محاضرات قسم الهندسة الميكانيكية

Subject Title: Control and Measurements
Class: Fourth Year
Instructor Name: Dr. Ahmed Fattah Ahmed

Lecture 1	General introduction on the subject of control & measurements in different branches of engineering, mech. Elect. Chem. Civil prod. ..etc
	Basic requirements for the subject and the connection between control and measurements (measurements here means sensors) giving some example& the contents of the subject.
Lecture 2&3	Definitions of terms and the meaning of transfer function, and why Laplaces transforms. Open loop and closed loop systems. Giving some examples in different applications.
Lecture 4&5	Representation of control systems components, mechanical rotational, fluidic, thermal and electrical. First-order system and time constants for different sub-systems.
Lecture 6&7	Dynamic equations and block-diagram representation of some actuators normally used in control systems, hydraulic integrator and hydraulic actuator, field-controlled D.C motors and armature-controlled D.C motors.
Lecture 8&9	Linearization of non-linear relationships and why it is needed in control system representation. Hydraulic actuator with load as an example on linearization and other examples.
Lecture 10&11	Block diagram algebra and simplification rules, solving an example on simplification. Examples on complete control and building block diagrams with reference input and disturbances.
Lecture 12,13&14	Steady state operation and the evaluation of steady state block diagram constants. Steady s. equation of operation, controller and system to be controlled characteristic curves. Given examples
Lecture 15	Lecture on measurements &sensors, temperature, pressure & rotational speed.

Control engineering

References

1. **Automatic control engineering by francis H. Raven Third Edition**
2. **Modern Control Engineering by Katsuhiko Ogata Fifth Edition**
3. **Automatic Control System by Benjamin C. Kuo Eighth Edition**

Introduction:

Engineering is concerned with understanding and controlling the materials and forces of nature for the benefit of humankind.

Control system engineers are concerned with **understanding and controlling** segments of their environment, often called systems, to provide useful economic products for society.

The twin goals of understanding and controlling are complementary because effective systems control requires that the systems be **understood and modeled**. Furthermore, control engineering must often consider the control of poorly understood systems such as chemical process systems.

Control engineering is based on the foundations of **feedback theory and linear system analysis**, and it integrates the **concepts of network theory and communication theory.**

Therefore control engineering is not limited to any engineering discipline but is equally applicable to **aeronautical, chemical, mechanical, environmental, civil, and electrical engineering.**

For example, a control system often includes **electrical, mechanical, and chemical components**. Furthermore, as the understanding of the **dynamics of business**, **social, and political systems** increases, the ability to control these systems will also increase.

Automation

The control of an **industrial process** (manufacturing, production, and so on) by automatic rather than manual means is often called automation. **Automation** is prevalent in the chemical, electric power, paper, automobile, and steel industries, among others. The concept of automation is central to our industrial society.

Automatic machines are used to increase the production of a plant per worker in order to offset rising wages and inflationary costs. Thus industries are concerned with the productivity per worker of their plants. **Productivity** is defined as the ratio of physical output to physical input [26]. In this case, we are referring to labor productivity, which is real output per hour of work.

Definitions

- **A system** is a combination or an arrangement of different physical components which act together as an entire unit to achieve certain objective.

  Also can be defined as: a system is a set of interacting components connected together in such a way that the variation or response in the state of one component affects the states of the others.

- **Static systems** have an output response to an input that does not change with time. (The output have the same instantaneous relationship with the input). Static system is described by algebraic equations. Present response is totally determined by the present value of the input figure (1-a).
- **Dynamic systems** have a response to an input that is not instantaneously proportional to the input or disturbance and that may continue after the input is held constant. Dynamic systems can respond to input signals, disturbance signals, or initial signals. See figure (1-b). Dynamic systems possesses memory (output signal depends on past values and future values of the input signal.

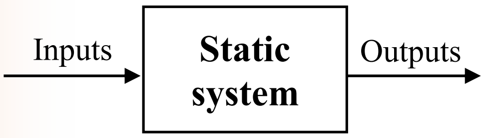


Figure (1-a)

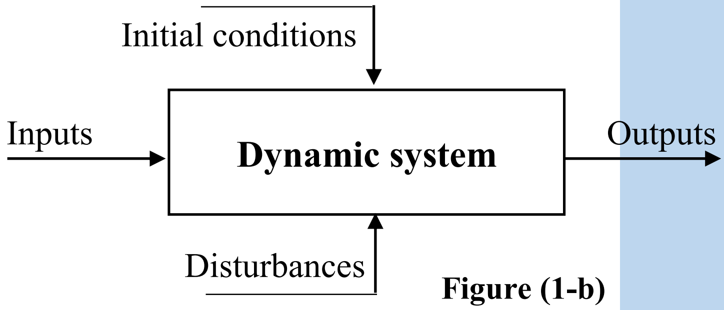
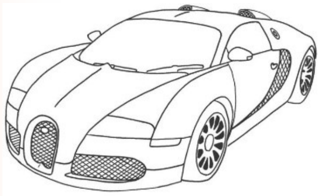


Figure (1-b)

- **Dynamic systems are found in:**

  1- All major **engineering disciplines** and include Mechanical, Electrical, Thermal, Fluid and mixed systems (electro-mechanical, fluid-mechanical, thermo-mechanical and electro-thermal). See figure (2)

  2- **Natural systems** include ecological, biological (human body), economic traffic, etc. see figure (3)

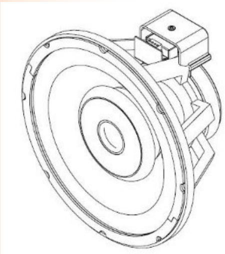


Auto

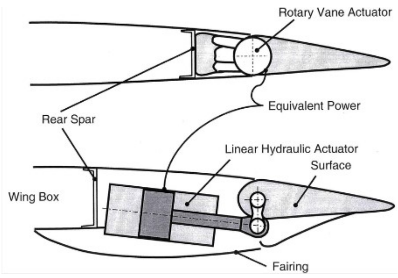
Boiling Pot

Water

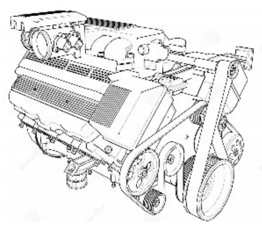
Figure (2) Dynamic systems (engineering disciplines)



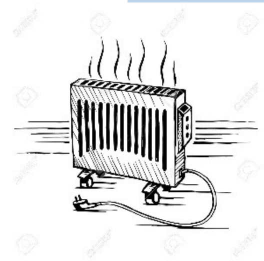
Loud speaker



Actuator



Engine



Space heater

Figure (3) Mixed systems

- **A Control system** is an arrangement of a physical components connected in a manner to regulate (command or direct) itself or another system.

<u>Control systems are classified in to different categories:</u>

1- **Open loop control system** is a system in which the output signal has no effect on the input signal. See figure (4).

<u>**Or:**</u> is one in which the control action (quantity responsible) for activating the system to produce the output.

Note: open loop control **system** is sometimes refer to as a passive control

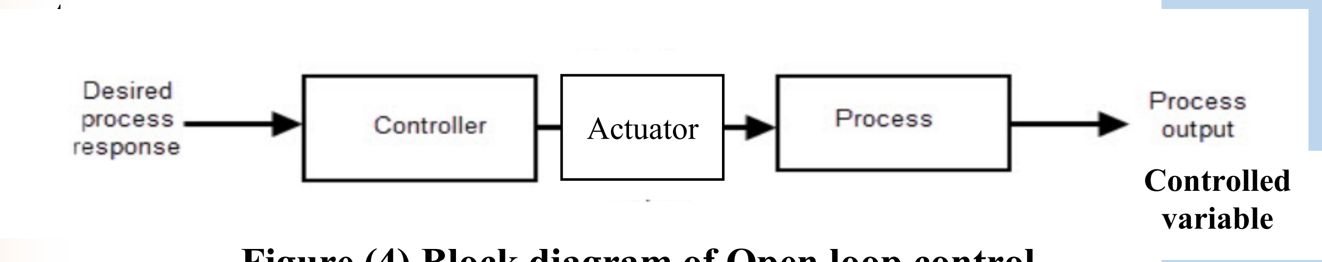
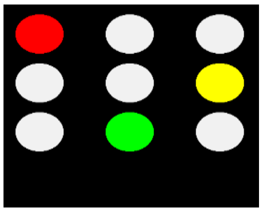
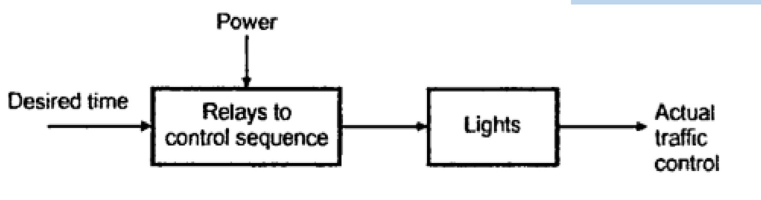


Figure (4) Block diagram of Open loop control

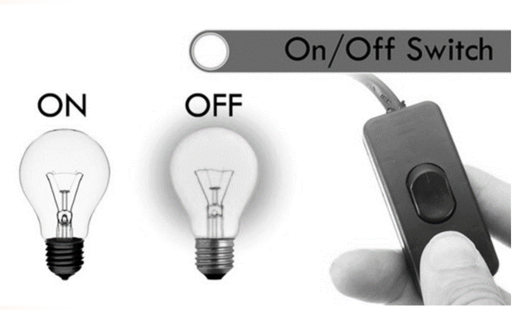
<u>Examples:</u>



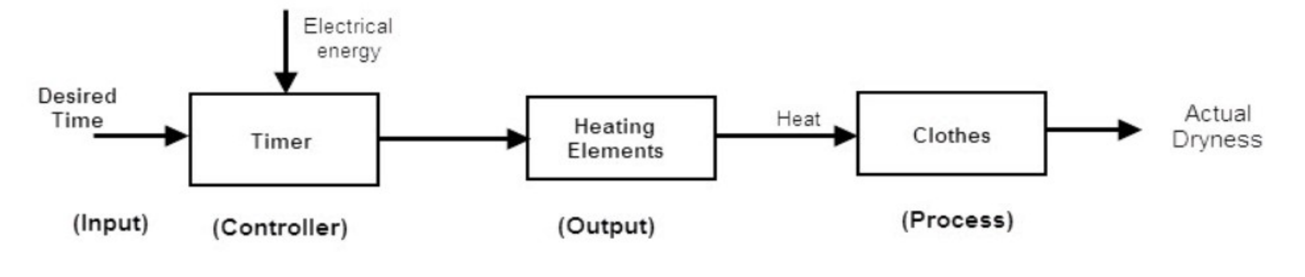


Automatic Washing machine. **Traffic lights.**

<u>**Note: [Any control systems that operates on a time basis are open loop]**</u>



On-off switch (light switch)



Clothes drier

<u>H.W…. Give three types of open loop control systems (physical or natural system with block diagram).</u>

2- **Closed loop control system** is a system in which the output signal effects the input signal through the feedback signal. Figure (5) shows the general block diagram of the closed loop feedback control system.

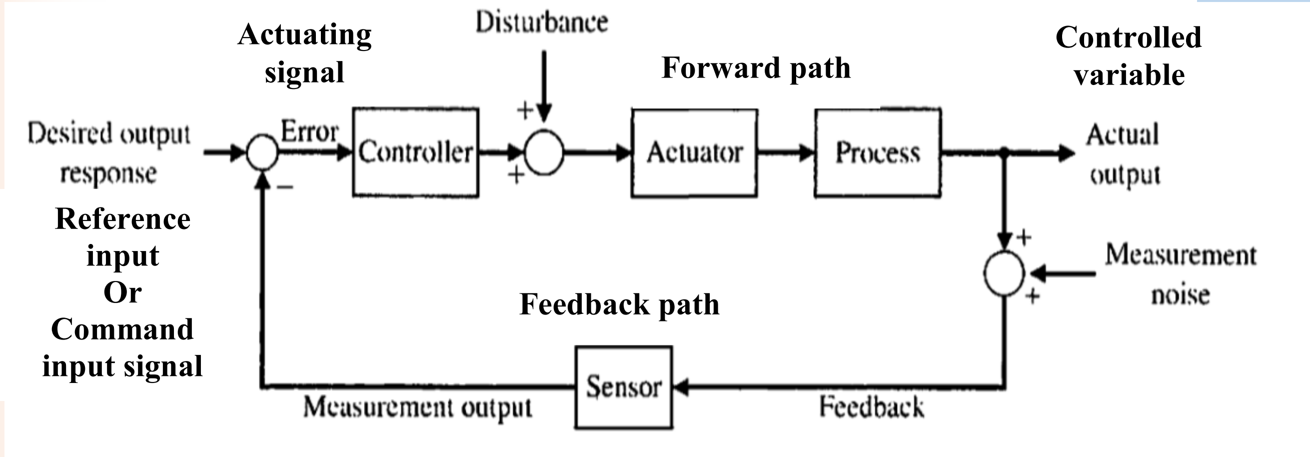


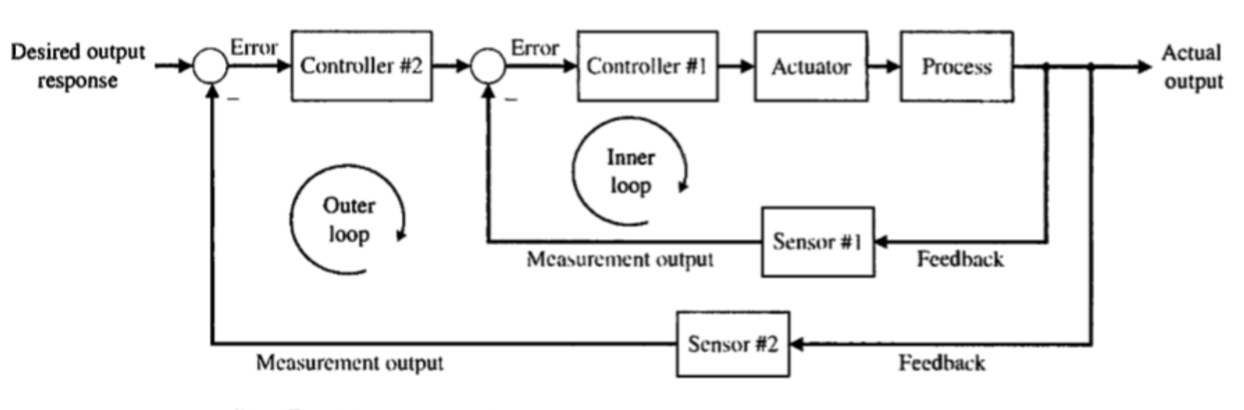
Figure (5) Negative feedback control system

- **Input signal** is the excitation or command applied to a control system, typically from an external energy source, usually in order to produce a specified response from the control system. It is an independent variable.
- **Output signal** is the actual response obtained from a control system. It may or may not be equal to specified response implied by the input. It is dependent variable.

Input and outputs can have many different forms. Input, for example, may be physical variables, or more abstract quantities such as reference, set point, or desired values for the output of the control system.

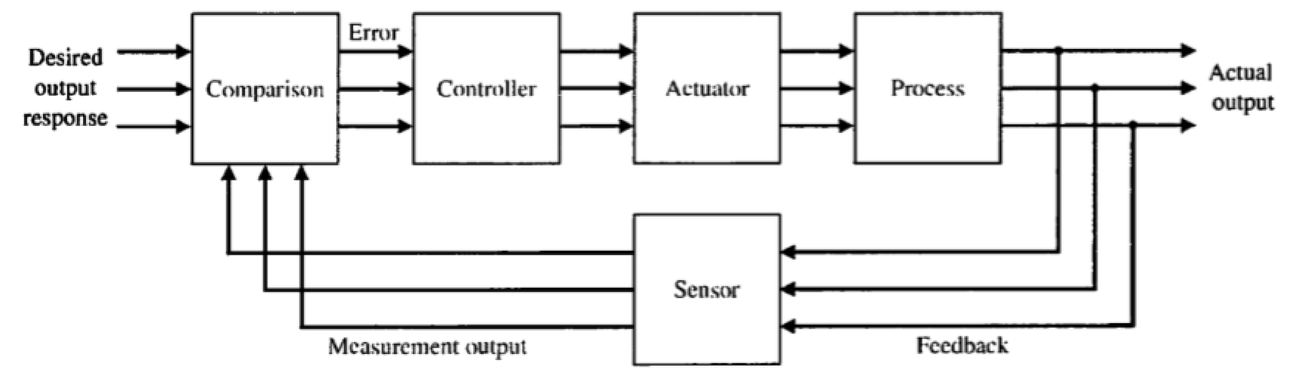
- **Actuating signal** (error signal) the difference between input and the output signal is the actual signal which actuates the control.
- **Comparator** is used to compare the input signal with the feedback signal. The deference between these two signals (+ve or –ve) is the error signal which performs the control action.

3- Multiloop feedback system with an inner loop and an outer loop



Multiloop feedback system with an inner loop and an outer loop.

4- Multivariable control system



Multivariable control system.

Applications of closed loop systems

1- Human being: if a person wants to reach for a book on the table, close loop system can be represented as shown in figure (6).

- ➢ Position of the book is given as a reference.
- ➢ Feedback signal from eyes
- ➢ The comparator used to compare the actual position of hands with reference position.
- ➢ Error signal is given to brain.
- ➢ Brain manipulates this error and gives signal to the hands.
- ➢ This process continues still the position of the hands get achieved appropriately.

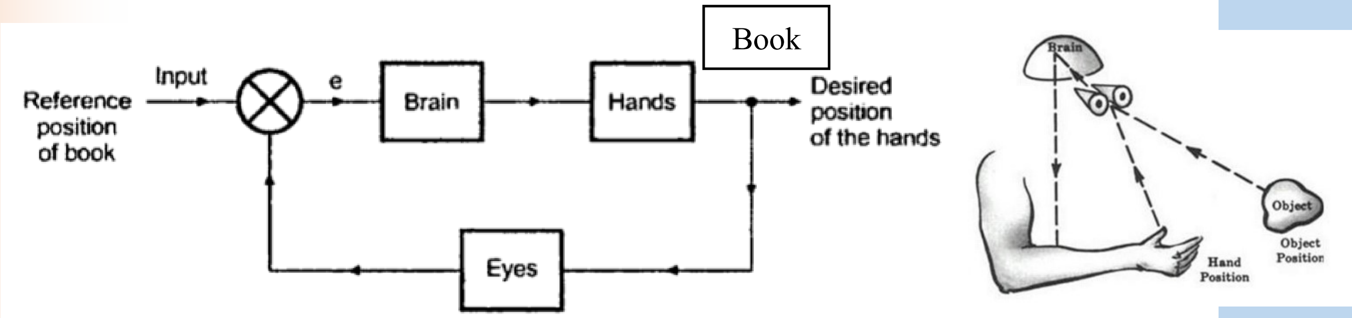


Figure (6) Block diagram of the human being control system

2- Automobile steering control system, see figure (7).

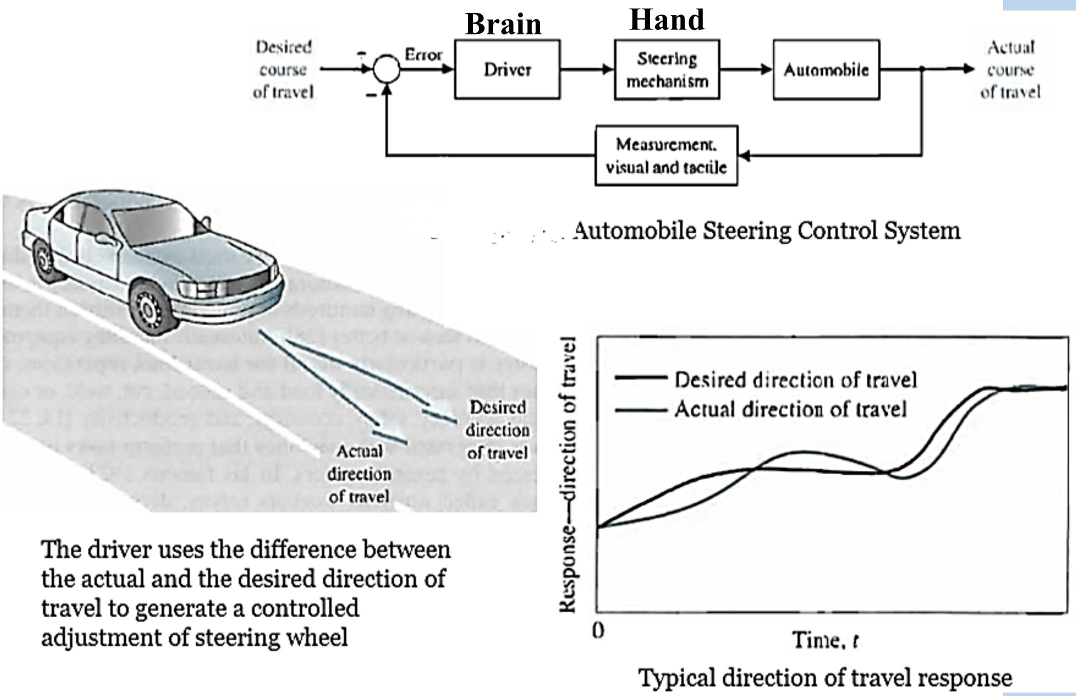


Figure (7) block diagram model of Automobile steering control system

3- Rotating disk speed control, see figure (8).

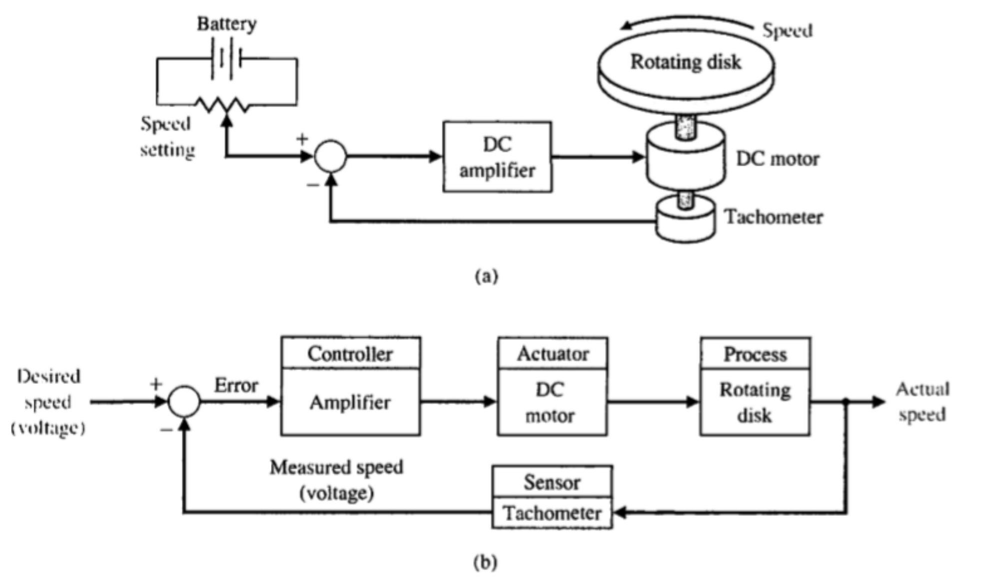


Figure (8)

➢ F (a) Closed-loop control of the speed of a rotating disk. (b) Block diagram model.
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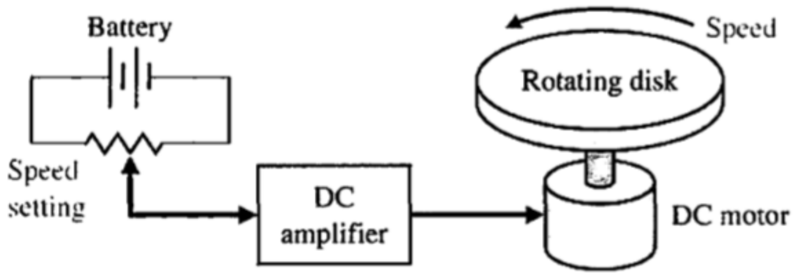
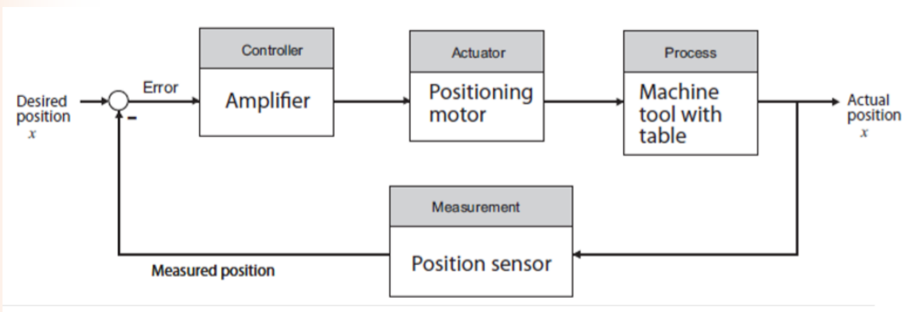


Figure (9)

4- Machine tool with table (CNC machine) control system, see figure (10).



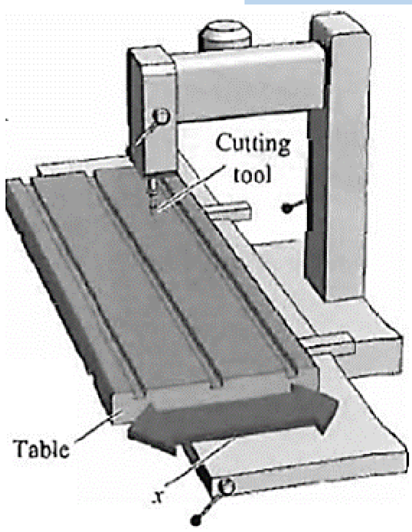


Figure (10)

➢ Comparison of Open Loop and Closed Loop Control System

Sr. No.	Open Loop	Closed Loop
1.	Any change in output has no effect on the input i.e. feedback does not exists.	Changes in output, affects the input which is possible by use of feedback.
2.	Output measurement is not required for operation of system.	Output measurement is necessary.
3.	Feedback element is absent.	Feedback element is present.
4.	Error detector is absent.	Error detector is necessary.
5.	It is inaccurate and unreliable.	Highly accurate and reliable.
6.	Highly sensitive to the disturbances.	Less sensitive to the disturbances.
7.	Highly sensitive to the environmental changes.	Less sensitive to the environmental changes.
8.	Bandwidth is small.	Bandwidth is large.
9.	Simple to construct and cheap.	Complicated to design and hence costly.
10.	Generally are stable in nature.	Stability is the major consideration while designing
11.	Highly affected by nonlinearities.	Reduced effect of nonlinearities.

Controlled Variable and Control Signal or Manipulated Variable.

The Controlled Variable is the quantity or condition that is measured and controlled. The control signal or manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of controlled variable. Normally, the controlled variable **is the output of the system**. Control means measuring the value of the controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value.

Plants. A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

Disturbance A disturbance is a signal that tends to adversely affect the value of the output of the system.

Feedback Control. Feedback control refers to an operation that, in the presence of disturbance, tends to reduce the difference between the output of a system and some reference input.

Open loop control system

Advantages

The advantages of open loop control system are,

1) Such systems are simple in construction.
2) Very much convenient when output is difficult to measure.
3) Such systems are easy from maintenance point of view.
4) Generally these are not troubled with the problems of stability.
5) Such systems are simple to design and hence economical.

Disadvantages

The disadvantages of open loop control system are,

1) Such systems are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate precalibration of the controller.
2) Such systems give inaccurate results if there are variations in the external environment i.e. such systems cannot sense environmental changes.
3) Similarly they cannot sense internal disturbances in the system, after the controller stage.
4) To maintain the quality and accuracy, recalibration of the controller is necessary from time to time.

Closed loop control system

Advantages

The advantages of closed loop system are,

1) Accuracy of such system is always very high because controller modifies and manipulates the actuating signal such that error in the system will be zero.
2) Such system senses environmental changes, as well as internal disturbances and accordingly modifies the error.
3) In such system, there is reduced effect of nonlinearities and distortions.
4) Bandwidth of such system i.e. operating frequency zone for such system is very high.

Disadvantages

The disadvantages of closed loop system are,

1) Such systems are complicated and time consuming from design point of view and hence costlier.
2) Due to feedback, system tries to correct the error from time to time. Tendency to overcorrect the error may cause oscillations without bound in the system. Hence system has to be designed taking into consideration problems of instability due to feedback. The stability problems are severe and must be taken care of while designing the system.

Other classification of control systems

Broadly control systems can be classified as:

1) **Natural control systems:** the biological systems, system inside human being are of natural type.

   For example: the **perspiration system** inside the human being is a good example of natural control system. This system activates the **secretion glands**, secreting sweat and regulate the temperature of human body figure (11).

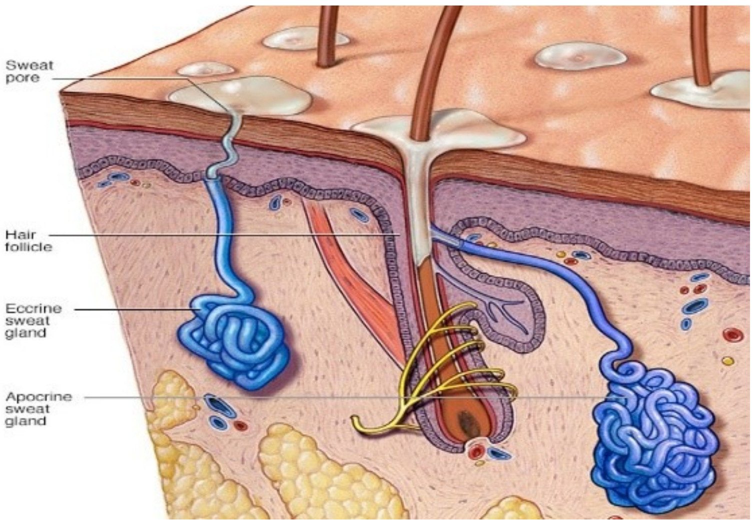


Figure (11)

2) **Manmade control systems:** different types of control systems which are used in our day to day life are designed and implemented by human beings. Such systems like vehicles, switches, various controllers and industrial automobile factory, figure (12).

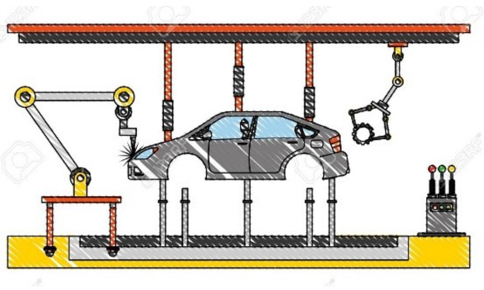


Figure (12)

3) **Combination control systems:** combinational control system is one, having combination of natural and manmade together i.e. **driver driving a tower crane** figure (13).

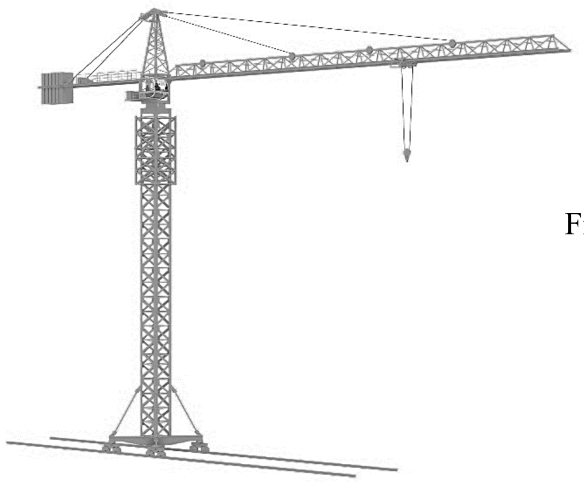
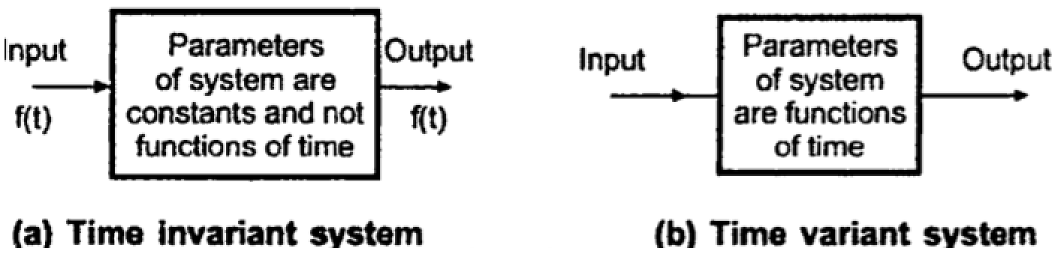


Figure (13)

4) **Time varying and time and invariant systems:** time varying control systems are those in which parameters of the systems are varying with time. It is not dependent on whether input and output are functions of time or not. For example space vehicle whose mass decreases with time, as it leave earth. The mass is a parameter of space vehicle system. Also, in case of a rocket, aerodynamic damping can change with time as the air density changes with the altitude, see figure (14). As against this if even though the inputs and outputs are functions of time but the parameters of system are independent of time, which are not varying with time and are constants, then system is said to be time invariant system.



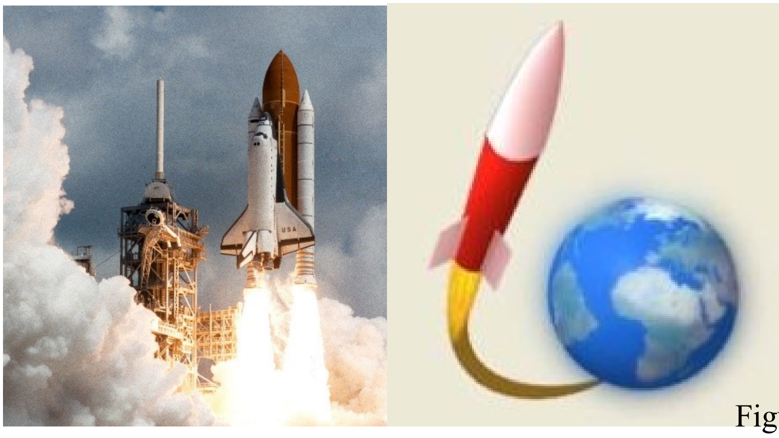


Figure (14)

5) **Single input single output (SISO) and multiple input and multiple output (MIMO) systems.**
6) **Open loop and closed loop systems.**
7) **Traditional and intelligent control systems.**

Representation of Control System Components

Transfer function:

It's a mathematical representation which relates the input and the output of an element, a subsystem or a complete system. It may be denoted by T(D) or T(s), where (D) and (s) are the time derivative operator and the Laplace operator respectively. The transfer function (T.F) is T(D) $= \frac{output}{input}$. T(D) may be represented by a block as shown in figure (15).

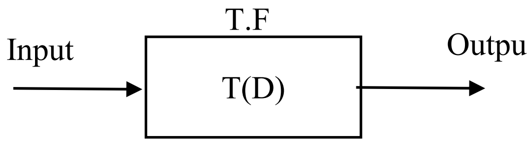


Figure (15)

Mechanical Components

Spring: the load deflection characteristics for a mechanical spring as shown in figure (16). The spring force F_s required to deflect a spring a distance x from its free length is given by the equation:

F = kx where k = spring rate is a constant which is equal to the slop of the curve of the load F_s versus deflection x.

$$\therefore \frac{output}{input} = \frac{x}{F_s} = \frac{1}{k}$$

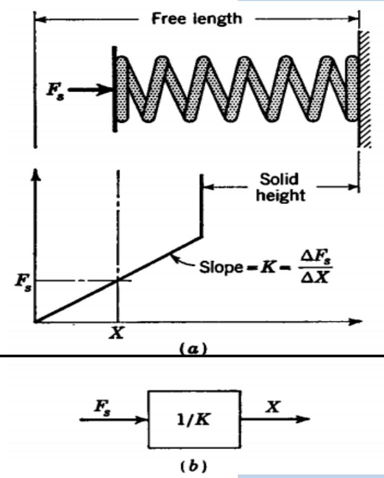
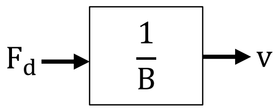
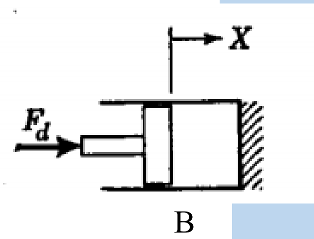


Figure (16)

Damper (Viscous Function): it is assumed that the damper will produce a counter effect to the applied force such that the force is proportional to the velocity (v), see figure (17).

$$F_d = Bv = B\dot{x} = BDx = Bsx$$





$$F(t) = B\dot{x}(t)$$

1. derivate operator: $F(D) = BDx(D)$ G(D) (transfer function) $= \frac{x(D)}{F(D)} = \frac{1}{BD}$

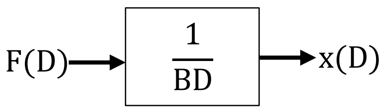
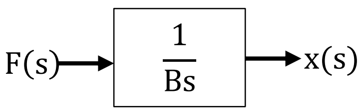


Figure (17)

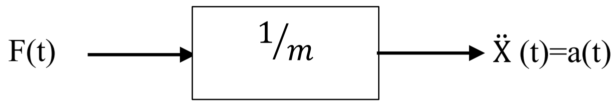
2. Laplace operator: $G(s) = \frac{x(s)}{F(s)} = \frac{1}{Bs}$



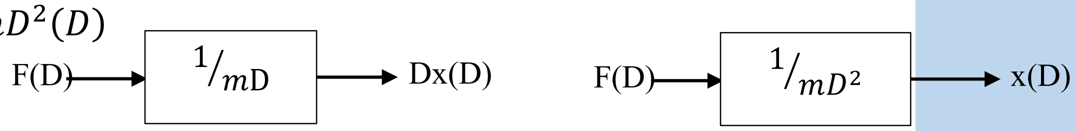
1. <u>Inertia (Mass):</u>

Using Newton second law, we have $f = ma = m\dot{v} = m\ddot{x} = mD^2x = ms^2x$.

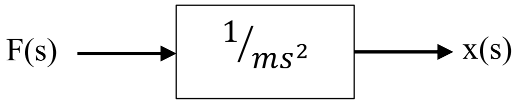
$$F(t) = m * \ddot{X}(t)$$



$$F(D) = mD^2(D)$$

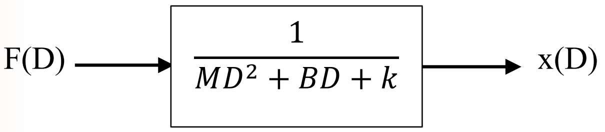


$$F(s) = ms^2X(s)$$



Example: find the transfer function of the translation motion of the mass- spring-Damper system as shown in figure (18).

$$\sum F = Ma = M\frac{d^2X}{dt^2} = MD^2X$$



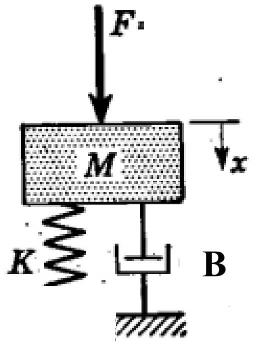
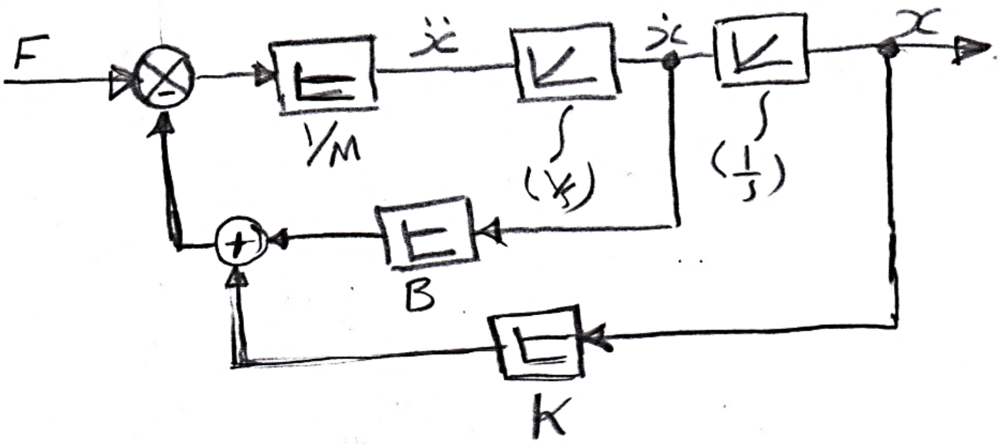


Figure (18)

$$M\ddot{x} + B\dot{x} + Kx = F$$

$$\ddot{x} = \frac{F - B\dot{x} - Kx}{M}$$

Control Block diagram.

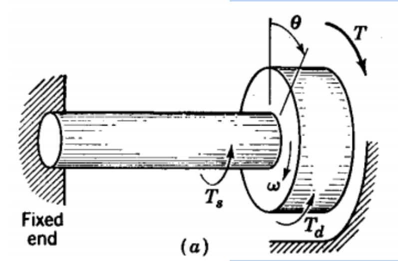


2. Rotational Mechanical System:

For torsional mechanical system, the forcing function is the torque (T) and the output is (θ) or ($\dot{\theta}$) or ($\ddot{\theta}$). In this case the mass moment of inertia (I), the torsional damping coefficients (B_t) and the torsional stiffness (K_t) have to be used.
For a torsional subsystem shown in figure (19), we can find the transfer function between the applied torque (T) and the angle of rotation (θ) if required as an output.

Solution:

$$T = k_t * \theta$$



Where; T is a twisting torque.
K_t is the torsional spring.
θ is the angular displacement of spring.

$$\sum T = J\alpha = J\ddot{\theta}(t)$$
$$T - T_d - T_s = J\ddot{\theta},$$
$$T = I\ddot{\theta} + T_d + T_s$$

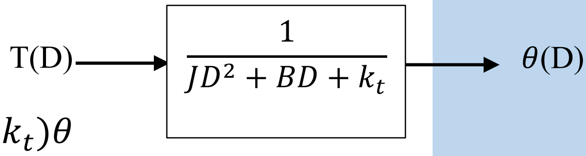


Figure (19)

$$T = JD^2\theta + BD\theta + k_t\theta = (JD^2 + BD + k_t)\theta$$
$$G(D) = \frac{\theta(\mathrm{D})}{\mathrm{T(D)}} = \frac{1}{JD^2 + BD + k_t}$$

Example: Calculate the transfer function (T.F) of the translation motion for each of the following mechanical systems as shown in figure (20-a, and 20-b).

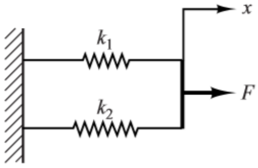


Fig. (20-a) two springs connected in parallel

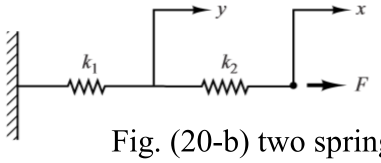


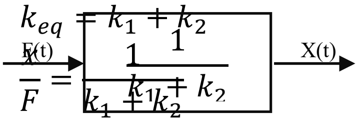
Fig. (20-b) two springs connected in series

a) F=F1+F2

$$k_1 x + k_2 x = F = k_{eq} x$$

$$k_{eq} = k_1 + k_2$$

$$\frac{x}{F} = \frac{1}{k_1 + k_2}$$



b) X = y+(x-y)

$k_1 y = F,$

$y = F/k_1,$ …. (1)

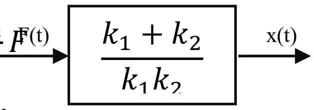
$k_2(x - y) = F,$ …. (2)

Sub. (1) in (2) we get:

$$k_2 x = F + \frac{k_2}{k_1} F = \frac{k_1 + k_2}{k_1} F$$

$$F = k_{eq} x = \left(\frac{k_1 k_2}{k_1 + k_2}\right) x$$

$$\frac{x}{F} = \frac{1}{\frac{k_1 k_2}{k_1 + k_2}} = \frac{k_1 + k_2}{k_1 k_2}$$



Example: Calculate the transfer function (T.F) for the each of the translation motion of the damper systems (as shown in figure (21-a, and 21-b).

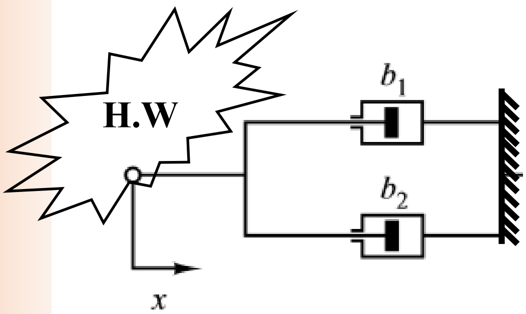


Fig. (21-a) two dampers connected in parallel

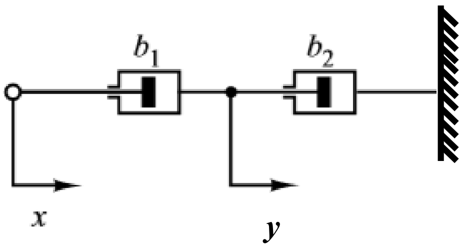
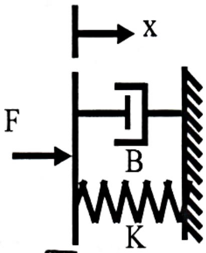


Fig. (21-b) Two dampers connected in series

Example: calculate the T.F of the following systems.

a)



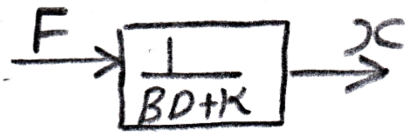
$$F = F_s + F_B$$

$$F = Kx + B\dot{x}$$

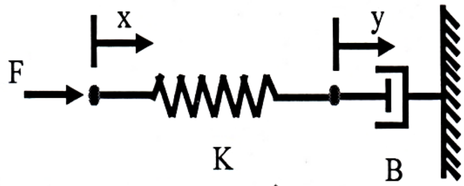
$$F = BDx + Kx$$

$$F = (BD + K)x$$

$$\frac{x}{F} = \frac{1}{BD+K} = T(D)$$



b)



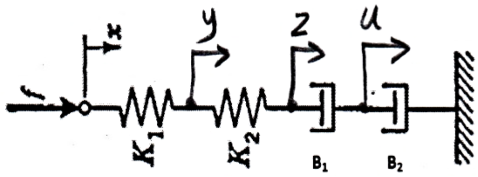
$$x = (x-y) + y$$

$$x = \frac{F}{K} + \frac{F}{BD}$$

$$x = F\left(\frac{1}{K} + \frac{1}{BD}\right)$$

$$\frac{x}{F} = \left(\frac{1}{K} + \frac{1}{BD}\right) = T(D)$$

c)

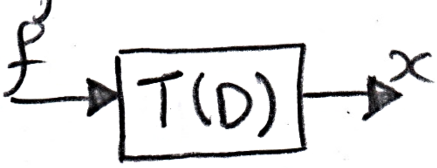


$$x = (x-y) + (y-z) + (z-u) + u$$

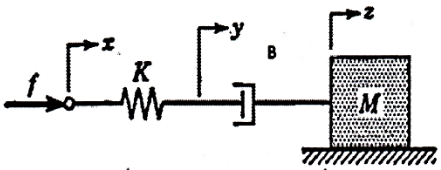
$$x = f\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{B_1D} + \frac{1}{B_2D}\right)$$

$$\frac{x}{f} = \frac{1}{B_1D} + \frac{1}{B_2D} + \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{x}{f} = T(D)$$



d)



$$x = (x-y) + (y-z) + z$$

$$x = \frac{f}{K} + \frac{f}{BD} + \frac{f}{MD^2} \quad (1)$$

$$\frac{x}{f} = \left(\frac{1}{K} + \frac{1}{BD} + \frac{1}{MD^2}\right) = T(D) \quad \text{ok} \quad (2)$$

* Note the eq. (1) Can be written as:

$$\dot{x} = \left(\frac{D}{K} + \frac{1}{B} + \frac{1}{MD}\right)f$$

OR

$$\ddot{x} = \left(\frac{D^2}{K} + \frac{D}{B} + \frac{1}{M}\right)f$$

Solutions based Equivalent impedance for parallel mechanical system (mass move in one coordinate only).

b) $f = \dfrac{1}{\dfrac{1}{K} + \dfrac{1}{BD}} x \Rightarrow f = \dfrac{1}{\left(\dfrac{K+BD}{KBD}\right)} x \Rightarrow f = Zx$

$f = \dfrac{KBD}{K+BD} x$

$x/f = \dfrac{K+BD}{KBD} = T(D).$ ok

c) $f = \dfrac{1}{\dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{1}{B_1 D} + \dfrac{1}{B_2 D}} x$

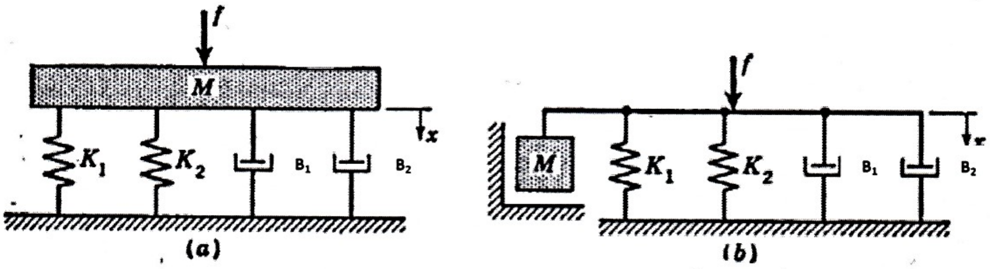
$f = \dfrac{1}{\left(\dfrac{(K_2 B_1 D B_2 D) + (K_1 B_1 D B_2 D) + (K_1 K_2 B_2 D) + (K_1 K_2 B_1 D)}{(K_1 K_2 B_1 D B_2 D)}\right)} x$

$f = Zx$

$\dfrac{x}{f} = \dfrac{(K_2 B_1 D B_2 D) + (K_1 B_1 D B_2 D) + (K_1 K_2 B_2 D)(K_1 K_2 B_1 D)}{K_1 K_2 B_1 D B_2 D}$

$= T(D).$ ok

e)



$+\downarrow \sum F = ma$

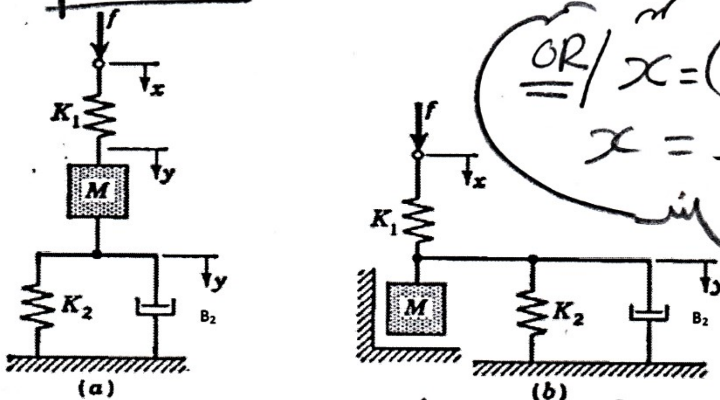
$M\ddot{x} = f - B_1\dot{x} - B_2\dot{x} - K_1 x - K_2 x$

$(MD^2 + (B_1 + B_2)D + K_1 + K_2)x = f$

$$\frac{x}{f} = \frac{1}{MD^2 + (B_1+B_2)D + K_1 + K_2} = T(D)$$

$f \rightarrow$ [$T(D)$] $\rightarrow x$

f)



Solution①

OR/ $x = (x - y) + y$

$$x = \frac{f}{K_1} + \frac{f}{MD^2 + B_2D + K_2}$$

compare the solution with ② & ③

Solution②

z: equivalent impedance for mechanical elements in parallel.

$f = Zx$

$$f = \frac{1}{\frac{1}{K_1} + \frac{1}{mD^2 + B_2D + K_2}}\, x$$

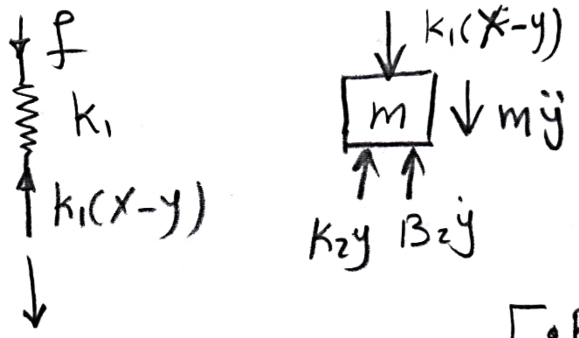
$$f = \frac{1}{\frac{(mD^2 + B_2D + K_2) + K_1}{(mD^2 + B_2D + K_2)K_1}}\, x$$

$$f = \frac{(mD^2 + B_2D + K_2)K_1}{(mD^2 + B_2D + K_2) + K_1}\, x$$

$$\frac{x}{f} = \frac{mD^2 + B_2D + K_1 + K_2}{(mD^2 + B_2D + K_2)K_1} = T(D)$$

$f \rightarrow$ [$T(D)$] $\rightarrow x$ OK//

Solution ③



F.B.D

$$f = K_1(x-y) \quad —①$$

$$m\ddot{y} = -B_2\dot{y} - k_2 y + k_1(x-y)$$

$$m\ddot{y} + B_2\dot{y} + (k_2+k_1)y - k_1 x = 0 \quad —②$$

To find x/f → eleminate y coordinate

∴ from eq.②

$$[mD^2 + B_2 D + (k_2 - k_1)]y - k_1 x = 0$$

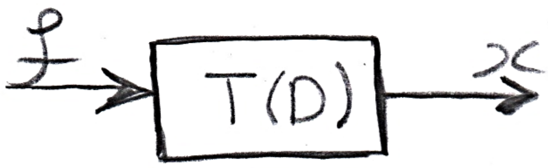
$$y = \frac{K_1}{mD^2 + B_2 D + K_1 + K_2} \quad —③ \quad \text{sub in ①}$$

$$f = K_1 x - \frac{K_1^2}{mD^2 + B_2 D + K_1 + K_2}x$$

$$= \left[\frac{(mD^2 + B_2 D)K_1 + K_1^2 + K_1 K_2 - K_1^2}{mD^2 + B_2 D + K_1 + K_2}\right]x$$

$$f = \frac{(mD^2 + B_2 D + K_2)K_1}{mD^2 + B_2 D + K_1 + K_2}x$$

$$\frac{x}{f} = \frac{mD^2 + B_2 D + K_1 + K_2}{(mD^2 + B_2 D + K_2)K_1} = T(D)$$



* y/f ?

at y coordinate we can find the y/f (from eq. ②)

$$(mD^2+B_2D+K_2)y = K_1(x-y)$$

sub eq ① in eq ②

$$f = K_1(x-y)$$

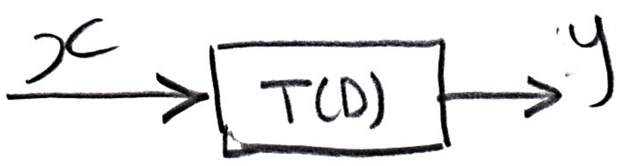
$$\therefore (mD^2+B_2D+K_2)y = f$$

$$\frac{y}{f} = T(D) = \frac{1}{mD^2+B_2D+K_2} \quad \text{ok}$$

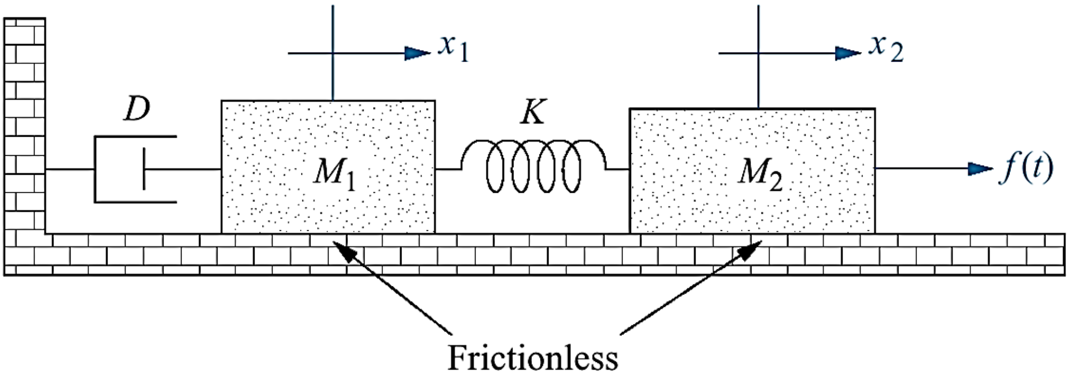
* $$y/x = y/f \cdot f/x$$

$$y/x = \frac{1}{(mD^2+B_2D+K_2)} \cdot \frac{(mD^2+B_2D+K_2)K_1}{(mD^2+B_2D+K_2)+K_1}$$

$$y/x = \frac{K_1}{mD^2+B_2D+K_2+K_1} = T(D)$$



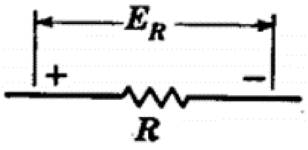
g)

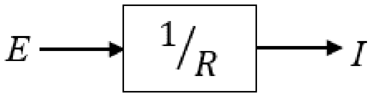


Electrical components:

1. **Resistance (R):** The relationship between the current and the applied voltage (or voltage difference) is given by:

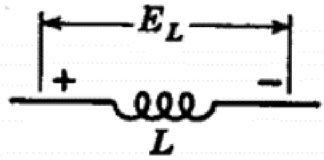
$$I = \frac{E}{R}$$

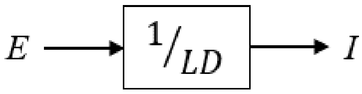




2. **Inductance (L):** In this case the relationship is given by:

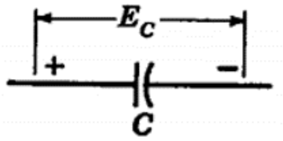
$$E = L\frac{dI}{dt} = LDI$$

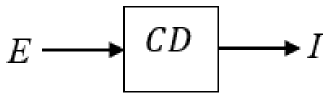




3. **Capacitance (C):** The relationship is given by:

$$I = C\frac{dE}{dt} = CDE$$

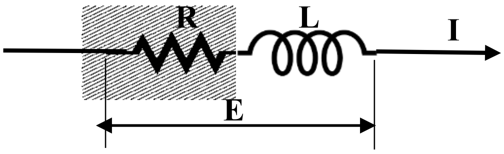




It may be noticed that when the applied voltage is constant, no current is induced in the capacitance. We have to know that the current (I) is related to the electrical charge (C) by:

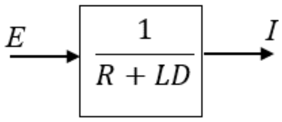
$$I = \frac{dC}{dt} = DC$$

Example: Determine the transfer function ($^I/_E$) for the following circuit.



Solution:

$$E = E_R + E_L$$
$$E = RI + LDI = I(R + LD)$$
$$\therefore \quad \frac{I}{E} = \frac{1}{R+LD}$$
$$or \ = \frac{1}{R}\left(\frac{1}{1+\tau D}\right)$$

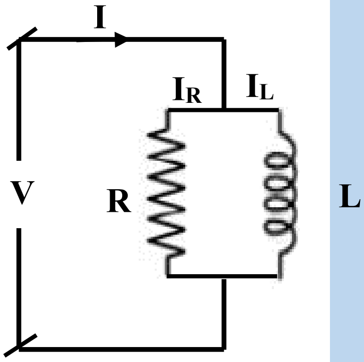


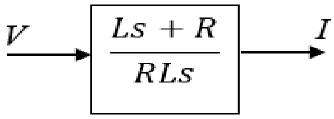
Where; $\tau = {}^L/_R$

Electrical components connected in parallel:

1. $I = I_R + I_L$

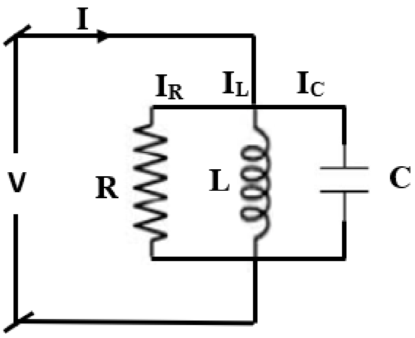
$$I(s) = \frac{V(s)}{R} + \frac{V(s)}{Ls}$$
$$\therefore \quad I(s) = V(s)\left[\frac{1}{R} + \frac{1}{Ls}\right]$$
$$I(s) = V(s)\left[\frac{Ls+R}{RLs}\right]$$

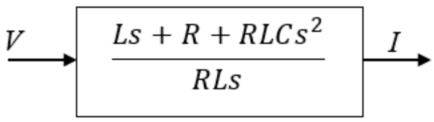




2. $I = I_R + I_L + I_C$

$$I(s) = V(s)\left[\frac{1}{R} + \frac{1}{Ls} + Cs\right]$$
$$I(s) = V(s)\left[\frac{Ls+R+RLCs^2}{RLs}\right]$$





Example:

Let e_o the voltage drops and i be the current, then

1- Resistance R $\quad e_o = i\,R$

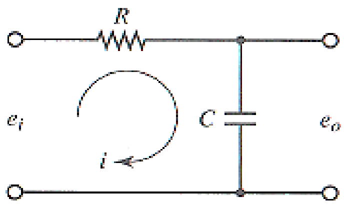
2- Capacitance C $\quad e_o = \frac{1}{C}\int i\,dt$

3- Inductance L $\quad e_o = \frac{di(t)}{dt}$

Basic laws governing electrical circuits are Kirchhoff's current law and voltage law. **Kirchhoff's current law (node law)**. Which states that the sum of currents entering a node is equal to the sum of currents leaving the same node.
Kirchhoff's voltage law (loop law). Which states that the sum of the voltage drops is equal to the sum of the voltage rises around a loop.

Procedures for Drawing a Block Diagram

Consider the RC circuit shown in the Figure (a). Applying Kirchhoff's voltage law to the system, we obtain



(a)

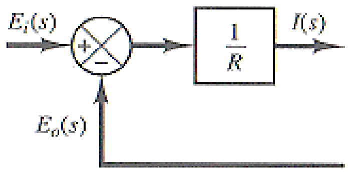
$$V_R + V_C = e_i$$

$$iR + e_o = e_i \quad \Longrightarrow \quad i = \frac{1}{R}(e_i - e_o) \quad \cdots\cdots\cdots (1)$$

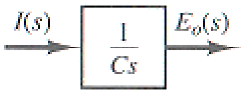
$$e_o = \frac{1}{C}\int i\,dt \quad \cdots\cdots\cdots (2)$$

The L.T of equations (1) and (2), with zero initial conditions, becomes

$$I(s) = \frac{1}{R}\left[E_i(s) - E_o(s)\right] \quad \Longrightarrow$$

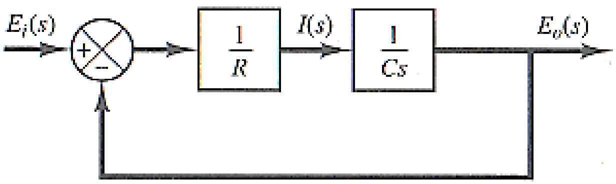


$$E_o(s) = \frac{I(s)}{Cs} \quad \Longrightarrow$$



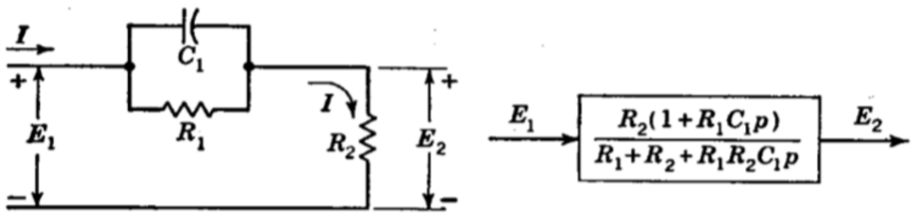
(c)

Assembling these two elements, we obtain the overall l the system as shown in Figure (d)



(d)

Example:



Electrical circuit.

SOLUTION. The parallel combination of R_1 and C_1 is in series with R_2, so that the total impedance Z is

$$Z = Z_1 + R_2 = \frac{1}{1/R_1 + C_1p} + R_2 = \frac{R_1}{1 + R_1C_1p} + R_2$$

The voltage E_1 is given by the equation

$$E_1 = ZI = \frac{R_1 + R_2 + R_1R_2C_1p}{1 + R_1C_1p} I \qquad \text{...1}$$

and similarly E_2 is

$$E_2 = R_2I \qquad \text{...2}$$

The substitution of I from Eq. (1) into Eq. (2) yields the desired answer

$$E_2 = \frac{R_2(1 + R_1C_1p)}{R_1 + R_2 + R_1R_2C_1p} E_1$$

Thermal Components:

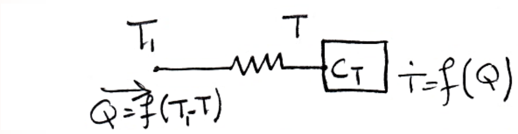
Thermal resistance: for small temperature difference, the rate of heat transferred into a body is proportional to the temperature difference across the body.

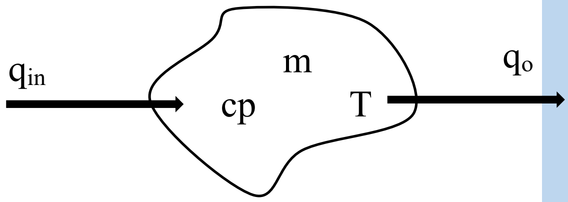
$$q = hA(T_1 - T) = \frac{T_1 - T}{R_T}$$

Where:
q= rate of heat flow
h= coefficient of heat transfer of the surface of the body
A= surface area
T= the temperature of the body
T_1=temperature of the surrounding medium
R_T= 1/hA equivalent thermal resistance (analog to the electrical resistance).

Thermal capacitance: the rate of change of temperature of the body (dT/dt =DT) is related to the rate of heat transfer in to the body by the expression:

$$q = mcp\frac{dT}{dt} = C_T DT$$





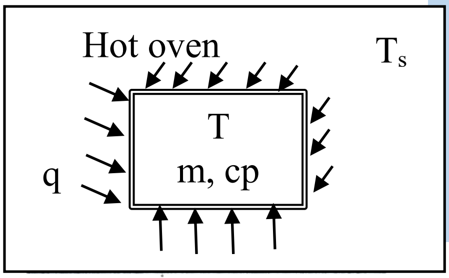
In this case, the mass (m) and its specific heat (cp) are combined to form the equivalent thermal capacitive effect such that: $C_T = mcp$.
However, thermal components appear combined in practice like in the following example.

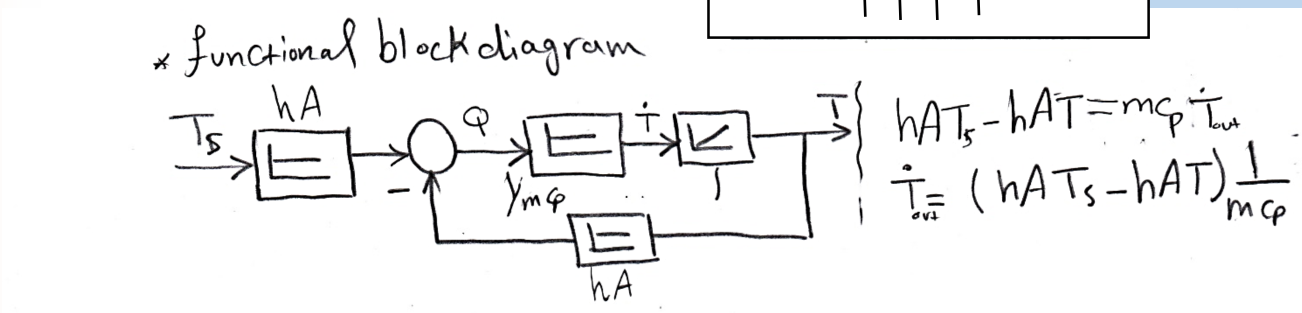
Example: A mass (m) with some (cp) is located in a hot oven of temperature (T_s), assuming that the mass is heated by convection only. Find the transfer function ($\frac{T}{T_s}$)

We have; $q = h.A(T_s - T)$

Also, $q = m.cp\frac{dT}{dt}$

$\therefore \frac{T}{T_s} = \frac{1}{1+\tau D}$ where $\tau = \frac{m.cp}{h.A} = m.cp.R_T$

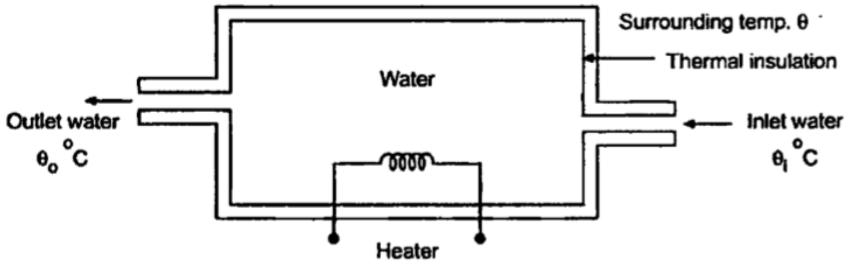




Example:

A thermal system used for heating flow of water as shown. Electric element is used in the tank to heat the water. The tank is insulated to reduce heat to the surroundings. Find the transfer function of outlet water temperature (θ_o) to the rate of heat flow from heat element (q).

Solution:



θ_o = Outlet water temp. (°C)

θ = Surrounding temp.

q = heat flow rate from heating element (J/sec)

q = heat flow rate to the water

q_o = heat flow rate through tank insulation

ok

C = Thermal capacity (J/°C)

R = Resistance of thermal insulation

Note/ * The tank is insulated to reduce heat to the surroundings. (There is q_o)

- $q - q_o = C\dfrac{d\theta_o}{dt} = C D\theta_o$ — (1)

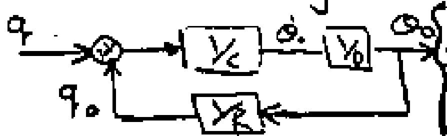
$q_o = \dfrac{\theta_o - \theta}{R} \simeq \dfrac{\theta_o}{R}$ (Value of $\theta_o >> \theta$) ∴ The term θ/R can be neglected.

$\therefore q = DC\theta_o + \dfrac{\theta_o}{R}$

$q = \dfrac{(DCR+1)}{R}\theta_o$

$\therefore \dfrac{\theta_o}{q} = \dfrac{R}{1+DCR} = \dfrac{R}{1+\tau D}$

$\tau = RC$ (Time constant)

- 

Reduction Block

$\dfrac{\theta_o}{q} = \dfrac{1/DC}{1+1/DC \cdot 1/R} = \dfrac{(1/DC)}{(DCR+1)/DCR} = \dfrac{R}{CDR+1}$

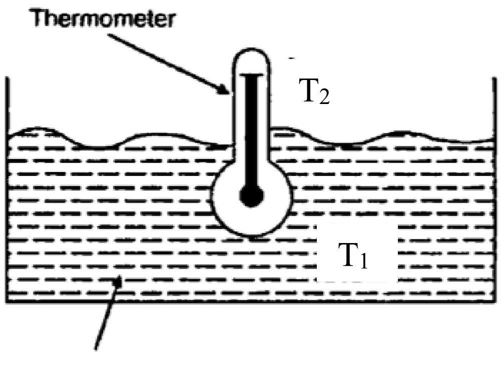
* $\left(q - \dfrac{\theta_o}{R}\right)\left(\dfrac{1}{DC}\right) = \theta_o \Rightarrow q - \dfrac{\theta_o}{R} = DC\theta_o$

$\therefore q = DC\theta_o + \theta_o/R = \dfrac{(RDC+1)}{R}\theta_o$

Example:

Example: Consider a thermometer placed in a water bath having temperature T_1, as shown, and T_2 is the temperature indicated by the thermometer. Calculate the transfer function of T_2/T_1.

q



Solution

$T_2/T_1 = ?$

* heat flow rate into the thermometer through its wall is:

$$q_R = \frac{T_1 - T_2}{R} \quad (1)$$

ok

* Temperature rises at a rate of

$$q_c = c\frac{dT_2}{dt} \quad (2)$$

$q_R = q_c \quad \therefore \quad c\frac{dT_2}{dt} = \frac{T_1 - T_2}{R}$

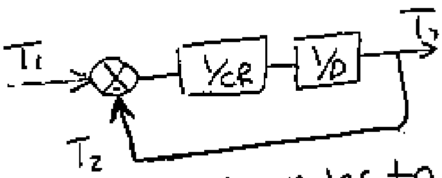
$$cDT_2 = \frac{T_1 - T_2}{R}$$

$$cDT_2R = T_1 - T_2$$

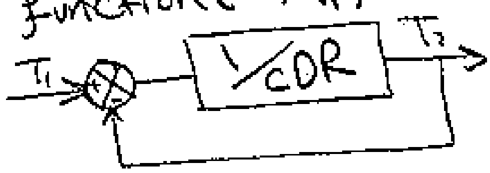
$$T_1 = (CDR + 1)T_2$$

$$\frac{T_2}{T_1} = \frac{1}{1+CDR} = \frac{1}{1+\tau D}$$

$\tau = RC$ (tim constant)



* use Reduction rules to find over all transfer function (T_2/T_1)



$$\frac{T_2}{T_1} = \frac{1/CDR}{1 + 1/CDR}$$

$$= \frac{1/CDR}{\frac{CDR+1}{CDR}}$$

$$\frac{T_2}{T_1} = \frac{1}{CDR+1} = \frac{1}{1+\tau D}$$

Fluid system components:

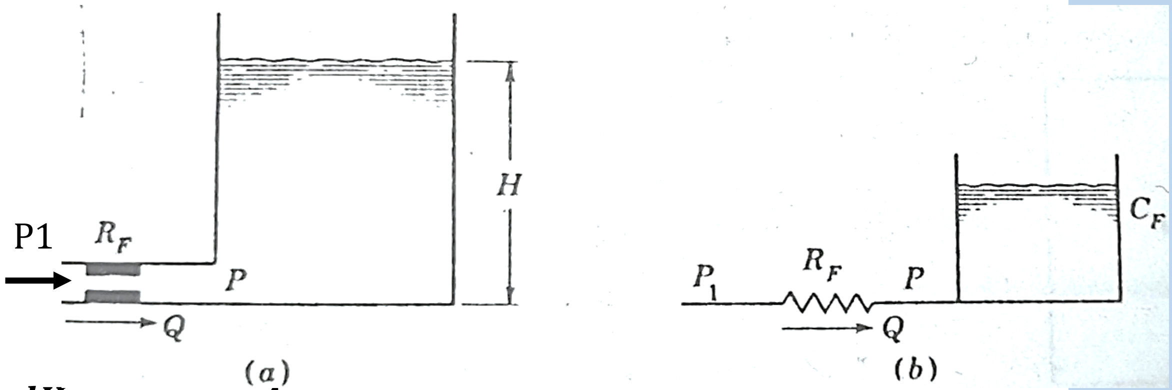
In working with fluid systems. It is necessary to distinguish if the fluid is incompressible (work with **volume rate** of flow) or compressible (work with **mass rate** of flow).

A) **Incompressible fluids:** when the pressure difference across a flow restriction is small, the volume rate of flow Q is proportional to the pressure drop ($P_1 - P$) across the restriction.

Q = f(P₁ – P) then….. $Q = \frac{P1 - P}{R_F}$

Where: R_F is the equivalent fluid resistance.

The rate of flow into a tank, such as that shown if figure (1), is equal to cross sectional area A of the tank times the rate of change of height.



$$Q = A\frac{dH}{dt} = ADH = \frac{A}{\rho g}DP = C_F DP$$

Where:

$$P(kg.m/(s.s.m.m) = \rho g H(kg.m.\frac{m}{m.m.m.s.s})$$, in which ρ is the density of the fluid

H = the head (m).

$C_F = A/\rho g$ = equivalent fluid capacitance.

The equation of operation for fluid system of figure (1) is:

$$Q = \frac{P1 - P}{R_F} = C_F DP$$

Solving for P gives:

$$P = \frac{P1}{1 + (R_F C_F)D}$$

$$\frac{P}{P1} = \frac{1}{1 + (R_F C_F)D} = \frac{1}{1 + \tau D}$$

$$\left(\frac{P1 - P}{R_F}\right)\left(\frac{1}{C_F}\right) = DP$$

Draw the block diagram
H.W

In general:

$$\sum(flowrate)in - \sum(flowrate)out = rate\ of\ accumulation.$$

B) **Compressible fluids:**

For small pressure differences, the mass rate of flow M (gas or air) through a restriction is proportional to the pressure difference P1 − P.

$$\mathbf{M} = \frac{\mathbf{P1 - P}}{R_F}$$

M = mass flow rate

Figure (2) shown a tank of constant volume V. the equation of state for the fluid in the tank is:

PV = WRT, where:

P=pressure, V=volume of tank, T isothermal temperature, R= heat transfer coefficient, M= mass flow rate, W= weight

The flow into such a tank is usually isothermal. Thus, differentiating both side of the equation of state with respect to time and solving for M = dW/dt gives:

$$\mathbf{M} = \frac{\mathbf{dW}}{dt} = \frac{\mathbf{V}}{RT}\frac{\mathbf{d}}{dt}P = \frac{\mathbf{V}}{RT}DP = C_F DP$$

$$\left(\frac{\mathbf{P1 - P}}{R_F}\right)\left(\frac{1}{C_F}\right) = DP$$

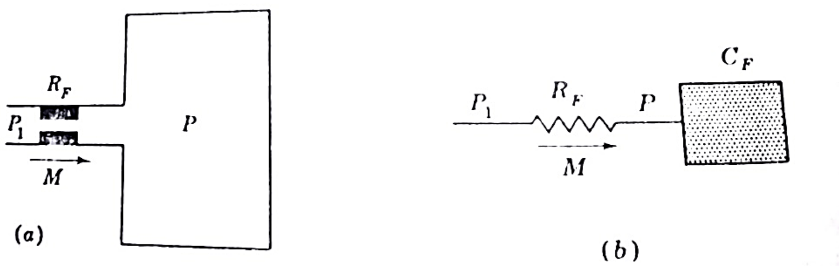
Draw the block diagram

H.W

$C_F = V/RT$ **Is the equivalent fluid capacitance.**

The equation of operation is:

$$P = \frac{P_1}{1 + (R_F C_F)D}$$



Resistance and Capacitance of Liquid- Level Systems

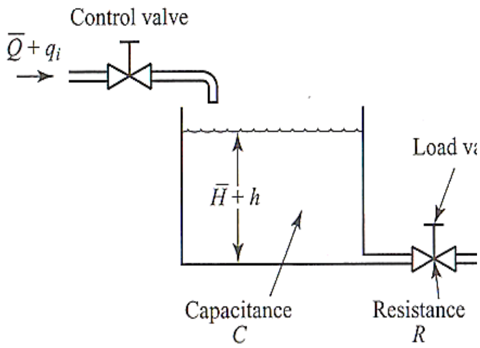
It is convenient to introduce the concept of **resistance and capacitance** to describe dynamic characteristics of liquid-level systems.

1) Consider the flow through a short pipe connecting two tanks. The resistance R for liquid flow in such a restriction is defined as the change in the level difference (the difference of the liquid levels of the two tanks) necessary to cause a unit change in flow rate; namely,

$$R = \frac{\text{Change in level difference, m}}{\text{Change in flow rate, m}^3/\text{s}}$$

The relationship between the **flow rate and level difference** differs for the laminar flow and turbulent flow. The flow regimes are divided into laminar flow and turbulent flow, according to the magnitude of the Reynolds number.

Consider the liquid-level system shown below.



- In this system the liq … n the side of tank. If the flow through this res …

$$Q = K\,I\ldots$$

Where, Q = Steady-state liquid flow rate, m^3/s
K = Coefficient, m^2/s
H = Steady-state head, m

For **laminar** flow, the resistance R_l is obtained as

$$R_l = \frac{dH}{dQ} = \frac{H}{Q}$$

The laminar-flow, resistance is constant and is analogous to the electrical resistance.

- If flow through the restriction is **turbulent**, the steady-state liquid flow rate is given by,

$$Q = K\sqrt{H} \qquad \cdots\cdots\cdots (1)$$

Where K is a coefficient having units of $m^{2.5}/s$. The resistance R_t for turbulent flow is obtained from equation (1)

$$dQ = \frac{K}{2\sqrt{H}}dH$$

We have

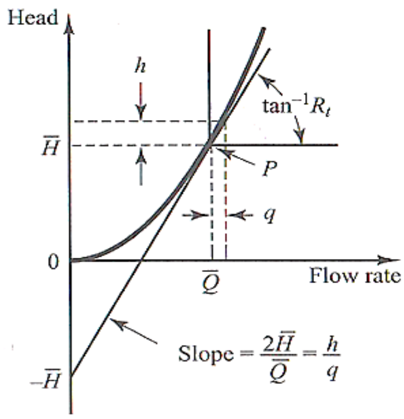
$$\frac{dH}{dQ} = \frac{2\sqrt{H}}{K} = \frac{2\sqrt{H}\sqrt{H}}{Q} = \frac{2H}{Q}$$

$$R_t = \frac{dH}{dQ} = \frac{2H}{Q}$$

- The value of the turbulent flow resistance R_t depends upon the flow rate and head.
- Linearization can be applied, provided that changes in the head and flow rate from their **respective steady-state values** are **small**.

- In many cases, the value of the coefficient K, which depends upon the **flow coefficient and the area of restriction**, **is not known**. Then the resistance may be determined by plotting the head versus flow rate curve based on experimental data and measuring the slope of the curve at the **operating condition**.

An example of such a plot is shown in Figure (b). In the Figure, point P is the steady-state operation point. The tangent line to the curve at point P intersects the ordinate at point $(0, -\bar{H})$. Thus, the slope of this tangent line is $2\bar{H}/\bar{Q}$. Since the resistance R_t at the operation point P is given by $2\bar{H}/\bar{Q}$, the resistance R_t is the slope of the curve at the operation point.



(b)

Consider the operation condition in the neighborhood of point P. Define a small deviation of the head from the steady-state value as h and the corresponding small change of the flow rate q. Then the slope of the curve at point P can be given by

$$\text{Slope of the curve at point } P = \frac{h}{q} = \frac{2\bar{H}}{\bar{Q}} = R_t$$

2) The capacitance C of a tank I defined to be the change in quantity of **stored liquid** necessary to cause a unit change in the **potential (head).**

$$C = \frac{\text{Change in liquid stored, } m^3}{\text{Change in head, m}}$$

- The differential equation of the tank system shown can be obtained based on the assumption that the system is either linear or linearized.
- Consider a small disturbance from certain steady state operating point where $\bar{Q}$ and $\bar{H}$ are steady state flow rate and head, respectively.
- A small deviation in inflow q_i will cause a small deviation in out flow q_o and head h. Since the inflow minus outflow during the small time interval dt is equal to the additional amount stored in the tank, then

$$C\,dh = (q_i - q_o)\,dt$$

From the definition of resistance, the relationship between q_o and h is given by

$$q_o = h / R$$

The differential equation for this system for a constant value of R becomes

$$RC\frac{dh}{dt} + h = Rq_i$$

Note that RC is the time constant of the system. Taking Laplace transform of both sides of the differential equation we obtain the transfer function of the system

$$\frac{H(s)}{Q_i(s)} = \frac{R}{1+RCs}$$

If q_o is taken as the output with q_i as the input, then

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{1+RCs} \qquad \text{Where } \frac{H}{R} = Q_o$$

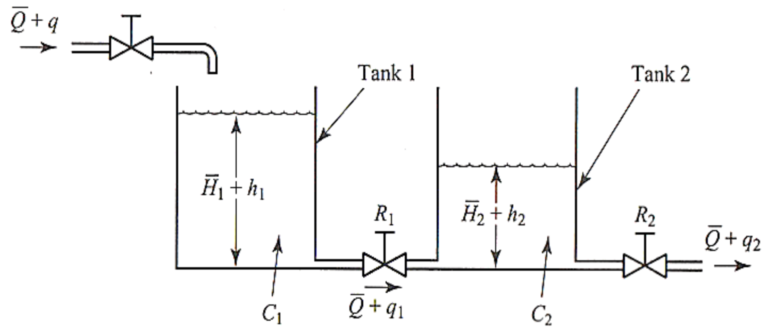
Liquid-Level System with Interaction

If there is more than one tank, then the dynamics of the two (or more) tanks will interact. The transfer function is NOT the PRODUCT of the two first-order transfer functions. For the two tanks system shown, assume only small variations of the variables from the steady-state values. Using the symbols

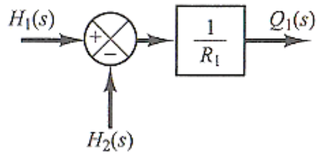
Q = Steady-state liquid flow rate, m^3/s

$\bar{H}_1$= Steady-state liquid level of tank 1, m

$\bar{H}_2$= Steady-state liquid level of tank 2, m

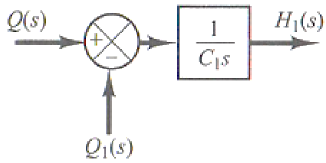


For tank I



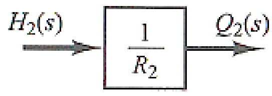
$$\frac{h_1-h_2}{R_1}=q_1 \quad Q_1(s)=[H_1(s)-H_2(s)]\,\frac{1}{R_1}$$

$$C_1\frac{dh_1}{dt}=q-q_1 \quad H_1(s)=[Q(s)-Q_1(s)]\frac{1}{C_1 s}$$

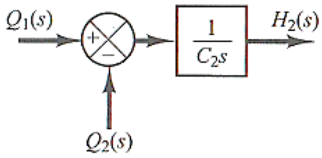


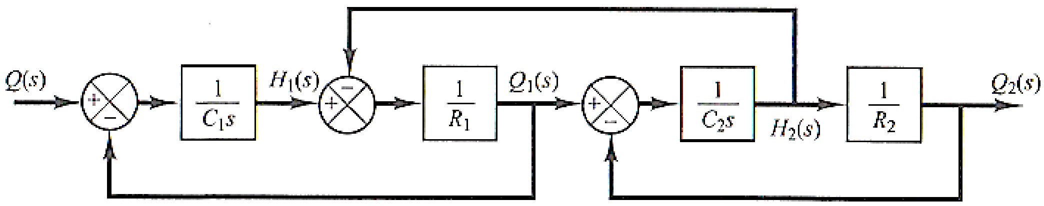
For tank II

$$\frac{h_2}{R_2}=q_2 \qquad Q_2(s)=\frac{1}{R_2}\,H_2(s)$$



$$C_2\frac{dh_2}{dt}=q_1-q_2 \quad H_2(s)=[Q_1(s)-Q_2(s)]\frac{1}{C_2 s} \quad \Rightarrow$$



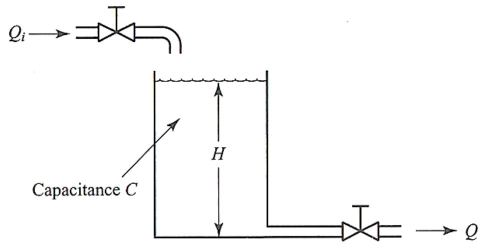


Example

In the liquid-level system shown in Figure, assume that the outlet flow rate Q (m^3/sec) through the out-flow valve is related to the head H (m) by

$$Q = K\sqrt{H} = 0.01\sqrt{H}$$

Assume also that when the inflow rate Q_i is 0.015 m^3/sec **the head stays constant**. For $t < 0$ the system is at steady state ($Q_i = 0.015\ m^3/sec$). At $t = 0$ the inflow valve is closed and so there is no inflow for $t \geq 0$. Find the time necessary to empty the tank to half the original head. The capacitance C of the tank is 2 m^2.



Solution

When the head is stationar rate. Thus head H_o at time $t = 0$ can obtained as

$$Q = 0.015 = 0.01\sqrt{H_o} \quad \Rightarrow \quad H_o = 2.25\ m$$

The equation for the system for $t > 0$ is

$$-CdH = Qdt \qquad \frac{dH}{dt} = -\frac{Q}{C} = -\frac{0.01\sqrt{H}}{2}$$

or

$$\frac{dH}{\sqrt{H}} = -0.005\ dt$$

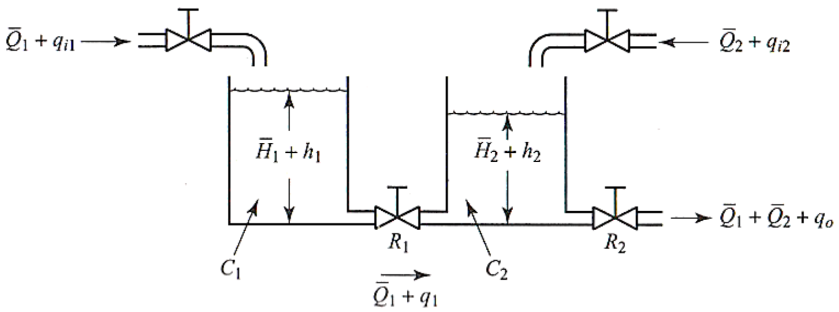
Then by integrating both sides of this equation $\int_{2.25}^{1.125} \frac{dH}{\sqrt{H}} =$

$$\int_0^{t_1} (-0.005)\ dt$$

$$2\sqrt{H}\Big|_{2.25}^{1.125} = -0.005\ t_1 \quad \Rightarrow \quad t_1 = 175.7\ sec$$

Example

Consider the liquid-level system shown in Figure. In the system, $\bar{Q}_1$ and $\bar{Q}_2$ are steady-state inflow rates and $\bar{H}_1$ and $\bar{H}_2$ are steady-state heads. The quantities q_{i1}, q_{i2}, h_1, h_2, and q_o are considered small. Obtain the dynamic equations of the system

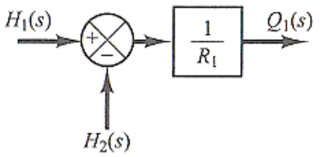


Solution

The equations for the system are

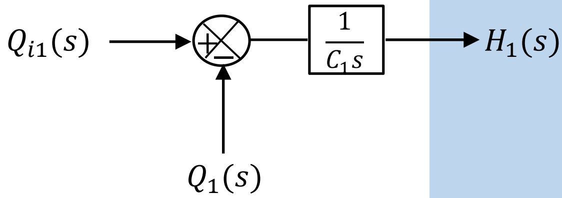
For tank I

$$\frac{h_1-h_2}{R_1} = q_1 \quad Q_1(s) = [H_1(s) - H_2(s)]\ \frac{1}{R_1}$$



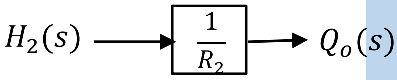
$$C_1\frac{dh_1}{dt} = q_{i1} - q_1$$

$$H_1(s) = [Q_{i1}(s) - Q_1(s)]\frac{1}{C_1 s} \quad \Rightarrow$$



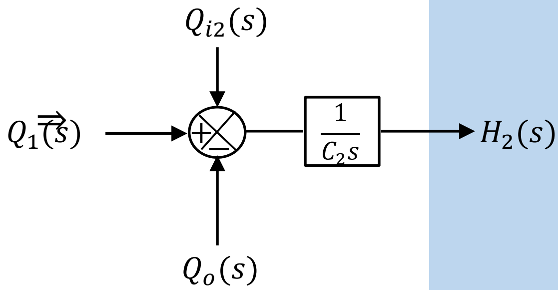
For tank II

$$\frac{h_2}{R_2} = q_o \qquad Q_o(s) = \frac{1}{R_2}\ H_2(s) \Rightarrow$$

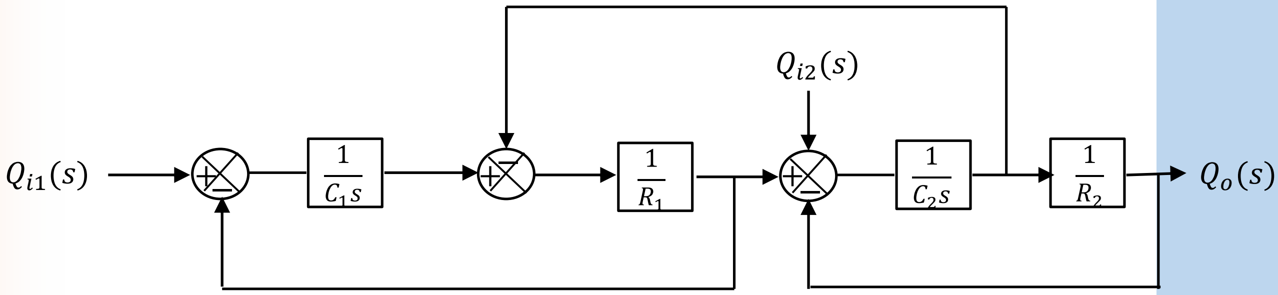


$$C_2\frac{dh_2}{dt} = q_1 + q_{i2} - q_o$$

$$H_2(s) = [Q_1(s) + Q_{i2}(s) - Q_o(s)]\frac{1}{C_2 s}$$



The overall block diagram of the system is



Pneumatic Systems

Resistance and Capacitance of Pressure Systems. Many industrial processes and pneumatic controllers involve the flow of a gas through connected pipelines and pressure vessels (خزانات).

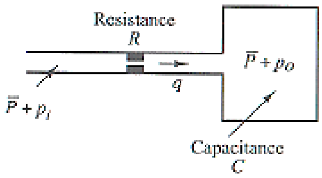
Consider the pressure system shown in Figure (a). The gas flow through the restriction is a function of the gas pressure difference $p_i - p_o$. Such a pressure system may be characterized in terms of a resistance and a capacitance.

Definitions

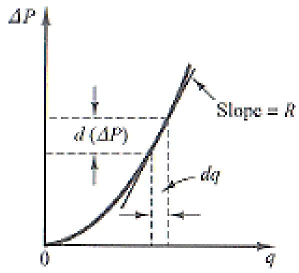
- The gas flow resistance R may be defined as follows

$$R = \frac{\text{change in gas pressure difference}}{\text{change in gas flow rate}} = \frac{d(\Delta P)}{dq} \quad \cdots\cdots\cdots (1)$$

The value of the gas flow resistance R can be determine experimentally from a plot of the pressure difference versus flow rate by calculating the slope of the curve at a given operating condition as shown in Figure (b).



(a)



(b)

- The capacitance of the pressure vessel may b

$$C = \frac{\text{change in gas stored}}{\text{change in gas pressure}}$$

or

$$C = \frac{dm}{dp} = V\frac{d\rho}{dp} \quad \cdots\cdots\cdots (2)$$

let us define

$\bar{P}$ = gas pressure in the vessel at steady state (before changes in pressure have occurred

p_i = small change in inflow gas pressure

p_0 = small change in gas pressure in the vessel

V = volume of the vessel

m = mass of the gas pressure in the vessel

q = gas flow rate

ρ = density of the gas

For small change of p_i and p_0, the resistance R given by equation (1) becomes constant and may be written as

$$q = \frac{P_i - P_o}{R} \quad \cdots\cdots\cdots (3)$$

The capacitance C is given by equation (2)

$$C = \frac{dm}{dp} \quad \Rightarrow \quad C\,dp = dm$$

Since the pressure change dp_o times the capacitance C is equal to the gas added to the vessel during dt seconds, we obtain

$$C\, dp_o = q\, dt \qquad \Rightarrow \qquad C\frac{dp_o}{dt} = q \quad \cdots\cdots\cdots (4)$$

Now, from equations (3) and (4)

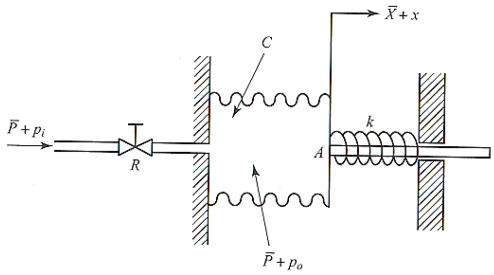
$$C\frac{dp_o}{dt} = \frac{P_i - P_o}{R} \qquad \Rightarrow \qquad RC\frac{dp_o}{dt} + p_o = p_i$$

Taking L.T of both sides $\qquad RCs\, P_o(s) + P_o(s) = P_i(s)$

$$\frac{P_o}{P_i} = \frac{1}{RCs+1}$$

<u>Example</u>

For the pneumatic system shown in Figure, assuming the steady-state values of the air pressure and the displacement of the bellows are $\bar{P}$ and $\bar{X}$, respectively. Assume also that the input pressure is changed from $\bar{P}$ to $\bar{P} + p_i$, where p_i is a small change in the input pressure. This change will cause the displacement of the bellows to change a small amount. Assuming that the capacitance of the bellows is C and the resistance of the valve is R, obtain the transfer function relating x and p.



<u>Solution</u>

For the bellows and spring $\qquad A\, p_o = kx \quad \Rightarrow \quad A\, P_o(s) = k\, X(s)$

Where A = Area of bellows $\qquad k$ = is the stiffness of the spring

Then, from the transfer function of pressure system

$$P_o(s) = \frac{1}{RCs+1} P_i(s) \qquad \frac{k}{A} X(s) = \frac{1}{RCs+1} P_i(s) \qquad \Rightarrow \qquad \frac{X(s)}{P_i(s)} = \frac{A}{k}\,\frac{1}{RCs+1}$$

Linearization of Nonlinear Functions:

Actual control systems usually contain some nonlinear elements. Such elements would in turn yield nonlinear differential equations for the system. In the following it is shown how the equations for nonlinear elements may be linearized. Thus, the resulting differential equation of operation for the system becomes linear. A plot of the nonlinear relationship:

$$Y = X^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Is shown in the figure (1). In the vicinity of the point (operating point) (X_i, Y_i), the function is closely approximated by the tangent.

For example, consider the point (X, Y) on the curve of the nonlinear function.

- The abscissa X is displaced a distance (x) from (X_i).
- This abscissa X intersects the nonlinear function a vertical distance (y+ε) from Y_i, and it intersects the tangent a distance (y) from (Y_i). the equation for Y is;

$$Y = Y_i + y + \varepsilon \approx Y_i + y \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Lowercase letters indicate the variation of the capital-letter parameters from the **reference point**. From figure (1), it is seen that the slope of the tangent line is;

$$\frac{y}{x} = \left.\frac{dY}{dX}\right|_i = slpoe\ at\ point\ (X_i, Y_i)$$

The symbol ($|_i$) means that the derivative is to be evaluated at the **reference (operating) point.** Thus,

$$y = \left.\frac{dY}{dX}\right|_i x = \left.\frac{d}{dX}(X^2)\right|_i x = 2X_i x \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

Substitution of (y) from eq. (3) into eq. (2) yields the following linear approximation for Y.

$$Y \approx Y_i + 2X_i x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

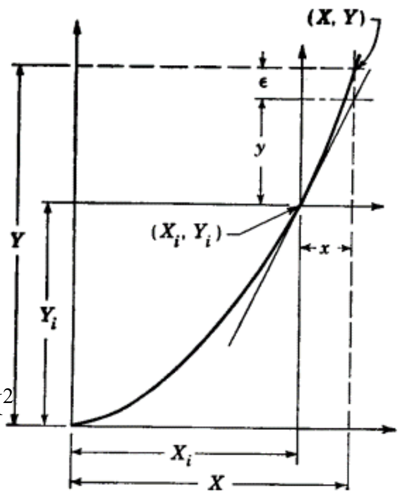


Figure (1): Graphs of function Y=X²

Example 1:

Effect a linear approximation for the equation $Y=X^2$ for values of X in the neighborhood of (10), and find the error when using this approximation for (X=11).

Solution: The reference values are (X_i =10) and (Y_i =X_i^2 = 100). The variation from the reference value is (x=X-X_i = 11-10=1). Substitution of these values into equation (4) yields;

$$Y \approx Y_i + 2X_i x = 100 + 2(10)(1) = 120$$

The exact value is ($Y=X^2$ = 121), thus, the error is (1) part in (121), or less than (1) percent.

A general procedure for obtaining a linear approximation is to use the expression derived in calculus for approximating the variation (ΔY) for a function:

$Y = Y(X_1, X_2, \dots \dots \dots, X_n)$ for (n) independent variables. That is:

$$\Delta Y = \left.\frac{\partial Y}{\partial X_1}\right|_i \Delta X_1 + \left.\frac{\partial Y}{\partial X_2}\right|_i \Delta X_2 +, \dots \dots \dots + \left.\frac{\partial Y}{\partial X_n}\right|_i \Delta X_n$$

Using lowercase letters to represent variations from the reference values, then:

$$y = \Delta Y = Y - Y_i$$
$$x_1 = \Delta X_1 = X_1 - X_{1i}$$
$$x_2 = \Delta X_2 = X_2 - X_{2i}$$
$$\vdots$$
$$x_n = \Delta X_n = X_n - X_{ni}$$

Thus, the general expression for obtaining a linear approximation for a nonlinear function is:

$$y = C_1 x_1 + C_2 x_2 + \cdots + C_n x_n \quad \dots\dots\dots\dots\dots\dots\dots\dots (5)$$

Where: $C_1 = \left.\frac{\partial Y}{\partial X_1}\right|_i$ and $C_2 = \left.\frac{\partial Y}{\partial X_2}\right|_i$ …... etc.

Example 2:

Effect the linear approximation for P in the equation of state PV=WRT. The reference conditions are (P_i =100 Ib_f /ft^2), (V_i =100 ft^3), (W_i =10/53.3 Ib_m) and (T_i =1000^0R). Determine the percent error in using this approximation for P when (V=110 ft^3), (T=1200^0R) and W remains the same. The constant R is (53.3 ft.Ib_f / Ib_m/^{o}R).

Solution: From the equation of state and the fact that W remains constant, it is seen that P as a function of the independent variables T and V, or P=P (T, V):

$$p = \left.\frac{\partial P}{\partial T}\right|_i t + \left.\frac{\partial P}{\partial V}\right|_i v$$

The partial derivative are evaluated from the equation of state as follows:

$$\left.\frac{\partial P}{\partial T}\right|_i = \frac{\partial}{\partial T}\left.\left(\frac{WRT}{V}\right)\right|_i = \left.\frac{WR}{V}\right|_i = \frac{(10)(53.3)}{(53.3)(100)} = 0.10$$

$$\left.\frac{\partial P}{\partial V}\right|_i = \frac{\partial}{\partial V}\left.\left(\frac{WRT}{V}\right)\right|_i = -\left.\frac{WRT}{V^2}\right|_i = -1.0$$

The linearized approximation for P is:

$$P \approx P_i + p = P_i + 0.1t - v$$

From the given information, it follows that:

$$v = V - V_i = 110 - 100 = 10$$
$$t = T - T_i = 200$$

Thus: $P \approx 100 + (0.1)(200) - (10) = 110\,\frac{Ib_f}{ft^2}$

The exact value of P is: $P = \frac{WRT}{V} = \frac{(10)(53.3)(1200)}{(53.3)(110)} = 109.1$

Therefore, the percent error is:

$$\frac{(110 - 109.1)}{109.1} * 100 = 0.82$$

Example 3: For sonic flow of air through a restriction, the mass:

$$M = \frac{0.53}{\sqrt{T}} AP$$

Where: M = mass rate of flow Ib_m/s.

T = the inlet temperature ^0R.

A = area of restriction ft^2.

P = inlet pressure Ib_f/ft^2.

Determine the linearized approximation for the variation M when the inlet temperature T is constant.

Solution: because the temperature is constant, the mass rate of flow is a function of A and P. Thus:

$$m = \left.\frac{\partial M}{\partial A}\right|_i a + \left.\frac{\partial M}{\partial P}\right|_i p$$

The partial derivatives are:

$$\left.\frac{\partial M}{\partial A}\right|_i = \left.\frac{0.53}{\sqrt{T}} P\right|_i = \left.\frac{M}{A}\right|_i \quad \text{AND} \quad \left.\frac{\partial M}{\partial P}\right|_i \left.\frac{0.53}{\sqrt{T}} A\right|_i = \left.\frac{M}{P}\right|_i$$

Thus: $m = M_i \left(\frac{a}{A_i} + \frac{p}{P_i}\right)$

Linearization of Operating Curves:

For many components encountered in control systems, the operating characteristics are given in the form of general operating curves rather than equations. For example, in figure (2) is shown a family of curves of constant values of Z (i.e. Z=15, 20 and 25). For X =1000 and Z =20, the corresponding value of Y is 60. This point is indicated by A. the general functional relationship is:

$Y = Y(X, Z)$

Linearization about a reference point of operation gives:

$$y = \left.\frac{\partial Y}{\partial X}\right|_Z x + \left.\frac{\partial Y}{\partial Z}\right|_X z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

$$\left.\frac{\partial Y}{\partial X}\right|_Z = \left.\frac{\Delta Y}{\Delta X}\right|_Z = \frac{Y_B - Y_C}{X_B - X_C} = \frac{40 - 80}{1200 - 800} = -0.10$$

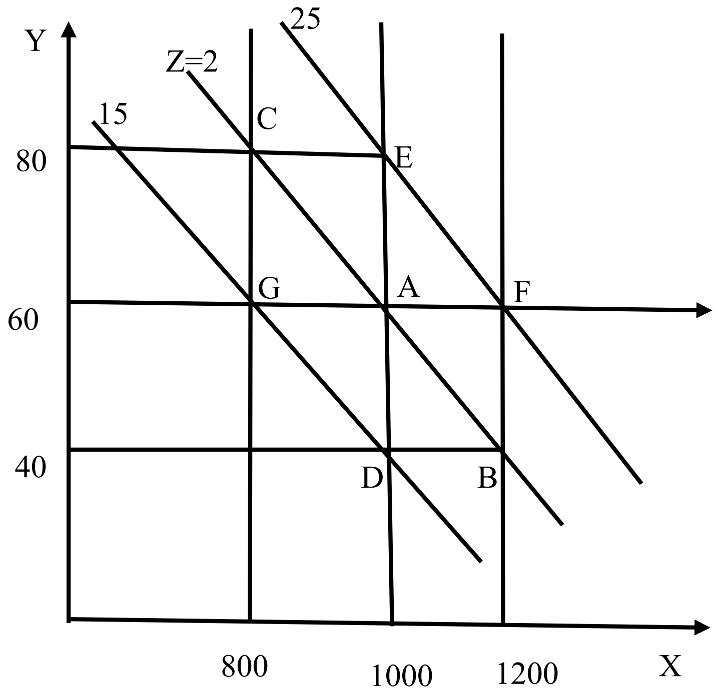
This partial derivative is the slope of the curve

$Z_i = 20$ at the reference operating point.

$$\left.\frac{\partial Y}{\partial Z}\right|_X = \left.\frac{\Delta Y}{\Delta Z}\right|_X = \frac{Y_D - Y_E}{Z_D - Z_E} = \frac{40 - 80}{15 - 25} = 4$$

Substituting in equation (6) we get:

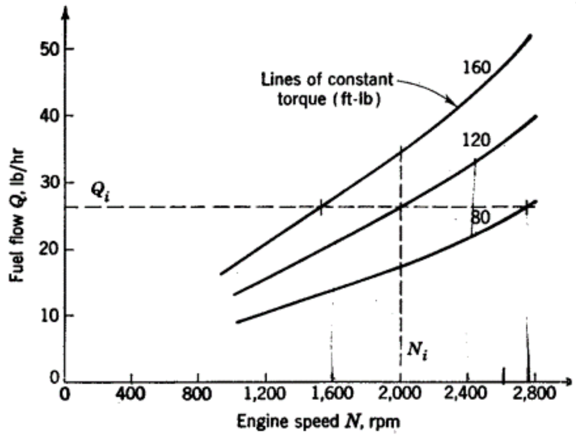
$y = -0.1x + 4z$



Example (2):

The linearized equation for the operating of the engine about some reference operating point is obtained as follows: from figure (3) it is seen that the speed N is a function of the rate of fuel flow Q and the engine torque T. Thus:

$$n = \left.\frac{\partial N}{\partial Q}\right|_i q + \left.\frac{\partial N}{\partial T}\right|_i t$$



The term $\left.\frac{\partial N}{\partial Q}\right|_i$ is the change in speed per Change if fuel flow with all other parameters Held constant (in this case with T constant). This partial derivative is equal to the reciprocal Of the slope of the line of constant torque at reference point:

$$\left.\frac{\partial N}{\partial Q}\right|_i = \frac{2400 - 1600}{32 - 20} = 66.7$$

The partial derivative $\left.\frac{\partial N}{\partial Q}\right|_i$ is the change in speed per change if torque with Q held constant. This is evaluated from a horizontal interpolation of the characteristic operating curves as follows:

$$\left.\frac{\partial N}{\partial T}\right|_i = \frac{2730 - 1530}{80 - 160} = -15$$

The minus sign indicates that for a constant Q the speed decreases as the torque increases. The linearized approximation for N is:

$$N \approx N_i + n = 2000 + 66.7q - 15t$$

Hydraulic Systems

A schematic diagram of a hydraulic amplifier is shown in figure (1). The position of the valve is designated by (x), and the position of large piston which moves the load is (y). When the valve is moved upward, the supply pressure admit oil to the upper side of the piston, and the fluid in the lower side of the piston is returned to the drain. For the reverse process, the valve is moved downward so that the supply pressure is connected to the bottom side of the big piston. When the mass of the load is negligible, the pressure drop across the valve remains constant. For this case, the rate of flow is proportional to the distance (x). Thus:

$$q = Cx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

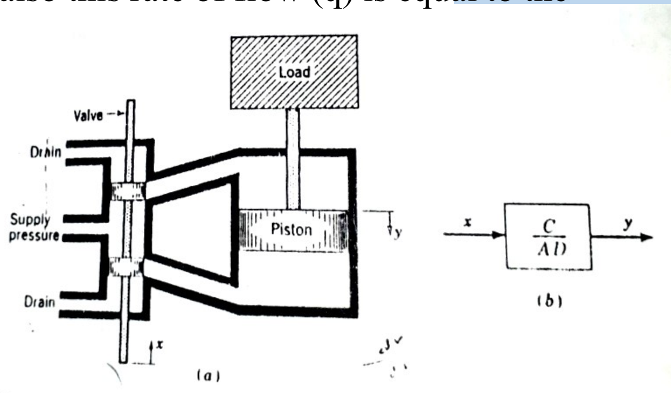
Where (q) is the rate of flow through the valve, also this rate of flow (q) is equal to the rate of change of volume of the chamber:

$$q = ADy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By equating (1) and (2), we get:

$$y = \frac{C}{AD}x = \frac{C}{A}\frac{1}{D}x$$

The (1/D) indicate that this hydraulic valve and piston combination in effect integrates hydraulically.



The rate of flow (Q) to the cylinder is a function of the piston valve (X) and the pressure drop (P), that is:

$$q = \left.\frac{\partial Q}{\partial X}\right|_p x + \left.\frac{\partial Q}{\partial P}\right|_x p = C_1 x - C_2 p \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where $C_1 = \left.\frac{\partial Q}{\partial X}\right|_p$ and $C_2 = \left.\frac{\partial Q}{\partial P}\right|_x p$

The force transmitted to the load by the power piston is equal to the product of the pressure drop across the piston and the cross-sectional area (A) of the piston, thus:

$$PA = M\frac{d^2y}{dt^2} = MD^2y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Substitution of P from equation (3) into (4), gives:

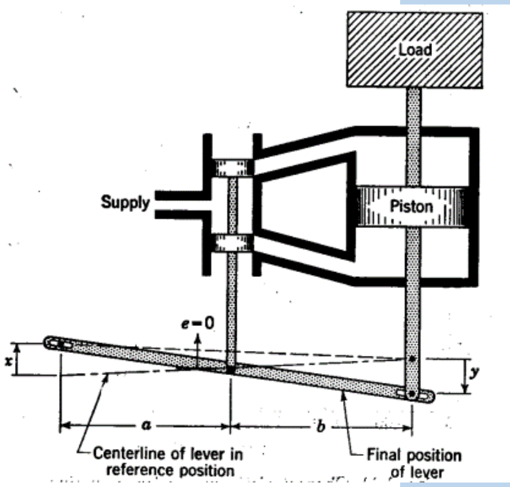
$$q = C_1 x - \frac{C_2 M}{A}D^2 y \quad \text{But : } q = ADy$$

$$\therefore y = \frac{\left(\frac{C_1}{A}\right)x}{D(1+\tau D)}$$

$where: (\tau) is\ time\ constant\ is\ equal\ to = \frac{C_2 M}{A^2}.$

Hydraulic Servomotor

A hydraulic servomotor is shown in figure (2). A linkage called a ***walking beam*** connects the input position (x), the valve position (e) and the piston position (y). The operation of this servomotor may be visualized as follows: when the input (x) is changed from the reference position, the walking beam first pivots about the connection at (y) because the large forces acting on the piston hold it in place temporarily. The relationship between the input (x) and the output (y) is shown in figure (2-a):



$\frac{y}{b} = \frac{x}{a}$ or $y = \frac{b}{a}x$

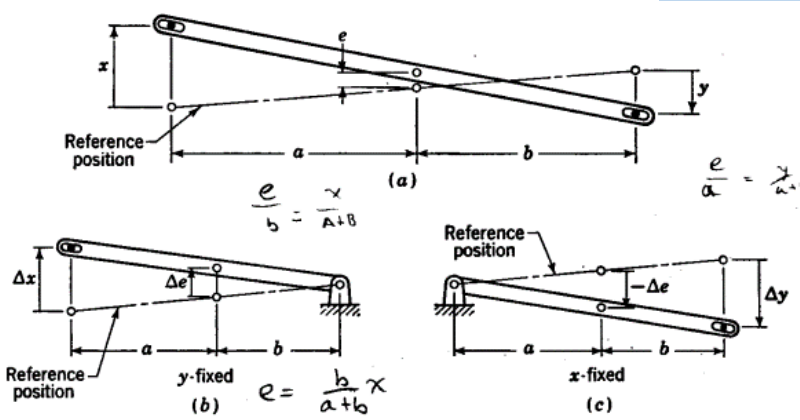
Figure (2-b) illustrate the linkage with (y) fixed in the reference position. From similar triangles:

$$ex = \frac{b}{a+b}.x$$

Similarly, from figure (2-c), in which (X) is fixed in the reference position:

$$ey = \frac{-a}{a+b}.y$$

The minus sign indicates that (e) decreases as (y) increases. The overall walking beam linkage movement is:



$$e = \frac{b}{a+b}ex - \frac{a}{a+b}ey$$

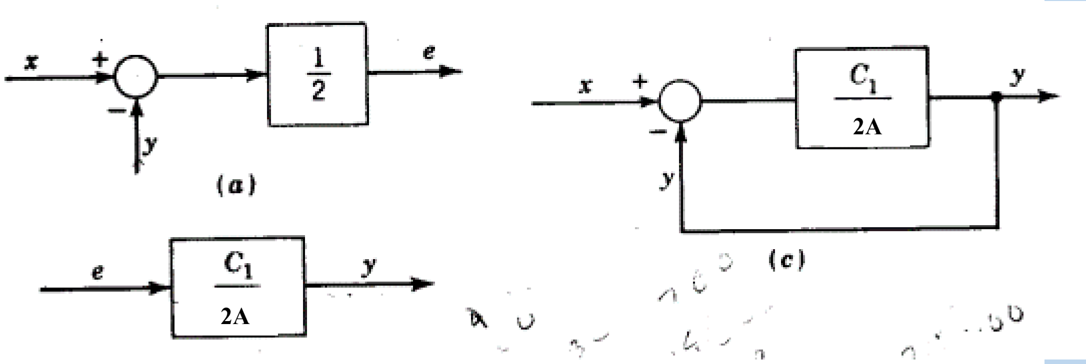
For the case in which a=b:

$$e = \frac{x-y}{2}$$

The equation for the valve and piston combination is given by:

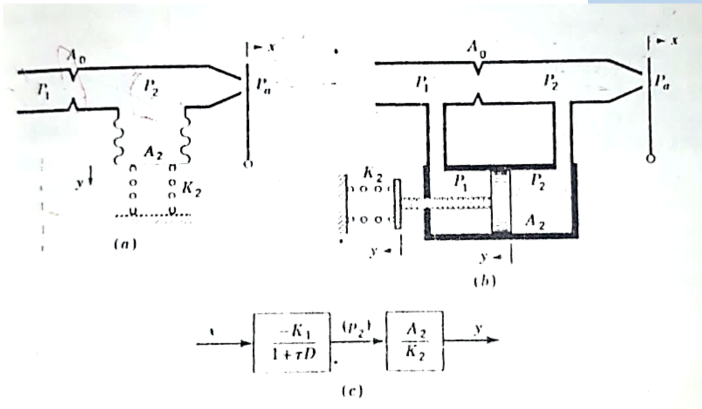
$$y = \frac{C}{AD}e$$

The block diagram representation for the preceding is shown in figure (3).



Pneumatic Systems

A flapper valve, as shown in figure (4-a), is one in which small changes in the position (X) of the flapper cause large variations in the controlled pressure (P_2) in the chamber. When the flapper is closed off, the pressure (P_2) in the chamber is equal to the supply pressure (P_1). If the flapper is opened wide, the chamber pressure approaches the ambient pressure (P_a).



For constant supply pressure (P_1) and fixed inlet orifice, the mass rate of flow in to the chamber (M_{in}) is a function of the chamber pressure (P_2) only. Linearization gives:

$$m_{in} = \left.\frac{\partial Min}{\partial P_2}\right|_i p_2 = -C_1 p_2$$

The minus sign indicates that as (P_2) increases, (m_{in}) decreases. Also, the mass rate of flow out from the chamber (M_o) is a function of (X) and (P_2). Thus:

$$m_o = \left.\frac{\partial Mo}{\partial X}\right|_i x + \left.\frac{\partial Mo}{\partial P_2}\right|_i p_2 = C_2 x + C_3 p_2$$

The change in mass (w) of air in the chamber is the integral of ($m_{in} - m_o$):

$$w = \frac{m_{in} - m_o}{D} = -\frac{C_1 p_2 - (C_2 x + C_3 p_2)}{D}$$

Multiplying through by D shows that:

$$-C_1 p_2 + C_2 x - C_3 p_2 = Dw$$

From the equation of state, the total mass W of air in the chamber is:

$$W = \frac{P_2 V_2}{RT_2}$$

Where (V_2) is the volume of the chamber and (T_2) is the stagnation temperature of air in the chamber. Linearization yields for the change in mass (w) of air in the chamber:

$$w = \left.\frac{\partial W}{\partial V_2}\right|_i v_2 + \left.\frac{\partial W}{\partial P_2}\right|_i p_2 = C_4 v_2 + C_5 p_2$$

The change in volume of the chamber is equal to the area (A_2) times the change in position:

$$v_2 = A_2 y$$

The summation of forces acting on the bellows is:

$$P_2 A_2 = K_2 Y$$

The summation of forces acting on the piston is:

$$P_2 A_2 - P_1 A_1 = K_2$$

Because (P_1A_1) is constant, linearization of either of the preceding gives the same result. That is:

$$y = \frac{A_2}{K_2} P_2$$

Substitution (w) from equation (1) in to equation (2), then using the preceding expression to eliminate (v_2) and (y):

$$p_2 = -\frac{K_1}{1 + \tau D} x$$

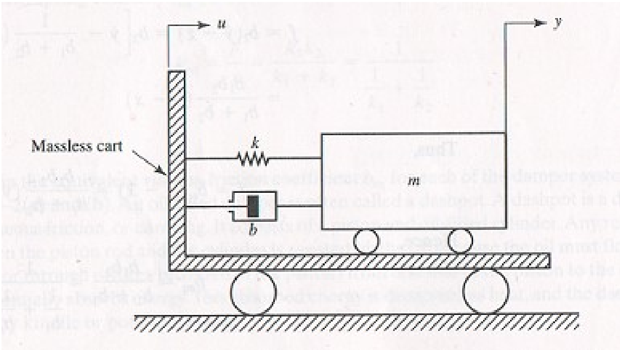
Where ($K_1 = \frac{C_2}{C_1 + C_3}$) and ($\tau = \frac{C_5 + {A_2}^2 C_4 / K_2}{C_1 + C_3}$)

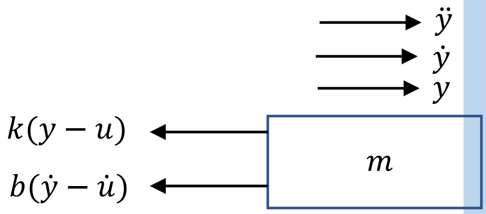
Because ($y = (A_2/K_2)$ p2), it follows that:

$$y = \frac{A_2(-K_1)}{K_2(1 + \tau D)} x$$

Mathematical Modeling of Mechanical Systems and Electrical Systems

<u>Example</u> Consider the spring – mass – dashpot system mounted on a massless cart as shown in Figure. Let us obtain mathematical models of this system by assuming that the cart is standing still for $t < 0$ and the spring – mass – dashpot system on the cart is also standing still for $t < 0$. In this system, $u(t)$ is the displacement of the cart and is the input to the system. At $t = 0$, the cart is moved at a constant speed, or $\dot{u} =$ constant. The displacement $y(t)$ of the mass is the output.
(The displacement is relative to the ground). In this system, m denotes the mass, b denotes the viscous-friction coefficient, and k denotes the spring constant. We assume that the friction force of the dashpot is proportional to $\dot{y} - \dot{u}$ and force is d that the spring is a linear spring; that is, the spring force is proportional to $y - u$.





<u>Solution</u>

Applying Newton's second law

Let $\quad y > u$

$$m\frac{d^2y}{dt^2} = -b(\dot{y} - \dot{u}) - k(y - u)$$

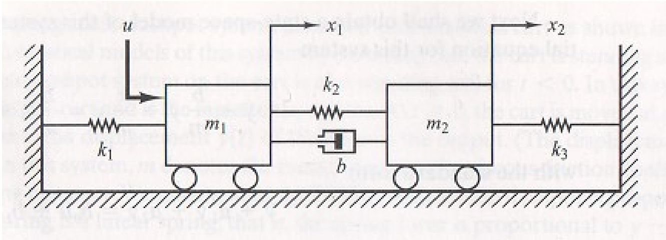
Or $\quad m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = b\frac{du}{dt} + ku \quad \Rightarrow \quad$ mathematical model of the system

To find the transfer function of the system, take L.T of both sides of last equation, assuming zero initial conditions:

$$(ms^2 + bs + k)Y(s) = (bs + k)U(s)$$

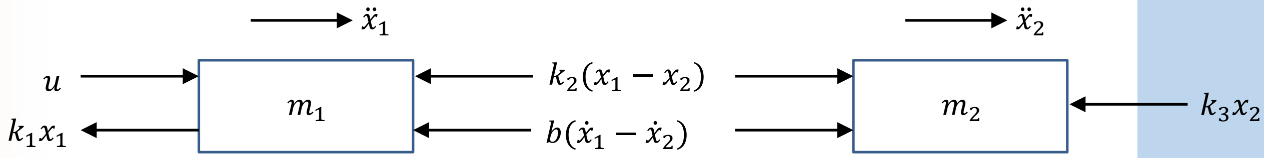
$$\Rightarrow \quad G(s) = \frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k}$$

Example Obtain the transfer function $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the mechanical system shown in Figure



Solution

Let $x_1 > x_2$ applying Newton's second law to FBDs we get



$$m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) + u$$

$$m_2\ddot{x}_2 = -k_3x_2 - k_2(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$$

Simplifying, we obtain

$$m_1\ddot{x}_1 + b\dot{x}_1 + (k_1 + k_2)x_1 = b\dot{x}_2 + k_2x_2 + u$$

$$m_2\ddot{x}_2 + b\dot{x}_2 + (k_2 + k_3)x_2 = b\dot{x}_1 + k_2x_1$$

Then, taking L.T of these two equations [initial conditions = zero]

$$[\, m_1s^2 + bs + (k_1 + k_2)]\mathrm{X}_1(s) = (bs + k_2)\mathrm{X}_2(s) + U(s) \quad \cdots\cdots\cdots\cdots (1)$$

$$[\, m_2s^2 + bs + (k_2 + k_3)]\mathrm{X}_2(s) = (bs + k_2)\mathrm{X}_1(s) \quad \cdots\cdots\cdots\cdots (2)$$

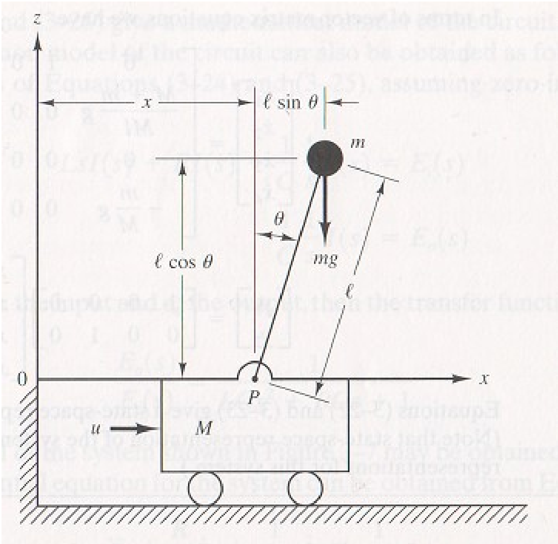
Solving equation (2) for $\mathrm{X}_2(s)$ and substituting it into equation (1)

$$\frac{X_1}{U} = \frac{m_2s^2+bs+(k_2+k_3)}{[\, m_1s^2+bs+(k_1+k_2)]\,[\, m_2s^2+bs+(k_2+k_3)]-(bs+k_2)^2} \quad \cdots\cdots\cdots\cdots (3)$$

Form equations (3) and (2)

$$\frac{X_2}{U} = \frac{bs+k_2}{[\, m_1s^2+bs+(k_1+k_2)]\,[\, m_2s^2+bs+(k_2+k_3)]-(bs+k_2)^2}$$

Example Consider the inverted-pendulum system shown in Figure.



Since in this system the mass is concentrated at the top of the rod, the center of gravity is the center of the pendulum ball. For this case, the moment of inertia of the pendulum about its center is small, and we assume $\bar{I} = 0$ in the equation $(**)$. Then the mathematical model for this system becomes as follows:

$$\boldsymbol{(M+m)\ddot{x} + ml\ddot{\theta} = u} \quad \cdots\cdots\cdots\cdots (\mathbf{1})$$

$$\boldsymbol{ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta} \quad \cdots\cdots\cdots\cdots (\mathbf{2})$$

By multiplying both sides of equation (1) by l and rearranging yield

$$ml^2\ddot{\theta} = ul - (M+m)l\ddot{x} \quad \cdots\cdots\cdots\cdots (3)$$

and equation (2) can be written as

$$\ddot{x} = g\theta - l\ddot{\theta} \quad \cdots\cdots\cdots\cdots (4)$$

By substituting equation (4) in to (1) and (3) into (2) we obtain the following

$$Ml\ddot{\theta} = (M+m)g\theta - u \quad \cdots\cdots\cdots\cdots (5)$$

$$M\ddot{x} = u - mg\theta \quad \cdots\cdots\cdots\cdots (6)$$

The plant transfer function can be obtained by taking L.T of equation (5)

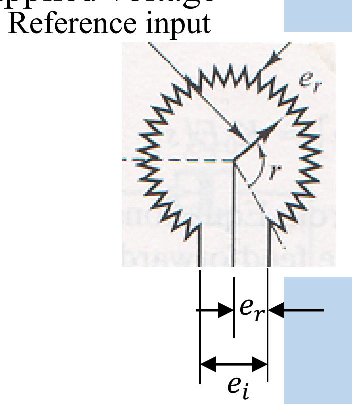
$$\frac{\Theta(s)}{-U(s)} = \frac{1}{Mls^2 - (M+m)\mathrm{g}} = \frac{1}{Ml\left(s+\sqrt{\frac{M+m}{Ml}\mathrm{g}}\right)\left(s-\sqrt{\frac{M+m}{Ml}\mathrm{g}}\right)}$$

The inverted pendulum plant has one pole on the negative real axis and another on the positive real axis. Hence, the plant is <u>open loop unstable</u>.

Sensors in Control Systems

جهاز قياس

Potentiometer A potentiometer s an electromechanical transducer that convert mechanical energy to electrical energy. The input to the device is in the form of a mechanical displacement, either linear or rotation. If e_i is applied voltage



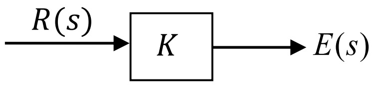
$$e_r = K\ r$$

Where

e_r :- is output voltage

r :- shaft position

K :- potentiometer constant (volt/rad)

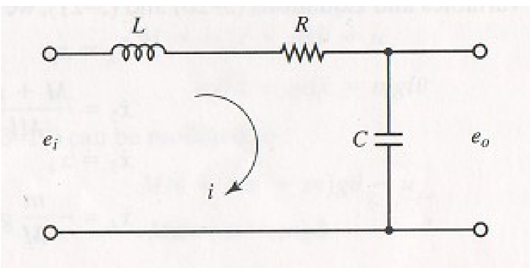


Tachometer This device works essentially as a voltage generator, with the output voltage proportional to the magnitude of the angular velocity of the input shaft, or

$$e_t = K_t \frac{d\theta}{dt} = K_t\ \omega$$

Where K_t:- is tachometer constant $\left(\frac{volt}{rad/sec}\right)$

LRC Circuit Consider the electrical circuit shown in Figure. The circuit consists of an inductance L (henry), a resistance R (ohm), and a capacitance C (farad).



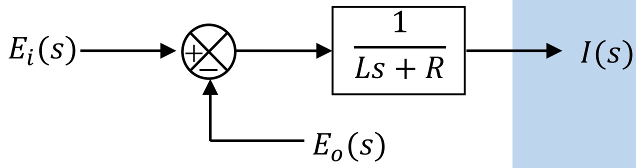
Applying Kirchhoff's voltage law to the system

$$L\frac{di}{dt} + Ri + e_o = e_i \qquad \Longrightarrow \qquad e_i - e_o = L\frac{di}{dt} + Ri \qquad \cdots\cdots\cdots (1)$$

$$e_o = \frac{1}{C}\int i\, dt \qquad \cdots\cdots\cdots (2)$$

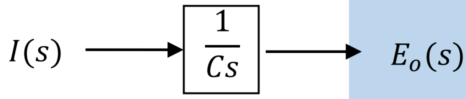
Taking L.T of both sides of equations (1) and (2)

$$E_i - E_o = (Ls + R)I(s) \implies$$

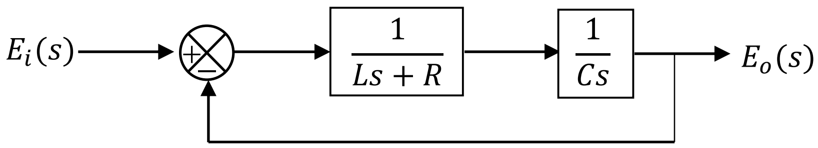


and,

$$E_o = \frac{I(s)}{Cs} \implies$$



The overall block diagram for E_i as input and E_o as output is



$$\frac{E_o}{E_i} = \frac{1}{LCs^2+RCs+1}$$

H.W Consider e_i is the input and i is the output in the previous example. Draw the block diagram of the system and then determine the transfer function $I(s)/E_i(s)$

Electric DC Motor

The basic equation for a DC motor are obtained from Maxwell's electromagnetic theory. The torque produced by a motor is proportional to the product of the magnetic flux ϕ and the armature current i_a , i.e

$$T = k_1 \phi i_a \qquad \cdots\cdots\cdots(1)$$

where k_1 is a constant for a motor. The voltage produced by a motor is proportional to the product of the magnetic flux ϕ and the shaft angular velocity ω , i.e

$$e_b = k_2 \phi \omega \qquad \cdots\cdots\cdots(2)$$

This voltage e_b is 180° out of phase with the applied armature voltage e_a and is therefore called back e.m.f. A DC motor with separate field winding is shown in Figure (1). The motor is controlled by varying the armature voltage $e_a(t)$ with the field current i_f kept constant.

J_O :-The combined load and armature mass moment of inertia. and

b:- Is the viscous friction coefficient.

L_a :- The inductance of the armature

R_a :- The resistance of the armature

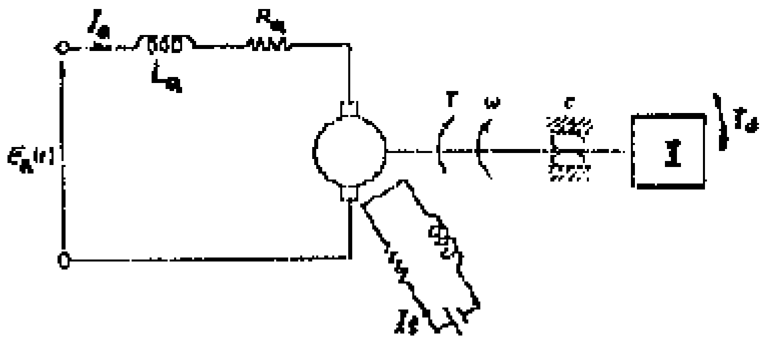


Figure (1) Armature controlled DC motor with fixed field

Let T_d be a disturbance load torque acting on the motor

Balancing the voltages and torques we obtain:-

$$e_a - e_b = L_a \frac{di_a}{dt} + R_a i_a \qquad \cdots\cdots\cdots(3)$$

$$T = J_O \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + T_d \qquad \cdots\cdots\cdots(4)$$

The magnetic flux ϕ is a function of the field current i_f as shown in Figure (2). If saturation is avoided, we obtain $\phi = k_3\, i_f$. Then equations (1) and (2) takes the following forms

$$T = (k_1 k_3 i_f)\, i_a = k_t\, i_a \qquad \cdots\cdots\cdots(5), \quad \text{where} \quad k_t = k_1 k_3 i_f$$

$$e_b = (k_2 k_3 i_f)\,\omega = k_b\,\omega = k_b \frac{d\theta}{dt} \qquad \cdots\cdots\cdots(6), \quad \text{where} \quad k_b = k_2 k_3 i_f$$

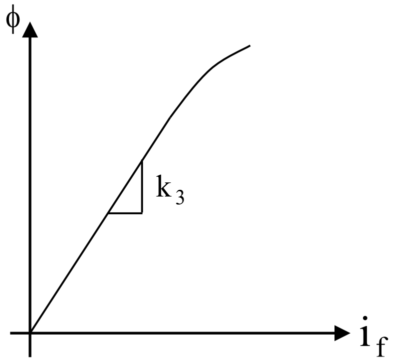


Figure 2 Magnetic flux versus field current

Substituting for T and e_b from equations (5) and (6), we can express equations (3) and (4) as

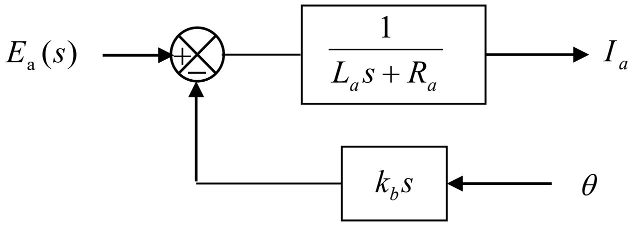
$$e_a = L_a \frac{di_a}{dt} + R_a i_a + k_b \frac{d\theta}{dt} \qquad \cdots\cdots\cdots(7)$$

$$J_O \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + T_d = k_t i_a \qquad \cdots\cdots\cdots(8)$$

Then taking L.T of both sides of equation (7) and rearranging

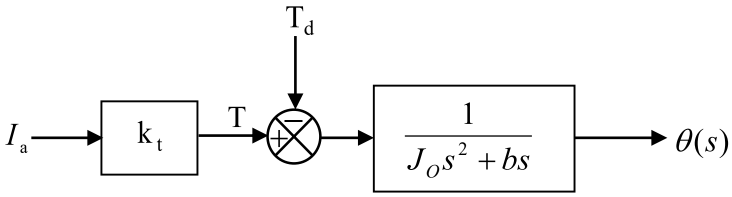
$$L_a sI_a(s) + R_a I_a(s) + k_b s\theta(s) = E_a(s)$$

$$(L_a s + R_a)I_a(s) = E_a(s) - k_b s\theta(s)$$

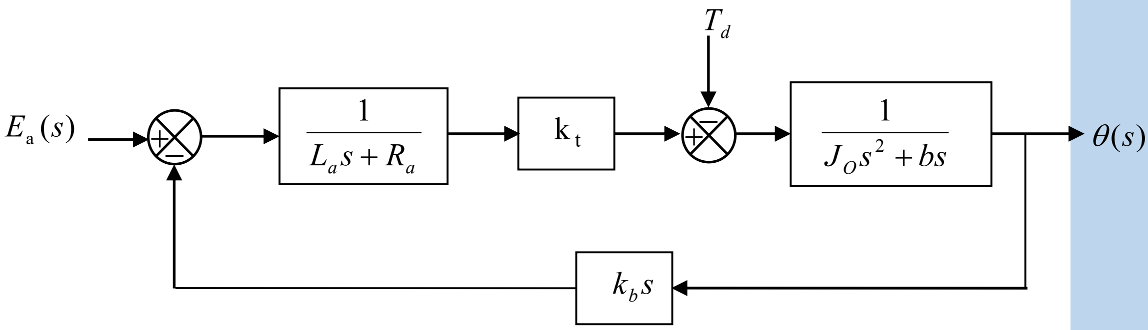


And taking L.T of both sides of equation (8) and rearranging

$$(J_O s^2 + bs)\theta(s) + T_d(s) = k_t I_a(s)$$
$$(J_O s^2 + bs)\theta(s) = k_t I_a(s) - T_d(s)$$

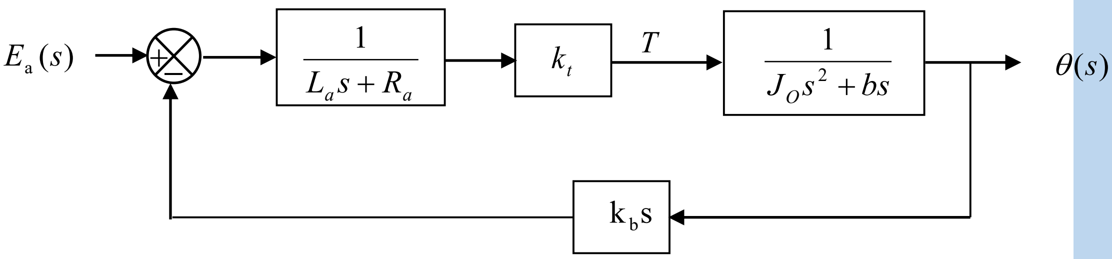


The overall block diagram of equations (7) and (8) is shown in Figure , where

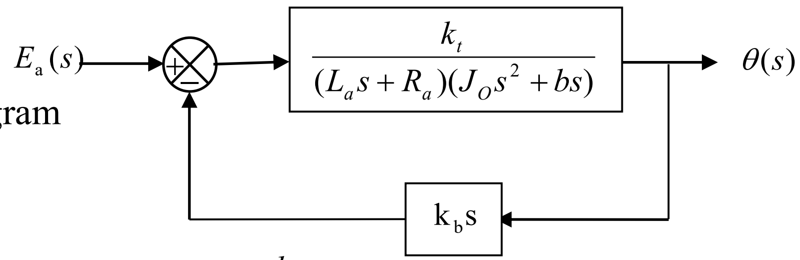


The DC motor driving a load is an open – loop system with an inherent feedback loop caused by the back emf. In a permanent – magnetic DC motor, the field winding is absent and the magnetic field ϕ is provided by the permanent magnet and is constant. Since in the preceding development, the magnetic flux is a constant equations (7) and (8) are also valid for a permanent – magnetic DC motor.

If disturbance T_d is zero



From last block-diagram



$$\theta(s) = (E_a - k_b s\theta) \cdot \frac{k_t}{(L_a s + R_a)(J_O s^2 + bs)}$$

Or
$$\boxed{\frac{\theta(s)}{E_a(s)} = \frac{k_t}{(L_a s + R_a)(J_O s^2 + bs) + k_t k_b s}}$$ (T.F of DC motor)

Example

Consider the servo system shown in Figure. The motor shown is a servomotor, a DC motor designed specifically to be used in a control systems. The operation of this system is as follows: A pair of potentiometers acts as an error-measuring device. They convert the input and output positions into proportional electric signals. The command input signal determines the angular position r wiper arm of the input potentiometer. The angular position r is the reference input to the system, and the electric potential of the arm is proportional to the angular position of the arm. The output shaft position determines the angular position of c of the wiper arm of the output potentiometer. Now, let

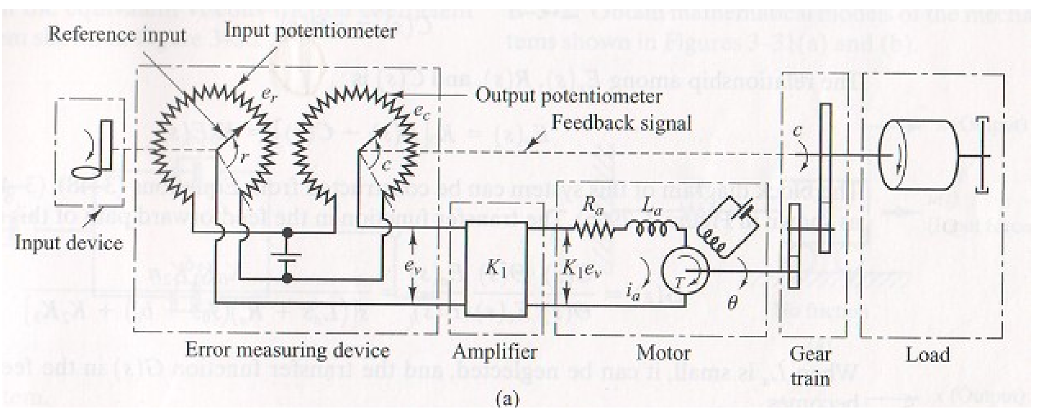
The difference between the input angular position r and the output angular position c is the error signal e, or

$$e = r - c$$

The potential difference $e_r - e_c = e_v$ is the error voltage, where

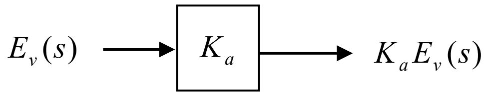
$$e_r = K_o r \qquad \text{and} \qquad e_c = K_o c$$

where K_o is a proportionality constant of potentiometers.



Let K_a is the gain constant of the amplifier then the voltage input to the DC motor is

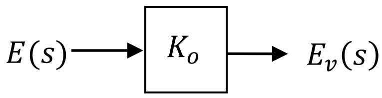
$e_a = K_a e_v$



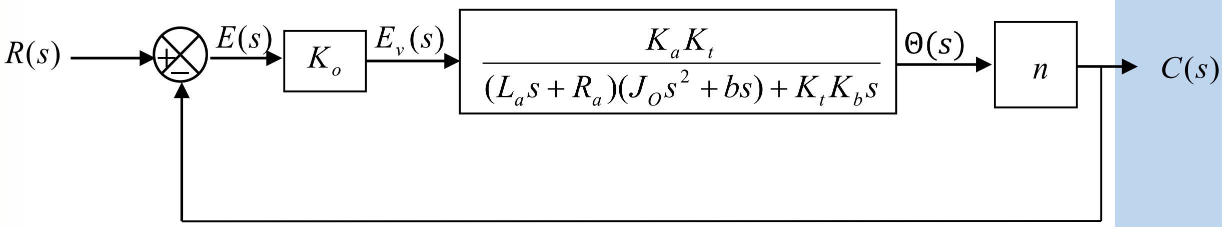
The output shaft rotates *n* times for each revolution of the motor shaft, or $C(s) = n\,\Theta(s)$

The relationship $E_v(s)$, $R(s)$, and $C(s)$ is

$$E_v(s) = K_o[R(s) - C(s)] = K_o E(s)$$



The block diagram of this system can be constructed from equations as shown in Figure (b)



−b −

The transfer function in the feedforward path of this system is

$$G(s) = \frac{C(s)}{E(s)} = \frac{K_o K_a K_t n}{s[(L_a s + R_a)(J_o s + b) + K_t K_b]}$$

When L_a is small, it can be neglected, and the transfer function $G(s)$ in the feedforward path becomes

$$G(s) = \frac{K_o K_a K_t n}{s[R_a(J_o s+b)+K_t K_b]} \quad \text{or} \quad G(s) = \frac{K_o K_a K_t n/R_a}{J_o\, s^2+\left(b+\frac{K_t K_b}{R_a}\right)s} \cdot \frac{n^2}{n^2}$$

Which can be simplified to

$$G(s) = \frac{K}{s(Js+B)}$$

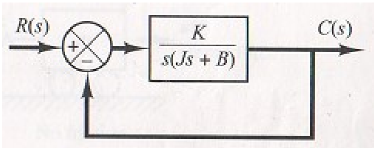
Where

$$J = J_o/n^2$$

$$B = [b + (K_t K_b/R_a)]/n^2$$

$$K = K_o K_a K_t/n\, R_a$$

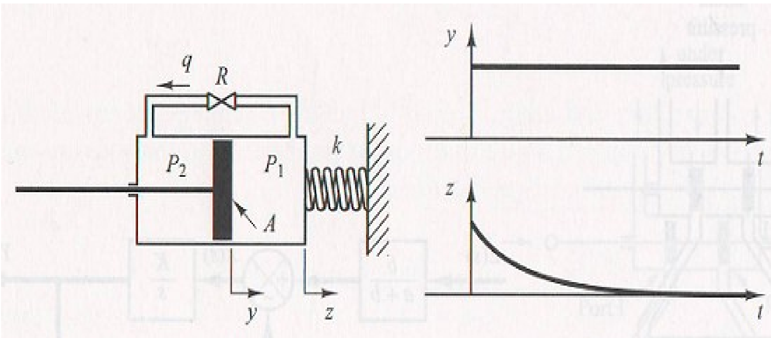
The block diagram of the system shown in Figure (b) can thus be simplified as shown in Figure (c)



(c)

Dashpots

Suppose that we introduce a step displacement to the piston position *y*. Then the displacement *z* becomes equal to *y* momentarily. Because of the spring force, however, the oil will flow through the resistance *R* and the cylinder will come back to the original position. The curves *y* versus *t* and *z* versus *t* are shown in Figure (b)



(a) (b)

Let

y be the input and *z* is the output of the system

P_1 , P_2 pressure existing on the right and left sides of the piston respectively

Suppose that the inertia force involved is negligible ($m = 0$). Thus

$$A(P_1 - P_2) = kz \Rightarrow \quad P_1 - P_2 = \frac{kz}{A} \quad \cdots\cdots\cdots (1)$$

where

$A =$ piston area, in^2

$k =$ spring constant

The flow rate q is given by

$$q = \frac{P_1 - P_2}{R} \quad \cdots\cdots\cdots (2)$$

where

$q =$ flow rate through the restriction.

$R =$ resistance to flow at the restriction.

Since the flow through the restriction during dt seconds must equal the change in the mass of oil to the left of the piston during the same dt seconds

$$qdt = A\,\rho(dy - dz) \quad \Rightarrow \quad \frac{dy}{dt} - \frac{dz}{dt} = \frac{q}{A\rho} \quad \cdots\cdots\cdots (3)$$

Where $\rho =$ density of oil (We assume that the fluid is incompressible or $\rho =$ constant)

Now, from equations (2) and (3)

$$\frac{dy}{dt} - \frac{dz}{dt} = \frac{P_1 - P_2}{RA\rho}$$

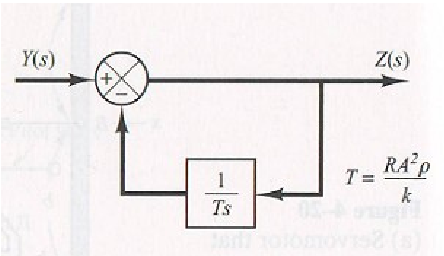
Then, from equations (1) $\frac{dy}{dt} = \frac{dz}{dt} + \frac{kz}{RA^2\rho}$ $\quad \cdots\cdots\cdots (4)$

Let $\frac{RA^2\rho}{k} = T$ (Note that $\frac{RA^2\rho}{k}$ has a dimension of time)

then the equations (4) becomes to $\frac{dy}{dt} = \frac{dz}{dt} + \frac{z}{T}$

taking L.T of both sides $sY(s) = sZ(s) + \frac{Z(s)}{T} \quad \Rightarrow \quad Z(s) = Y(s) - \frac{Z(s)}{Ts}$

the block diagram of this system is shown in Figure (c)



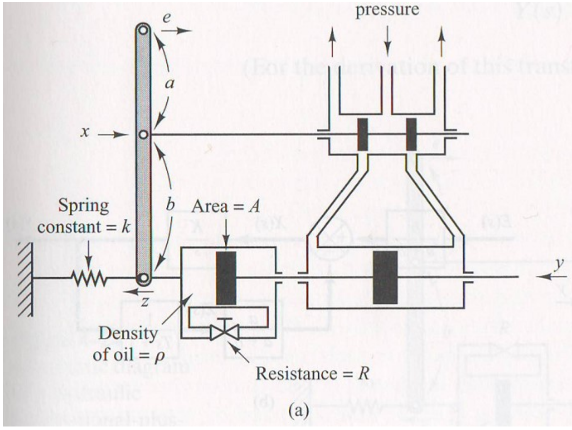
(c)

Then the transfer function (T.F) of the this system becomes to

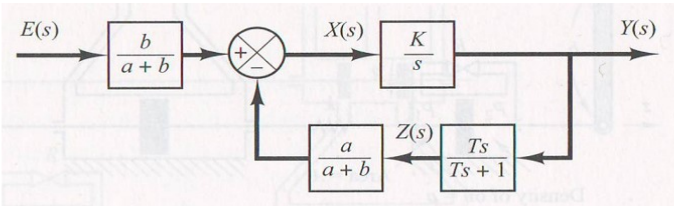
$$\frac{Z(s)}{Y(s)} = \frac{Ts}{Ts+1} = \frac{1}{1+\frac{1}{Ts}}$$

Obtaining Hydraulic Proportional-Plus-Integral Control Action

If we the dashpot to the hydraulic system as shown in Figure (a) below.



A block diagram of this controller is shown in Figure (b)



then, the transfer function $Y(s)/E(s)$ is given by

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\frac{K}{s}}{1+\frac{Ka}{a+b}\frac{T}{Ts+1}}$$

under normal operation $|KaT/[(a+b)(Ts+1)]| \gg 1$, then $1+\frac{Ka}{a+b}\frac{T}{Ts+1}$ can be replaced by $\frac{Ka}{a+b}\frac{T}{Ts+1}$

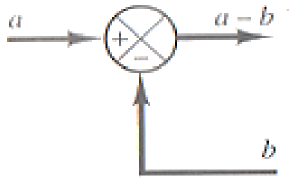
$$\frac{Y(s)}{E(s)} = K_p\left(1+\frac{1}{T_i s}\right)$$

Where

$$K_p = \frac{b}{a} \quad , \quad T_i = T = \frac{RA^2\rho}{k}$$

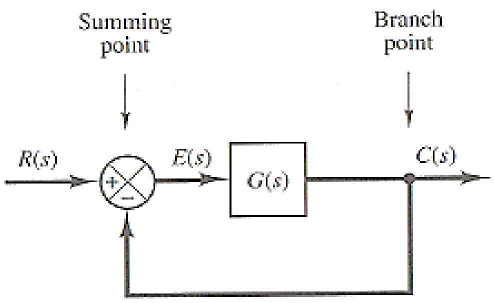
Thus the controller is proportional-plus-integral controller (PI controller)

Summing Point. Referring to the Figure below, a circle with a cross is the symbol that indicates a summing operation. The plus or minus sign at each arrowhead indicates whether that signal is to be ad or subtracted. It is important that the quantities being added or subtracted have the same **dimensions and same units**.

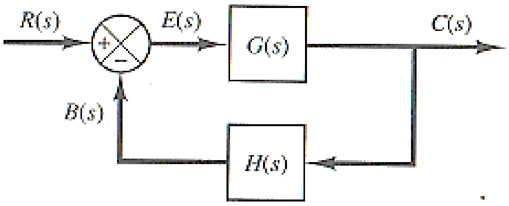


Branch Point. A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.

Block Diagram of a Closed-Loop System. Figure shows an example of a block diagram of a closed-loop system. The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$. The output of the block, $C(s)$ in this case, is obtained by multiplying the transfer function $G(s)$ by the input to the block, $E(s)$.



When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal. For example, in a temperature control system, the output signal is usually the controlled temperature. The output signal, which has the dimension of temperature, must be converted to a force or position or voltage before it can be compared with the input signal. This conversion is accomplished by the feedback element whose transfer function is $H(s)$. (In most cases the feedback element is a sensor that measures the output of the plant. The output of the sensor is compared with the system input, and the actuating error signal is generated.). In this example, the feedback signal $B(s) = H(s)C(s)$



Open-Loop Transfer Function

The ratio of the feedback signal $B(s)$ to the actuating error signal $E(s)$. Is called *open-loop transfer function*. That is

Closed –Loop System

Open-loop transfer function $= \frac{B(s)}{E(s)} = \frac{C(s)H(s)}{E(s)} = \frac{E(s)G(s)H(s)}{E(s)} = G(s)H(s)$

Feed forward Transfer Function

The ratio of the output $C(s)$ to the actuating error signal $E(s)$. is called *feed forward transfer function*. That is

$$\text{Feed forward transfer function } \frac{C(s)}{E(s)} = \frac{E(s)G(s)}{E(s)} = G(s)$$

Closed- Loop Transfer Function

For the closed loop system, the output $C(s)$ and input $R(s)$ are related as follows: since

$$C(s) = G(s)E(s) \qquad \cdots\cdots\cdots\cdots(1)$$

But $$E(s) = R(s) - B(s) = R(s) - H(s)\,C(s) \qquad \cdots\cdots\cdots\cdots(2)$$

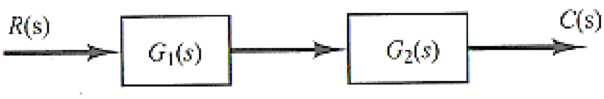
Substituting equation (2) into equation (1) gives

$$C(s) = G(s)[R(s) - H(s)\,C(s)]$$

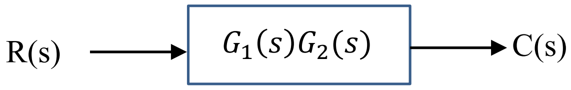
$$\Longrightarrow \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad \textit{Closed-loop transfer function.}$$

Cascaded, Parallel Transfer Functions

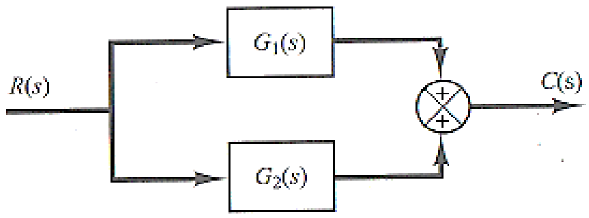
(1) Cascaded



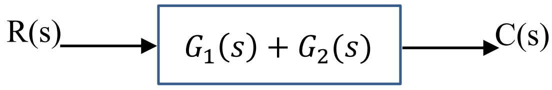
$$C(s) = R(s)\,G_1(s)\,G_2(s) \qquad \Longrightarrow \qquad \frac{C(s)}{R(s)} = G_1(s)\,G_2(s)$$



(2) Parallel

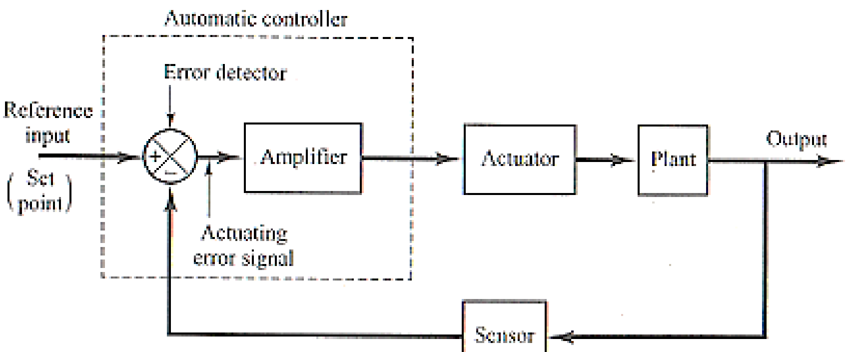


$$C(s) = R(s)\,G_1(s) + R(s)G_2(s) \qquad \Longrightarrow \qquad \frac{C(s)}{R(s)} = G_1(s) + G_2(s)$$



Automatic Controllers

An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value. The manner in which the automatic controller produces the control signal is called the *control action*, Figure below is a block diagram of an industrial control system, which consists of an automatic controller, an actuator, a plant, and a sensor (measuring element). The controller detects the actuating error signal, which is usually at a very low power level, and amplifies



it to a sufficiently high level. The output of an automatic controller s fed to an actuator,

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such as an electric motor, a hydraulic motor, or a pneumatic motor or valve. (The actuator is a power device that produces the input to the plant according to the control signal so that the output signal will approach the reference input signal.)

The sensor or measuring element is a device that converts the output variable into another suitable variable, such as a displacement, pressure, voltage, etc. that can be used to compare the output to the reference input signal.

Classifications of Industrial Controllers

Industrial controller may be classified according to their control action as

1. Two – position or on – off controllers.
2. Proportional controllers.
3. Integral controllers.
4. Proportional- plus- integral controllers.
5. Proportional- plus- derivative controllers.
6. Proportional- plus- integral – plus - derivative controllers.

Two – Position or On – Off Controller Action

In a two – position control system, the actuating element has only two fixed positions, simply on and off.

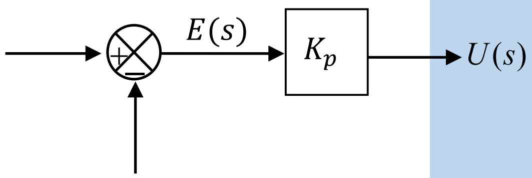
Proportional Control Action

Let the output signal from the controller be $u(t)$ and the actuating error signal be $e(t)$, for controller with proportional control action, the relationship between $u(t)$ and $e(t)$ is

$$u(t) = K_p e(t)$$

or, in L.T quantities

$$\frac{U(s)}{E(s)} = K_p$$



Where K_p is termed *proportional gain*

Integral Control Action. In a controller with integral control action, the value the value of controller output $u(t)$ is changed at <u>a rate</u> proportional to the actuating error signal $e(t)$. That is

$$\frac{du(t)}{dt} = K_i\, e(t) \qquad \text{or} \qquad u(t) = K_i \int_0^t e(t)\, dt$$

or, in L.T quantities

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

Proportional – Plus - Integral Control Action. The control action of a proportional – plus - integral controller is defined by

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t)\, dt$$

where T_i is called *integral time*.

or, in L.T quantities

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s}\right)$$

Proportional – Plus - Derivative Control Action. The control action of a proportional – plus – derivative controller is defined by

$$u(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt}$$

and the transfer function is

$$\frac{U(s)}{E(s)} = K_p (1 + T_d s)$$

Where T_d is called the *derivative time*

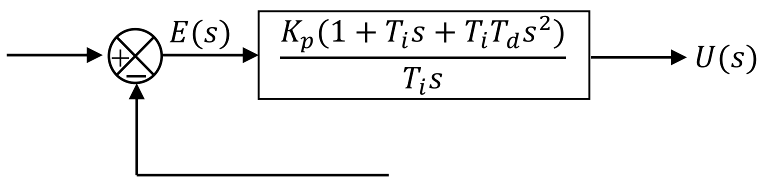
Proportional – Plus - Integral - Plus - Derivative Control Action. Is the combination of proportional control action, and integral control action, and derivative control action. The equation of a controller with this combine is given by

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t)\, dt + K_p T_d \frac{de(t)}{dt}$$

or the transfer function is

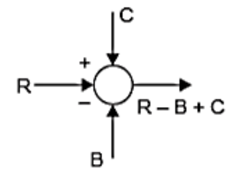
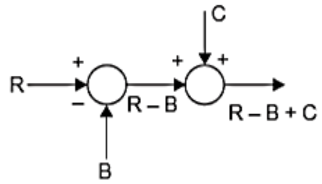
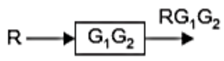
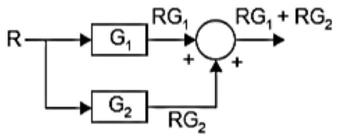
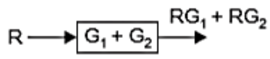
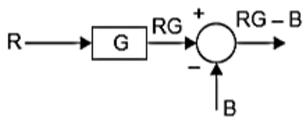
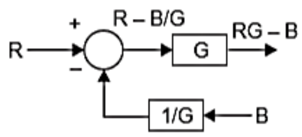
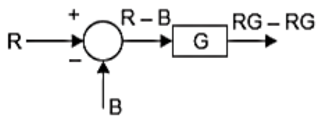
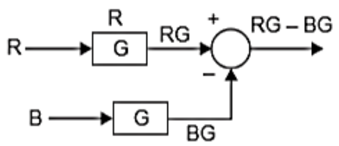
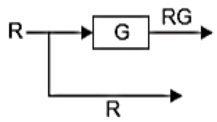
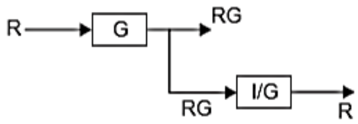
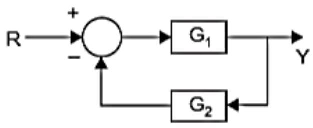
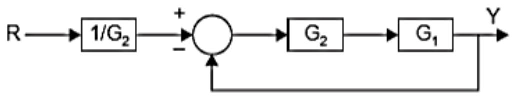
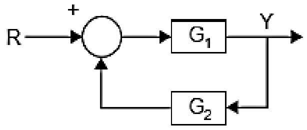
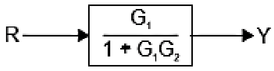
$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s\right)$$

The block diagram of a proportional – plus - integral - plus - derivative Controller is shown in Figure

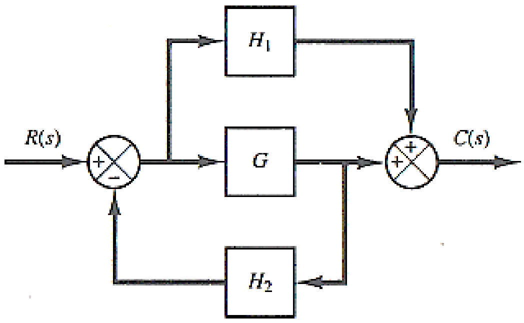


Block Diagram Reduction

A complicated block diagram involving many feedback loops can be simplified by a step-by-step rearrangement. Simplification of the block diagram by rearrangements considerably reduces the labor needed for subsequent mathematical analysis as shown in the table below:

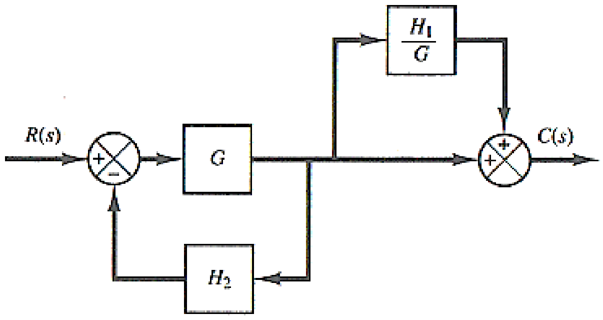
	Original Block Diagram	*Equivalent Block Diagram*
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		

Example Simplify the block diagram shown in Figure below.

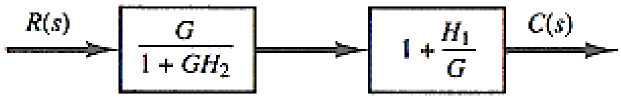


Solution

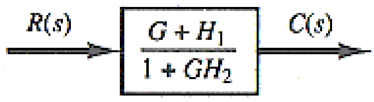
(a)



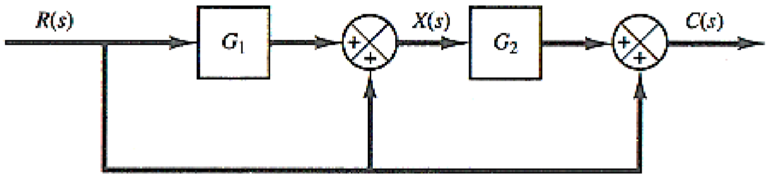
(b)



(c)

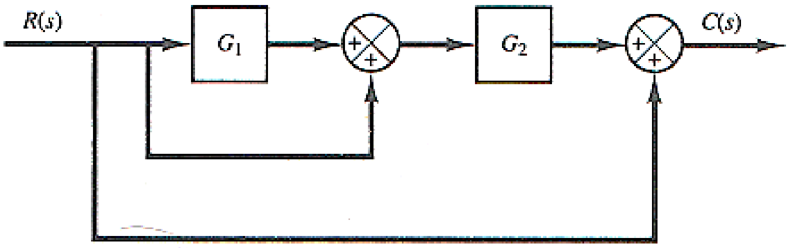


Example Simplify the block diagram shown in Figure. Obtain the transfer function related $C(s)$ and $R(s)$



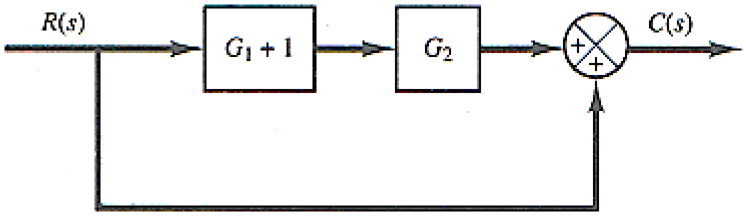
Solution

The block diagram can be modified to that shown in Figure (a).



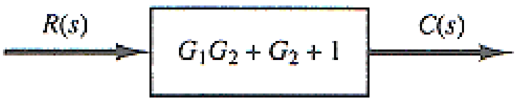
(a)

Eliminating the minor feed forward path, we obtain Figure (b).



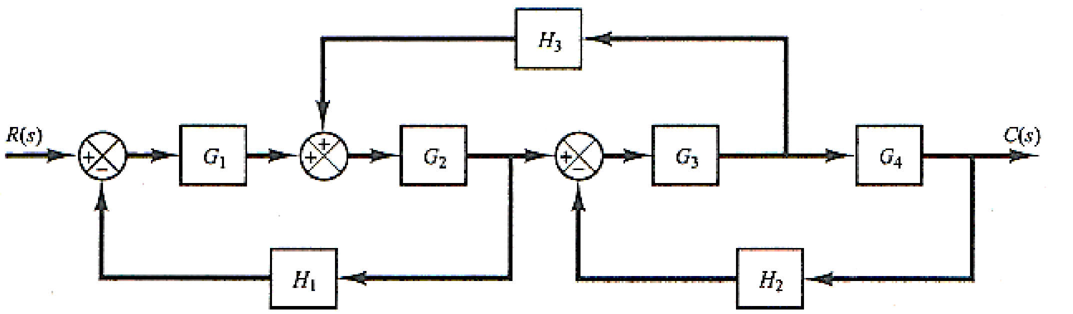
(b)

Simplified to Figure (c).

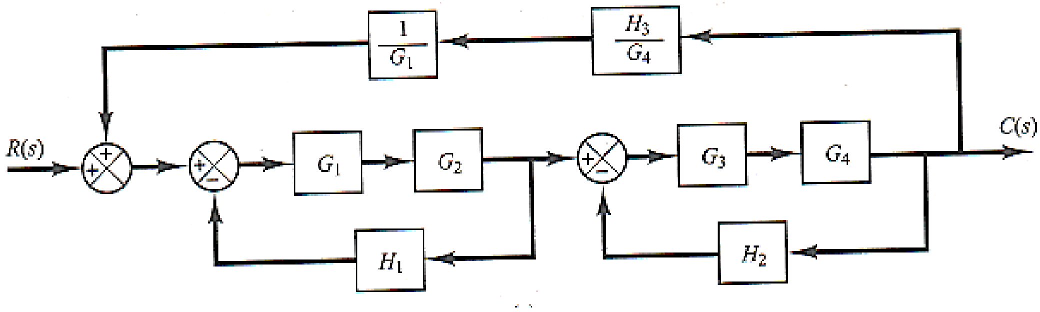


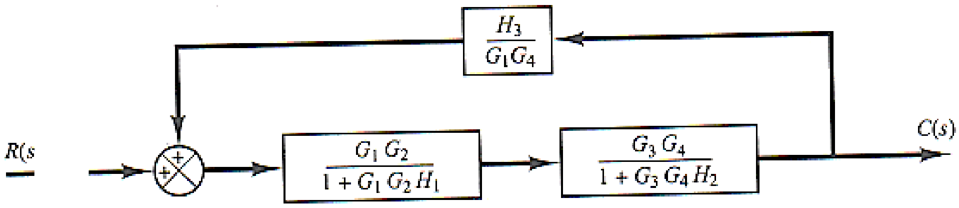
(c)

Example Simplify the block diagram shown in Figure. Obtain the transfer function related $C(s)$ and $R(s)$

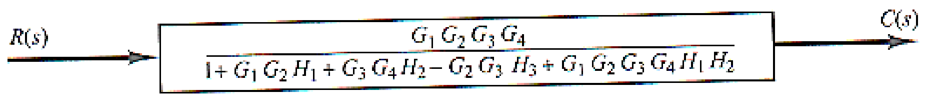


Solution



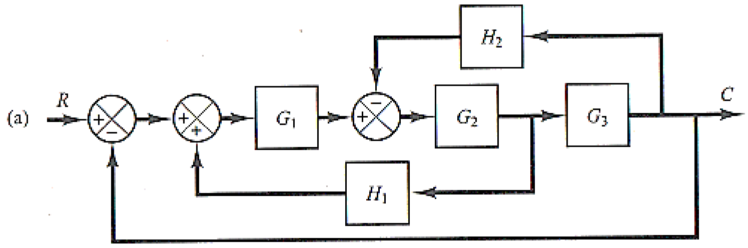


(b)



(c)

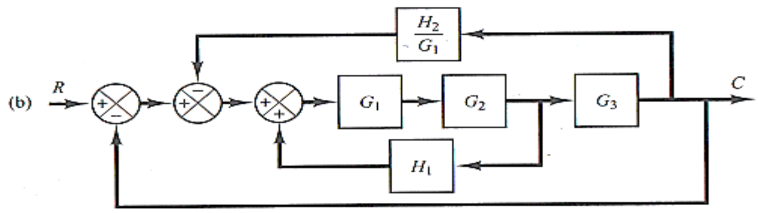
Example Consider the system shown in Figure (a). Simplify this diagram.



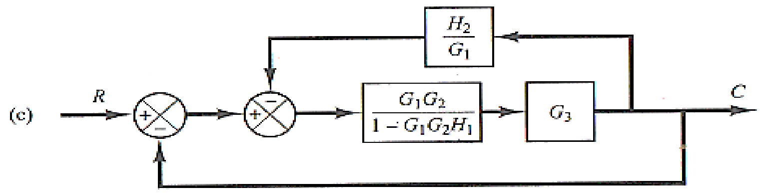
Solution

By moving the summing point of the negative feedback loop containing H_2 outside the positive

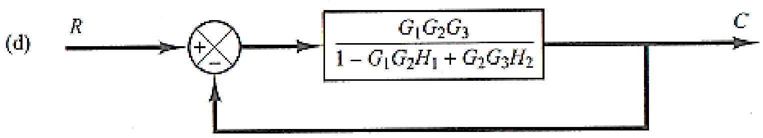
Feedback loop containing H_1, we obtain Figure (b).



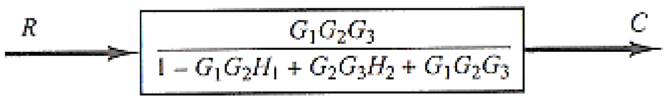
Eliminating the positive feedback loop we have Figure (c).



The Elimination



Finally eliminat



NOTE:

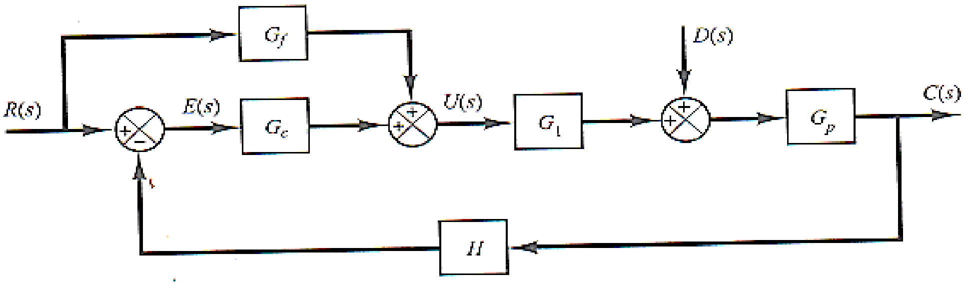
The numerator of the closed-loop transfer function $C(s)/R(s)$ is the product of the transfer functions of the feed forward path. The denominator $C(s)/R(s)$ is equal to

$$1 + \sum(\text{product of the transfer functions around each loop})$$

$$= 1 + (-G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3)$$

(The positive feedback loop yields a negative term in the denominator)

Example Obtain the transfer function $C(s)/R(s)$ and $C(s)/D(s)$ of the system shown in the Figure.



Solution

$$U(s) = G_f R + G_c\, E(s) \qquad \cdots\cdots\cdots\cdots (1)$$

$$C(s) = G_p[U(s)G_1 + D(s)] \qquad \cdots\cdots\cdots\cdots (2)$$

$$E(s) = R - HC \qquad \cdots\cdots\cdots\cdots (3)$$

Now, by substitution equation (1) into equation (2), we get

$$C(s) = G_p\, D(s) + G_p G_1 (G_f R + G_c\, E) \qquad \cdots\cdots (4)$$

Substitution equation (3) into equation (4),

$$C(s) = G_p\, D(s) + G_p G_1 [G_f R + G_c\,(R - HC)]$$

Solving for

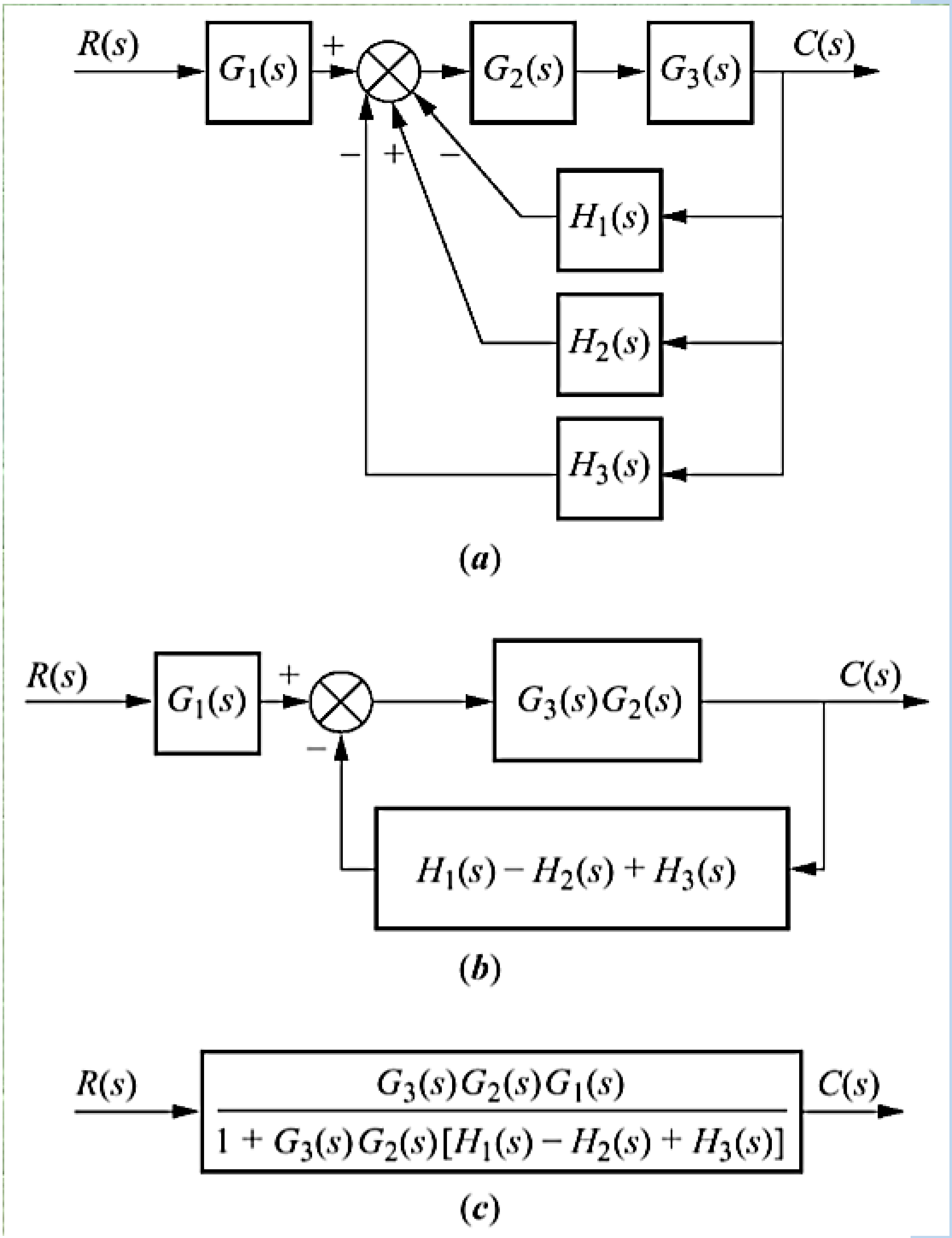
$$C(s) = \frac{G_p\, D + (G_f + G_c)R}{1 + G_1 G_p G_c\, H} \qquad \cdots\cdots\cdots\cdots (5)$$

Note that the equation (5) gives the response C when both inputs R and D are present, now

$$\frac{C}{R} = \frac{G_f + G_c}{1 + G_1 G_p G_c\, H} \qquad \text{[By letting } D = 0]$$

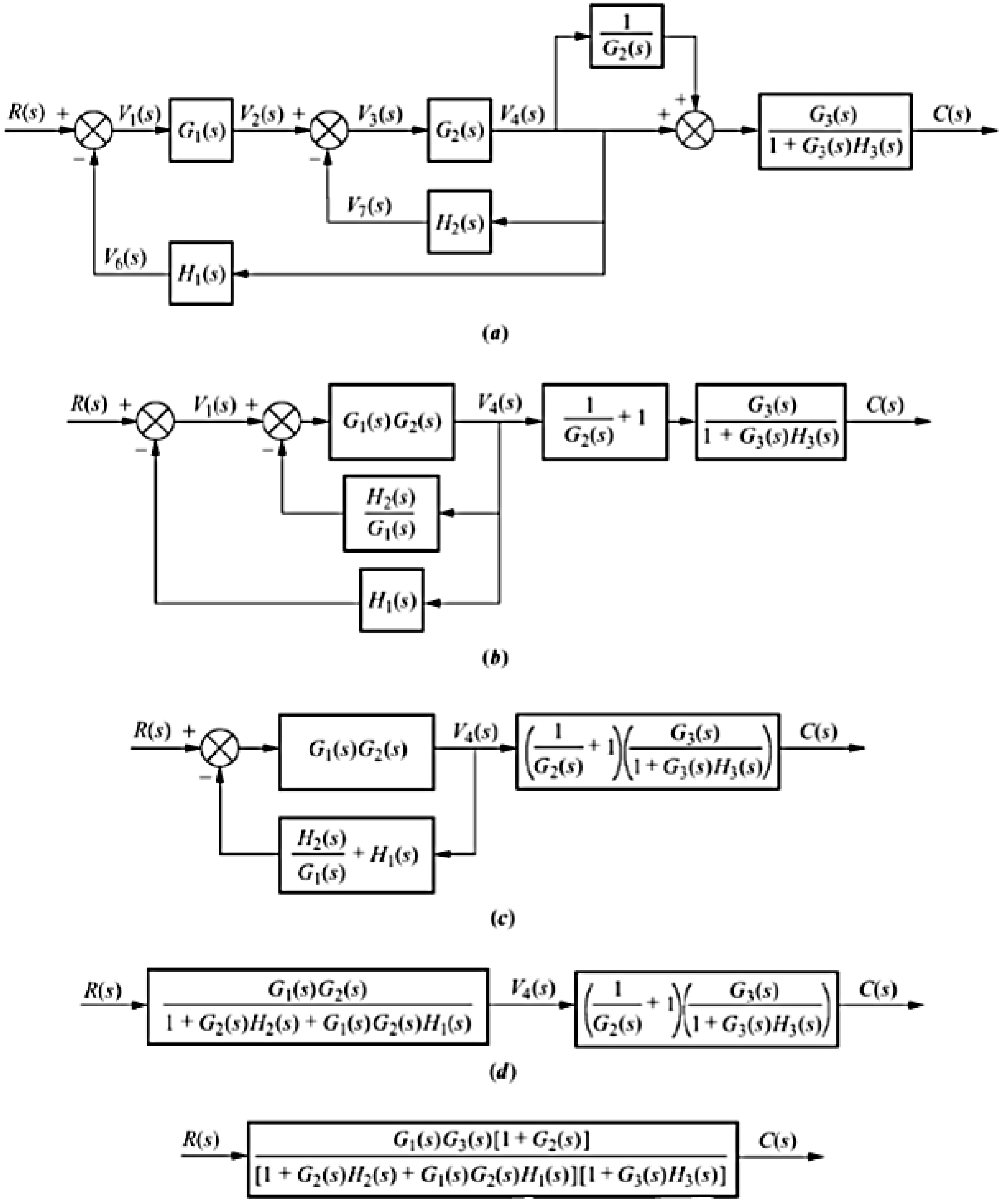
$$\frac{C}{D} = \frac{G_p}{1 + G_1 G_p G_c\, H} \qquad \text{[By letting } R = 0]$$

Example:



R(s)
G1(s)
G2(s)
G3(s)
C(s)
H1(s)
H2(s)
H3(s)
(a)
G3(s)G2(s)
H1(s) − H2(s) + H3(s)
(b)
G3(s)G2(s)G1(s) / 1 + G3(s)G2(s)[H1(s) − H2(s) + H3(s)]
(c)

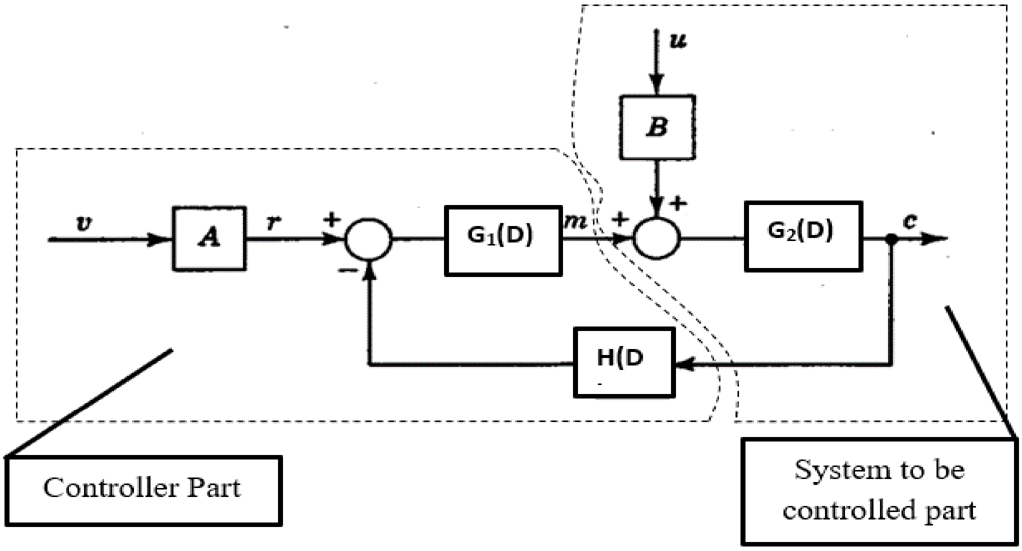
Example:



R(s)
V1(s)
G1(s)
V2(s)
V3(s)
G2(s)
V4(s)
1/G2(s)
G3(s)/(1 + G3(s)H3(s))
C(s)
V7(s)
H2(s)
V6(s)
H1(s)
(a)
G1(s)G2(s)
1/G2(s) + 1
H2(s)/G1(s)
(b)
H2(s)/G1(s) + H1(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
(c)
G1(s)G2(s)/(1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
(d)
G1(s)G3(s)[1 + G2(s)] / [1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)]

Steady-State Operation:

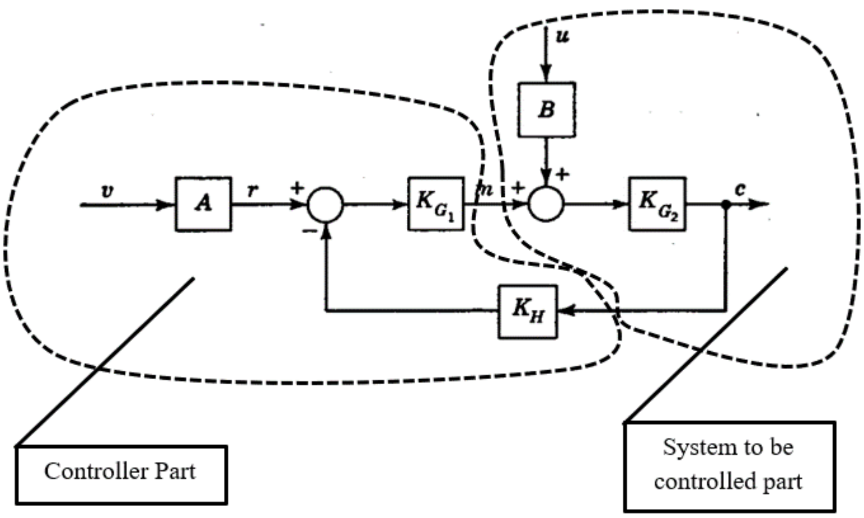
In steady-state operation, the control system variables remain unchanged. Their happens when there is no change in the requires input or disturbance with time.
The general block diagram of a standard feed back control system is as follows:



$G_1(D)$ is the controller transfer function.
$G_2(D)$ is the system to be controlled (Plant), transfer function.
H(D) is the feedback element transfer function.
A is a constant reflecting how the input signal is inserted to the system.
B is a constant reflecting how the disturbance affecting the system.
v is the change on the required input signal.
u is the change on the disturbance signal.
c is the change on the controlled variable (or output).
m is the change on the manipulated variable (or the variable out of a power device.

$$c = \Delta C, \qquad v = \Delta V, \qquad m = \Delta M, \qquad u = \Delta U$$

(C) and (m) are change due to changes (v) and/or (u).
In the steady-state operation, the variables (V, M, U and C) are not changing and the system remains in equilibrium. The block diagram will be as follows:



It may notice that the transfer function in the block diagram of fig. (1), become constants after making the operator (D) in the transfer function approaches to zero, so that:

$K_{G1} = G_1(D)|_{D=0}$, $K_{G2} = G_2(D)|_{D=0}$, $K_H = H(D)|_{D=0}$

It is noticed that the value of (c) in the steady-state depends on (v) and (u), so that the

steady-state equation is obtained as:

$$c = \frac{AK_{G1}K_{G2}}{1+K_{G1}K_{G2}K_H}.v + \frac{BK_{G2}}{1+K_{G1}K_{G2}K_H}.u$$

$$= \frac{\partial C}{\partial V}\Big|_U * v + \frac{\partial C}{\partial U}\Big|_V * u \qquad \text{(since C=C(V,U))}$$

The constant (A) in the above equation is, in effect, the scale factor for the input dial. It is required to have the coefficient of (V) equal unity, i.e.:

$$\frac{AK_{G1}K_{G2}}{1+K_{G1}K_{G2}K_H} = 1 \longrightarrow \boxed{A = \frac{1}{K_{G1}K_{G2}} + K_H}$$

Choosing (A) as given before makes the steady-state equation of operation as the following:

$$c = v + \frac{BK_{G2}}{1 + K_{G1}K_{G2}K_H}.u$$

The error in the control is, (e=v-c). to make this error equals to zero, the coefficient of (u) in the above equation has to be zero, i.e. the coefficient $\frac{BK_{G2}}{1+K_{G1}K_{G2}K_H} = 0$. Practically, this can only be achieved by making the coefficient ($K_{G1} = \infty$), which can be obtained by having an integrator term in the controller $[G_1(D)]$ to yield a ($\frac{1}{D}$) term to give infinity (∞), when we make (D=0). However, by increasing the slope of the controller lines, the error can be reduced.

In the left portion of the control system block diagram which represent the controller, it follows that:

$(Av - K_H.c)K_{G1} = m$ ***OR*** $c = -\frac{1}{K_{G1}.K_H}.m + \frac{A}{K_H}.v$

The lines of command signal are plotted on the (C-M) plane so that the controller lines slope is given by:

$$-K_{G1}.K_H = \frac{m}{c}\Big|_{v=0} = \frac{\Delta m}{\Delta c}\Big|_{\Delta V=0} = \frac{\partial M}{\partial C}\Big|_V$$

For the case when ($c = \Delta C = 0$), we have:

$$AK_{G1} = \frac{m}{v}\Big|_{c=0} = \frac{\Delta m}{\Delta v}\Big|_{\Delta C=0} = \frac{\partial M}{\partial V}\Big|_C$$

For the case (C=Constant) or (returned to the same value). This case gives vertical line of constant (C), the term ($\frac{m}{v}|_{c=0} = \frac{\partial M}{\partial V}|_{C}$) determine the vertical spacing between lines of (V). In the steady equilibrium equation, we get:

$\frac{K_H}{A} = \frac{v}{c}|_{m=0} = \frac{\Delta V}{\Delta C}|_{\Delta M=0} = \frac{\partial V}{\partial C}|_{M}$, which determines the horizontal spacing (since M=Constant), between (V) lines of the command signal.

Considering the system to be controlled portion, we can (on the same base) determine the equilibrium equation:

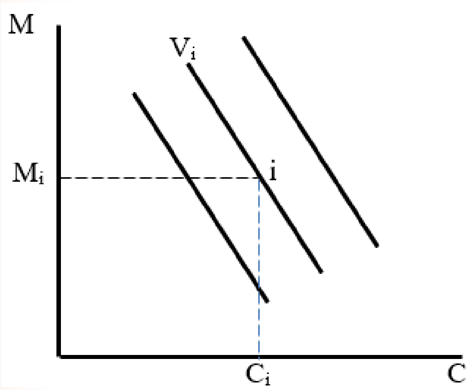
$(m + Bu)K_{G2} = c$, to give:

$$\frac{m}{c}|_{u=0} = \frac{\Delta M}{\Delta C}|_{\Delta U=0} = \frac{\partial M}{\partial C}|_{U} = \frac{1}{K_{G2}}$$

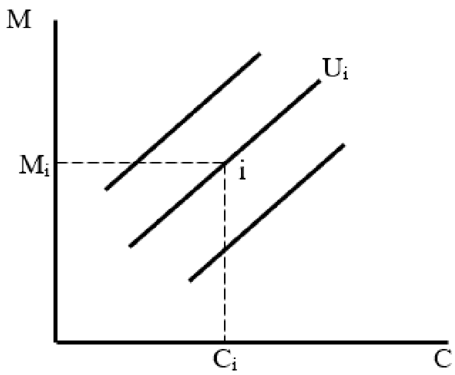
And $\frac{m}{u}|_{c=0} = \frac{\Delta M}{\Delta U}|_{\Delta C=0} = \frac{\partial M}{\partial U}|_{C} = -B$

And $\frac{c}{u}|_{m=0} = \frac{\Delta C}{\Delta U}|_{\Delta M=0} = \frac{\partial C}{\partial U}|_{M} = BK_{G2}$

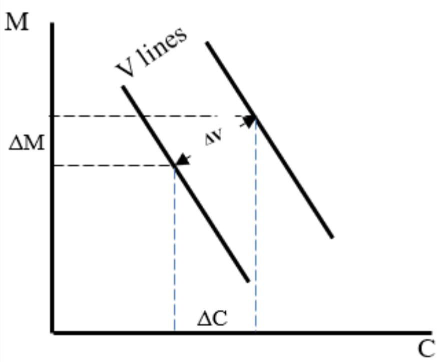
Thus, from the above relationships the slope of controller and the system to be controlled lines can be determined also the horizontal and vertical spacing between constant lines of (V) and between lines of constant (U) in the system to be controlled. The following figures gives the steady-state operation characteristic of the controller and the system to be controlled.



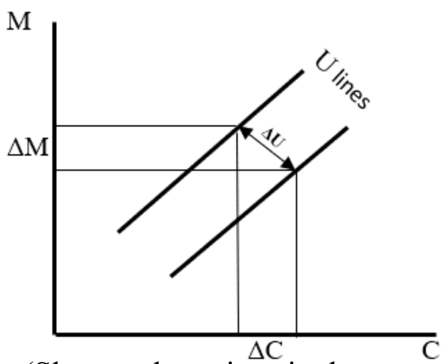
(The controller characteristic lines)



(The system to be controlled characteristic lines)



(Slope and spacings in the controller characteristic)



(Slope and spacings in the system to be controlled characteristic lines)