equation $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$. By differentiation, $\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

Relative Acceleration Due to Rotation.

Let A and B are two points located on the same rigid body in plane motion. The distance r between them remains constant.
A is observed by B as moving in circular motion about B (Art. 5/4 Relative Velocity)

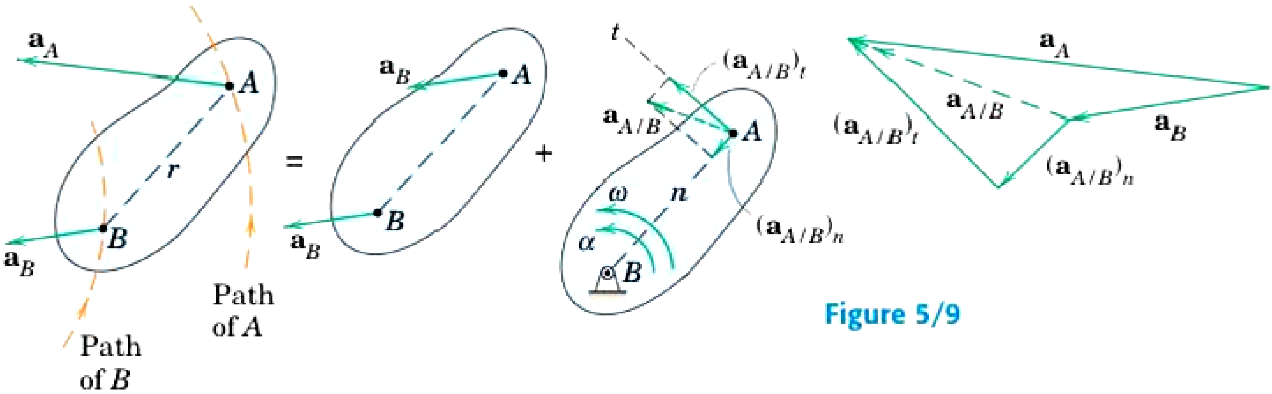


Figure 5/9

As the relative motion is circular, the relative acceleration term has two components:

1. Normal component directed from A toward B (due to the change in direction).
2. Tangential component normal to the line $\overline{AB}$ due to the change in the magnitude of $\vec{v}_{A/B}$.

These two components are mentioned in Eqs. 5/2 and earlier in Art. 2/5.

$$\vec{a}_A = \vec{a}_B + \left(\vec{a}_{A/B}\right)_n + \left(\vec{a}_{A/B}\right)_t \quad \text{........ 5/8}$$

$$\left(a_{A/B}\right)_n = \frac{\left(v_{A/B}\right)^2}{r} = r\omega^2$$

$$\left(a_{A/B}\right)_t = \left(\dot{v}_{A/B}\right) = r \propto \quad \text{... ... 5/9}$$

In vector notation: $\left(\vec{a}_{A/B}\right)_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$$\left(\vec{a}_{A/B}\right)_t = \vec{\propto} \times \vec{r}$$

where:
$\vec{\omega}$ *is the angular velocity of the body.*

$\vec{\propto}$ *is the angular acceleration of the body.*
$\vec{r}$ *is the vector locating A from B*
Hence, acceleration terms depend on ω *and* $\propto$ *.*

Interpretation of the Relative Acceleration Equations

The figure 5/9 shows a rigid body in plane motion and A and B are any two points moving in separate curved paths with absolute $\vec{a}_A$ *and* $\vec{a}_B$

Note that the acceleration of A is composed of two parts: the acceleration of B and the acceleration of A with respect to B.

To know the correct sense of the two components of the relative acceleration, the reference is shown as fixed.

Alternatively, the acceleration of B may be expressed in terms of the acceleration of A which puts the non-rotating reference axis on A rather than B. This order gives.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Here $\vec{a}_{B/A}$ and its n- and t- components are the negatives of $\vec{a}_{A/B}$ and its n- and t- components.

Solution of the Relative Acceleration Equation.

1. Scalar Geometric
2. Vector Algebra
3. Graphical solution

Making a sketch of vector polygon according to the vector equation.
Paying attention to the head- to - tail combination of vectors so that it agrees with the relative equation.
known vectors should be drawn first and the unknown vectors will become the closing legs of the vector polygon.
Since the normal components of acceleration depend on velocity, it's necessary to solve for velocities before acceleration.
Choose the reference point (in the relative equation) as a point on the body whose acceleration is known or can be found easily

Problems

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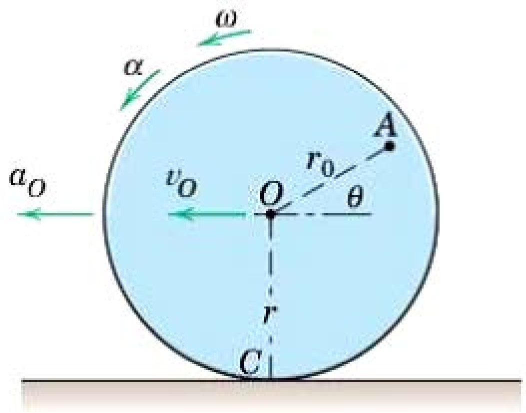
Sample Problem 5/13

The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\vec{v}_0$ and an acceleration $\vec{a}_0$ to the left. Determine the acceleration of point A and C on the wheel for the instant considered.



Solution

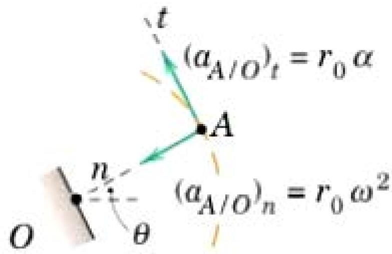
From sample problem 5/4 we know:

$$\omega = v_O/r \quad and \quad \alpha = a_O/r$$

$$\vec{a}_A = \vec{a}_O + \vec{a}_{A/O}$$

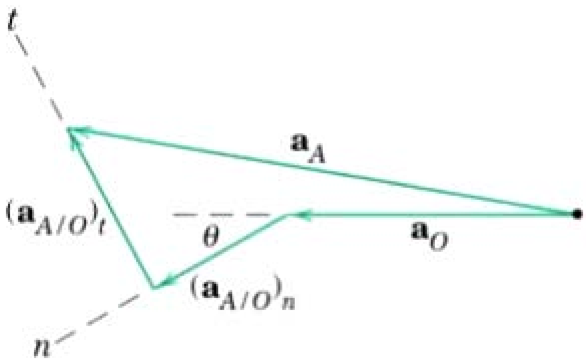
$$\vec{a}_A = \vec{a}_O + \left(\vec{a}_{A/O}\right)_n + \left(\vec{a}_{A/O}\right)_t$$

$$\left(a_{A/O}\right)_n = r\omega^2 = r_O\,(v_O/r)^2$$



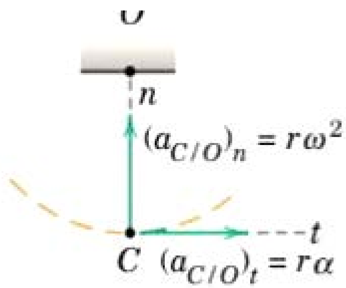
$$\left(a_{A/O}\right)_t = r \propto = \ r_O\,(a_O/r)$$

Adding the vectors head - to -tail gives $\vec{a}_A$ as shown.



$$a_A = \sqrt{(a_A)_n{}^2 + (a_A)_t{}^2}$$

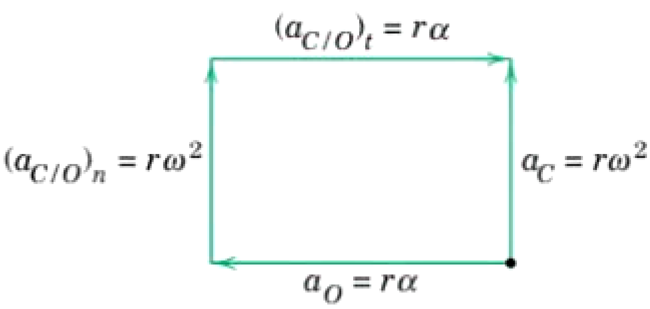
$$a_A = \sqrt{\left[a_O\cos\theta + \left(a_{A/O}\right)_n\right]^2 + \left[a_O\sin\theta + \left(a_{A/O}\right)_t\right]^2}$$



$$a_A = \sqrt{(r\alpha\cos\theta + r_O\omega^2)^2 + (r\alpha\sin\theta + r_O\alpha)^2} \qquad Ans.$$

$$\vec{a}_C = \vec{a}_O + \vec{a}_{C/O}$$

$$a_C = r\omega^2 \quad Ans.$$



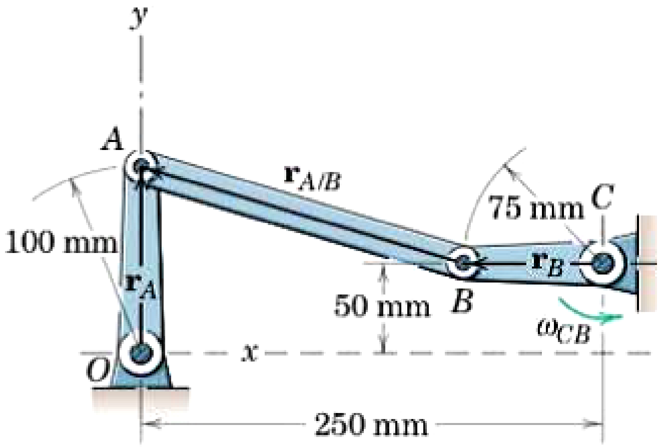
Sample Problem 5/14

The linkage of sample problem 5/8 is repeated here. Crank CB has a constant counterclockwise angular velocity of 2 rad/s in the position shown during a short interval of its motion. Determine the angular acceleration of links AB and OA for this position. Solve by using vector algebra.

Solution

(Refer sample problem 5/8 for angular velocities)

$$\omega_{AB} = -\frac{6}{7} rad/sec \;\; and \; \omega_{OA} = -\frac{3}{7} rad/sec$$



$$\vec{a}_A = \vec{a}_B + \left(\vec{a}_{A/B}\right)_n + \left(\vec{a}_{A/B}\right)_t$$

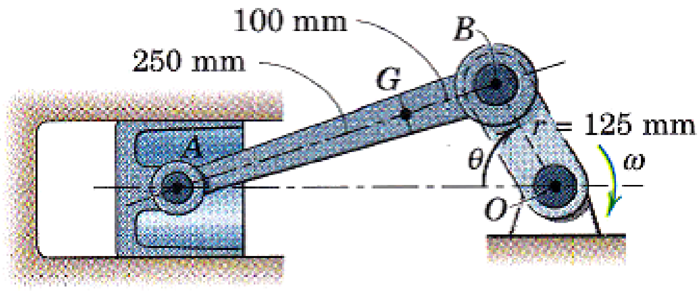
$$\vec{a}_A = \vec{\alpha}_{OA} \times \vec{r}_A + \vec{\omega}_{OA} \times (\vec{\omega}_{OA} \times \vec{r}_A)$$

-
-
-

Sample Problem 5/15

The slider -crank mechanism of sample problem 6/9 is repeated here. The crank OB has a constant clockwise angular speed of 1500 rev/min. For the instant when the crank angle θ is 60^o , determine the acceleration of the piston A and the angular acceleration of the connecting rod AB.

Solution



$$\vec{a}_A = \vec{a}_B + \left(\vec{a}_{A/B}\right)_n + \left(\vec{a}_{A/B}\right)_t$$

$[a_n = r\omega^2]$ $\quad a_B = 0.125(\frac{2\pi 1500}{60})^2$

$a_B = 3080\ m/s^2$

From Sample Problem 5/9 $\quad \omega_{AB} = 29.5\ rad/s$

$[a_n = r\omega^2]$ $\quad (a_{A/B})_n = 0.35(29.5)^2 = 305\,\mathrm{m/s^2}$

With the law of sine, the angle between AB and the horizontal becomes 18.02^o

Equating separately the horizontal components and the vertical components of the terms , gives

$$a_A = 3080\ cos60^o + 305\ cos18.02^o - (a_{A/B})_t\ sin18.02^o$$

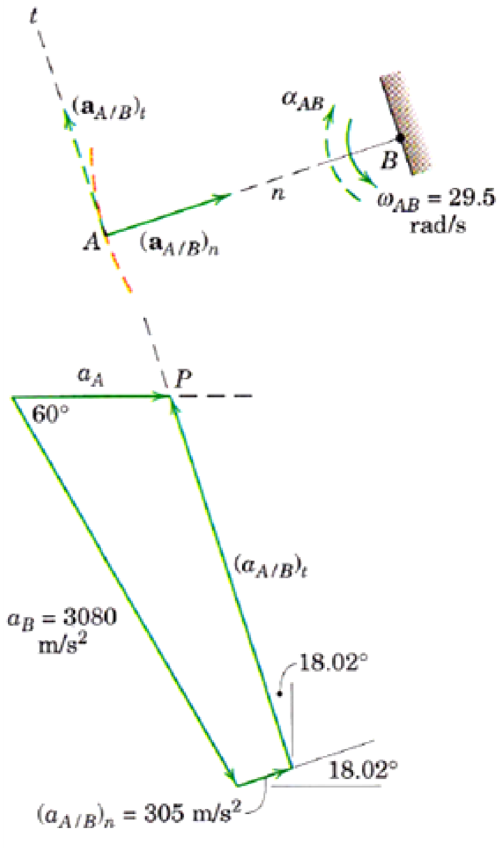
$$0 = 3080\ sin60^o - 305\ sin18.02^o - (a_{A/B})_t\ cos18.02^o$$

The solutions to these equations gives the magnitudes

$(a_{A/B})_t = 2710\ m/s^2$

$a_A = 994\ m/s^2 \quad Ans$

$[\alpha = a_t/r]$ $\quad \alpha_{AB} = 7742.8\ rad/s^2\ CW \quad Ans.$



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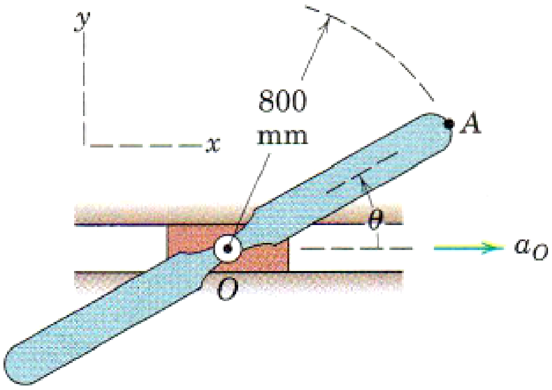
The two rotor blades of 800-mm radius rotate counterclockwise with a constant angular velocity $\omega = \dot{\theta} = 2$ rad/s about the shaft at O mounted in the sliding block. The acceleration of the block is $a_O = 3$ m/s^2. Determine the magnitude of the acceleration of the tip A of the blade when (*a*) $\theta = 0$, (*b*) $\theta = 90°$, and (*c*) $\theta = 180°$. Does the velocity of O or the sense of ω enter into the calculation?



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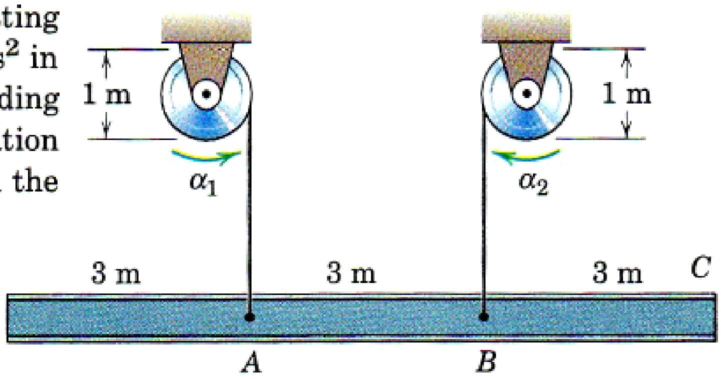
The 9-m steel beam is being hoisted from its horizontal position by the two cables attached at A and B. If the initial angular accelerations of the hoisting drums are $\alpha_1 = 0.5$ rad/s^2 and $\alpha_2 = 0.2$ rad/s^2 in the directions shown, determine the corresponding angular acceleration α of the beam, the acceleration of C, and the distance b from B to a point P on the beam centerline which has no acceleration.

Ans. $\alpha = 0.05$ rad/s^2 CW, $a_C = 0.05$ m/s^2 down
$b = 2$ m right of B



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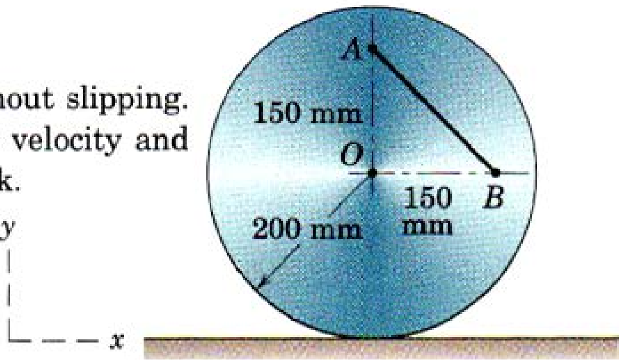
The circular disk rolls to the left without slipping. If $\mathbf{a}_{A/B} = -2.7\mathbf{j}$ m/s^2, determine the velocity and acceleration of the center O of the disk.



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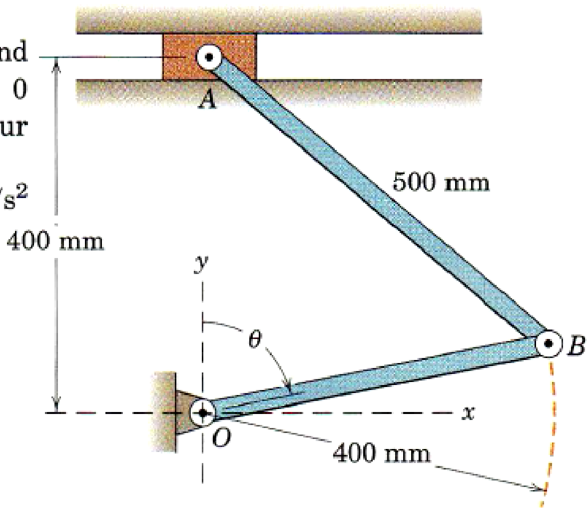
Determine the angular acceleration of link AB and the linear acceleration of A for $\theta = 90°$ if $\dot{\theta} = 0$ and $\ddot{\theta} = 3$ rad/s^2 at that position. Carry out your solution using vector notation.

Ans. $\boldsymbol{\alpha}_{AB} = -4\mathbf{k}$ rad/s^2, $\mathbf{a}_A = 1.6\mathbf{i}$ m/s^2



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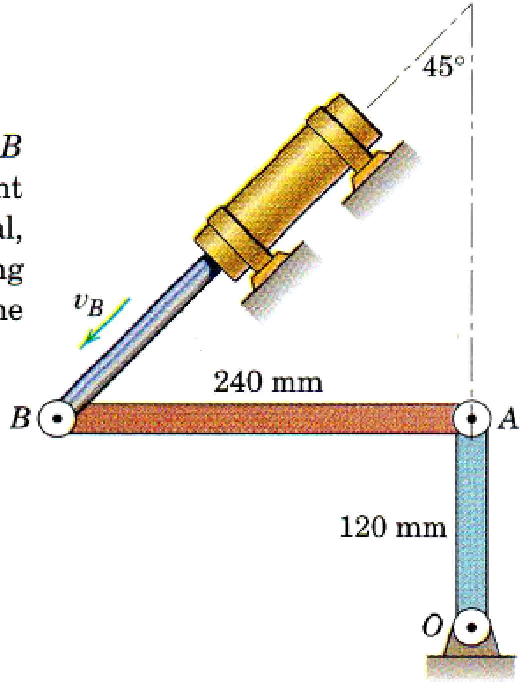
The hydraulic cylinder imparts motion to point *B* which causes link *OA* to rotate. For the instant shown where *OA* is vertical and *AB* is horizontal, the velocity v_B of pin *B* is 4 m/s and is increasing at the rate of 20 m/s^2. For this position determine the angular acceleration of *OA*.



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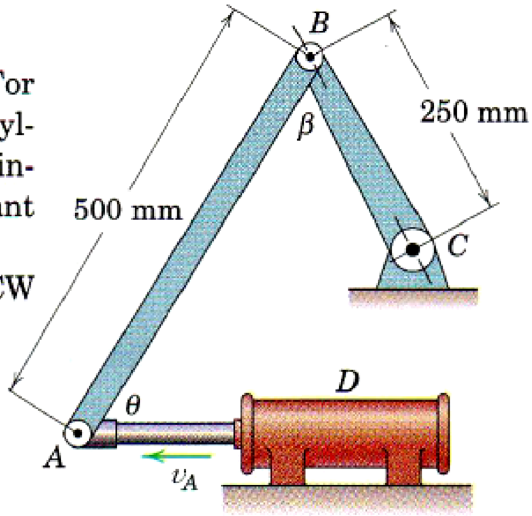
The linkage of Prob. 5/69 is shown again here. For the instant when $\theta = \beta = 60°$, the hydraulic cylinder gives *A* a velocity $v_A = 1.2$ m/s which is increasing by 0.9 m/s each second. For this instant determine the angular acceleration of link *BC*.

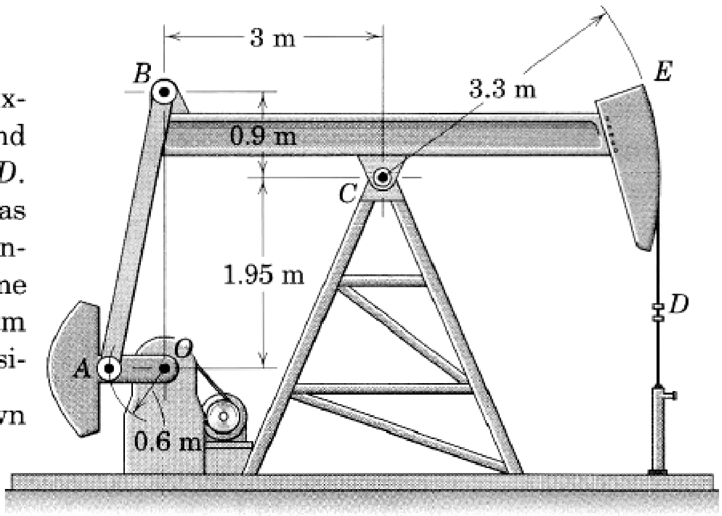
Ans. $\alpha_{BC} = 2.08$ rad/s^2 CCW



▶5/149

An oil pumping rig is shown in the figure. The flexible pump rod *D* is fastened to the sector at *E* and is always vertical as it enters the fitting below *D*. The link *AB* causes the beam *BCE* to oscillate as the weighted crank *OA* revolves. If *OA* has a constant clockwise speed of 1 rev every 3 s, determine the acceleration of the pump rod *D* when the beam and the crank *OA* are both in the horizontal position shown.

Ans. $a_D = 0.568$ m/s^2 down



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

Lecture Contents	Lecture sequences:	20th lecture	Bakr Noori Alhasan/Lecturer
	The major contents **6/4 Fixed Axis Rotation**		
	The detailed contents **6/4 Fixed Axis Rotation** **Sample Problem 6/3** **Sample Problem 6/4**		

6/4 Fixed Axis Rotation

In fixed axis rotation, all points in the body describe circles about the rotation axis.

All lines of the body in the plane motion have the same angular velocity ω and angular acceleration α .

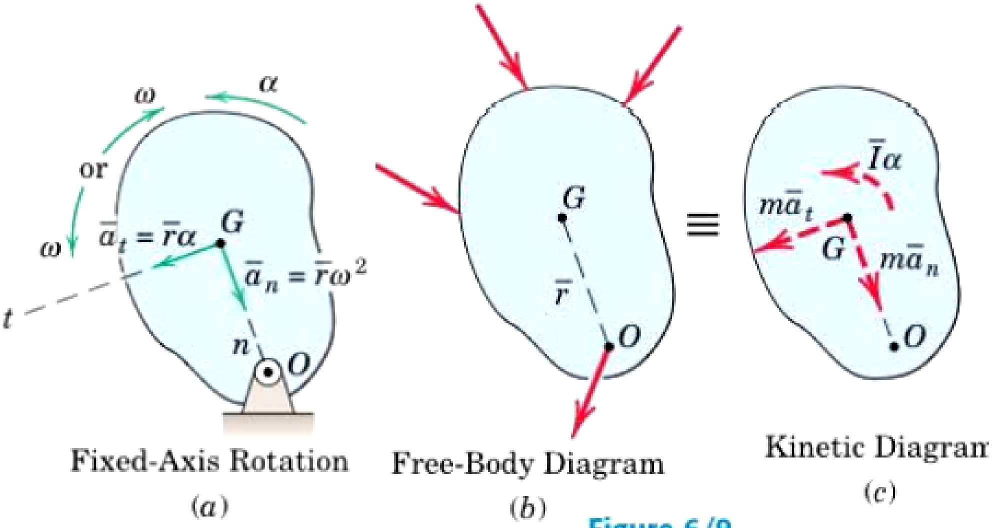


Figure 6/9

In this type of motion, the solution is easily expressed in n-t coordinates.

Recall

$$\sum \vec{F} = m\vec{\bar{a}}$$
$$\sum M_G = \bar{I}\alpha \qquad \text{........ 6/1}$$

Scalars:

$$\sum F_n = m\bar{a}_n = m\bar{r}\omega^2 \qquad \text{and} \qquad \sum F_t = m\bar{r}\alpha$$

In fixed axis rotation, it's generally useful to apply a moment equation about "O"

$$\sum M_O = I_O\alpha \qquad \text{.......... 6/4}$$

Equation 6/4 can be derived with the aid of Fig. 6/9 c ,

$$\sum M_O = \bar{I}\alpha + m\,\bar{a}_t\bar{r} \qquad (I_O = \bar{I} + m\bar{r}^2 \quad leads\ to \sum M_O = I_O\alpha)$$

In the case of a rigid body rotates about a fixed axis G,

$$\bar{a} = 0 \;\rightarrow\; \sum \bar{F} = 0 \qquad and \; \sum M_G = \bar{I}\alpha$$

To find a point say "Q" have the unique property that the resultant of all forces applied to the body must pass through, the $m\bar{r}\alpha$ be moved to this point :

$$\sum M_G = \bar{I}\alpha \quad \rightarrow \quad m\bar{r}\alpha(q - \bar{r}) = \bar{I}\alpha$$

$$m\bar{r}\alpha q = \bar{I}\alpha + m\bar{r}\alpha(\bar{r})$$

$$m\bar{r}\alpha q = I_o\alpha$$

$$m\bar{r}\alpha q = k_O^2 m\alpha$$

So $q = \frac{k_O^2}{\bar{r}}$

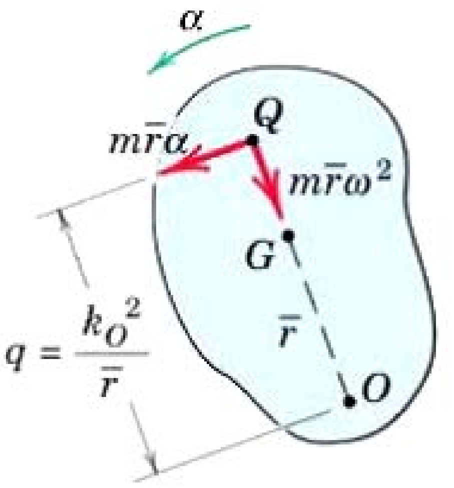


Figure 6/10

Q is called the center of percussion and $\sum M_Q = 0$ always.

Problems

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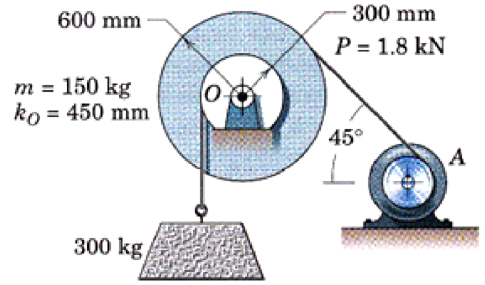
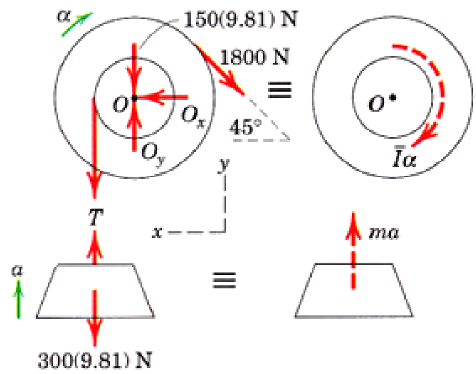
Sample Problem 6/3



The 300-kg concrete block is elevated by the hoisting mechanism shown, where the cables are securely wrapped around the respective drums. The drums which are fastened together and turn as a single unit about their mass center at O, have a combined mass of 150-kg and a radius of gyration about O of 450 mm. If a constant tension P of 1.8 kN is maintained by the power unit at A, determine the vertical acceleration of the block and the resultant force on the bearing at O.



Solution I:

$[I = k^2 m]$ $\quad \bar{I} = I_O = (0.450^2)150 = 30.4\ kg.m^2$

$[\sum M_G = \bar{I}\alpha]$ $\quad 1800(0.6) - T(0.300) = 30.4\alpha \qquad (a)$

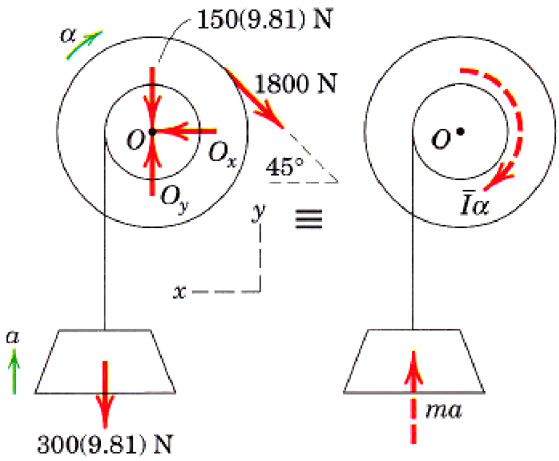
The acceleration of the block is described by

$[\sum F_y = ma_y]$ $\quad T - 300(9.81) = 300\ a \qquad (b)$

From $a_t = r\alpha$, we have $a = 0.3\ \alpha$. With this substitution, Eqs. (a) and (b) are combined to give

$T = 3250\ N \qquad \alpha = 3.44\ rad/s^2 \qquad a = 1.031\ m/s^2 \qquad Ans.$

The bearing reaction is computed from its components. Since $\bar{a} = 0$, we use the equilibrium equations



$[\sum F_x = 0]$ $\quad O_x - 1800\cos 45^o = 0 \quad O_x = 1273\ N$

$[\sum F_y = 0]$ $\quad O_y - 150(9.81) - 3250 - 1800\sin 45^o =$
0

$O_y = 6000\ N$

$O = \sqrt{1273^2 + 6000^2} = 6130\ N \qquad Ans$

Solution II

By drawing the free-body diagram of the entire system and from the kinetic diagram for this system we have

$[\sum M_O = \bar{I}\alpha + m\bar{a}d]$ $\quad 1800(0.6) - 300(9.81)(0.3) = 30.4\alpha + 300a(0.3)$

With $a = 0.3\alpha$, *the solution gives, as before,* $a = 1.031\ m/s^2$

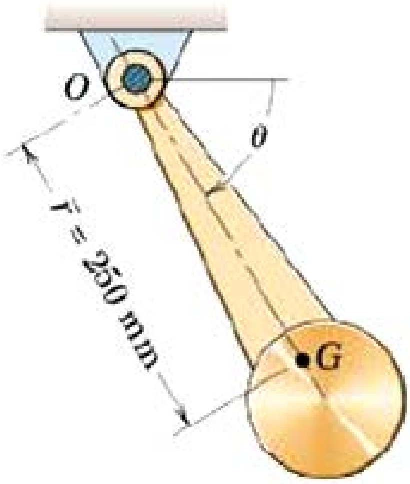
$[\sum F_y = \sum m\,\bar{a}_y]$ $\quad O_y - 150(9.81) - 300(9.81) - 1800\sin 45^o = 150(0) + 300(1.031)$

$O_y = 6000\ N$

$[\sum F_x = \sum m\,\bar{a}_x]$ $\quad O_x - 1800\cos 45^o = 0 \qquad O_x = 1273\ N$

Sample Problem 6/4

The pendulum has a mass of 7.5-kg with center of mass at G and has a radius of gyration about the pivot O of 295 mm. If the pendulum is released from rest at $\theta = 0$, determine the total force supported by the bearing at the instant when $\theta = 60^o$. Friction in the bearing is negligible.



Solution

The moment equation about O gives

$[\sum M_O = I_O\alpha]$ $\quad 7.5(9.81)0.25\cos\theta = (0.295)^2(7.5)\alpha$

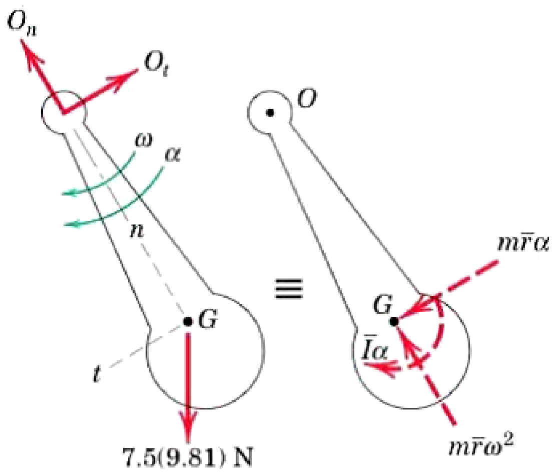
$$\alpha = 28.2\cos\theta \quad rad/s^2$$

and for $\theta = 60^o$

$[\omega d\omega = \alpha d\theta]$

$\int_0^{\omega} \omega d\omega = \int_0^{\frac{\pi}{3}} 28.2\cos\theta\, d\theta$

$\omega^2 = 48.8 \quad (rad/s)^2$



$\left[\sum F_n = m\bar{r}\omega^2\right]$ $\quad O_n - 7.5(9.81)\sin 60^o$

$$= 7.5(0.25)(48.8)$$

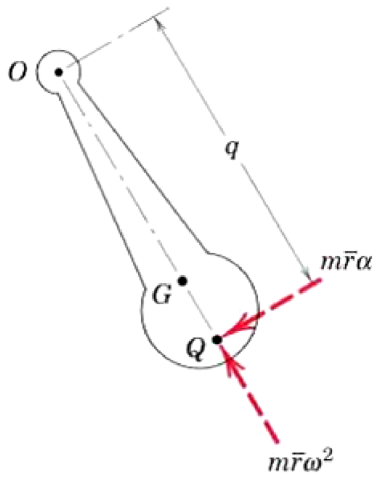
$$O_n = 155.2\ N$$

$[\sum F_t = m\bar{r}\alpha]$

$-O_t + 7.5(9.81)\cos 60^o =$
$7.5(0.25)(28.2)\cos 60^o$

$$O_t = 10.37\ N$$

$\sqrt{155.2^2 + 10.37^2} = 155.6\ N \quad Ans.$



The force O_t may also be obtained initially by a moment equation about the center of percussion Q

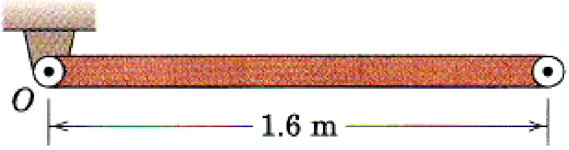
$[q = k_O{}^2/\bar{r}]$ $\quad q = \frac{(0.295)^2}{0.250} = 0.348\ m$

$[\sum M_Q = 0]$ $\qquad Q_t(0.348) - 7.5(9.81)(\cos 60^o)(0.348 - 0.25) = 0$

$$O_t = 10.37\ N \qquad Ans.$$

6/33

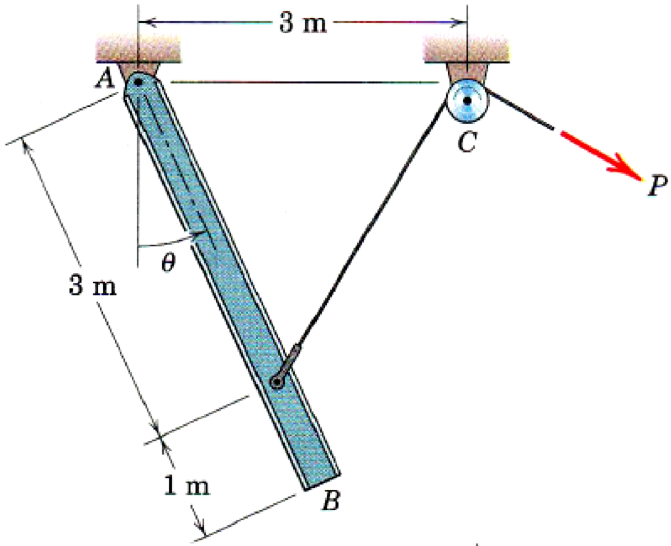
The uniform 20-kg slender bar is pivoted at O and swings freely in the vertical plane. If the bar is released from rest in the horizontal position, calculate the initial value of the force R exerted by the bearing on the bar an instant after release. *Ans.* $R = 49.0$ N



6/35

The uniform 100-kg beam is freely hinged about its upper end A and is initially at rest in the vertical position with $\theta = 0$. Determine the initial angular acceleration α of the beam and the magnitude F_A of the force supported by the pin at A due to the application of a force $P = 300$ N on the attached cable.

Ans. $\alpha = 1.193$ rad/s^2, $F_A = 769$ N



6/36

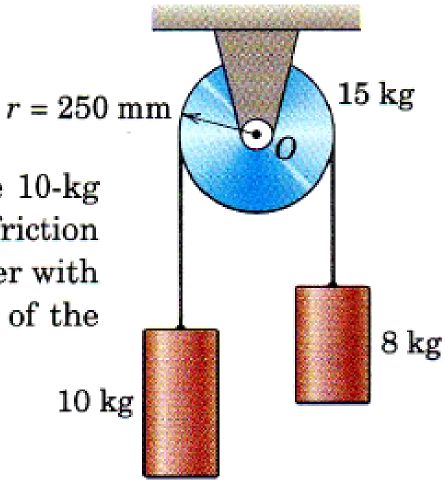
Explain why a baseball bat should be swung so that the impact of the ball with the bat does not occur opposite the mass center G of the bat as shown. Where should impact occur?

6/39

Calculate the downward acceleration a of the 10-kg cylinder. The drum is a uniform cylinder, and friction at the pivot is negligible. Compare your answer with that obtained by ignoring the inertia effects of the drum.

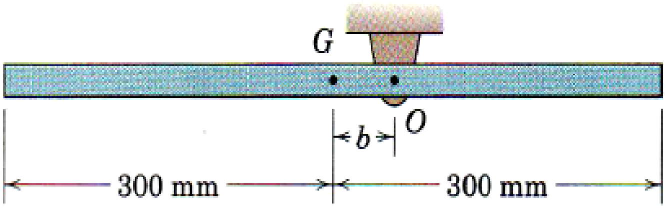
Ans. $a = 0.769$ m/s^2

With $I_O = 0$: $a = 1.090$ m/s^2



6/42

The uniform 8-kg slender bar is hinged about a horizontal axis through O and released from rest in the horizontal position. Determine the distance b from the mass center to O which will result in an initial angular acceleration of 16 rad/s^2, and find the force R on the bar at O just after release.



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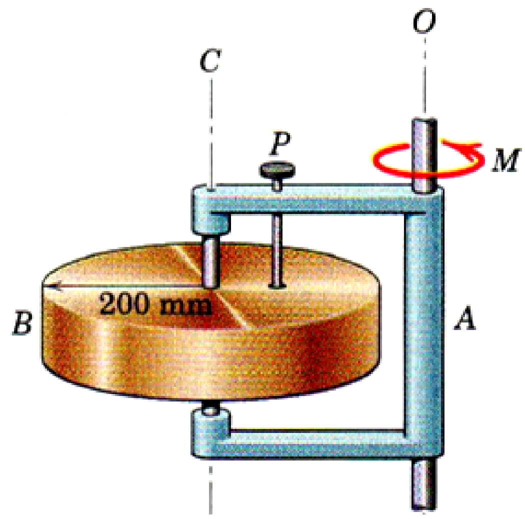
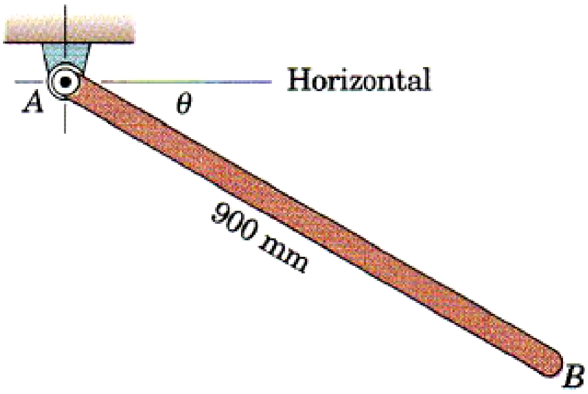
The solid cylindrical rotor B has a mass of 43 kg and is mounted on its central axis C-C. The frame A rotates about the fixed vertical axis O-O under the applied torque $M = 30$ N·m. The rotor may be unlocked from the frame by withdrawing the locking pin P. Calculate the angular acceleration α of the frame A if the locking pin is (a) in place and (b) withdrawn. Neglect all friction and the mass of the frame.



6/47

The uniform slender bar AB has a mass of 8 kg and swings in a vertical plane about the pivot at A. If $\dot{\theta} = 2$ rad/s when $\theta = 30°$, compute the force supported by the pin at A at that instant.

Ans. $A = 56.3$ N



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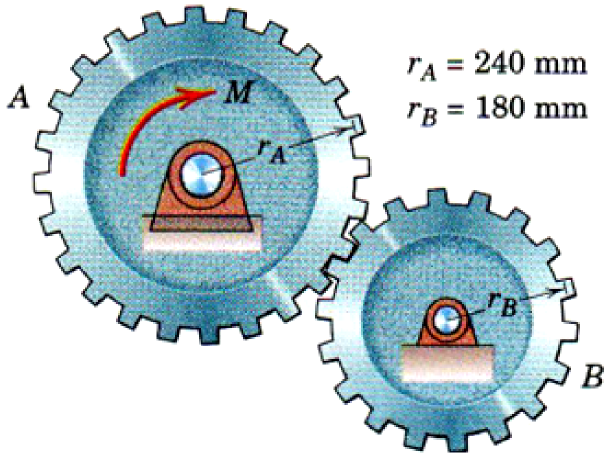
The mass of gear A is 20 kg and its centroidal radius of gyration is 150 mm. The mass of gear B is 10 kg and its centroidal radius of gyration is 100 mm. Calculate the angular acceleration of gear B when a torque of 12 N·m is applied to the shaft of gear A. Neglect friction.

Ans. $\alpha_B = 25.5$ rad/s^2 CCW



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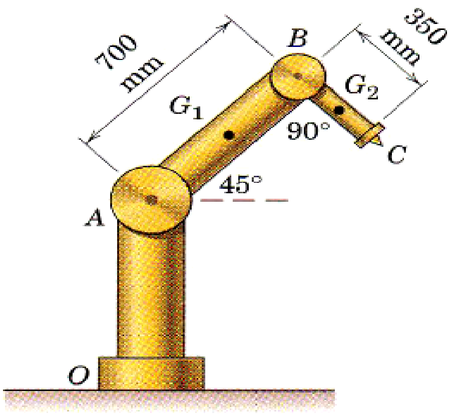
The robotic device consists of the stationary pedestal OA, arm AB pivoted at A, and arm BC pivoted at B. The rotation axes are normal to the plane of the figure. Estimate (a) the moment M_A applied to arm AB required to rotate it about joint A at 4 rad/s^2 counterclockwise from the position shown with joint B locked and (b) the moment M_B applied to arm BC required to rotate it about joint B at the same rate with joint A locked. The mass of arm AB is 25 kg and that of BC is 4 kg, with the stationary portion of joint A excluded entirely and the mass of joint B divided equally between the two arms. Assume that the centers of mass G_1 and G_2 are in the geometric centers of the arms and model the arms as slender rods.



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

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<tr><td rowspan="3">Lecture Contents</td><td>Lecture sequences:</td><td>21st lecture</td><td>Bakr Noori Alhasan/Lecturer</td></tr>
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Constrained Versus Unconstrained Motion</td></tr>
<tr><td colspan="3">The detailed contents
6/5 General Plane Motion
Constrained Versus Unconstrained Motion
Number of Unknowns
Identification of the Body or System
Kinematics
Consistency of Assumptions
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Sample Problem 6/6
Sample Problem 6/7</td></tr>
</table>

6/5 General Plane Motion

The general plane motion combines translation and rotation motion.

$$\sum \vec{F} = m\vec{\bar{a}}$$
$$\sum M_G = \bar{I}\alpha \quad \text{……….} \quad 6/1$$

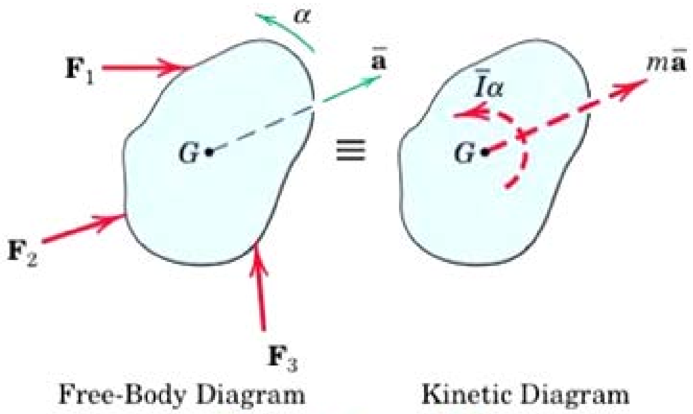


Figure 6/4, repeated

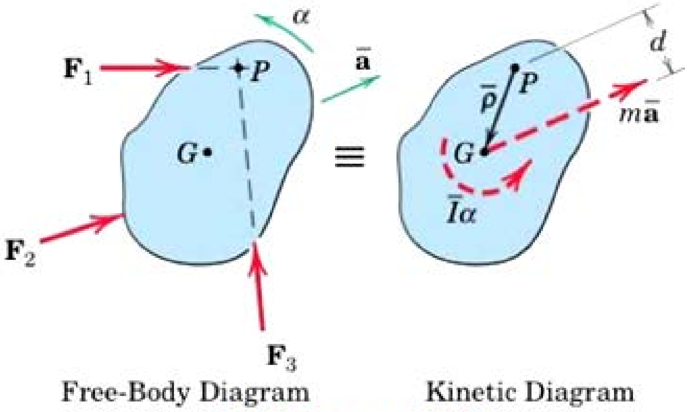


Figure 6/5, repeated

Solving Plane Motion Problems:

1. Choice of Coordinates:

Choose coordinates that describes the acceleration of the center of the body; rectangular, normal, tangential, … etc.

2. Choice of Moment Equation: $\sum M_P = \bar{I}\alpha + m\bar{a}d$

The other alternative moment equation about the non-accelerating point 'O' can be used as well.

Constrained Versus Unconstrained Motion

In constrained motion, the kinematic relationship between the linear acceleration and angular acceleration should be constructed and be incorporated into force and moment equations of motion.

In unconstrained motion, the acceleration can be determined independently by the three equations of motion.

Number of Unknowns

At the most, we have three scalar equations of motion plus two scalar components of the relative acceleration equation for constrained motion.

Identification of the Body or System

Choosing the body to be isolated, drawing the FBD, and Kinetic diagram.

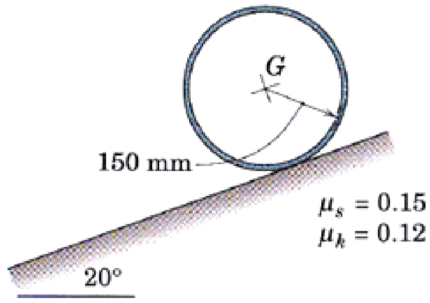
Kinematics

Consistency of Assumptions

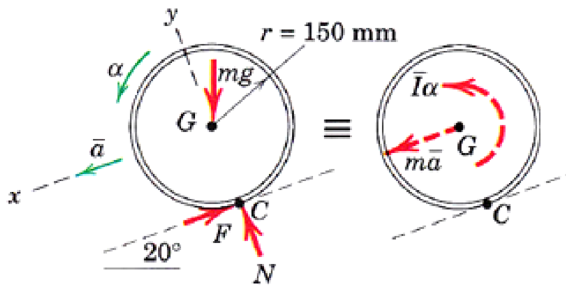
Problems 6/75 6/76 6/79 6/83 6/90 6/93 6/95 6/97 6/99 6/101

Sample Problem 6/5

A metal hoop with a radius $r = 150\ mm$ is released from rest on the 20^o incline. If the coefficients of static and kinetic friction are $\mu_s = 0.15$ and $\mu_k = 0.12$, determine the angular acceleration α of the hoop and the time t for the hoop to move a distance of 3 m down the incline.



Solution



$$\left[\sum F_x = ma_x\right]$$

$$mg\ sin\ 20^o - F = m\bar{a}\ ...\ \ (1)$$

$$\left[\sum F_y = ma_y = 0\right]$$

$$N - mg\ cos\ 20^o\ = 0\ ...\ \ (2)$$

$$\left[\sum M_G = \bar{I}\alpha\right]\quad F\ r = m\bar{r}^2\alpha \quad F = mr\alpha\ ...\ (3)$$

$$(\bar{I} = m\bar{r}^2)$$

First, assume pure rolling(no slipping), $\bar{a} = r\alpha$ (sample problem 5/4)

Equation (3) becomes $F = m\bar{a}$ in (1); $mg\ sin\ 20^o - m\bar{a} = m\bar{a}$

$$\bar{a} = \frac{g}{2}\ sin\ 20^o\ = 1.687\ \ m/s^2$$

From (3); $F = 1.687m$

And from (2); $N = mg\ cos\ 20^o$, $N = 9.22m\ \ N$

$$F_{max}\ = \mu_s\ N\ ,\ \ F_{max}\ = 0.15(9.22m) = 1.38m < F$$

Therefore, the hoop slips as it rolls and $\bar{a} \neq \bar{r}\alpha$

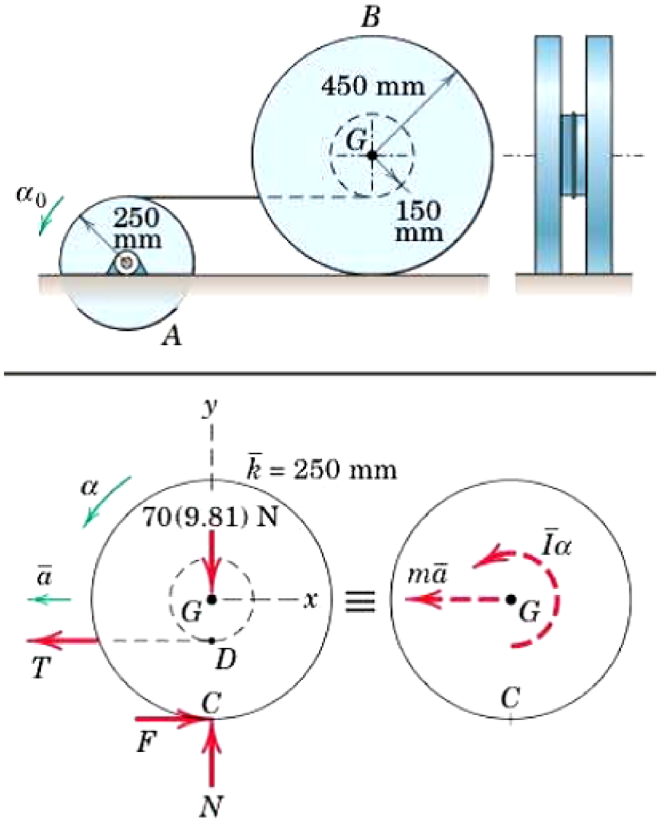
$$F = \mu_k N = 0.12(9.22)m = 1.106m$$

Refer to (1) ; $mg\ sin\ 20^o - 106m = m\bar{a}$, $\bar{a} = 2.25\ \ m/s^2\ \ Ans.$

Refer to (3); $F = m\bar{r}\alpha$, $1.106m = m(0.15)\alpha,\quad \alpha = 7.37\ \ rad/s^2\quad Ans.$

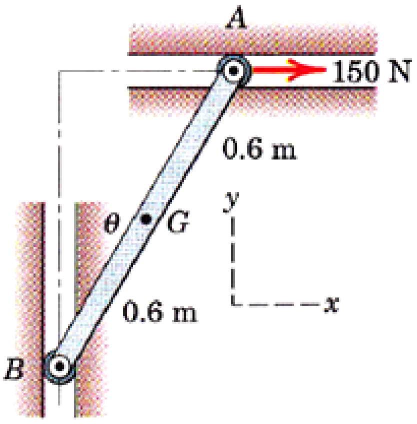
Sample Problem 6/6

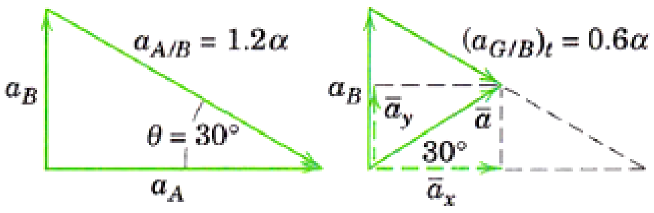
The drum A is given a constant angular acceleration α_0 of 3 rad/s^2 and causes the 70-kg spool B to roll on the horizontal surface by means of the connecting cable, which wraps around the inner hub of the spool. The radius of gyration $\bar{k}$ of the spool about its mass center G is 250 mm, and the coefficient of static friction between the spool and the horizontal surface is 0.25. Determine the tension T in the cable and the friction force F exerted by the horizontal surface on the spool.



Sample Problem 6/7

The slender 30-kg bar AB moves in the vertical plane, with its ends constrained to follow the smooth horizontal and vertical guides. If the 150 N force is applied at A with the bar initially at rest in the position for which $\theta = 30^o$, calculate the resulting angular acceleration of the bar and the forces on the small end rollers at A and B.

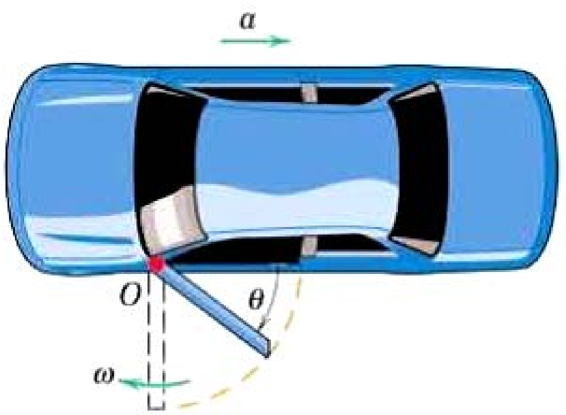


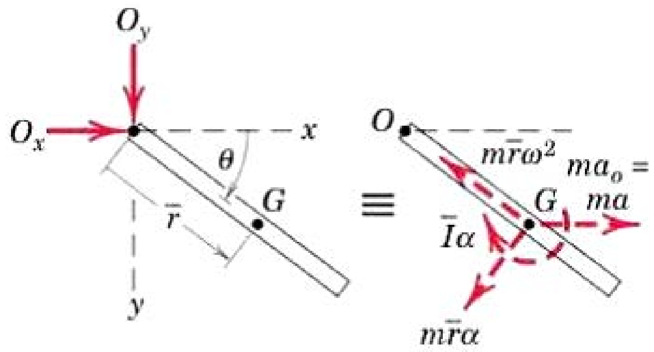


Solution

Sample Problem 6/8

A car door is inadvertently left slightly open when the brakes are applied to give the car a constant rearward acceleration a. Derive expressions for the angular velocity of the door as it swings past the 90° position and the components of the hinge reactions for any value of θ. The mass of the door is m, its mass center is a distance $\bar{r}$ from the hinge axis O, and the radius of gyration about O is k_O.

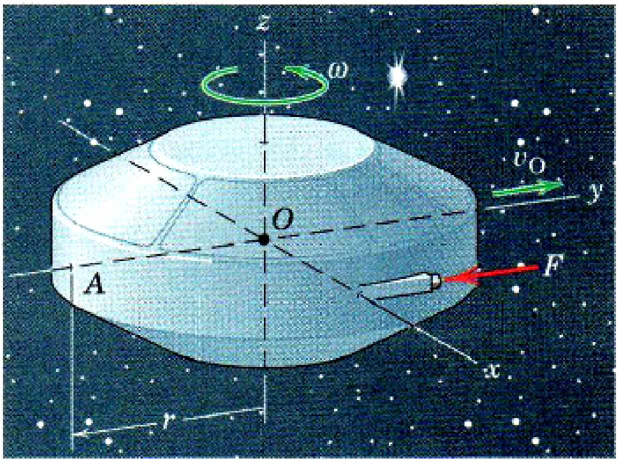


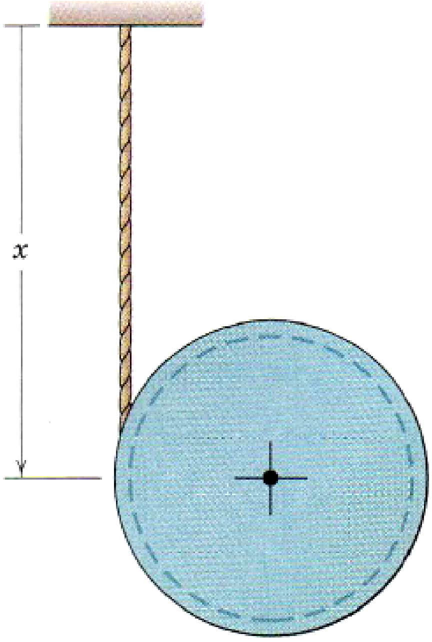


6/75

The spacecraft is spinning with a constant angular velocity ω about the z-axis at the same time that its mass center O is traveling with a velocity v_O in the y-direction. If a tangential hydrogen-peroxide jet is fired when the craft is in the position shown, determine the expression for the absolute acceleration of point A on the spacecraft rim at the instant the jet force is F. The radius of gyration of the craft about the z-axis is k, and its mass is m.

Ans. $\mathbf{a}_A = -\dfrac{Fr^2}{mk^2}\mathbf{i} - \left(\dfrac{F}{m} - r\omega^2\right)\mathbf{j}$





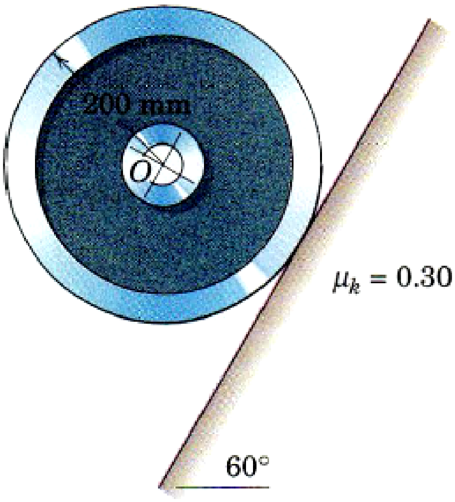
6/76

A long cable of length L and mass ρ per unit length is wrapped around the periphery of a spool of negligible mass. One end of the cable is fixed, and the spool is released from rest in the position shown. Find the initial acceleration a of the center of the spool.

6/79

The 10-kg wheel with a radius of gyration of 180 mm about its center O is released from rest on the 60° incline and slips as it rolls. If the kinetic coefficient of friction is $\mu_k = 0.30$, calculate the acceleration a_O of the center O of the wheel and its angular acceleration α.

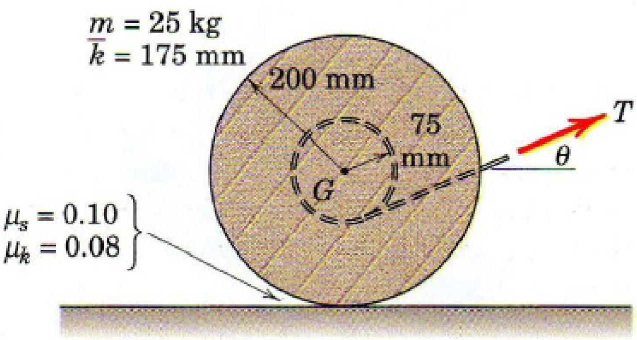
Ans. $a_O = 7.02 \text{ m/s}^2$, $\alpha = 9.08 \text{ rad/s}^2$



6/83

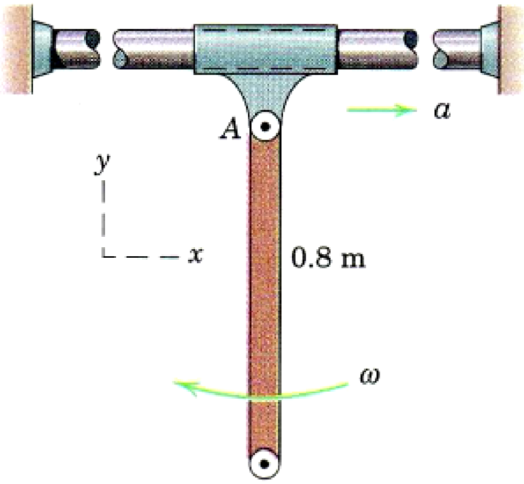
The circular disk of 200-mm radius has a mass of 25 kg with centroidal radius of gyration $\bar{k}$ = 175 mm and has a concentric circular groove of 75-mm radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If T = 30 N, θ = 0, μ_s = 0.10, and μ_k = 0.08, determine the angular acceleration α of the disk, the acceleration a of its mass center G, and the friction force F which the surface exerts on the disk.

Ans. α = 2.12 rad/s^2 CW, a = 0.425 m/s^2 right
F = 19.38 N



6/90

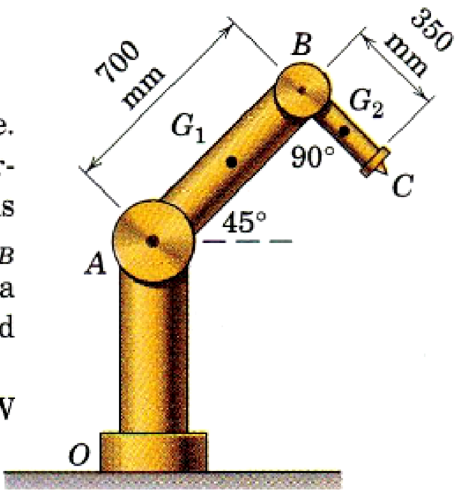
End A of the uniform 5-kg bar is pinned freely to the collar, which has an acceleration a = 4 m/s^2 along the fixed horizontal shaft. If the bar has a clockwise angular velocity ω = 2 rad/s as it swings past the vertical, determine the components of the force on the bar at A for this instant.



6/93

The robotic device of Prob. 6/54 is repeated here. Member AB is rotating about joint A with a counterclockwise angular velocity of 2 rad/s, and this rate is increasing at 4 rad/s^2. Determine the moment M_B exerted by arm AB on arm BC if joint B is held in a locked condition. The mass of arm BC is 4 kg, and the arm may be treated as a uniform slender rod.

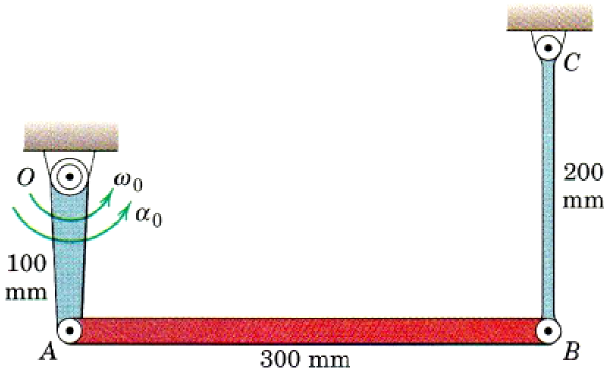
Ans. M_B = 3.55 N·m CCW



6/95

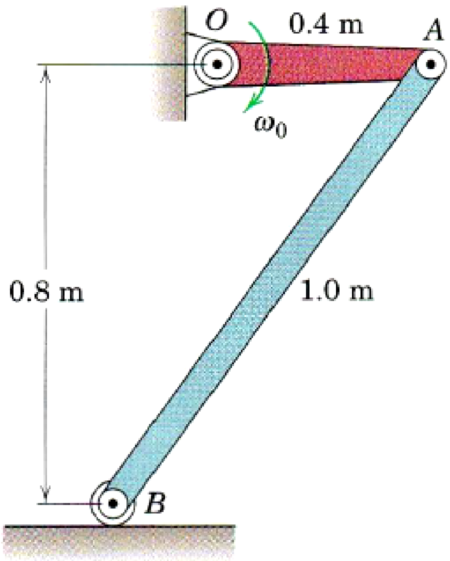
The uniform slender bar AB has a mass of 0.8 kg and is driven by crank OA and constrained by link CB of negligible mass. If OA has an angular acceleration α_0 = 4 rad/s^2 and an angular velocity ω_0 = 2 rad/s when both OA and CB are normal to AB, calculate the force in CB for this instant. (*Suggestion:* Consider the use of Eq. 6/3 with A as a moment center.)

Ans. BC = 4.03 N (tension)



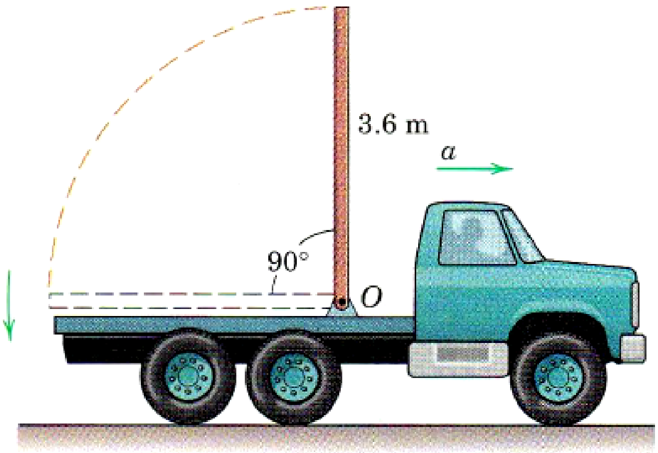
6/97
The crank OA rotates in the vertical plane with a constant clockwise angular velocity ω_0 of 4.5 rad/s. For the position where OA is horizontal, calculate the force under the light roller B of the 10-kg slender bar AB.

Ans. $B = 36.4$ N



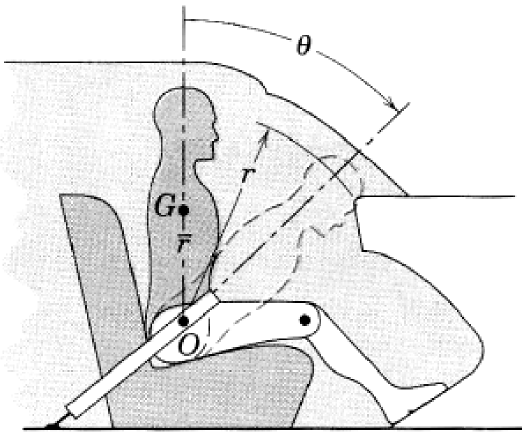
6/99
The uniform 3.6-m pole is hinged to the truck bed and released from the vertical position as the truck starts from rest with an acceleration of 0.9 m/s^2. If the acceleration remains constant during the motion of the pole, calculate the angular velocity ω of the pole as it reaches the horizontal position.

Ans. $\omega = 2.99$ rad/s



6/101 In a study of head injury against the instrument panel of a car during sudden or crash stops where lap belts without shoulder straps or airbags are used, the segmented human model shown in the figure is analyzed. The hip joint O is assumed to remain fixed relative to the car, and the torso above the hip is treated as a rigid body of mass m freely pivoted at O. The center of mass of the torso is at G with the initial position of OG taken as vertical. The radius of gyration of the torso about O is k_O. If the car is brought to a sudden stop with a constant deceleration a, determine the velocity v relative to the car with which the model's head strikes the instrument panel. Substitute the values $m = 50$ kg, $\bar{r} = 450$ mm, $r = 800$ mm, $k_O = 550$ mm, $\theta = 45°$, and $a = 10g$ and compute v.

Ans. $v = 11.73$ m/s



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

	Lecture sequences:	19th lecture	Bakr Noori Alhasan/Lecturer
Lecture Contents	*The major contents* Chapter 6 Plane Kinetics of Rigid Bodies Section A: Force, Mass and Acceleration Plane Motion Equations System of Interconnected Bodies 6/3 Translation		
	The detailed contents Chapter 6 Plane Kinetics of Rigid Bodies Background for the Study of Kinetics Section A: Force, Mass and Acceleration Plane Motion Equations Alternative Derivation Alternative Moment Equations Unconstrained and Constrained Motion System of Interconnected Bodies Analysis Procedure 6/3 Translation Sample Problem 6/1 Sample Problem 6/2		

Chapter 6
Plane Kinetics of Rigid Bodies

Kinetics of rigid bodies treats the relationships between the external forces acting on a body and the corresponding translational and rotational motions of the body.

The body can be approximated as a thin slab with its motion confined to the plane of the slab will be considered to be in plane motion.

Background for the Study of Kinetics

In Ch.3, two force equations of motion were required. For the plane motion of a rigid body, an additional equation is needed to specify the state of rotation of the body.

You should master the necessary kinematics, including the calculation of relative accelerations, before proceeding.

For problems involving the instantaneous relationships among force, mass, and acceleration or momentum, the body should be isolated with its free body diagram. In the principles of work and energy, an active force diagram may be used.

In the kinetics of rigid bodies which have angular motion, we must introduce the mass moment of inertia of the body " I".

Section A: Force, Mass and Acceleration

6/2 General Equations of Motion

With a general rigid body in three dimensions,

$\sum \vec{F} = m\vec{\bar{a}}$ $\bar{a}$ is the acceleration of the body mass center.

$\sum \vec{M}_G = \dot{\vec{H}}_G$ The moment and the angular momentum taken about the mass center.

To visualize the action of the forces and the corresponding dynamic response of the body, the equivalence between the FBD and Kinetic diagram enables us to

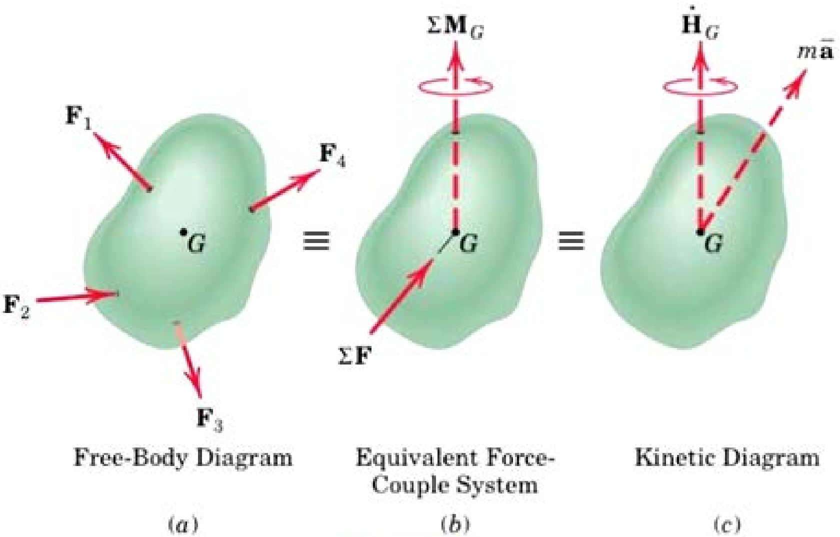


Figure 6/1

visualize the separate translational and rotational effects of the forces applied to a rigid body.

Plane Motion Equations

Consider a rigid body moving with plane motion in the x-y plane, From Ch.4

$\vec{H}_G = \sum \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i \qquad \dot{\vec{\rho}}_i = \vec{\omega} \times \vec{\rho}_i$

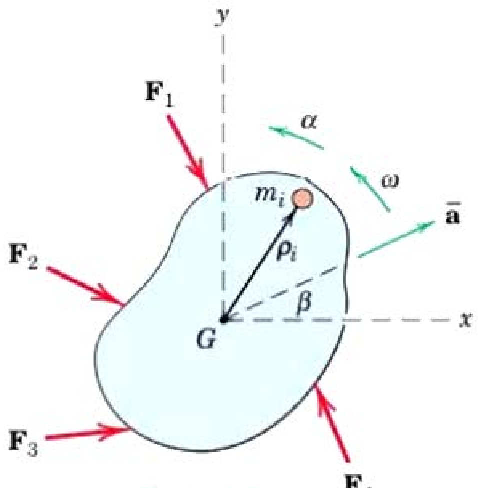


Figure 6/2

Scalar: $\quad H_G = \sum \rho_i \, m_i \omega \rho_i \qquad = \sum m_i \; \omega \; {\rho_i}^2$

$H_G = \bar{I}\omega$

$\sum M_G = \dot{H}_G = \bar{I}\alpha$

$\sum M_G = \bar{I}\alpha$

$$\boxed{\begin{aligned} \sum \vec{F} &= m\vec{\bar{a}} \\ \sum M_G &= \bar{I}\alpha \end{aligned}} \qquad \text{----------------} \quad 6/1$$

Equations 6/1 are the general equations of motion for a rigid body in plane motion.

Alternative Derivation

By referring to the forces which act on the m_i, the moment equation can be derived as well.

Refer to the figure 6/3:

$$\vec{a}_{m_i} = \vec{\bar{a}} + \rho_i \omega^2 \; \vec{e}_n + \rho_i \alpha \;\; \vec{e}_t$$

(The second term on the right hand is the relative term and G is the reference point).

The forces are:

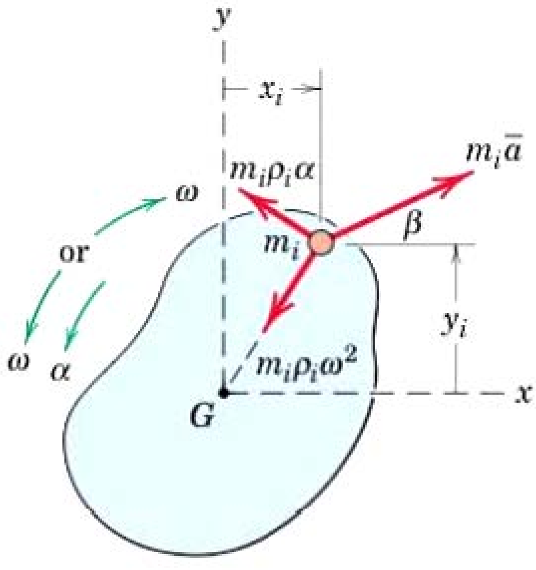


Figure 6/3

$m_i\bar{a}$, $m_i\rho_i\omega^2$, and $m_i\rho_i\alpha$

$M_{G_i} = m_i\rho_i^2\alpha + m_i\bar{a}\, sin\beta \; x_i - m_i\bar{a}\, cos\beta \; y_i$

For all particles in the body:

$$\sum M_G = \sum m_i\rho_i^2\alpha + \bar{a}\, sin\beta \sum m_i \, x_i - \bar{a}\, cos\beta \sum m_i \; y_i$$

The origin of the coordinates is taken at the mass center, hence:

$\sum m_i \, x_i = m\bar{x} = 0 \quad and \quad \sum m_i \; y_i = m\bar{y} = 0$

$\sum M_G = \sum m_i\rho_i^2\alpha = \bar{I}\alpha$

$\sum M_G = \bar{I}\alpha \qquad$ as before where, $\sum M_G$, represents the sum of moments about the mass center G of only the external forces acting on the body

The contribution of internal forces is , of course, zero (actions and reactions). Note ω has no influence on the moment equation about G because the term $m_i \rho_i^2 \omega^2$ has no moment about G.

The equations of motion $\sum \vec{F} = m\vec{\bar{a}}$ and $\sum M_G = \bar{I}\alpha$ in two dimensions (parts a and c of Fig. 6/1) are represented diagrammatically in Fig. 6/4.

The kinetic diagram discloses the resulting dynamic response in terms of translational term $m\vec{\bar{a}}$ and the rotational term $\bar{I}\alpha$.

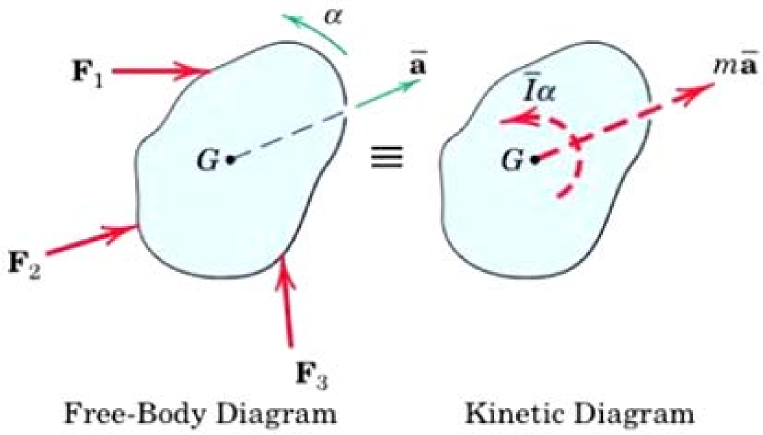


Figure 6/4

Alternative Moment Equations

The general equations for moments about an arbitrary point P,

$$\sum \vec{M}_P = \dot{\vec{H}}_G + \vec{\bar{\rho}} \times m\vec{\bar{a}}$$

$$\boxed{\sum M_P = \bar{I}\alpha + m\bar{a}d} \quad \ldots 6/2$$

All terms are positives CCW (refer to α). Point P is chosen in the way that eliminates forces as possible.

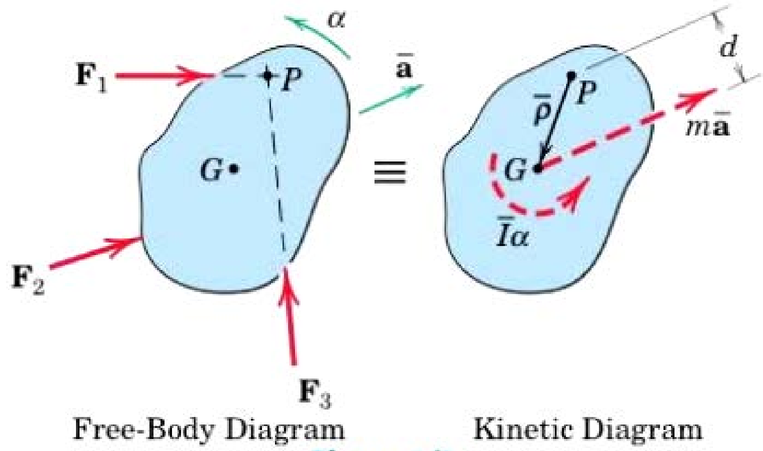


Figure 6/5

There is another moment equation about point P.

$$\boxed{\sum \vec{M}_P = I_P \vec{\alpha} + \vec{\bar{\rho}} \times m\vec{a}_P} \quad \ldots\ldots\ldots\ldots 6/3$$

When $\vec{\bar{\rho}} = 0$ point P becomes G and $\sum M_G = \bar{I}\alpha$ as before.
When point P becomes a point O fixed in an inertial reference system and attached to the body (or body extended), then $\vec{a}_P = 0$

$$\boxed{\sum \vec{M}_O = I_O \vec{\alpha}} \quad \ldots\ldots\ldots \; 6/4$$

Eq. 6/4 then applies to the rotation of a rigid body about a non-accelerating point O fixed to the body.

Unconstrained and Constrained Motion

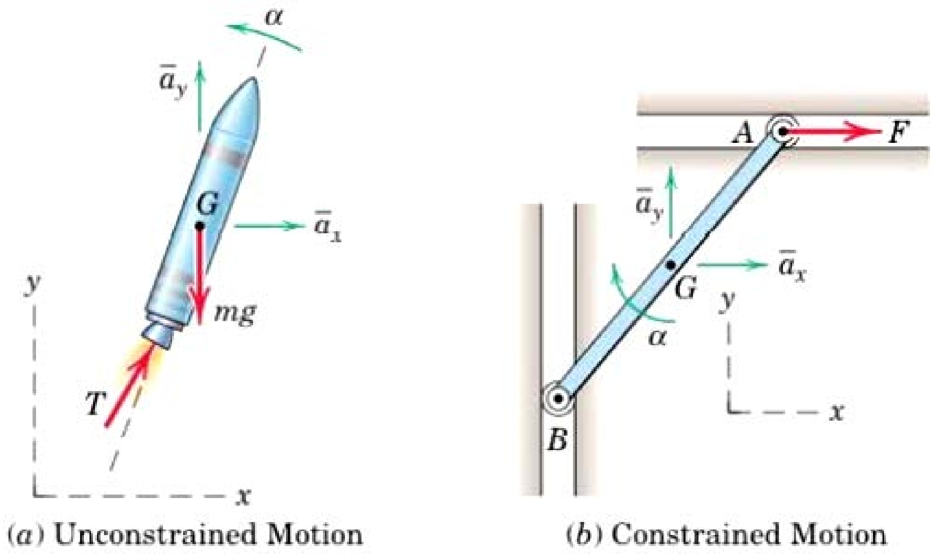


Figure 6/6

a. *Unconstrained Motion:* There are no physical confinements to the body motion.
$\bar{a}_x , \bar{a}_y , and\ \alpha\ may\ be\ determined\ independently\ of\ one\ another\ by\ direct\ application\ of Eqs. 6/1$

b. *Constrained Motion:* The kinematic analysis relating the acceleration components of the mass center to angular acceleration is combined with the force and moment equations of motion . The ends of the bar are constrained by the horizontal and vertical guides. This construction impose a kinematic relationship between $\bar{a}_x , \bar{a}_y , and\ \alpha.$

System of Interconnected Bodies

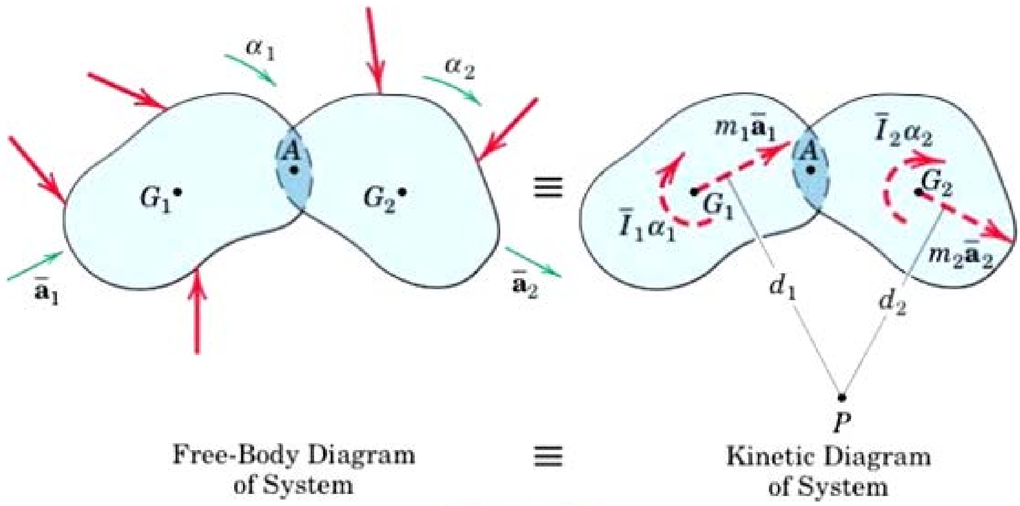


Figure 6/7

In two or more interconnected bodies whose motions are related kinematically, its convenient to analyze the bodies as an entire system.

$$\sum \vec{F} = \sum m\vec{\bar{a}}$$

$$\sum M_P = \sum \bar{I}\alpha + \sum m\bar{a}d \qquad \text{............ 6/5}$$

If there are more than three unknowns in a system, more advanced methods such as virtual work (Art. 6/7) or Lagrange's equation could be employed, or else the system could be dismembered and each part analyzed separately with the resulting equations solved simultaneously.

Analysis Procedure

The following steps should be taken once you understand the conditions and requirements of the problem:

1. First, identify the class of motion and then solve for linear and angular accelerations which can be determined from kinematics. In the case of constrained motion, it is necessary to establish the relation between the linear acceleration of the mass center and the angular acceleration of the body by first solving the relative-velocity and relative acceleration equations. The frequent review of Chapter 5 is recommended.

2. Diagrams
Draw the complete FBD of the body. Assign a convenient inertial coordinate system and label all known and unknown quantities. The kinetic diagram should be constructed to clarify the equivalence between the forces and the dynamic response.

3. Equations of Motion.
Apply the three equations of motion, being consistent with the choice of reference axes. Eqs. 6/2 or 6/3 may be employed as an alternative to the second of Eqs. 6/1. These added to the kinematic analysis. The maximum unknowns are no more than 5 unknowns that should be determined from the three scalar equations of motion obtained from Eqs. 6/1, and the two scalar component relations which come from the relative-acceleration equation.

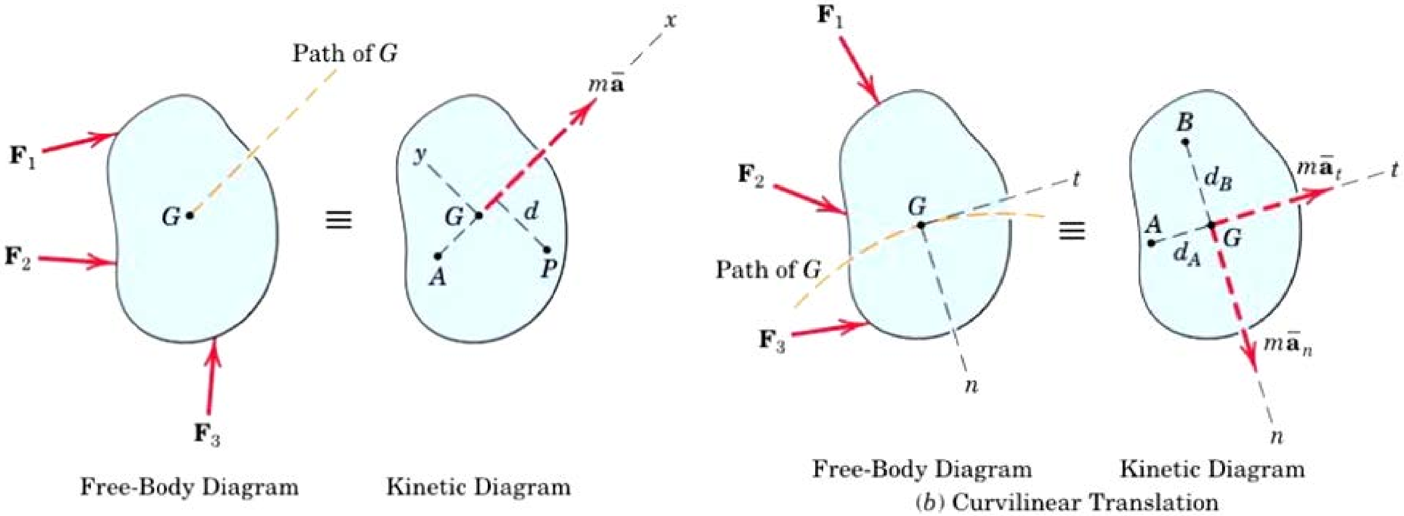
6/3 Translation

In translation of rigid body in plane motion, every line in a translating body remains parallel to its original position at all times.

1. Rectilinear translation: All points move in straight lines.
2. Curvilinear translation: All points move in congruent curved paths.

In translation motion; $\omega = 0 \;\; and \;\; \alpha = 0$

1. Rectilinear Translation: $\omega = \ \alpha = 0$ 2. Curvilinear Translation: $(\omega = \ \alpha = 0)$



(a) Rectilinear Translation
$(\alpha = 0, \ \omega = 0)$

(b) Curvilinear Translation
$(\alpha = 0, \ \omega = 0)$

Figure 6/8

If the x-axis is chosen in
the direction of the acceleration:

$\sum F_x = ma_x$

$\sum F_y = ma_y = 0$

$\sum M_G = 0$

Alternatives:
$\sum M_P = m\,\bar{a}d$
$\sum M_A = 0$

$\sum F_n = m\bar{a}_n$

$\sum F_t = m\bar{a}_t$

$\sum M_G = 0$

Alternatives:
$\sum M_A = m\,\bar{a}_n d_A$
$\sum M_B = m\,\bar{a}_t d_B$

$$\sum \vec{F} = m\vec{\bar{a}}$$

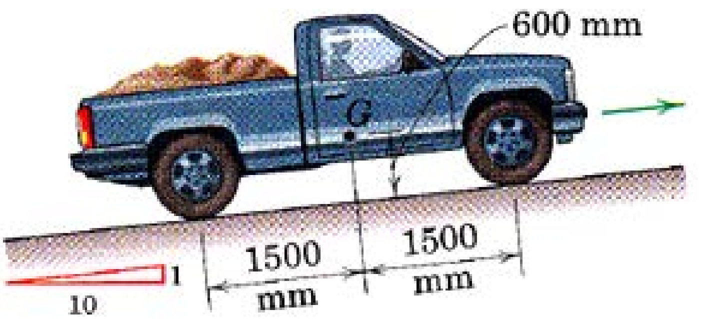
$$\sum \vec{M}_G = \bar{I}\alpha = 0 \qquad \text{.......... 6/6}$$

Problems: 6/2 6/6 6/8 6/9 6/10 6/12 6/15 6/19

Sample Problem 6/1

The 1500-kg pickup truck reaches a speed of 50 km/hr from rest in a distance of 60 m up the 10 percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.8.

Solution



We will assume that the mass of the wheels is negligible compared with the total mass of the truck. The truck may now be simulated by a single rigid body in rectilinear translation.

$[v^2 = 0 + 2as]$

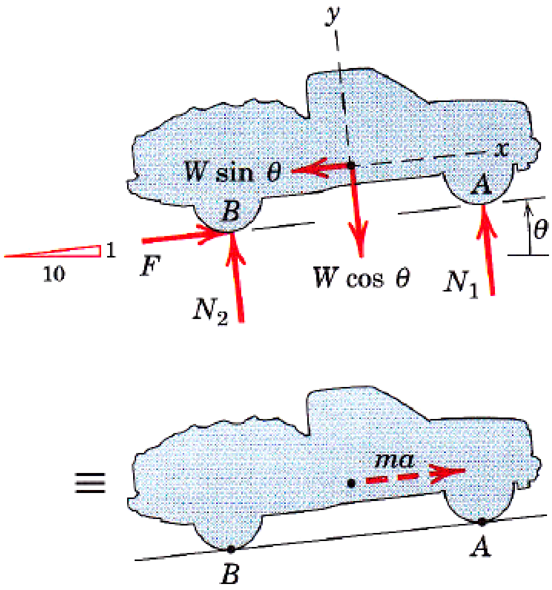
$\bar{a} = \frac{(50/3.6)^2}{2(60)} = 1.608\ m/s^2$

$\theta = tan^{-1} 1/10 = 5.71^o$

$W cos\ \theta = 1500(9.81)\ cos\ 5.71^o = 14.64(10)^3\ N$

$W sin\ \theta = 1500(9.81)\ sin\ 5.71^o = 1464\ N$

$m\bar{a} = 1500(1.608) = 2410\ N$



Applying the three equations of motion, Eqs. 6/1, for the three unknowns gives

$[\sum F_x = m\bar{a}_x]$ $\quad F - 1464 = 2410$

$F = 3880\ N \quad Ans.$

$[\sum F_y = m\bar{a}_y = 0] \quad N_1 + N_2 - 14.64(10^3) = 0 \quad (a$

$[\sum M_G = \bar{I}\alpha = 0] \quad 1.5\ N_1 + 3880\ (0.6) - N_2(1.5) = 0 \quad (b$

Solving a and b simultaneously gives

$N_1 = 6550\ N \qquad N_2 = 8100\ N \qquad Ans.$

In order to support a friction force of 3880 N, a coefficient of friction of at least $F/N_2 = 3880/8100 = 0.48$ *is required* < 0.8, the surfaces are rough enough to support the calculated value of F so that our result is correct.

Alternative Solution

From the kinetic diagram

$\sum M_A = m\bar{a}\ d \qquad 3N_2 - 1.5(14.64)10^3 - 0.6(1464) = 2410(0.6)$

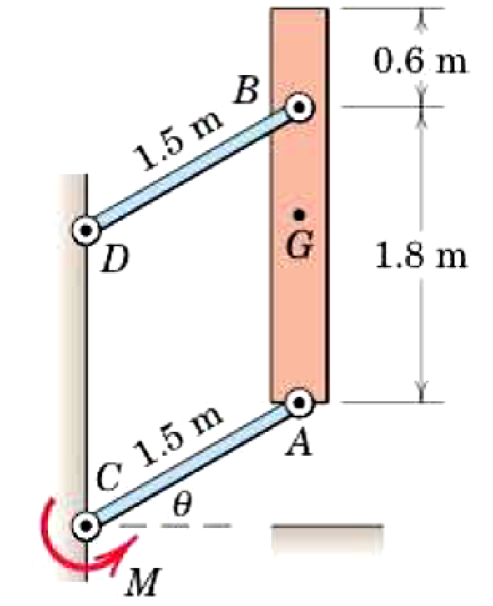
$N_2 = 8100\ N \quad Ans.$

$\sum M_B = m\bar{a}\ d \qquad 14.64(10^3)(1.5) - 1464(0.6) - 3N_1 = 2410(0.6)$

$N_1 = 6550\ N \quad Ans.$

Sample Problem 6/2

The vertical bar AB has a mass of 150-kg with center of mass G midway between the ends. The bar is elevated from rest at $\theta = 0$ by means of the parallel links of negligible mass, with a constant couple $M = 5\ kN.m$ applied to the lower link at C. Determine the angular acceleration α of the links as a function of θ and find the force B in the link DB at the instant when $\theta = 30^o$.



Solution

The motion of the bar is seen to be curvilinear translation
With the circular motion of the mass center G, we choose n- and t-coordinates as the most convenient description.

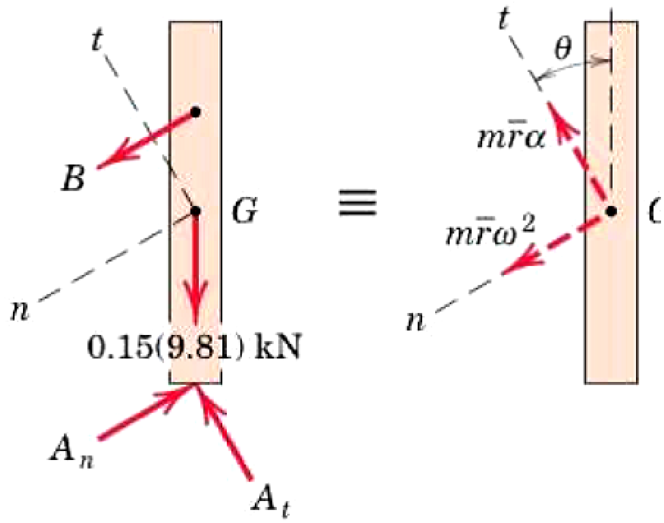
With negligible mass of the links, and from the FBD of AC

$$\sum M_C = 0 \qquad M - A_t \times AC$$

$$A_t = 5/1.5 = 3.33\ kN$$

The force at B is along the link
For the link AC,

$$(\vec{a}_A)_t = AC\ \alpha \quad and \quad (\vec{a}_A)_n = AC\ \omega^2$$



For the FBD of the bar

$$\left[\sum F_t = m\bar{a}_t\right] \qquad 3.33 - 0.15(9.81)\cos\theta$$

$$= 0.15(1.5\ \alpha)$$

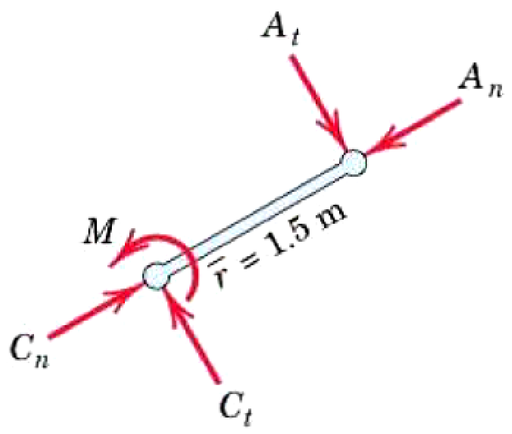
$$\alpha = 14.81 - 6.54\cos\theta \quad rad/s^2 \qquad Ans.$$

$$[\omega d\omega = \alpha d\theta]$$

$$\int_0^{\omega} \omega d\omega = \int_0^{\theta} 14.81 - 6.54\cos\theta)\ d\theta$$

$$\omega^2 = 29.6\ \theta - 13.08\sin\theta$$

Substitution of $\theta = 30^o$ gives
$(\omega^2)_{30^o} = 8.97\ (rad/s)^2$
and $\alpha_{30^o} = 9.15\ rad/s^2$



$m\bar{r}\omega^2 = 0.15(1.5)(8.97) = 2.02\ kN$
$m\bar{r}\alpha = 0.15(1.5)(9.15) = 2.06\ kN$

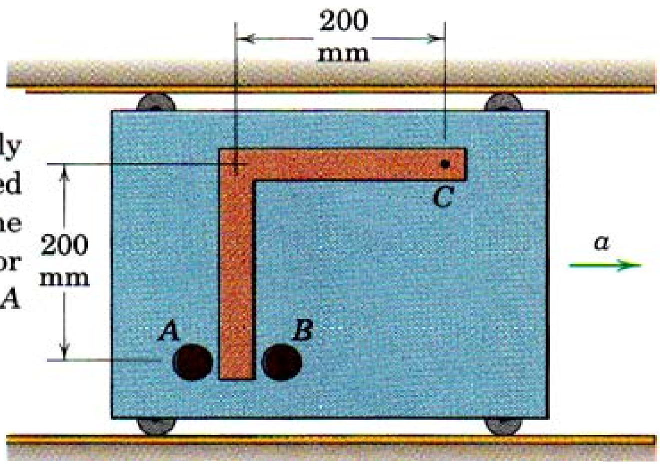
Using A as a moment center gives

$[\sum M_A = m\bar{a}d]$

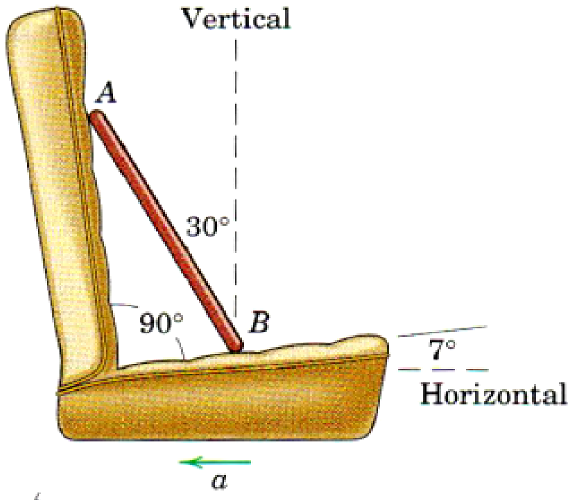
$B \times 1.8\ cos\ 30^o = 2.02(1.2)cos\ 30^o + 2.06(1.2)\sin 30^o$

B=2.14 kN   Ans.

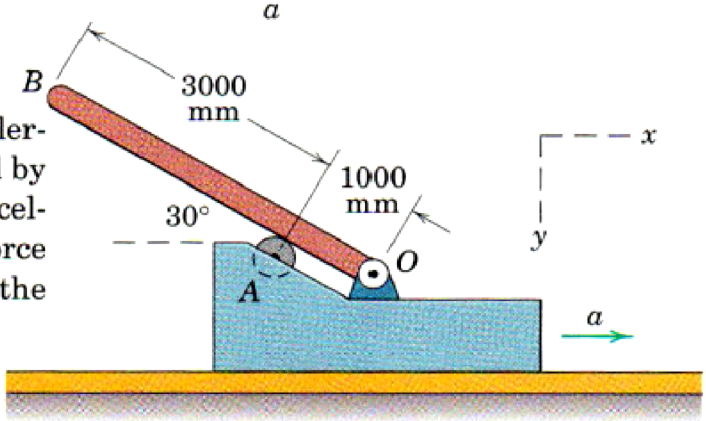
6/2
The right-angle 3-kg bar with equal legs is freely hinged to the vertical plate at C. The bar is prevented from rotating by the two pegs A and B fixed to the plate. Determine the acceleration a of the plate for which no force is exerted on the bar by either peg A or B.



6/6
A uniform slender rod rests on a car seat as shown. Determine the deceleration a for which the rod will begin to tip forward. Assume that friction at B is sufficient to prevent slipping.

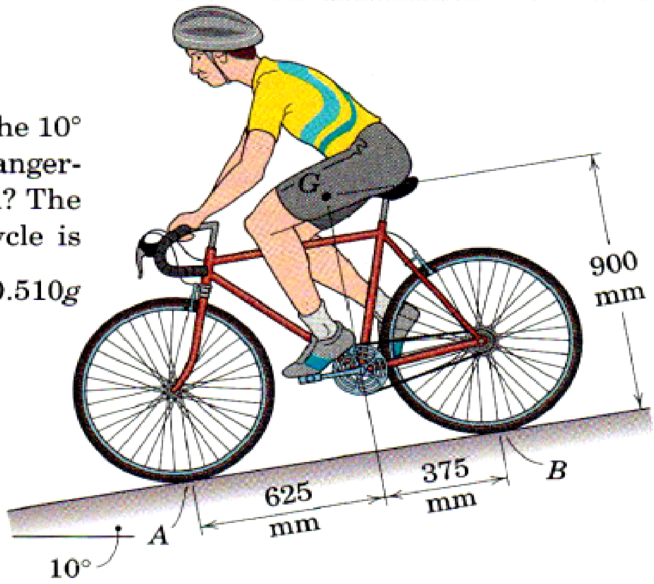


6/8
The uniform 30-kg bar OB is secured to the accelerating frame in the 30° position from the horizontal by the hinge at O and roller at A. If the horizontal acceleration of the frame is $a = 20\ \text{m/s}^2$, compute the force F_A on the roller and the x- and y-components of the force supported by the pin at O.



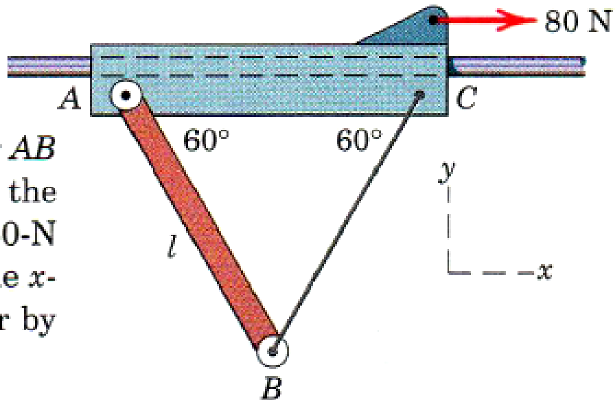
6/9
The bicyclist applies the brakes as he descends the 10° incline. What deceleration a would cause the dangerous condition of tipping about the front wheel A? The combined center of mass of the rider and bicycle is at G.

Ans. $a = 0.510g$



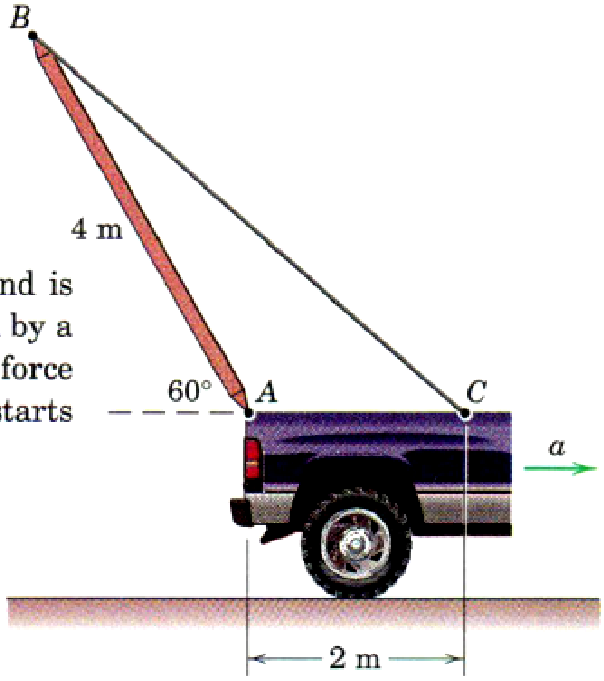
6/10

The 6-kg frame AC and 4-kg uniform slender bar AB of length l slide with negligible friction along the fixed horizontal rod under the action of the 80-N force. Calculate the tension T in wire BC and the x- and y-components of the force exerted on the bar by the pin at A. The x-y plane is vertical.



6/12

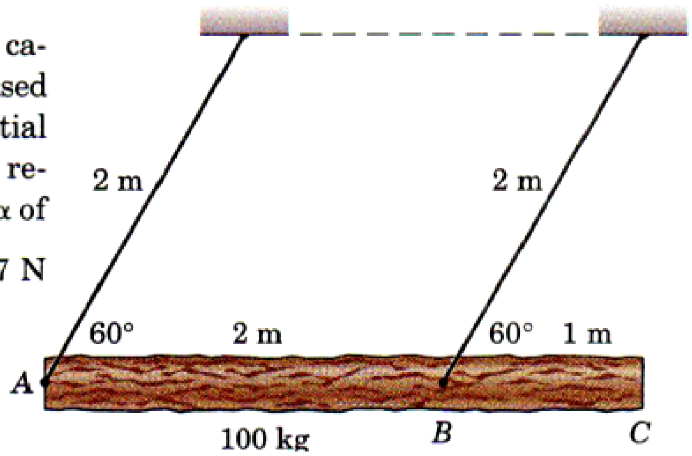
The uniform 4-m boom has a mass of 60 kg and is pivoted to the back of a truck at A and secured by a cable at C. Calculate the magnitude of the total force supported by the connection at A if the truck starts from rest with an acceleration of 5 m/s^2.



6/15

The uniform 100-kg log is supported by the two cables and used as a battering ram. If the log is released from rest in the position shown, calculate the initial tension induced in each cable immediately after release and the corresponding angular acceleration α of the cables.

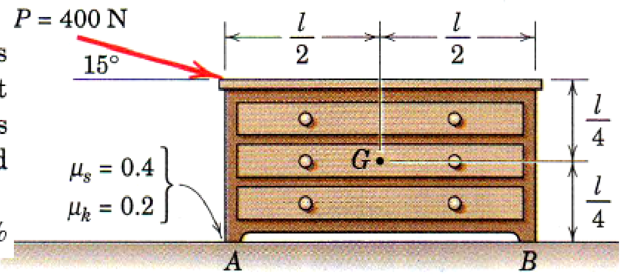
Ans. $T_A = 212$ N, $T_B = 637$ N
$\alpha = 2.45$ rad/s^2



6/19

The force $P = 400$ N is applied to the 75-kg chest as shown. The mass center G of the chest is located at its geometric center. Determine the percent changes n_A and n_B in the normal forces at A and B compared with the static values when $P = 0$.

Ans. $n_A = -9.52\%$, $n_B = 37.7\%$



Lectures of the Department of Mechanical Engineering

Subject Title: Engg. Mechanics 'DYNAMICS'

Class: 2nd Year

<table>
<tr><td rowspan="3">Lecture Contents</td><td>Lecture sequences:</td><td>22nd lecture</td><td>Bakr Noori Alhasan/Lecturer</td></tr>
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Appendix B

B/1 Mass Moment of Inertia About an Axis

In a rigid body, all particles move in parallel planes which are normal to the rotation axis O - O.

We may choose any one of the planes as the plane of motion (c.g. plane is preferred)

$$dF = r\ \alpha\ dm \quad (F = ma)$$
$$dM = r^2\ \alpha\ dm$$
$$M = \alpha \int r^2\ dm$$

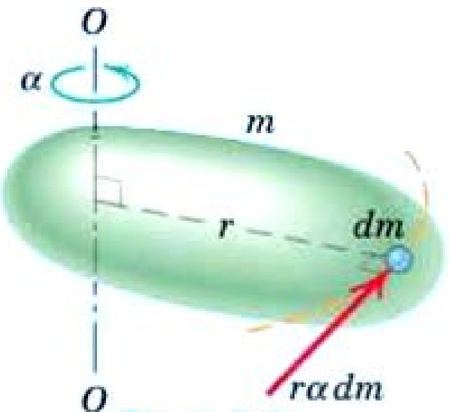


Figure B/1

$\int r^2\ dm$ is called the mass moment of inertia I of the body about the axis O-O.

$$I = \int r^2\ dm \quad \ldots\ldots B/1$$

The comparison between mass and mass moment of inertia

Mass	Mass Moment of Inertia
Is a measure of the resistance to translational acceleration.	Is a measure of resistance to rotational acceleration of a body.

Alternatively: $I = \sum r_i^2\ m_i \ \ldots\ldots\ B/1\ a$

r_i is the radial distance from the inertia axis to the particle m_i

$I = \rho \int r^2\ dV \quad (\rho\ is\ constant)$ In this case the integral defines a purely geometrical property of the body.

Units

SI Units	US Customary Units
$kg.m^2$	$Ib - ft - \ sec^2$

Radius of Gyration

$$I = k^2 m \quad or \quad k = \sqrt{\frac{I}{m}} \ldots\ldots B/2$$

Where k is a measure of the distribution of mass of a given body about the axis in question.

Transfer of Axis

$$I = \int r^2 dm \; = \; \int (r_o^2 + d^2 + 2r_o d\cos\theta) dm$$

$$= \int {r_o}^2\, dm + \int d^2\, dm + 2d \int u dm$$

$I = \bar{I} + md^2$ …… $B/3$ Parallel axis theorem

Equation B/3 applied provided the two axes are parallel and one of them passes through center of gravity.

Substituting B/2 in B/3 ,

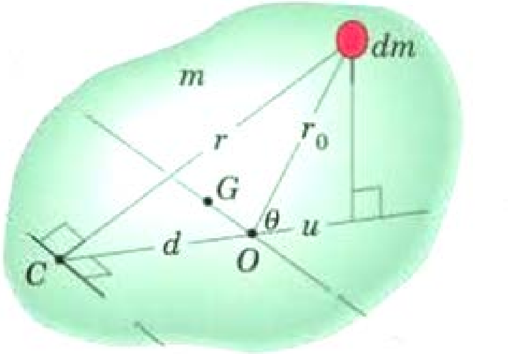


Figure B/3

$k^2 = \bar{k}^2 + d^2$ …… $B/3\,a$ (parallel axis theorem)

For plane motion problem, a single subscript for I is sufficient.

In Fig. B/4, the moment of inertia of the plate about z- axis through O is designated I_O.

In three dimensional motion, the moments of inertia about the x-, y-, and z- axis are labeled $I_{xx}, I_{yy}, and\ I_{zz}$ respectively.

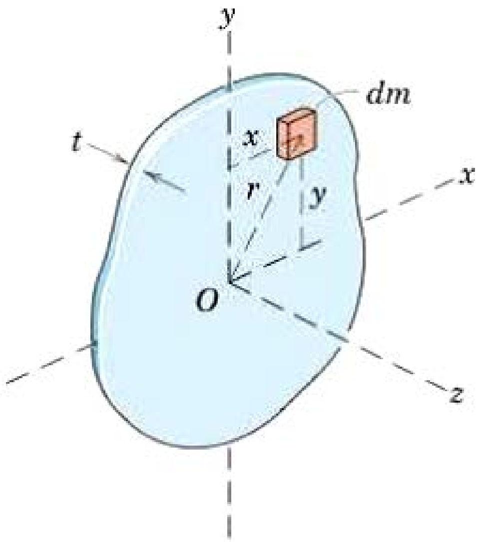


Figure B/4

From Fig. B/5:

$I_{xx} = \int {r_x}^2\, dm = \int (y^2 + z^2) dm$

$I_{yy} = \int {r_y}^2\, dm = \int (x^2 + z^2) dm$ …… B/4

$I_{zz} = \int {r_z}^2\, dm = \int (x^2 + y^2) dm$

The relationship between the mass moment of inertia and area moment of inertia exists in the case of flat plate of uniform thickness, Fig. B/4.

$$I_{zz} = \int r^2\, dm = \rho t \int r^2\, dA = \rho t\, I_z \text{ …… } B/5$$

If t is small compared with the dimensions of the plate in its plane.

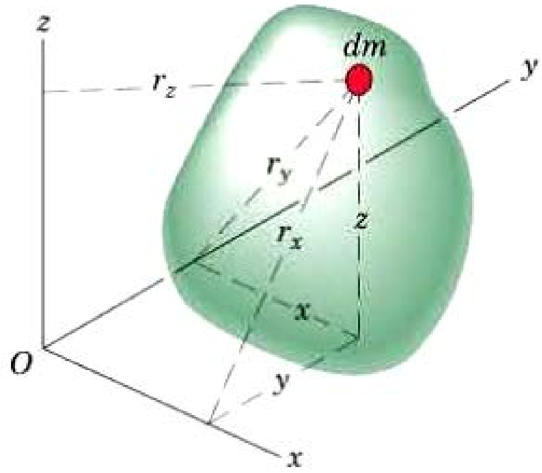


Figure B/5

$I_{xx} = \int y^2\, dm = \rho t \int y^2\, dA = \rho t\, I_x$

$I_{yy} = \int x^2\, dm = \rho t \int x^2\, dA = \rho t\, I_y$ …… $B/6$ (B/4: z = 0)

Also $I_{zz} = I_{xx} + I_{yy}$ …… $B/7$ (Only for a thin flat plate, t and z are neglected compared with the dimensions of plate x, y flat slice)

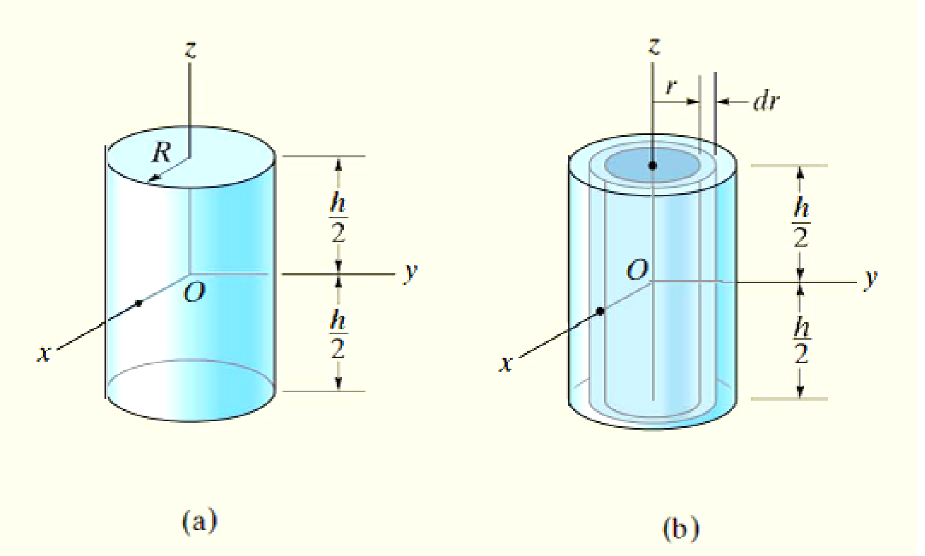
$$dI_{zz} = dI_{xx} + dI_{yy} \;\; \ldots\ldots \;\; B/7 \;\; (\text{thick } dz)$$

Composite Bodies

The mass moment of inertia of a composite body is the sum of the moments of inertia of the individual parts about the axis. There are positive volumes and negative volumes. Useful formulas for mass moments of inertia of various masses of common shapes given in table D/4 App. D

Example (Hibbeler 12nd ed.)

Determine the mass moment of inertia of the cylinder shown in Fig. 17.3 a about the z-axis. The density of the material ρ is constant.



Shell element is used
The entire element lies at the same distance from the z- axis.

$$dI_z = r^2 dm$$

$$I_z = \int r^2 \, dm$$

$$I_z = \int r^2 \, 2\pi r \, dr \, \rho h$$

$$I_z = 2\pi \, \rho h \int_0^R r^3 \, dr$$

$$I_z = 2\pi \, \rho h \left(\frac{R^4}{4}\right) = \frac{\pi}{2} \rho h R^4$$

$$I_z = \frac{\pi}{2} \rho h R^4 \, \frac{1}{\rho \pi R^2 h}$$

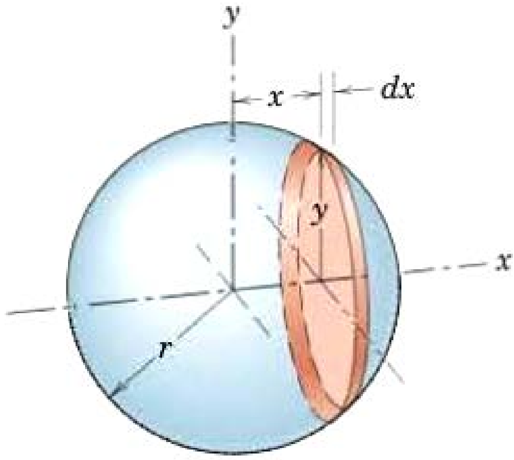
$$I_z = \frac{1}{2} m R^2$$

(For any disc or cylinder)

Sample Problem B/2 (Merriam P668 5th ed.)

Determine the moment of Inertia and radius of gyration of a homogenous solid sphere of mass m and radius r about a diameter.

Solution



$$dI_{xx} = \frac{1}{2} dm\, y^2$$

$$dI_{xx} = \frac{1}{2} \rho\pi\, y^2 dx\, y^2$$

$$\frac{\rho\pi}{2}\, y^4 dx$$

$$I_{xx} = \int_{-r}^{r} \frac{\rho\pi}{2} (r^2 - x^2)^2 dx$$

$$= \frac{\rho\pi}{2} \int_{-r}^{r} r^4 - 2r^2x^2 + x^4 dx$$

$$= \frac{\rho\pi}{2} \left[r^4 x - 2r^2 \frac{x^3}{3} + \frac{x^5}{5} \right]_{-r}^{r}$$

$$I_{xx} = \frac{8\rho}{15} \pi r^5$$

$$I_{xx} = \frac{8\rho}{15} \pi r^5 \frac{m}{\rho \frac{4}{3} \pi r^3}$$

$$I_{xx} = \frac{2}{5} m r^2 \quad Ans.$$

$$I = mk^2$$

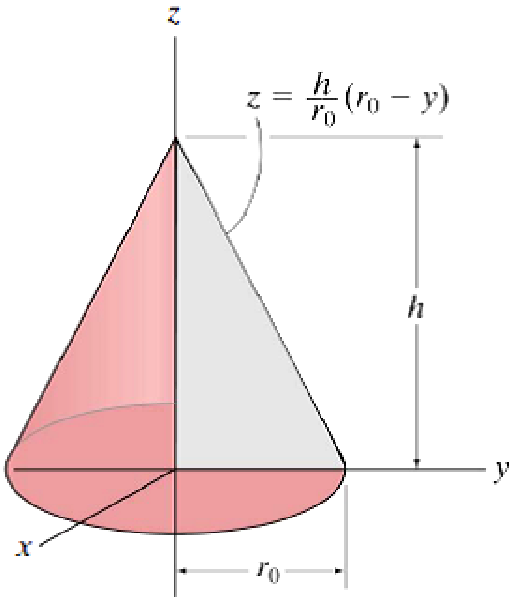
$$\frac{2}{5} m r^2 = mk^2$$

$$k = \sqrt{\frac{2}{5}} r \quad Ans.$$

Note: For The sphere $dm = \rho\pi y^2 dx$ and $r^2 = x^2 + y^2$

17/8 Hibbeler 12nd ed.

Determine the mass moment of inertia I_z of the cone formed by revolution of the shaded area around the z-axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



Solution

$$dI_z = \frac{1}{2}dm\, r^2$$

$$I_z = \int \frac{1}{2}\rho\pi r^4 dz$$

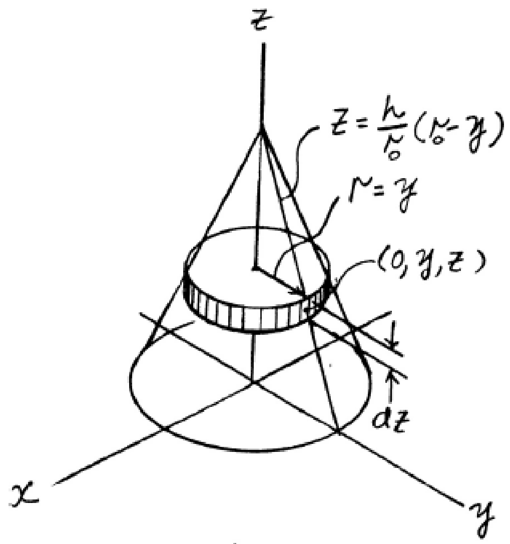
$$I_z = \frac{\rho\pi}{2}\int_0^h \frac{r_o{}^4}{h^4}(h-z)^4 dz$$

$$I_z = \frac{\rho\pi\, r_o{}^4}{2h^4}\left[\frac{-(h-5)^5}{5}\right]_0^h$$

$$I_z = \frac{\rho\pi\, r_o{}^4}{2h^4}\left[\frac{h^5}{5}\right]$$

$$I_z = \frac{\rho\pi\, r_o{}^4}{10}\frac{m}{\frac{1}{3}\rho\pi\, r_o{}^2 h}$$

$$I_z = \frac{3}{10}m\, r_o{}^2 \quad Ans.$$



Note

$$dV = \pi\, r^2\, dz$$

$$dm = \rho\pi\, r^2\, dz$$

$$m = \int \rho\pi\, r^2\, dz$$

$$m = \rho\pi \int \frac{r_o{}^2}{h^2}(h-z)^2\, dz$$

$$= \frac{\rho\pi r_o{}^2}{h^2}\int_o^h h^2 - 2hz + z^2 dz$$

$$m = \frac{1}{3}\rho\pi r_o{}^2$$

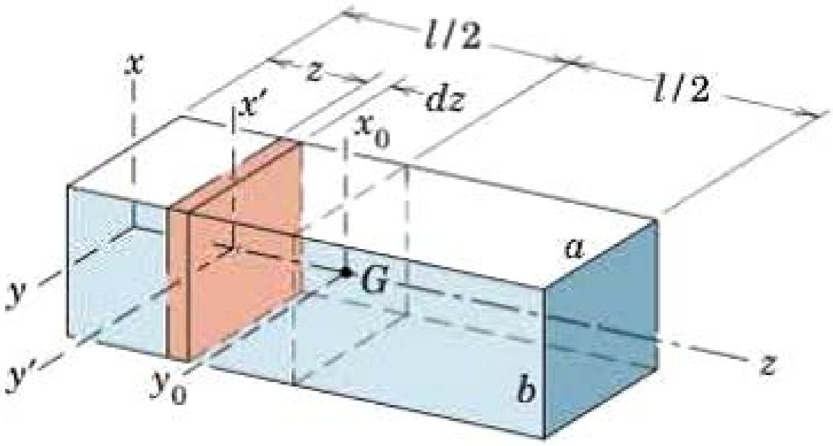
For cone $\frac{r_o}{h} = \frac{r}{h-z}$ *and* $r = \frac{r_o}{h}(h-z)$

Sample Problem B/3 (Merriam 5th ed.)

Determine the moment of inertia of the homogenous rectangular parallepiped of mass m about the centroidal $x_o -$ and $z - axis$ and about the $x - axis$ through one end.

Required

$$I_{x_o x_o} = ?$$
$$I_{zz} = ?$$
$$I_{xx} = ?$$



Solution

The moment of inertia of the slice element

$$dI_{\bar{x}\bar{x}} = \rho dz\, I_{\bar{x}} = \rho dz \frac{ba^3}{12}$$

$$dI_{\bar{y}\bar{y}} = \rho dz\, I_{\bar{y}} = \rho dz \frac{ab^3}{12}$$

$$dI_{zz} = dI_{\bar{x}\bar{x}} + dI_{\bar{y}\bar{y}} = \frac{\rho dz}{12}(ba^3 + ab^3) = \frac{\rho ab}{12}(a^2 + b^2)dz$$

$$I_{zz} = \frac{\rho ab}{12}(a^2 + b^2)\int_0^L dz$$

$$I_{zz} = \frac{\rho abL}{12}(a^2 + b^2) \times \frac{m}{\rho abL} = I_{zz} = \frac{m}{12}(a^2 + b^2) \ldots Ans.$$

Interchanging of symbols

$$I_{x_o x_o} = \frac{m}{12}(L^2 + a^2) \ldots Ans.$$

$$I_{xx} = I_{x_o x_o} + md^2$$

$$I_{xx} = \frac{m}{12}(L^2 + a^2) + m\left(\frac{L}{2}\right)^2 = \frac{m}{12}(a^2 + 4L^2) \ldots Ans.$$

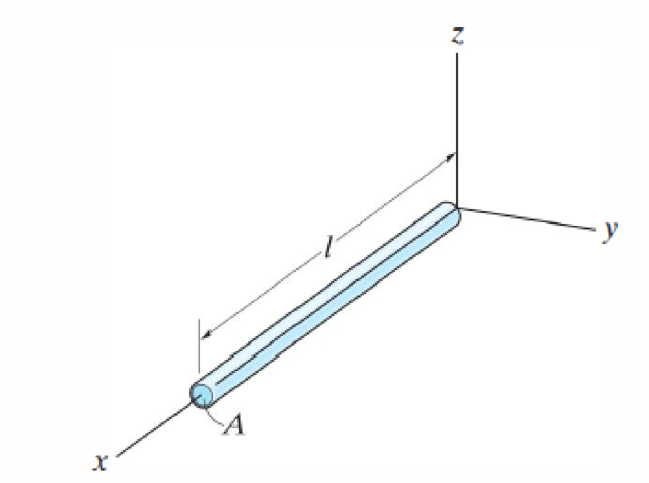
For slender bar $a = 0$ $\quad I_{xx} = \frac{ml^2}{3} \quad and\ I_{x_o x_o} = \frac{ml^2}{12}$

or

$$dI_{xx} = dI_{\bar{x}\bar{x}} + dm\, z^2$$

17.1 Hibbeler 12d ed.

Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross sectional area A are constant. Express the result in terms of rod's total mass m.



Solution

$$I_{yy} = \int r^2 dm$$

$$= \int x^2 dm$$

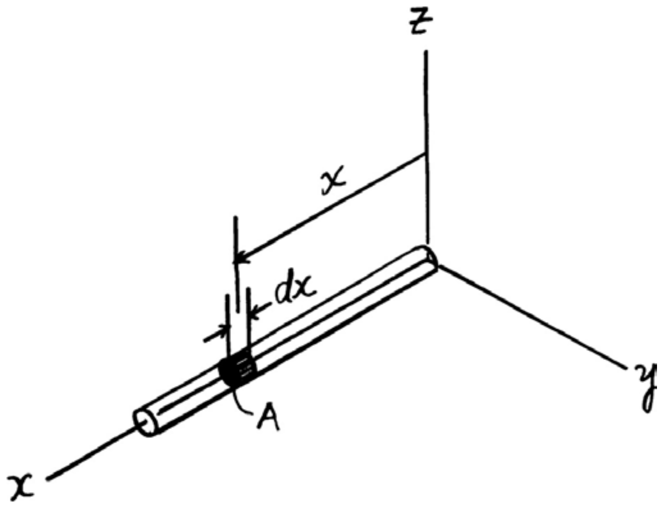
$$= \int x^2 \rho \, A dx$$

$$= \rho A \int_0^l x^2 dx$$

$$\rho A \frac{l^3}{3}$$

$$I_{yy} = \rho A \frac{l^3}{3} \times \frac{m}{\rho A l}$$

$$I_{yy} = \frac{m l^2}{3} \quad \ldots \quad Ans.$$



17/21 Hibbeler

Determine the mass moment of Inertia of the pendulum about an axis perpendicular to the page and passing through point O, the slender rod has a mass of 10 kg and the sphere has a mss of 15 kg.

Solution

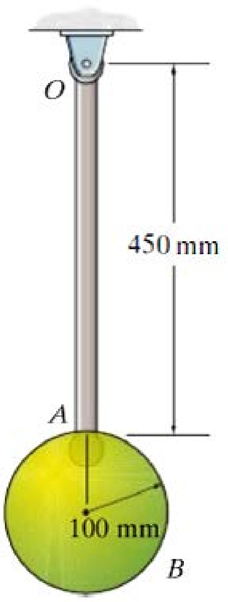
$I_{G_1} = \frac{ml^2}{12}$

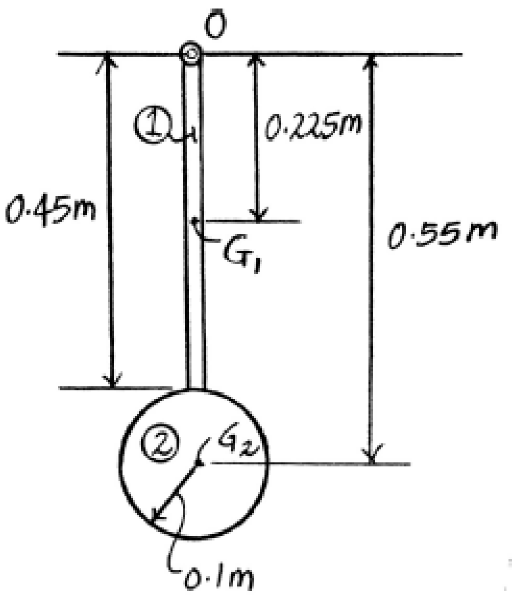
$$I_{G_2} = \frac{2}{5}mr^2$$

$$I_o = \sum I_G + md^2$$

$$I_o = \left[\frac{10 \times 0.45^2}{12} + 10(0.225)^2\right] + \left[\frac{2}{5}15 \times 0.1^2 + 15 \times 0.55^2\right]$$

$$I_o = 5.27\ kg.m^2$$





17/23 Hibbeler

Determine the mass moment of Inertia of the thin plate about an axis perpendicular to the page and passing through point O, the material has a mass per unit area 20 kg/m^2 .

Solution

$$m_1 = \pi \times 0.2^2 \times 20$$
$$= 0.8\,\pi \;\; kg$$
$$m_2 = 0.2 \times 0.2 \times 20 = 0.8\, kg$$
$$I_0 = \sum I_C + md^2$$
$$I_o = \left[\frac{0.8\pi 0.2^2}{2} + 0.8\pi \times 0.2^2\right] - \left[\frac{0.8}{12}(0.2^2 + 0.2^2) + 0.8 \times 0.2^2\right]$$

$$I_o = 0.13\, kg.m^2 \quad Ans.$$