

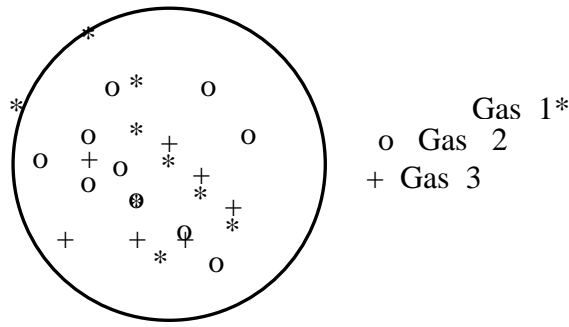


Lectures of the Department of Mechanical Engineering



Subject Title: Internal Combustion Engines
Class: Third year 2019-2020

Lecture Contents	Lecture sequences:	First lecture:Dr. Abidulrahman.Habbo	Instructor Name: A.R.Younis
	<p>The major contents:</p> <p>1-Mixture of ideal gases</p> <p>2- Combustion types , complete, incomplete and stoichiometric</p> <p>3- Air-Fuel Calculation</p>		
	<p style="text-align: center;">Mixture of Ideal Gases</p> <p>Consider a gas mixture composed of k components, then the total mass of the mixture is clearly the sum of the masses of each component; i.e</p> $m = m_1 + m_2 + m_3 + \dots + m_i = \sum_{i=1}^K m_i \quad (1)$ <p>and the total number of moles of the mixture can be obtained as follows:-</p> $n = \sum_{i=1}^K n_i \quad (2)$		



Obviously, there are two ways to describe the composition of mixture, either by specifying the number of moles of each components " Molar Analysis" or by specifying the mass of each components " Gravimetric Analysis" or it called "Mass Analysis".

$$X_i = \frac{n_i}{n} \quad (\text{Molar analysis}) \quad (3)$$

$$\bar{X}_i = \frac{m_i}{m} \quad (\text{Gravimetric analysis}) \quad (4)$$

Also, we have

$$\sum_{i=1}^K X_i = 1.0 \quad (5)$$

$$\sum_{i=1}^K \bar{X}_i = 1.0 \quad (6)$$

Number of moles of each components or gas can be obtained using the following formula:

$$n_i = \frac{m_i}{M_i} \quad (7)$$

Therefore , the mass of each constituent can be calculated as follows:-

$$m_i = n_i M_i \quad (8)$$

Hence, from the definition of total mass of mixture, we have

$$m = \sum_{i=1}^K n_i M_i \quad (9)$$

The molecular weight of the mixture can be obtained as follows

$$M = \frac{m}{n} \quad (10)$$

If equation (8) is substitute in equation 10 ,then

$$M = \frac{\sum_{i=1}^K n_i M_i}{n} \quad (11)$$

Which can be written as follows:-

$$M = \sum_{i=1}^K \frac{n_i}{n} M_i = \sum_{i=1}^K X_i M_i \quad (12)$$

The specific gas of the mixture may be calculated as follows:-

$$R = \frac{R_o}{M} \quad (13)$$

The relationship between the gravimetric analysis and volumetric analysis can be given as follows:-

$$\frac{m_i}{m} = \frac{X_i M_i}{\sum X_i M_i} \quad (14)$$

Partial pressure P_i

The partial pressure of each component is defined as:-

$$P_i V = n_i R_o T \quad (15)$$

While the total pressure of the mixture is given as follows:-

$$P V = n R_o T \quad (16)$$

Divide equation (15) by equation (16), we have

$$\frac{P_i}{P} = \frac{n_i}{n} = X_i \quad (17)$$

Equation (17) can be written as follows:-

$$\sum_{i=1}^K P_i = \sum_{i=1}^K X_i P \quad (18)$$

$$\text{i.e } \sum_{i=1}^K P_i = P \quad (19)$$

Now suppose the mixture of ideal gases is separated into its individual component as shown in figure (2). The volume occupied by each component in this case can be obtained as follows:-

$$P V_i = n_i R_o T \quad (20)$$

and the total volume of the mixture at given pressure and temperature can be written as follows:-

$$P V = n R_o T \quad (21)$$

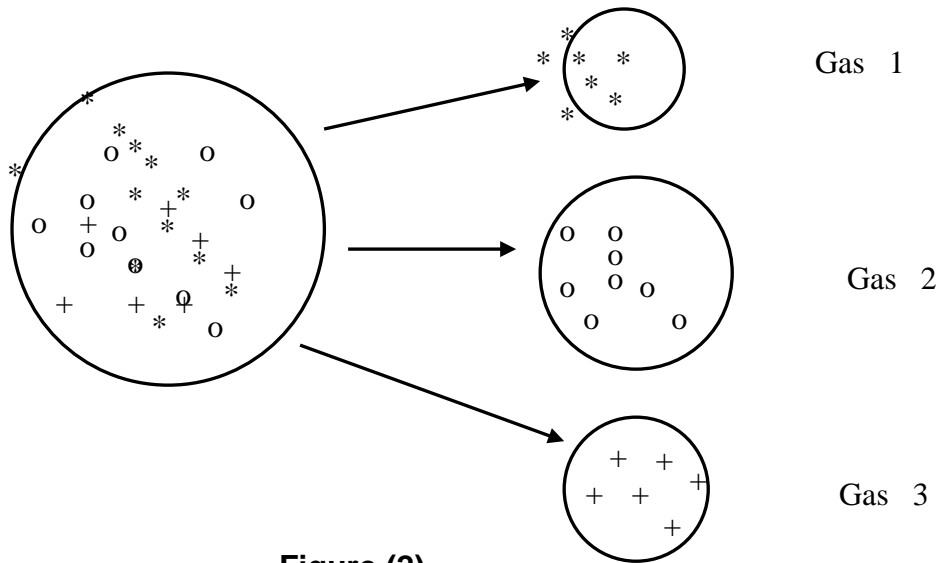


Figure (2)

From equation (20) and (21) , we can get

$$\frac{V_i}{V} = \frac{n_i}{n} = X_i \quad (22)$$

i.e
$$\sum_{i=1}^K V_i = \sum_{i=1}^K X_i V \quad (23)$$

$$\sum_{i=1}^K V_i = V \quad (24)$$

Equation (24) is called Amagat –Leduc law of additive volume

ENERGY PROPERTIES OF THE MIXTURE

The total internal energy of the mixture U

$$U = U_1 + U_2 + U_3 + \dots + U_K = \sum_{i=1}^K U_i \quad (25)$$

also can be written on mass analysis as follows:-

$$U = \sum_{i=1}^K m_i u_i \quad (26)$$

Or on molar analysis as follows:-

$$U = \sum_{i=1}^K n_i \bar{u}_i \quad (27)$$

Equation (26) and (27) can be written in the following forms

$$u = \sum_{i=1}^K \frac{m_i}{m} u_i = \sum_{i=1}^K \bar{X}_i u_i \quad (\text{mass basis}) \quad (28)$$

$$\bar{u} = \sum_{i=1}^K \frac{n_i}{n} \bar{u}_i = \sum_{i=1}^K X_i \bar{u}_i \quad (\text{molar basis}) \quad (29)$$

Relations for the enthalpy and entropy of the mixture can be developed in a similar manner as follows:-

$$H = \sum_{i=1}^K m_i h_i \quad \text{and} \quad h = \sum_{i=1}^K \frac{m_i}{m} h_i = \sum_{i=1}^K \bar{X}_i h_i \quad (\text{mass basis}) \quad (30)$$

$$\text{Also} \quad \bar{h} = \sum_{i=1}^K \frac{n_i}{n} \bar{h}_i = \sum_{i=1}^K X_i \bar{h}_i \quad (\text{molar basis}) \quad (31)$$

The entropy of a mixture can be obtained as follows:-

$$S = \sum_{i=1}^K m_i s_i \quad \text{and} \quad s = \sum_{i=1}^K \frac{m_i}{m} s_i = \sum_{i=1}^K \bar{X}_i s_i \quad (32)$$

And on molar basis :-

$$\bar{s} = \sum_{i=1}^K \frac{n_i}{n} \bar{s}_i = \sum_{i=1}^K X_i \bar{s}_i \quad (33)$$

The specific heats of the mixture can be obtained with aid of the following definitions:-

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (34)$$

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (35)$$

Substitute equation (28) in equation (34), we get

$$C_v = \left[\frac{\partial \left(\sum_{i=1}^K \frac{m_i}{m} u_i \right)}{\partial T} \right]_v \quad (36)$$

$$C_v = \frac{1}{m} \sum_{i=1}^K m_i \left(\frac{\partial u_i}{\partial T} \right)_v \quad (37)$$

Hence equation (38) can be written in term of C_v as follows:

$$C_v = \sum_{i=1}^K \frac{m_i}{m} C_{v_i} = \sum_{i=1}^K \bar{X}_i C_{v_i} \quad (38)$$

Or on molar basis :-

$$\bar{C}_v = \sum_{i=1}^K \frac{n_i}{n} \bar{C}_{v_i} = \sum_{i=1}^K X_i \bar{C}_{v_i} \quad (39)$$

The specific heat at constant pressure is developed in a similar manner which gives the following formula:-

$$C_p = \sum_{i=1}^K \frac{m_i}{m} C_{p_i} = \sum_{i=1}^K \bar{X}_i C_{p_i} \quad (40)$$

Or on molar basis :-

$$\bar{C}_p = \sum_{i=1}^K \frac{n_i}{n} \bar{C}_{p_i} = \sum_{i=1}^K X_i \bar{C}_{p_i} \quad (41)$$

The change of internal energy of the mixture can be calculated as follows:-

$$\Delta U = \sum_{i=1}^K \Delta U_i \quad \text{or} \quad \Delta U = \sum_{i=1}^K m_i \Delta u_i \quad \text{or} \quad \Delta U = \sum_{i=1}^K m_i C_{v_i} \Delta T_i \quad (42)$$

Similarly, the change in enthalpy for the mixture can be obtained as follows:-

$$\Delta H = \sum_{i=1}^K \Delta H_i \quad \text{or} \quad \Delta H = \sum_{i=1}^K m_i \Delta h_i \quad \text{or} \quad \Delta H = \sum_{i=1}^K m_i C_{p_i} \Delta T_i \quad (43)$$

Or on molar basis :-

$$\Delta U = \sum_{i=1}^K n_i \bar{C}_{v_i} \Delta T_i \quad \text{and} \quad \Delta H = \sum_{i=1}^K n_i \bar{C}_{p_i} \Delta T_i \quad (44)$$

The calculation of the change of entropy is a bit more complicated since the entropy is not a function of temperature alone. The change in entropy for an ideal gas can be presented as follows:-

$$S_2 - S_1 = m C_p \ln \frac{T_2}{T_1} - m R \ln \frac{P_2}{P_1} \quad (45)$$

Where the subscript 1 and 2 represent the initial and the final state respectively, thus for the mixture, we have:-

$$\Delta S = S_2 - S_1 = \sum_{i=1}^K m_i C_{p_i} \ln \frac{T_2}{T_1} - \sum_{i=1}^K m_i R_i \ln \frac{P_{i2}}{P_{i1}} \quad (46)$$

Where P_{i1} and P_{i2} are the initial and final pressure of the i^{th} component respectively. However, the change in entropy can be written on molar basis as follows:-

$$\Delta S = S_2 - S_1 = \sum_{i=1}^K n_i \bar{C}_{p_i} \ln \frac{T_2}{T_1} - \sum_{i=1}^K n_i R_{oi} \ln \frac{P_{i2}}{P_{i1}} \quad (47)$$

ADIABATIC MIXING

Two gases separated by diaphragm as shown in figure (3). The internal energy before mixing can be obtained as follows:-

$$U_1 = (U_A + U_B)_1$$

Which can be written as follows:-

$$U_1 = m_A C_{v_A} T_A + m_B C_{v_B} T_B$$

Which may be written as follows:-

$$U_1 = \sum m_i C_{v_i} T_i \quad (48)$$

If the diaphragm is removed and mixing take place adiabatically, therefore the internal energy can be obtained as follows:-

$$U_2 = (U_A + U_B) = m_A C_{v_A} T_2 + m_B C_{v_B} T_2$$

$$U_2 = T_2 \sum m_i C_{v_i} \quad (49)$$

$$\text{However } U_1 = U_2$$

$$\text{Hence } T_2 \sum m_i C_{v_i} = \sum m_i C_{v_i} T_i$$

$$\text{Which gives } T_2 = \frac{\sum m_i C_{v_i} T_i}{\sum m_i C_{v_i}} \quad (50)$$

Example 1.

A rigid vessel contain 5 mole of CO and 2 mole of air at 10 bar and 30 °C. Determine :-

- 1- the mass of each species
- 2- the volumetric and gravimetric analysis
- 3- the specific gas of the mixture
- 4- the molecular weight of the mixture
- 5- the total volume of the mixture

Example 2

Consider a gas mixture that consist of 5 kg of O₂ , 8 kg of N₂ and

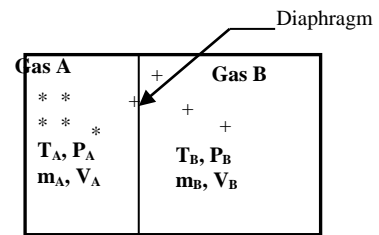


Figure (3)

12 kg of C_2H_6 . Determine :-

- 1- the mass fraction of each component
- 2- the mole fraction of each component
- 3- the specific gas of the mixture

Example 3

Two kg of CO_2 at $50^\circ C$ and 2.0 bar are mixed with five kg of N_2 at $200^\circ C$ and 1.5 bar to form a mixture at final pressure of 1.0 bar. The mixing process occurs adiabatically in steady flow apparatus. Calculate the final temperature of the mixture and the change in entropy.

H.W

Q1. A mixture of H_2 and N_2 is to be made that the ratio is Four to one by volume. Calculate the mass of N_2 required and the volume of the container per 1 Kg of H_2 , if the pressure and temperature are 1.0 bar and $15^\circ C$ respectively.

Q2. A rigid vessel of 1.5 m^3 capacity, contain oxygen at 7 bar and $40^\circ C$. the vessel is connected to an other vessel of 3 m^3 capacity containing carbon monoxide at 1 bar and $15^\circ C$. A connecting valve is opened and gases mixes adiabatically. Calculate the final temperature and pressure of the mixture and the change in entropy.

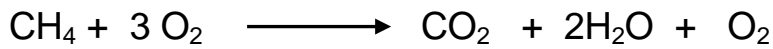
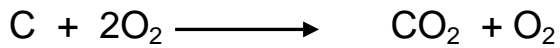
COMBUSTION

Combustion may be defined as a rapid chemical reaction between fuel and air (oxygen); its usually a companied by flame and heat is released

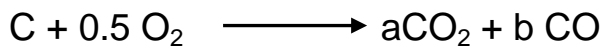
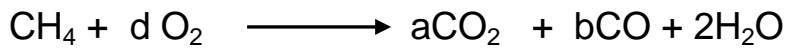
during the reaction. Combustion can be classified to the following types:-

- 1- Complete combustion
- 2- Incomplete combustion
- 3- Stiochiometric combustion

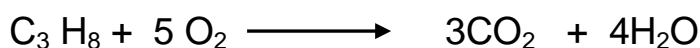
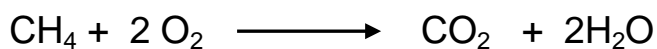
1- **Complete combustion**:- this type of combustion occurs when the amount of air (oxygen) available is more than enough to convert all carbon to carbon dioxide and all hydrogen to water.

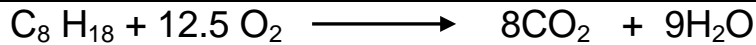


2- **Incomplete combustion**:- :- this type of combustion occurs when the amount of air (oxygen) available is not enough to convert all carbon to carbon dioxide , therefore a carbon monoxide (CO) is present in the products as a result of incomplete combustion.



3- **Stiochiometric combustion**:- this type of combustion occurs when the amount of air (oxygen) available is just enough to convert all carbon to carbon dioxide (CO₂) and all hydrogen to water (H₂O), with no oxygen is left over.





Note :- the maximum heat released from combustion of fuel can be obtained when the combustion is of a stiochiometric type.

FUEL TYPE

Fuel may be solid or liquid or gas

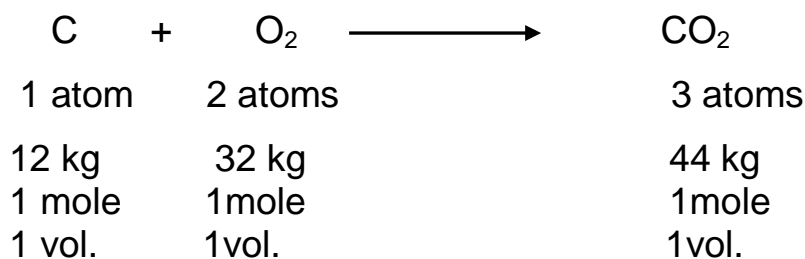
Solid fuel such as coal ,wood , solid waste etc

Liquid fuel such as petrol and all their derivatives

Gases such as natural gas, coal gas and methane

CHEMICAL EQUATION

The first step in combustion analysis is to setup a chemical equation which shows how atoms (moles, kg) of the reactants are arranged to form the products. It expresses the principle of conservation of mass. It can be written in the form of atoms , mass ,mole and volume.



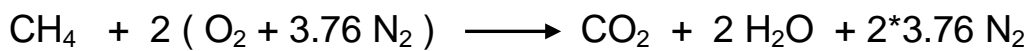
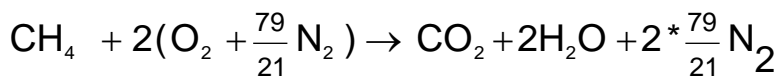
However, oxygen is usually available in air, and for atmospheric air the composition is approximately 78% N₂ and 21% O₂ and 1% Argon and other gases. For combustion calculation , argon is usually ignored and the air is assumed to be 79% N₂ and 21% O₂.

This means that for every mole of air there is 0.21 mole of O₂ and 0.79

mole of N₂. and for every 1kg of air there is 0.233 kg of O₂ and 0.767 kg of N₂.

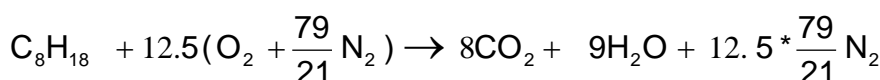
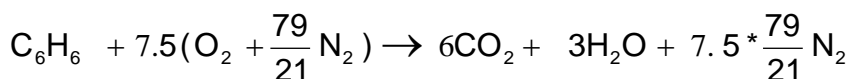
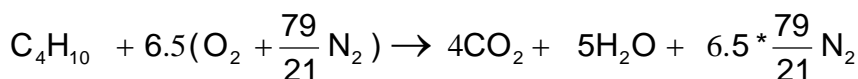
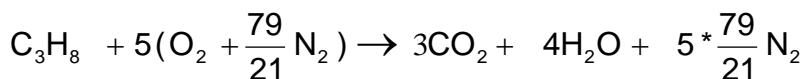
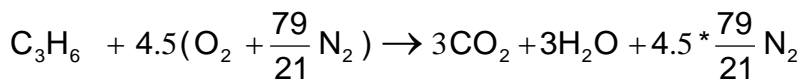
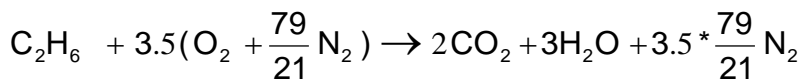
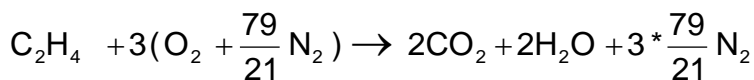
	Oxygen (O ₂)	Nitrogen (N ₂)
Volumetric analysis	0.21	0.79
Gravimetric analysis	0.233	0.767

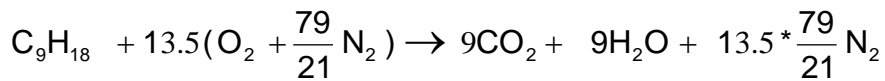
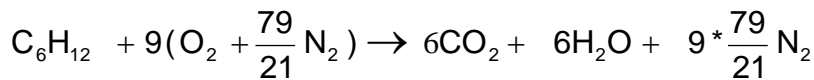
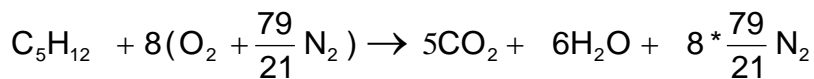
The stiochiometric combustion equation of methane with air can be written as follows:-



if one mole of oxygen is needed for combustion , 4.76 mole of air must be supplied (one mole of O₂ + 3.76 mole of N₂).

The stiochiometric combustion equations of some familiar hydrocarbon fuels are given below:-

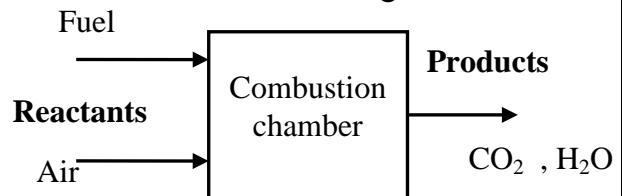




AIR-FUEL RATIO

A frequently used quantity in the analysis of combustion process is the air fuel ratio. It is usually expressed on a mass basis and its defined as the ratio of mass of air to the mass of fuel burned during combustion process.

$$\frac{A}{F} = \frac{m_{\text{air}}}{m_{\text{fuel}}} = \frac{m_a}{m_f}$$



The air-ratio can also expressed on a mole basis as the ratio of mole numbers of air to the mole numbers of fuel. However, combustion can occurs within limits, stiochiometric mixture, lean mixture and rich mixture.

a- stiochiometric mixture

b- lean mixture

c- rich mixture

for example natural gas can be burned at various type according to its mixture with air



Equivalence Ratio

For actual combustion in an engine , the equivalence ratio is a measure of the air-fuel mixture relative to stiochiometric conditions. It may be defined as the ratio of stiochiometric air-fuel ratio to actual air fuel ratio.

$$\phi = \frac{\left(\frac{A}{F}\right)_{\text{stoich}}}{\left(\frac{A}{F}\right)_{\text{actual}}} \quad \text{or} \quad \phi = \frac{\left(\frac{F}{A}\right)_{\text{actual}}}{\left(\frac{F}{A}\right)_{\text{stoichl}}}$$

Values of Φ can be classified as follows;_

$\Phi < 1.0$ for lean mixture { combustion products CO_2 , H_2O and O_2 }

$\Phi > 1.0$ for rich mixture { combustion products CO_2 , CO and H_2O }

$\Phi = 1.0$ for stiochiometric mixture { combustion products CO_2 , H_2O }

Example 1

Iso-octane C_8H_{18} is burned with 120% theoretical air in a small three cylinder turbocharged automobile engine, calculate:-

- 1- the stiochiometric air-fuel ratio
- 2- the equivalence ratio

Example2

A hydrocarbon fuel of C_4H_{10} is burned in steady flow combustion chamber with 85% theoretical air, determine:-

- 1- the actual air-fuel ratio.
- 2- the equivalence ratio

Example3

One mole of C_3H_8 is burned with an unknown amount of air during a combustion process. An analysis of the combustion products shows that the combustion is complete and there 3 moles of free O_2 in the products. Determine;-

- 1- the actual air-fuel ratio
- 2- the equivalence ratio
- 3- the percentage of theoretical air used during this process.

H.W

Q1. Gasoline C_7H_{15} is burned in steadily with air in spark ignition engine. If the air-fuel ratio is 21 kg of air/kg of fuel. Determine the percentage of theoretical air used during this process.

Q2. In a combustion chamber , a hydrocarbon fuel of C_6H_6 is burned at a rate of 8 kg/h with air that enters the combustion chamber at a rate of 138 kg/h. Determine the percentage of excess air used.

Exhaust and Flame Analysis

The product of combustion are mainly gases such as CO_2 , CO , SO_x , NO_2 and water vapor H_2O . The analysis of products with H_2O included is called “ wet analysis” . If the products are cooled to a temperature which is below the dew point temperature ,water vapor will condensed , and the analysis is called “ dry analysis”.

Example1.

A gasoline fuel C_7H_{17} is burned in a single cylinder spark ignition engine with air at an equivalence ratio of 0.9. determine :-

- 1- the actual fuel-air ratio
- 2- the wet volumetric exhaust analysis
- 3- the dry volumetric exhaust analysis

Example2.

A gaseous fuel the following volumetric composition : $CH_4 = 0.38$, $H_2 = 0.26$, $N_2 = 0.06$, $CO = 0.26$ and $CO_2 = 0.04$. Find the stiochiometric air-fuel ratio by volume and the corresponding wet and dry volumetric analysis of the product of combustion.

Ex3. Ethyl alcohol C_2H_6O . Calculate the stiochiometric air-fuel ratio, also determine the wet and dry volumetric and gravimetric analysis of the products when 10% excess air is supplied with the fuel.

H.W

Q1. two mole of C_6H_6 is burned with air that contains 20 mole of air of O_2 . Determine :-

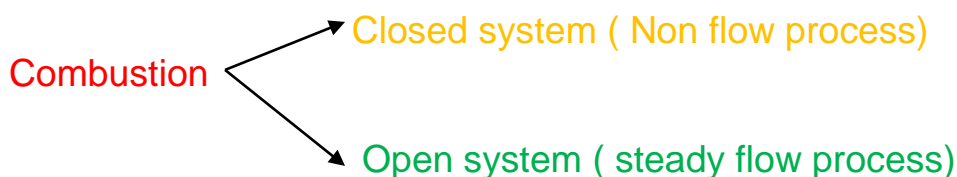
- 1- the number of mole of each gas in the products
- 2- the air-fuel ratio
- 3- the equivalence ratio
- 4- the wet and dry exhaust volumetric analysis

Q2. Iso-octane C_8H_{18} is burned with air in steady flow combustion process. The mole fraction of the combustion products on dry basis showing the following results:- $CO_2 = 10.02\%$, $O_2 = 5.62\%$, $CO = 0.88$ and $N_2 = 83.48$, determine:-

- 1- the air-fuel ratio
- 2- the percentage of theoretical used

Application of the first law of thermodynamics to combustion process

Combustion can take place in either closed or open system.



A) Non flow combustion process (closed system)

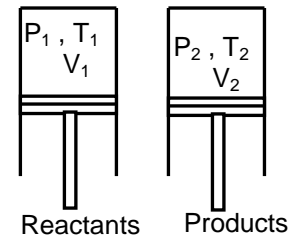
Applying the non flow energy equation

$$Q - W = \Delta U \quad (\text{NFEE})$$

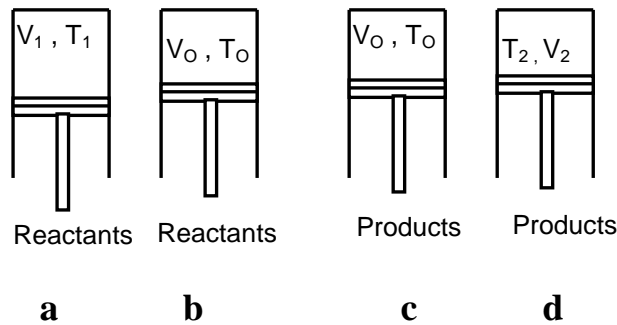
ΔU : depend on end states and not the process

$$\Delta U = U_2 - U_1 = U_{p2} - U_{R1}$$

$$\Delta U = U_{p2} - U_{R1} \quad (\text{total change in internal energy})$$



The process may be imagined to be carried in three stages as shown in figure below.



Process a-b and c-d are carried out on mixture of gases and vapor of the reactants and products respectively. The change in internal energy in these process is a function of temperature only.

$$\Delta U_{a-b} = U_{R0} - U_{R1}$$

And $\Delta U_{c-d} = U_{p2} - U_{p0}$

From the combustion itself, if it is assumed to be carried out at constant volume V_0 , then

$$Q - W = \Delta U_{c-b} \quad \text{and} \quad W = \int P dv \quad \text{and} \quad dv = 0, \quad \text{this means} \quad W = 0$$

$$Q = \Delta U_{c-b} = U_c - U_b$$

The change of internal ΔU_{c-b} due to combustion process can be written as follows:-

$$\Delta U_{c-b} = U_c - U_b = U_{p0} - U_{R0}$$

This change in internal energy ΔU_{c-b} is due to the differences in chemical energy in the bonds between molecules of the reactants and products (it is always have a negative value -).

The total change in internal energy $\Delta U = U_{p_2} - U_{R_1}$ can be obtained as follows:-

$$\Delta U = U_{p_2} - U_{R_1} = (U_{p_2} - U_{p_0}) + (U_{p_0} - U_{R_0}) + (U_{R_0} - U_{R_1})$$

However, the term $(U_{p_0} - U_{R_0})$ is called the internal energy of combustion at T_0 . It is determined experimentally by measuring the heat released in constant volume combustion process (heat released -ve). It is also called constant volume heat combustion and is denoted as ΔU_0 .

$$\Delta U_0 = (U_{p_0} - U_{R_0}) \quad (\text{experimentally measured})$$

If the specific heats of products and reactants are known and no phase change takes place during the process a-b and c-d, therefore, the internal energy change can be obtained as follows:-

$$U_{p_2} - U_{p_0} = \sum m_i c_{v_i} (T_2 - T_0)$$

$$\text{And } U_{R_0} - U_{R_1} = \sum m_i c_{v_i} (T_0 - T_1)$$

Where m_i and C_{v_i} are the mass and specific heat at constant volume of each constituent of reactants and products.

After finding the total change of internal energy:-

$$\Delta U = U_{p_2} - U_{R_1} = \sum m_i c_{v_i} (T_2 - T_0) + \Delta U_0 + \sum m_i c_{v_i} (T_0 - T_1)$$

The non flow energy equation can be applied according to the type of process:-

1- Constant volume combustion process

$$Q - W = \Delta U$$

And $W = 0$ which gives $Q = \Delta U$

2- Adiabatic combustion process

For adiabatic combustion no heat is added or rejected throughout the system i.e $Q = 0$

$$Q - W = \Delta U$$

i.e $-W = \Delta U$

3- Normal combustion with heat and work

$$Q - W = \Delta U$$

4- Constant pressure combustion

$$Q - W = \Delta U$$

In this case work can be obtained as follows:-

$$W = \int_1^2 P dV = P (V_2 - V_1)$$

$$Q = U_{p_2} - U_{R_1} + PV_2 - PV_1$$

$$\text{i.e. } Q = (U_{p_2} + PV_2) - (U_{R_1} + PV_1)$$

$$\text{or } Q = H_{p_2} - H_{p_1} = \Delta H$$

b) Steady flow combustion

introducing steady flow energy equation (SFEE)

$$Q - W = \Delta H + \Delta K.E + \Delta P.E$$

Where ΔH : is the total enthalpy change

$\Delta K.E$: total change of kinetic energy

$\Delta P.E$: total change of potential energy

Ignoring the change in kinetic and potential energy and willing to obtain the total change of enthalpy ΔH .

$$\Delta H = H_{P_2} - H_{R_1}$$

Which can be written as follows:-

$$\Delta H = H_{P_2} - H_{R_1} = (H_{P_2} - H_{P_0}) + (H_{P_0} - H_{R_0}) + (H_{R_0} - H_{R_1})$$

Where $(H_{P_2} - H_{P_0})$:- is the change of enthalpy of products

$(H_{R_0} - H_{R_1})$:- is the change of enthalpy of reactants

$(H_{P_0} - H_{R_0})$: is the enthalpy of combustion at T_0 (ΔH_0)

ΔH_0 :its measured experimentally under the condition that the reactants and products at T_0 (298 K) and no work is done so that

$$Q = \Delta H$$

The change of enthalpy of products and reactants can be obtained as follows:-

$$H_{p_2} - H_{p_0} = \sum m_i C_{p_i} (T_2 - T_0)$$

$$H_{R_0} - H_{R_1} = \sum m_i C_{p_i} (T_0 - T_1)$$

Now after the total enthalpy change have been resorted out, the steady flow energy equation can be applied for three different cases as follows:-

1- constant volume steady flow

In this case the SFEE equation can be in the following form:

$$Q = \Delta H$$

2- Adiabatic combustion

$$Q - W = \Delta H, \text{ but } Q=0$$

i.e $-W = \Delta H$

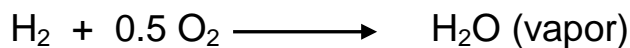
3- Normal combustion , both work and heat

$$Q - W = \Delta H$$

Note:- value ΔU_0 and ΔH_0 are quoted for a reference temperature T_0 of 25 °C for 1kg or 1mole of fuel. When quoting values of ΔU_0 and ΔH_0 , it is necessary to specify the phase of any individual reactants and products which might be in doubt.

For example, value of ΔH_0 for liquid hydrocarbon is less than that of hydrocarbon in gaseous form, and this differences is due to the latent heat of vaporization h_{fg} (kJ/kg).

The combustion of hydrogen can be performed as follows:-



Hence $\Delta H_0 = \Delta H_{25} = -120900 \text{ kJ/kg}$

Or $\Delta H_0 = \Delta H_{25} = -241800 \text{ kJ/mole}$

Normally ΔU_0 is determined for solid and liquid fuels

While ΔH_0 is determined for gaseous fuel

$$\Delta H_0 \text{ (vapor)} - m_f h_{fg} = \Delta H_0 \text{ (liquid)}$$

Relationship between ΔH_o and ΔU_o

The enthalpy of combustion at 25 °C (ΔH_o) is defined as follows:-

$$\Delta H_o = (H_{P_o} - H_{R_o})$$

$$\Delta H_o = [U_{P_o} + (P_{P_o} V_{P_o})] - [U_{R_o} + (P_{R_o} V_{R_o})]$$

Which can be written as follows:-

$$\Delta H_o = [U_{P_o} - U_{R_o}] + (P_{P_o} V_{P_o} - P_{R_o} V_{R_o})$$

Or $\Delta H_o = \Delta U_o + (P_o V_{P_o} - P_o V_{R_o})$

But $PV = nR_o T$

i.e $\Delta H_o = \Delta U_o + (n_p - n_R) R_o T$

where n_p : number of mole of products

n_R : number of mole of reactants

note for solid and liquid fuel the term pv is negligible (ignored)

Example1.

When all the products of hydrocarbon fuel ethane (C_2H_6) are in gaseous phase, the enthalpy of combustion at 25 °C is (- 47484 kJ/kg), find:-

- the internal energy of combustion at 25 °C
- the enthalpy of combustion at 540 °C
- calculate the heat transferred when 0.2 kg of ethane is burned at constant pressure in a cylinder containing 4 kg of air and the temperature of reactants and products being 40 °C and 540 °C respectively.

The relevant mean specific heats at constant pressure for the range 25 °C to 540 °C are quoted from tables at an average temperature of 555 K . ($C_2H_6 = 2.8$, $O_2 = 0.989$, $CO_2 = 1.049$, $H_2O(v) = 1.987$ and $N_2 = 1.059$ kJ/kg.K)

And for the range 25 °C to 40 °C ($C_2H_6 = 1.788$, $N_2 = 1.04$ kJ/kg.K)

Example2.

Benzene gas C_6H_6 at $25^\circ C$ is burned in a steady flow combustion chamber with 95% theoretical of air that enters the combustion chamber at $25^\circ C$. if the products at 1000 K , find

- a- the mole fraction of CO in the products
- b- the heat transfer from this process

H.W

A constant volume tank contain 1 mole of C_7H_{14} and 12 mole of O_2 gas at a temperature of $25^\circ C$ and 1 bar. The contents of the tank is ignited and C_7H_{14} is burned completely and final products temperature is found to be 1700 K . Determine the heat transfer during this process. (take $\Delta H_o = -47800\text{ kJ/kg}$).

Enthalpy of Formation h_f°

The enthalpy of formation is defined as the heat evolved when one mole of the substances is formed from its elements at standard conditions $25^\circ C$ and 1 atmosphere. The subscript f indicates the formation of the compound from element and the index $^\circ$ refers to all products and reactants in their standard states

By definition each pure element in its standard states is assigned an enthalpy of zero.

The enthalpy of combustion at $25^\circ C$ (ΔH_o) can be calculated with using of enthalpy of formation as follows:-

$$\Delta H_o = \sum_P n_i h_f^\circ - \sum_R n_i h_f^\circ$$

Where n_i ; number of mole

h_f° ; enthalpy of formation (kJ/mole)

Example 3.

Calculate the enthalpy of combustion of propane C_3H_8 at $25\text{ }^{\circ}\text{C}$ in both kJ/kg and kJ/mole under the following conditions:-

- 1- gaseous propane with H_2O liquid in the products
- 2- gaseous propane with H_2O vapor in the products
- 3- liquid propane with H_2O liquid in the products
- 4- liquid propane with H_2O vapor in the products

note: the enthalpy of evaporation of propane at $25\text{ }^{\circ}\text{C}$ is 425 kJ/kg

Example 4.

Calculate the enthalpy of products (ΔH_P) and reactants (ΔH_R) and the enthalpy of combustion at $25\text{ }^{\circ}\text{C}$ (ΔH_O) and the internal energy of combustion at $25\text{ }^{\circ}\text{C}$ (ΔU_O) of the combustion of methane CH_4 with air at an equivalence of ($\Phi=1.0$) assuming the reactants and products at temperature of $25\text{ }^{\circ}\text{C}$.

CALORIFIC VALUE

When the precise fuel composition is not known, the enthalpy of combustion at $25\text{ }^{\circ}\text{C}$ can not be determined from the enthalpy of formation h_f° of the reactants species. Therefore, the heating value (Calorific value) should be introduced.

The calorific value may be defined as the amount of heat is released when a unit mass of fuel is burnt completely in a calorimeter under specified condition and should all carbon converted to CO_2 and all hydrogen to water, it can be measured in (kJ/kg) or Kcal/kg)

There are two types of calorific values:-

- 1- calorific value at constant pressure $Q_{HPV} = (\Delta H_O)$

2- calorific value at constant volume $Q_{HVV} = (\Delta U_O)$

For solid and liquid fuel (constant volume) which is measured with aid of bomb calorimeter. For gaseous and volatile liquid fuel (constant pressure) flame calorimeter .

Two values of calorific values can be recognized :-

Higher calorific value HCV , when H_2O in products in liquid state

Lower calorific value LCV , when H_2O in products in vapor state

$$HCV - LCV = h_{fg}$$

h_{fg} : latent heat of evaporation of water (kJ/kg)

for engine analysis and calculation, lower calorific value should be used , the heat supplied can measured as follows:-

$$Q_{add} = \eta_c * m_f * LCV$$

Where η_c : is the combustion efficiency (98%- 99%)

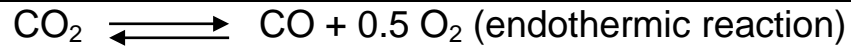
m_f : mass of fuel

ADIABATIC FLAME TEMPERATURE

Adiabatic flame temperature is the maximum theoretical temperature that can be obtained for a combustion of a given fuel. However, the maximum adiabatic flame temperature can achieved when the air-fuel mixture is of a stiochiometric and combustion efficiency 100%.

The actual value of adiabatic flame temperature is found to be several hundred degrees less than the theoretical and the reasons for that are:-

- 1- there is some heat loss occurs even in very short time of one cycle
- 2- combustion efficiency is less than 100%, so a small amount of fuel dose not burned
- 3- some component dissociate at high temperature such as carbon dioxide and water vapor



Example 5.

A stiochiometric mixture of hydrocarbon fuel CH_4 and air is admitted to a steady flow combustion chamber at 25°C . Combustion occurs at constant pressure and adiabatically. Determine the adiabatic flame temperature.

H.W

One mole of gaseous hexane C_6H_{12} is to be burned adiabatically in a rigid vessel with theoretical amount of air. if the temperature of fuel and air is assumed to be 45°C . Find the adiabatic flame temperature.

Note C_v for the fuel is 1.56 kJ/kg K .

THE SECOND LAW ANALYSIS OF COMBUSTION PROCESS

The second law of thermodynamics can be applied to combustion system. It can be seen that the second law gives us the sense of what direction a particular reaction proceeds in, and also specifies the equilibrium products of a given reactions.

However, from the definition of entropy, we have

$$\Delta S \geq \frac{Q_{\text{in}} - Q_{\text{out}}}{T_o} = \frac{Q_{\text{net}}}{T_o} \quad (1)$$

For non-reacting system, and single phase steady state, steady flow system, the second law can be stated on mass basis as follows:

$$0 = \frac{Q_{net}}{T_o} + \dot{m} (s_{in} - s_{out}) + \dot{S}_{GEN} \quad (2)$$

Hence $\dot{S}_{GEN} \geq 0$ (must equal or greater than zero)

Now , assume a reacting system (combustion occurs), the entropy at any pressure can be found using the following formula:

$$\bar{S}(T,P) = \bar{S}^o(T,P_{REF}) + [\bar{S}(T,P) - \bar{S}^o(T,P_{REF})] \quad (3)$$

Where $\bar{S}^o(T,P_{REF})$; is the absolute entropy at fixed reference pressure (1 atm.) is tabulated.

Equation 3 can be written in the following form:-

$$\bar{S}(T,P) - \bar{S}^o(T,P_{REF}) = -\bar{R}_o \ln \frac{P}{P_{REF}} \quad (4)$$

For combustion process, the entropy for each constituent can be obtained as;

$$\bar{S}_i(T,P) = \bar{S}_i^o(T,P_{REF}) - \bar{R}_o \ln \frac{P}{P_{REF}} \quad (5)$$

The change in entropy for a process with several species can be written as :

$$\Delta S = \sum_P n_p \bar{S}_p - \sum_R n_R \bar{S}_R \quad (6)$$

Hence, the entropy generation for steady flow combustion system is given as follows:

$$\dot{S}_{GEN} = \sum_P n_p \bar{S}_p - \sum_R n_R \bar{S}_R - \frac{Q_{net}}{T_o} \geq 0 \quad (7)$$

We may have a special such as an isothermal and constant pressure combustion system as might occurs in an isothermal steady flow reactor. i.e

$$Q_{net} = Q_{in} - Q_{out} = \Delta H_o$$

And
$$\Delta H_o = \sum_P n_i h_f^o - \sum_R n_i h_f^o$$

Hence, equation (7) can be written in the following form:

$$\dot{S}_{GEN} T_o = T_o [\sum_P n_p \bar{S}_p - \sum_R n_R \bar{S}_R] - \sum_P n_i h_f^o - \sum_R n_i h_f^o$$

Re-arranging the above equation, we get

$$S_{GEN} T_o = \sum_P n_p (T_o \bar{S}_p - h_f^\circ) - \sum_R n_p (T_o \bar{S}_R - h_f^\circ) \geq 0 \quad (8) \quad \text{Now}$$

, it is quite convenient to introduce a new function which is GIBBS FUNCTION (**G**)

$$G = H - TS \quad (9)$$

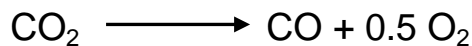
Or $g = h - Ts$

Dissociation

The maximum adiabatic flame temperature can be obtained for adiabatic combustion, but this limited by a phenomenon of dissociation. For the following reaction, heat is released and this reaction is called exothermic reaction.



At high temperature if sufficient energy is supplied to molecules of CO_2 , the process proceed in the opposite direction, i.e some of CO_2 undergo the following reaction:



Or can be written as follows:



The reversed reaction is accompanied by an absorption of energy, hence it is called endothermic reaction.

It has been found that at any particular temperature and pressure , the species of CO_2 , CO and O_2 adjust themselves until the reaction proceed at the same rate. This means until the number of molecules of CO_2 being formed is equal to the number of molecules of CO_2 dissociates.

Normally dissociation take place at high temperature approximately more than 1500 K. This applies to H_2O as well as to CO_2 and this explain why the temperature in adiabatic combustion is less than the maximum temperature predicted by simple calculation.

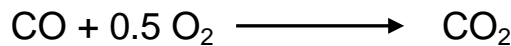
Assume we have a gas of CO_2 initially at 1 atmosphere and 300 K is heated up to 500 K, hence CO_2 is quite stable and there is very little tendency toward dissociation into CO and O_2 .



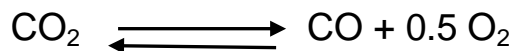
i.e $a \gg b$ and $a \gg d$

at a temperature of 500 K and a pressure of 1 atmosphere, values of b and d are very small which is less 10^{-15} mole and should be ignored.

Now if the temperature is raised to 2500 K, CO_2 will dissociate to CO and O_2 and in the same time CO recombine with O_2 till it reach an equilibrium state. And values of b and d are relatively have a significant magnitude.



Or usually written as:



At equilibrium, the rate of formation of CO_2 is the same as the rate of dissociation of CO_2 , i.e the number of moles of (a, b and d) of CO_2 , CO and O_2 remains constant.

However, as the temperature of product mixture is increased, the equilibrium value of (a) will decrease while (b and d) increase. In order to be able to determine products composition with dissociation, it is necessary to have a criterion for chemical equilibrium. Such a criterion for equilibrium at a given pressure and temperature is provided by thermodynamic property is called GIBBS FUNCTION (G)

$$G = H - TS \quad (10)$$

Or in differential form as:

$$dG = dH - TdS - SdT \quad (11)$$

from the definition of enthalpy H

$$H = U + PV$$

$$\text{Or } dH = dU + PdV + VdP \quad (12)$$

First law of thermodynamics can be presented as

$$dQ = dU + PdV \quad (13)$$

from equation (12), we get

$$dU = dH - PdV - VdP \quad (14)$$

substitute equation (14) in (13), we get

$$dQ = dH - VdP \quad (15)$$

from the definition of entropy we have

$$dQ = TdS$$

hence equation (15) can be written as

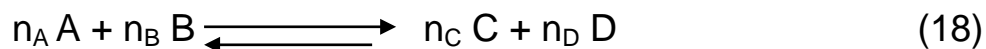
$$TdS = dH - VdP \quad (16)$$

Substitute equation (16) in equation (11), we get

$$dG = VdP - SdT \quad (17)$$

for a reacting system that has reached pressure and temperature equilibrium, this means ($dP=dT=0$), hence equation (17) gives us ($dG=0$).

To apply this criterion, let us consider a general case in which species A, B, C and D are in equilibrium at temperature T and pressure P. the chemical equation can be formed as:



since $dG=0$

$$\text{i.e } (n_C \bar{g}_C + n_D \bar{g}_D - n_A \bar{g}_A - n_B \bar{g}_B) * d\omega = 0 \quad (19)$$

where $d\omega$ is a measure of the extent of the reaction

\bar{g} : Gibbs free function per mole of constituent

Depending on the direction of reaction $d\omega$ can have positive or negative value but can not be zero, hence

$$(n_C \bar{g}_C + n_D \bar{g}_D - n_A \bar{g}_A - n_B \bar{g}_B) = 0 \quad (20)$$

And at constant temperature (i.e $dT=0$) equation (17) become

$$dG = VdP - SdT = VdP$$

which can be written as

$$d\bar{g} = V \frac{dP}{N} = R_o T \frac{dP}{P} \quad (\text{using } PV = N R_o T)$$

$$\text{Hence } \int d\bar{g} = R_o T \int_{1\text{atm}}^P \frac{dP}{P}$$

$$\text{Or } \bar{g} - \bar{g}_{\text{ref}} = R_o T \ln P$$

Applying this to equation (20), we get

$$n_C \bar{g}_C = n_C \bar{g}_{C_{\text{ref}}} + n_C R_o T \ln P_C$$

$$n_D \bar{g}_D = n_D \bar{g}_{D_{\text{ref}}} + n_D R_o T \ln P_D$$

$$n_A \bar{g}_A = n_A \bar{g}_{A_{\text{ref}}} + n_A R_o T \ln P_A$$

$$n_B \bar{g}_B = n_B \bar{g}_{B_{\text{ref}}} + n_B R_o T \ln P_B$$

Combining the above equation, we get

$$n_C \bar{g}_{C_{\text{ref}}} + n_D \bar{g}_{D_{\text{ref}}} - n_A \bar{g}_{A_{\text{ref}}} - n_B \bar{g}_{B_{\text{ref}}} = R_o T \ln \frac{P_A^{n_A} P_B^{n_B}}{P_C^{n_C} P_D^{n_D}}$$

Or can be written as follows;

$$\ln \frac{P_A^{n_A} P_B^{n_B}}{P_C^{n_C} P_D^{n_D}} = \frac{\Delta G_{\text{ref}}}{R_o T} \quad (21)$$

$$\text{The term } \ln \frac{P_A^{n_A} P_B^{n_B}}{P_C^{n_C} P_D^{n_D}} = K_P \quad (22)$$

Where K_P : is called the equilibrium constant

Equation (22) can be written as:

$$\ln K_P = \frac{\Delta G_{\text{ref}}}{R_o T} \quad (23)$$

It can be seen that K_P is function of temperature only , its values for various equilibrium reaction are taken from table.

Example 1.

The exhaust analysis from the combustion of a stiochiometric mixture of

CO and O₂ at a pressure of 2 atmosphere and a certain temperature shows that 20% of CO₂ dissociate. Determine the equilibrium constant at this temperature. Thereafter, find the percentage of dissociation when the products of combustion are kept at the same temperature but its compressed to 10 atmosphere.

Example 2.

A mixture of 2 mole of CO₂ , 3 mole of N₂ and 1 mole of O₂ initially at a temperature of 300 K and a pressure of 1 atmosphere. The mixture is heated up to 3000 K. Find the equilibrium composition of the mixture at this temperature if the final pressure is 12 atmosphere.

Example 3.

A mixture of 3 mole of CO and 2.5 mole of O₂ in an engine cylinder arrangement is ignited and combustion take place at constant pressure of 5.065 bar. Find the equilibrium composition of the products if the final temperature is 2800 K.

H.W

- Q1.** A stiochiometric mixture of a hydrocarbon fuel C₆H₁₂ and air is burned in steady flow combustion chamber. Both the fuel and air enters at 25 °C and products leaves at 2200K. Find:-
- 1- The enthalpy of combustion at 25 °C.
 - 2- The mass flow rate of fuel required to supply heat at a rate of 1000 KW, (Assume no dissociation).
 - 3- The percentage of CO₂ that would be dissociate at products temperature

ENGINE CLASSIFICATIONS

Internal combustion engines can be classified in a number of different ways:

1- Type of Ignition

- a) Spark ignition engine (SI): combustion process occurs by spark plug.
- b) compression ignition engine (CI): the combustion process starts when the air-fuel mixture self-ignites due to high temperature caused by high compression.

2- Engine Cycle

- a) four-stroke cycle
- b) two-stroke cycle

3- Valve Location

- a) valves in head (overhead valve)
- b) valves in block (flat head) also called L head engine

4- Basic Design

- a) Reciprocating engine
- b) Rotary engine

5- Position and Number of Cylinders of Reciprocating Engines

- a) single cylinder
- b) In-line
- c) V engine
- d) opposed cylinder engine

6- Air Intake Process

- a) Naturally Aspirated
- b) Supercharged
- c) Turbocharged
- d) crankcase compressed

7-Method of Fuel Input for SI Engines

- a) Carbureted
- b) Multiport fuel injection

c) Throttle body fuel injection

8- Fuel Used

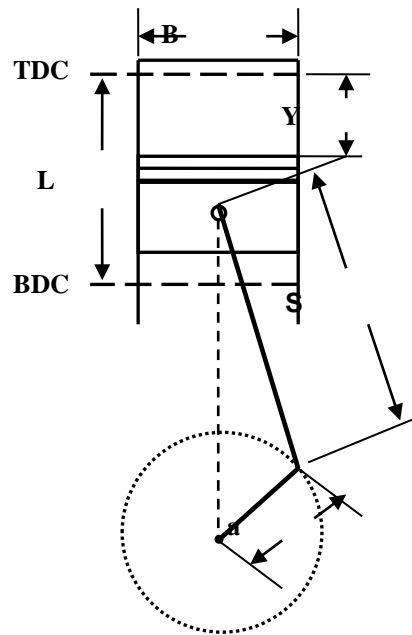
- a) Gasoline
- b) Diesel fuel
- c) Gas, Natural gas , Methane
- d) LPG
- e) Alcohol – Ethyl, Methyl

9- Type of Cooling

- a) Air cooled
- b) Water cooled

The Performance Characteristics of Internal Combustion Engines

The performance of internal combustion engines is characterized with several geometric and thermodynamic parameters. For any one cylinder, the crankshaft, connecting rod, piston and head assembly can be represented by mechanism shown in figure (1).



TDC: Top dead center

BDC: Bottom dead center

L : stroke length

S: connecting rod length

a: crankshaft radius

1- Compression ratio: may be defined as the ratio of total volume of engine cylinder to clearance volume.

$$r = \frac{V_{BDC}}{V_{TDC}} = \frac{V_t}{V_c}$$

2- displacement volume V_d : may be defined as the difference between the maximum and minimum volume for a single cylinder.

$$V_d = V_t - V_c = \frac{\pi B^2}{4} L$$

Also we have $L=2a$

For multi cylinder engine, the total displacement volume is given

$$\text{as : } V_d = \frac{\pi B^2}{4} L * n_c$$

Where n_c : is number of cylinders

The instantaneous volume at any crank angle is

$$V(\theta) = V_c + \frac{\pi B^2}{4} Y$$

Where Y: is the instantaneous stroke which can be obtained as:

$$Y = S + a - [(S^2 - a^2 \sin^2 \theta)^{\frac{1}{2}} + a \cos \theta]$$

3- Mean piston speed : the mean piston speed is an important parameter in engine design since stresses and other factors scale with piston speed rather than engine speed.

$$\overline{S_p} = 2LN$$

Where $\overline{S_p}$ is the mean piston speed (m/s)

N is the rotational speed of the crankshaft

L is the stroke length

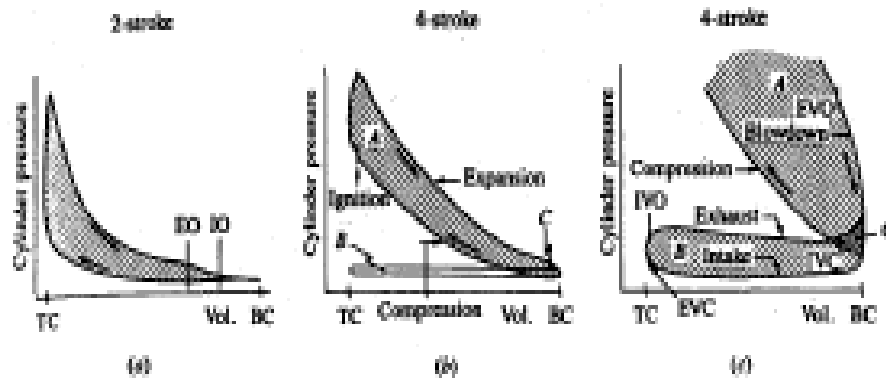
CRITERIA PERFORMANCE OF INTERNAL COMBUSTION ENGINES

In order to make a comparison for different type of engines of the same size or different size of engines of the same types, various operating parameters must be defined and compared such as :- indicated power , mean effective pressure (MEP), specific fuel consumption, mechanical efficiency and volumetric efficiency etc..

1- Indicated Power P_i (kW)

Indicated power may be defined as the actual rate of work rate of

work done by the gas on the piston as evaluated from an indicated diagram. Or may be defined as the higher than the power actually obtained from shaft of the engine since no mechanical loss is considered.



$P_i = (\text{area power loop} - \text{area of pumping loop})$

$$P_i = \text{imep} \cdot \dot{V}_s \cdot 100 \quad (\text{kW})$$

$$P_i = \dot{m}_f \cdot \text{LCV} \cdot \eta_{\text{ith}} \quad (\text{kW})$$

2- Mean Effective pressure

May be defined as a constant pressure which would have to act upon the piston for one stroke (power stroke) to give an equal work output for a complete cycle. Two types of mean effective pressure.:-

- brake mean effective pressure (bmep)
- indicated mean effective pressure (imep)

3- Brake Power P_b (kW)

The actual power output produced by the engine and its available on the engine shaft is called brake power.

$$P_b = \frac{2\pi NT}{60 * 1000} \quad (\text{kW})$$

Where

N: rotational speed (rpm)

T: torque (N.m)

Torque usually is measured with aid of dynamometer, various type of dynamometers are available such as:-

- 1- Friction dynamometer
- 2- Hydraulic dynamometer
- 3- Electric dynamometer
- 4- Eddy current dynamometer

Also brake power can be obtained using the following formula:

$$P_b = \text{bmep} * \dot{V}_s * 100 \quad (\text{kW})$$

$$P_b = \dot{m}_f * \text{LCV} * \eta_{\text{bth}} \quad (\text{kW})$$

Where

bmep : brake mean effective pressure

\dot{m}_f : mass flow rate of fuel (kg/s)

LCV: lower calorific value of fuel (kJ/kg)

\dot{V}_s : swept volume (m³/s)

The swept volume can be calculated as follows:

$$V_s = \frac{\pi B^2}{4} * L * N_c$$

$$\dot{V}_s = \frac{\pi B^2}{4} * L * N_c * \frac{N}{60n_R}$$

Where n_R : is the number of crankshaft revolution for complete cycle

$n_R = 2$ for four-stroke cycle

$n_R = 1$ for two-stroke cycle

4- Friction Power

The differences between the indicated power and brake power is the power lost in mechanical friction P_f

$$P_f = P_i - P_b$$

Friction power can be measured directly by recording torque required to run the engine with dynamometer acting as motor.

$$P_f = \frac{2\pi N T_f}{60 * 1000}$$

Where T_f : is the friction torque (N.m)

5- Mechanical efficiency

The mechanical efficiency can be defined as the ratio of brake power to indicated power

$$\eta_M = \frac{P_b}{P_i}$$

$$\text{Also } \eta_M = \frac{P_b}{P_i} = \frac{bmep}{imep} = \frac{\eta_{bth}}{\eta_{ith}}$$

6- Specific Fuel Consumption

A useful criterion of economical power is the specific fuel consumption. Two types of specific fuel consumption can be defined.

a) Indicated specific fuel consumption isfc (kg/kW h)

$$isfc = \frac{m_f}{P_i} \quad (\text{kg/kW h})$$

b) brake specific fuel consumption bsfc (kg/kW h)

$$bsfc = \frac{m_f}{P_b} \quad (\text{kg/kW h})$$

Typical values of specific fuel consumption lies between 0.25 0.35 (kg/kW h) .

7- Volumetric Efficiency

The power output of an engine depend on the amount of fresh charge

(air+ fuel) that would be inducted into the cylinder. In practice, the engine does not a complete cylinder full of air on each intake stroke, hence the volumetric efficiency can be defined as:-

$$\eta_v = \frac{(m_{air})_{actual}}{(m_{air})_{ideal}}$$

$$\eta_v = \frac{(m_{air})_{actual}}{\rho_{air} * \dot{V}_s}$$

Typical values of volumetric efficiency 0.8-0.9 in naturally aspirated engine. In turbo and supercharger volumetric efficiency may exceed 100%.

Example (1)

A spark ignition engine of four cylinder four-stroke cycle is running at a stiochiometric air-fuel ratio of C_7H_{18} of lower calorific value of 44000 kJ/kg . Its bore is 90 mm and its stroke length is 93 mm. The engine produce a torque of 250 N.m at a mean piston speed of 10- m/s. the air flow rate is measured to be 60 g/s, find

- 1- the brake power
- 2- the brake mean effective pressure
- 3- the brake specific fuel consumption
- 4- the volumetric efficiency

Example (2)

A single cylinder four-stroke cycle compression ignition engine of 12.9 cm bore and 15 cm stroke, running at 800 rpm and consumed 0.113 kg of fuel in 4 minutes, develops a torque of 76 N.m , the air-fuel ratio Of 30:1, the lower calorific value of the used fuel is 42000 kJ/kg, calculate :-

- 1- the brake power
- 2-the brake thermal efficiency
- 3- the brake specific fuel consumption
- 4- the volumetric efficiency

Example (3)

A 1.6 liter for-stroke cycle, four-cylinder spark ignition engine operating at 3000 rpm, produce 28 kW of brake power. The volumetric efficiency is 92%, the air-fuel ratio is 20:1, lower calorific value of fuel is 44000 kJ/kg and the atmospheric conditions are 1.0 bar and 25 °C, find

- 1- the brake power
- 2-the brake thermal efficiency
- 3- the brake mean effective pressure.

The Fuel-Air Cycles

Introduction

The basic problem in the air-cycle analysis is that it is based on highly simplified approximations. This is why the results obtained from such analysis are much greater than the actual performance. For example, an engine with $CR=7$ has a thermal efficiency (based on air cycle analysis) equals to 54% while the actual value does not exceed 30%. This is mainly due to the following reasons:

1. Non-instantaneous burning of the fuel.
2. Non-instantaneous operation of the valves.
3. Over simplifications in using the values of the properties of the working fluids.
4. Incomplete combustion of the fuel.
5. Assuming constant specific heat of the working fluid.
6. Assuming the working fluid to be only air.

Fuel-Air cycle: is defined as *the theoretical cycle that is based on the actual properties of cylinder gases* .

Fuel-Air cycle approximation

1. the variation in specific heat of gases with temperature (i.e C_p and

C_v are not constant)

2. the fact that the fuel and air do not completely combine chemically at high temperature
- 3- No change in the fuel or air chemical composition before combustion.
- 4- The variations in the number of molecules present in the cylinder as the temperature and pressure change (dissociation). The working fluid in a thermal equilibrium

Fuel-Air Cycle Assumptions

The following assumption are considered when dealing with fuel-air cycles calculations:

1. fuel is completely vaporized and perfectly mixed with air
2. combustion take place at top dead center, i.e highly idealized
3. assume no heat exchange occurs between the gases and cylinder walls
4. compression and expansion process are reversible and charge transfer happen but transfer work may be ignored

DEFINITIONS

Approximate method of fuel-air cycle calculation

If the fuel-air ratio is F , then for 1 kg of air will be $(1+F)$ kg of products formed. This $(1+F)$ kg of combustion products at P_1 and T_4 undergoes blow down process from P_4 to P_{exh} at constant volume V_1 ($T_4 = \text{constant}$). Fraction of this combustion products would be left in the engine cylinder, its volume equal to the clearance volume V_2 and its temperature is T_4 and its called residual gas and may be expressed as residual gas fraction f .

The following definitions are very important to be able to analyze

the fuel-air cycle and for their calculations.

1. Fresh air (\dot{m}_{air}): which is the fresh mass of air admitted to the cylinder engine for each cycle.

$$\dot{m}_{air} = (1-f) \quad (1)$$

2. Fresh fuel (\dot{m}_f): may be defined as the new mass of fuel supplied to the cylinder for each cycle.

$$\dot{m}_f = (1-f) * F \quad (2)$$

where

F: is the fuel-air ratio

f: residual gas fraction: may be defined as the ratio of mass fraction remained in an engine cylinder from the previous cycle to the total mixture

3. Total charge (\dot{m}_T) : is the total contents of cylinder at any time of the cycle.

$$\dot{m}_T = \dot{m}_{air} + \dot{m}_f + \dot{m}_r \quad (3)$$

where

\dot{m}_r : residual mass

$$f = \frac{\dot{m}_r}{\dot{m}_T}$$

4. Fresh mixture (\dot{m}_i) : this may be defined according to the type of engine , spark ignition engine or compression ignition engine.

For spark ignition engines \dot{m}_i can be obtained as follows:

$$\dot{m}_i = \dot{m}_{air} + \dot{m}_f \quad (4)$$

for compression ignition engines \dot{m}_i can be calculated as:

$$\dot{m}_i = \dot{m}_{air} \quad (5)$$

finally, the masses can be summarized as follows:

$$\dot{m}_{air} = (1-f)$$

$$\dot{m}_f = (1-f) * F$$

$$\dot{m}_r = f * (1+F)$$

$$\dot{m}_T = (1+F)$$

and $m_T = m_{air} + m_f + m_r$ at any time of the cycle

Calculation of Residual Gas Fraction (f)

The calculation of residual gas fraction (f) simply can be calculated by using the general law of perfect gas at point o and 4.

For point 4

$$P_4 * V_1 = m_1 R T_4$$

$$P_4 * V_1 = (1+F) R T_4 \quad (1)$$

And

$$P_{exh} V_2 = m_r R T_4$$

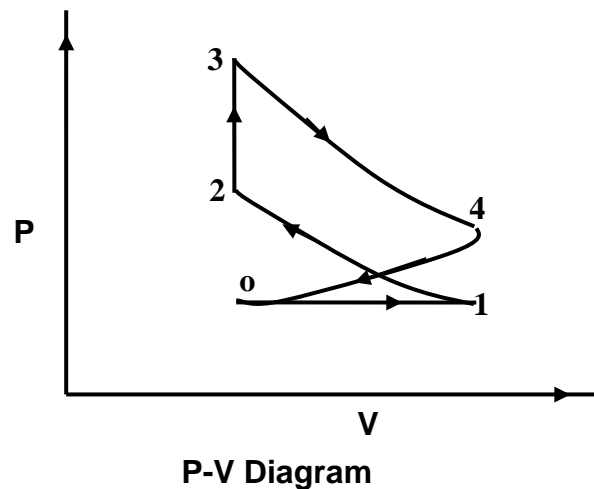
$$P_{exh} V_2 = (1+F)f R T_4 \quad (2)$$

Divide eq 1 and 2 , we get

$$\frac{P_{exh}}{P_4} * \frac{V_2}{V_1} = \frac{(1+F)f}{(1+F)}$$

Which gives

$$\boxed{\frac{P_{exh}}{P_4} * \frac{1}{r} = f}$$



Calculation of Trapped temperature T_1

In order to calculate the trapped temperature (T_1) at state 1 before the compression process begin, two kind of cycles are available:

1. constant volume cycle
2. constant pressure cycle

1- Constant volume cycle

in this cycle the total mass can be obtained as follows:

$$m_m = m_T = m_{air} + m_f + m_r$$

where

m_m is the mass of total mixture

however, the trapped temperature obviously can be obtained from the enthalpy balance

$$H_m = \sum H_i$$

Or $H_m = H_{air} + H_f + H_r$

Where H_m is the enthalpy of the mixture in the cylinder at state 1

And $H_m = m_m C_{P_m} (T_1 - 273)$

$$H_{air} = m_{air} C_{P_{air}} (T_{air} - 273)$$

$$H_f = m_f C_{P_f} (T_f - 273)$$

$$H_r = m_r C_{P_{exh}} (T_{exh} - 273)$$

Which can be written as follows:

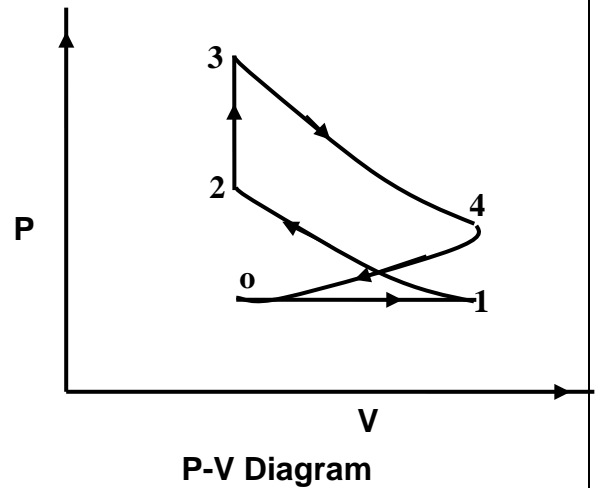
$$m_m C_{P_m} (T_1 - 273) = m_{air} C_{P_{air}} (T_{air} - 273) + m_f C_{P_f} (T_f - 273) + m_r C_{P_{exh}} (T_{exh} - 273)$$

Referring to constant volume cycle, the enthalpy of fuel H_f depend on the condition of the fuel supply at entry, gas or liquid.

1- If the fuel is gas, the H_f can be obtained as follows:

$$H_f = m_f C_{P_f} (T_f - 273)$$

2- If the fuel is liquid, then the enthalpy of fuel depend on the fuel temperature because the latent heat of evaporation is involved , (i.e h_{fg} is available at 25 °C). three cases can be present :-



- Case 1 : if the temperature of fuel is less than 25 °C ($T_f < 25 \text{ }^{\circ}\text{C}$), hence H_f

$$H_f = m_f [(CP_f)_L (T_f - 273) + (CP_f)_L (298 - T_f) - h_{fg25}]$$

- Case 2 : if the temperature of fuel is equal 25 °C ($T_f = 25 \text{ }^{\circ}\text{C}$), hence H_f

$$H_f = m_f [(CP_f)_L (298 - 273) - h_{fg25}]$$

- Case 2 : if the temperature of fuel is higher than 25 °C ($T_f > 25 \text{ }^{\circ}\text{C}$), hence H_f

$$H_f = m_f [(CP_f)_L (298 - 273) + (CP_f)_g (T_f - 298) - h_{fg25}]$$

2. Constant pressure cycle

Clearly , this is the typical cycle applied for compression ignition engines. At state 1 , the total mass (m_T) can be obtained as follows:-

$$m_T = m_{air} + m_r$$

or $m_m = m_{air} + m_r$

however , the trapped temperature T_1 also can be calculated in similar manner by introducing the enthalpy balance:

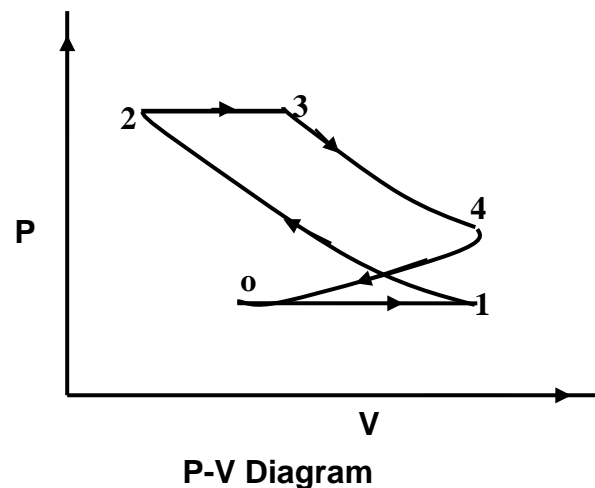
$$H_m = \sum H_i$$

$$H_m = H_{air} + H_r$$

Or can be written as

$$m_m CP_m (T_1 - 273) = m_{air} CP_{air} (T_{air} - 273) + m_r CP_{exh} (T_{exh} - 273)$$

and



$$m_r = (1+F) f$$

$$f = \frac{P_{\text{exh}}}{P_4} * \frac{1}{r}$$

Example 1

In a constant volume fuel air-cycle with a compression ratio of 9. The mixture of fuel –air ratio of 0.0785, fuel is C_8H_{17} of lower calorific value of 10500 Kcal/kg. Mixture pressure and temperature at the beginning of compression is 0.96 bar and 328 K. Calculate the thermal efficiency, indicated specific fuel consumption and mean effective pressure .

Assume residual gas fraction is 0.03, specific heat of gases at average temperature of 2800 K, and dropping in temperature due to dissociation is 10%. Also assume incomplete combustion which gives

$$\frac{n_{\text{co}}}{n_{\text{co}_2}} = 0.3 \quad \text{and} \quad \frac{n_{\text{H}_2}}{n_{\text{H}_2\text{O}}} = 0.14, \text{ relative to } \text{CO}_2 \text{ and } \text{H}_2\text{O} \text{ of complete}$$

combustion, and take the index of expansion and compression 1.3.

Example 2

A constant volume fuel-air cycle of compression ratio of 7 , operates on a hydrocarbon fuel of C_8H_{17} . The engine is running at 120% theoretical of air. The ambient condition is 1.01 bar and 30 °C. The temperature and pressure before the exhaust valve opened are 5 bar and 1100 K respectively. Calculate the maximum temperature in the cycle and mean effective pressure taking in account the following data :

For reactants $M = 28.5$, $C_p = 1.45 \text{ kJ/kg.K}$

For products $M = 30.5$, $C_p = 1.15 \text{ kJ/kg.K}$

And LCV = 40000 kJ/kg and $T_f = 20^\circ\text{C}$, $C_{p_f} = 1.8 \text{ kJ/kg.K}$

Example 3

A constant pressure fuel-air cycle of a compression ratio of 16. The ambient conditions are 1.0 bar and 25 °C. The fuel air ratio is given as 0.075, the residual gas fraction is 5%. If the pressure at the end of expansion stroke is 6 bar and the exhaust temperature is 1000 K, calculate the cutoff ratio and thermal efficiency of the cycle if the lower calorific value is 42000 kJ/kg . Assume C_p for products is 1.2 kJ/kg. K and $M = 30$, for reactants $M = 28$.

H.W

A spark ignition engine of a compression ratio of 8 , operates with stoichiometric fuel-air ratio of hydrocarbon fuel of C_8H_{18} . The pressure and temperature inside the cylinder at the start of compression are 1 atmosphere and 350 K respectively. Find the temperature and pressure per unit mass of air at the end of compression stroke. Also find the maximum temperature and pressure in the cycle assuming residual gas fraction is 0.08 and the exhaust temperature is 1800 K.
(note LCV= 44300 kJ/kg and $C_{p_f} = 1.979$ kJ/kg.K)

Gas Turbine

Introduction

A gas turbine, in its most common form, is a heat engine consisting of three components: a compressor, a combustion chamber and a turbine. A schematic diagram for a simple gas turbine is shown in figure (1). Air is drawn in by the compressor and delivered to the combustion chamber at a pressure of (8 – 12) bars depending on the design. Liquid or gaseous fuel is commonly used to increase the temperature of compressed air through a combustion process. Hot gases leaving the combustion chamber expands in the turbine, which produces work and finally discharges to the atmosphere. The simple gas turbine

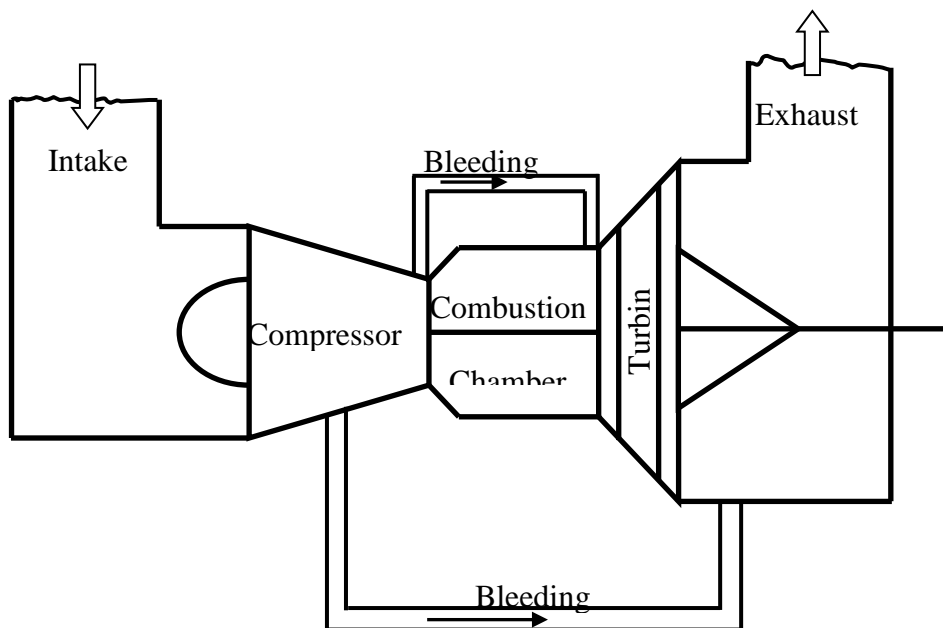
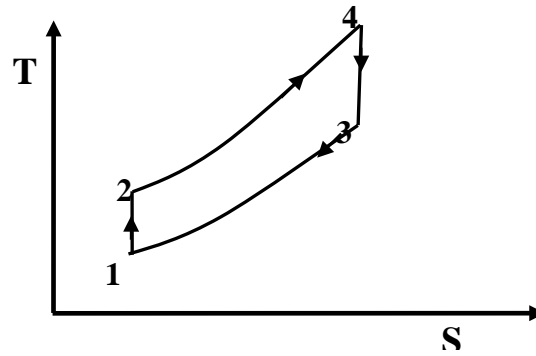


Figure (1): Pure Industrial Gas Turbine Schematic

cycle is shown on T-S diagram as in figure (2). The two major application areas of gas turbine are the aircraft propulsion and electric power generation. When it is used for aircraft propulsion, the turbine produce just enough power to drive the compressor and small generator to power the auxiliary equipment, the high velocity exhaust are reasonable for producing the necessary thrust to propel the aircraft.



Figure(2) T-S diagram for simple gas turbine cycle

In the power generation industry, gas turbine are particularly useful for peak-load or as a stand by units, as they are self-contained, require no additional steam capacity, can put in load with minimum of delay and occupy less room than other plant for the same capacity, their exhaust gases can be used for preheating air before entering the combustion, and or for heating as in combined heat and power plant.

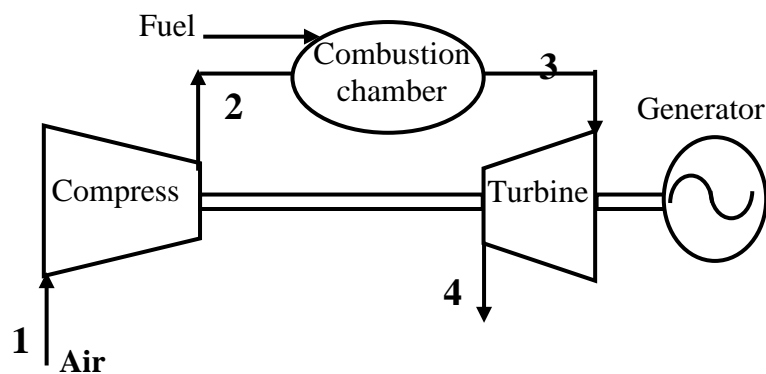


Figure (3): Simple single shaft gas turbine

2- Gas Turbine is The ultimate Choice For Power Generation System

Recently , gas turbine power plant become the most favorable type for producing electricity over the world and the main reasons for that as

follows:-

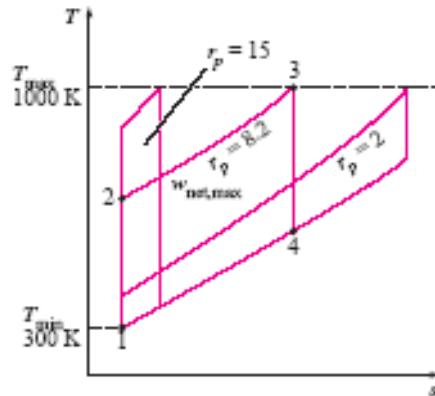
- 1- Low installation cost :-** because of its high output per unit weight and volume, both the building housing the plant and actual floor foundation required are much smaller and simpler than for the power plant of similar output.
- 2- Easy starting:-** gas turbine for large output for electric generation may requires minutes of warm-up time which is usually much less than the corresponding steam turbine.
- 3- Ability to consume wide range of fuels:-** a gas turbine is designed to utilize gaseous fuel or liquid hydrocarbon fuel. Some fuel system may require simple adjustments to cover the wide range of fuel type.
- 4- Low maintenance cost:-** at this point the gas turbines may be comparable to the steam turbine in lubricating and very superior to the lubricating type of internal combustion engine.
- 5- Clean exhaust:-** this aspect is one of the greatest importance and will undoubtedly play a crucial part in general acceptance.

3- Parameters Affect the Gas Turbine Performance

Gas turbine performance can be qualified with respect to its efficiency, work ratio, specific fuel consumption and back work ratio. However, there are several parameters affect its performance and as follows:

- 1- compressor pressure ratio:-** the thermal cycle efficiency increase as the pressure increase but this is limited by the maximum temperature that the turbine blabbed can withstand. Therefore for a fixed turbine inlet temperature, the net work output per cycle first increase as the pressure ratio increase, reach a maximum and then decrease as shown in figure (4).The optimum pressure ratio can be obtained by differentiate the net work with respect to the pressure , i.e the

$$\frac{dw_{net}}{dr_p} = 0, \text{ this gives that } r_{p_{op}} = \left(\frac{T_{max}}{T_{min}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$



2- Combustion Inlet Chamber:- as the air leaves the compressor it enters the combustion chamber where a fuel is injected, mixes with air and combustion occurs. Therefore, when the temperature of compressed air is high enough before entering the combustion chamber, then the amount of heat added to reach the maximum temperature in the cycle (TIT) slightly would be less, this cause a fairly increase in the efficiency of the cycle.

3- Turbine Inlet temperature (TIT):- it has been seen that the combustion chamber inlet temperature play a big role in increasing gas turbine efficiency (i.e less heat added) while the work net was not changed. Therefore, increasing the turbine inlet temperature will result in improve the net work and ultimateltly improving work ratio.

1-Simple Gas Turbine Cycle (Brayton Cycle)

Brayton cycle consist mainly of four processes as follows:-

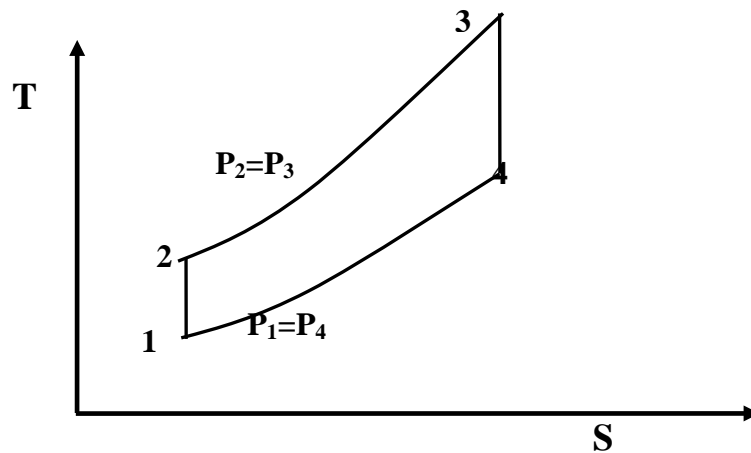


Figure (5) :T-S Diagram for Simple gas turbine cycle

1- process 1-2 take place in the compressor , isentropic compression

$$T_2 = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} \quad (1)$$

The compressor work can be obtained as follows:

$$W_c = m (h_2 - h_1)$$

$$W_c = m C_P (T_2 - T_1) \quad (\text{kJ}) \quad (2)$$

Or per unit mass

$$w_c = C_P (T_2 - T_1) \quad (\text{kJ/kg}) \quad (3)$$

2- process 2-3, combustion at constant pressure which take place in the combustion chamber.

$$Q_{23} = m C_P (T_3 - T_2) \quad (\text{kJ}) \quad (4)$$

$$\text{Or } q_{23} = C_P (T_3 - T_2) \quad (\text{kJ/kg}) \quad (5)$$

And $P_3 = P_2$ assuming no pressure drop in the combustion chamber.

$$\text{Hence } r_p = \frac{P_3}{P_4} = \frac{P_2}{P_1}$$

3- process 3-4, isentropic expansion in turbine

$$\text{hence } T_4 = T_3 \left(\frac{1}{r_p} \right)^{\frac{\gamma-1}{\gamma}} \quad (6)$$

the turbine work can be calculated as follows:-

$$W_t = m C_p (T_3 - T_4) \quad (7)$$

Or per unit mass

$$w_t = C_p (T_3 - T_4) \quad (\text{kJ/kg}) \quad (8)$$

if a closed gas turbine cycle is considered therefore process 4-1 must be take in account .

4- process 4-1 , heat rejection at constant pressure

$$Q_{34} = m C_p (T_4 - T_1) \quad (\text{kJ}) \quad (9)$$

$$q_{rej} = q_{34} = C_p (T_4 - T_1) \quad (\text{kJ/ kg}) \quad (10)$$

Gas turbine performance can be qualified with respect to its efficiency, work ratio and specific fuel consumption.

a- thermal efficiency

$$\text{thermal efficiency} = \frac{\text{output}}{\text{input}}$$

$$\text{Or in mathematical form } \eta = \frac{w_{net}}{q_{add}} = \frac{q_{add} - q_{rej}}{q_{add}} = 1 - \frac{q_{rej}}{q_{add}} \quad (11)$$

$$\text{And } w_{net} = w_t - w_c = C_p (T_3 - T_4) - C_p (T_2 - T_1)$$

$$\text{Or } w_{net} = q_{add} - q_{rej} = C_p (T_3 - T_2) - C_p (T_4 - T_1)$$

Hence equation 11 can be written as follows:

$$\eta = 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)} \quad (12)$$

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\text{Or } \eta = 1 - \frac{T_1 \left(\frac{T_4}{T_1} - 1 \right)}{T_2 \left(\frac{T_3}{T_2} - 1 \right)} \quad (13)$$

Equation 13 can be written in the following form :-

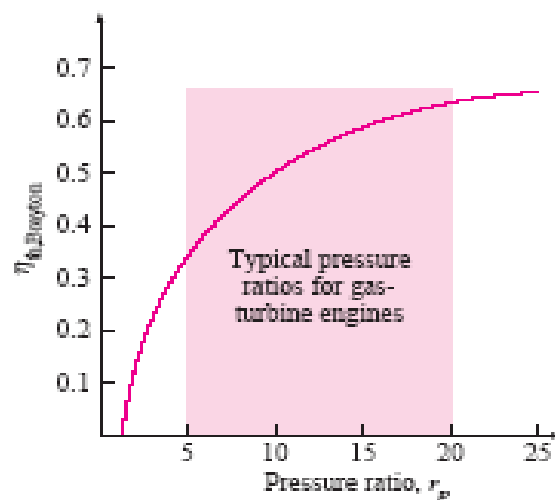
$$\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \quad (14)$$

Since $\frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$ and $\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}}$

This means that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ which gives that $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

Clearly, examining equation 14 , it concluded that thermal efficiency of Brayton cycle increase as the pressure ratio increase for fixed value of specific heat ratio γ .

For fixed values of T_1 and T_3 , the net work first increase with respect to pressure ratio r_p and then decrease.

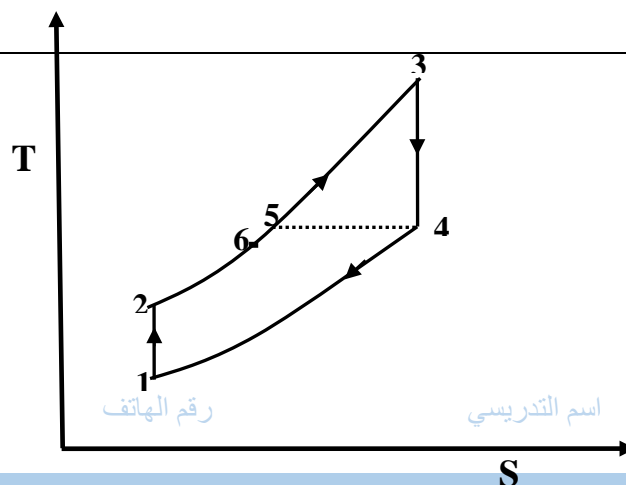
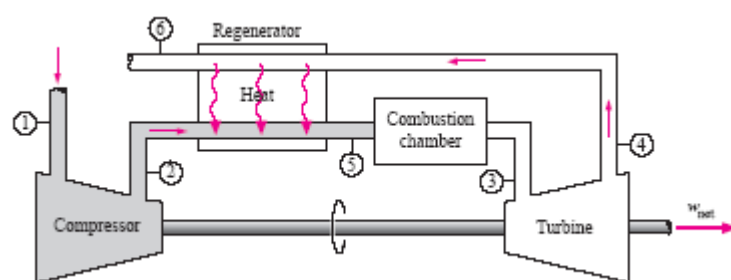


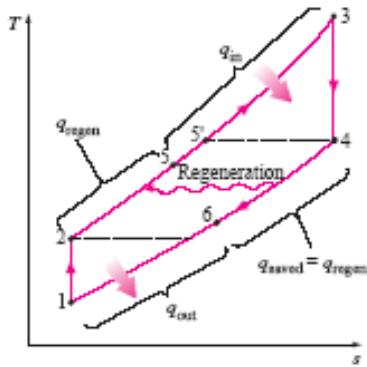
Example 1.

A stationary gas turbine power plant operate on Brayton cycle with a pressure ratio of 8, the maximum and minimum temperature of the cycle are 1300 K and 300 K respectively. Calculate the thermal efficiency and specific work output if the compressor and turbine efficiencies are 85% and 90% respectively. Thereafter determine the improvement in thermal efficiency when this unit is operating at optimum pressure ratio.

2- Gas Turbine With Regeneration (Brayton cycle with heat exchanger) .

Obviously, the temperature of exhaust gases leaving the turbine is often higher than the temperature leaving the compressor (i.e $T_4 > T_2$). Therefore, these exhaust gases can be used to heat up the compressed air leaving the compressor before entering to the combustion chamber of a gas turbine via a heat exchanger and ultimately improving the thermal efficiency.





Clearly, using a heat exchanger would not affect the net work since the net work as follows $w_{net} = w_t - w_c$

However, the amount of heat added in the combustion is reduced by using the heat exchanger in order to increase the combustion chamber inlet temperature from T_2 to T_5 or T_6 in actual case.

$$q_{add} = q_{35} = C_P (T_3 - T_5) \text{ for ideal heat exchanger } \varepsilon = 100\%$$

$$q_{add} = q_{36} = C_P (T_3 - T_6) \text{ for actual heat exchanger } \varepsilon < 100\%$$

where ε : is the effective of heat exchanger which can be defined as

$$\text{follows : } \varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$$

$$\text{or in mathematical form } \varepsilon = \frac{C_P (T_6 - T_2)}{C_P (T_5 - T_2)} = \frac{(T_6 - T_2)}{(T_5 - T_2)} \quad (15)$$

Most heat exchanger effectiveness value lies between 60% and 85%

The thermal efficiency of ideal gas turbine with refrigeration can be obtained by using the following formula:-

$$\eta = 1 - \frac{T_1}{T_2} (r_p)^{\frac{\gamma-1}{\gamma}} \quad (16)$$

Example 2.

Determine the thermal efficiency of the gas turbine plant in example 1 if the a heat exchanger is used with effectiveness of 80%.

3- Gas Turbine with Intercooling and Reheating

Using a heat exchanger(Regeneration) in gas turbine would improve its efficiency, however, this would not effect its work ratio. Therefore, we have to look for a way to improve its work ratio without affecting its thermal efficiency. This, simply can be done by increasing the turbine or decreasing the compressor work.

$$W_{net} = W_t - W_c$$

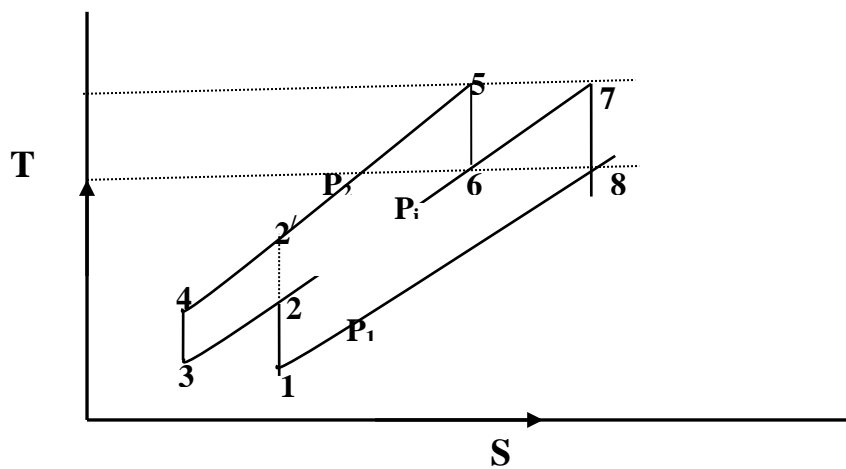


Figure (7) : T-S diagram for gas turbine with intercooling and reheating

It is clearly obvious that the work required to compress a gas between specified pressure can be decreased by carrying out the compression process in more than one stage and cooling the gas in between (i.e using multi-stage compression with intercooling). From figure (7), we get

$$T_2' - T_1 > (T_4 - T_3) + (T_2 - T_1)$$

$$\text{Hence } w_c = C_p (T_4 - T_3) + C_p (T_2 - T_1) < w_c = C_p (T_2' - T_1)$$

therefore $w_c = C_p (T_4 - T_3) + C_p (T_2 - T_1)$ can be differentiate with respect to the pressure, then we get: $P_i = \sqrt{P_1 P_4}$

this means that $\frac{P_i}{P_1} = \frac{P_4}{P_i} = r_{p_i}$ for minimum work.

For complete intercooling $T_1 = T_3$ and $T_2 = T_4$

And the compressor work is given as:

$$w_c = 2 C_P (T_2 - T_1)$$

in similar manner, reheating process can be used in two stage turbine to increase the turbine work and eventually improve the work ratio.

The turbine work can be found by using the following formula:

$$w_t = C_P (T_5 - T_6) + C_P (T_7 - T_8)$$

and for complete reheating:-

$$w_t = 2 C_P (T_5 - T_6)$$

the amount of heat added in this type of cycle can be found using the following formula:-

$$q_{add} = C_P (T_5 - T_4) + C_P (T_7 - T_6)$$

Example 3.

A gas turbine cycle with two stage of compression and two stage of expansion, the overall pressure ratio is 9, air enters each stage of compression at 298 K and each stage of turbine at 1200 K. Determine the thermal efficiency of the cycle under the following conditions:-

- 1- no heat exchanger is used
- 2- ideal regenerator is used

Pressure Losses

Fluid friction results in pressure losses in combustion chambers, heat exchanger and in the inlet and exhaust ducts. In the combustion chamber a loss in stagnation pressure is $\Delta P_{c.c}$ occurs due to the aerodynamics resistance of flame stability and mixing devices.

The following definition of losses are given:-

$\Delta P_{h.a}$: pressure losses in heat exchanger air side

ΔP_b : pressure losses in the combustion chamber

$\Delta P_{h,g}$: pressure losses in heat exchanger gas side

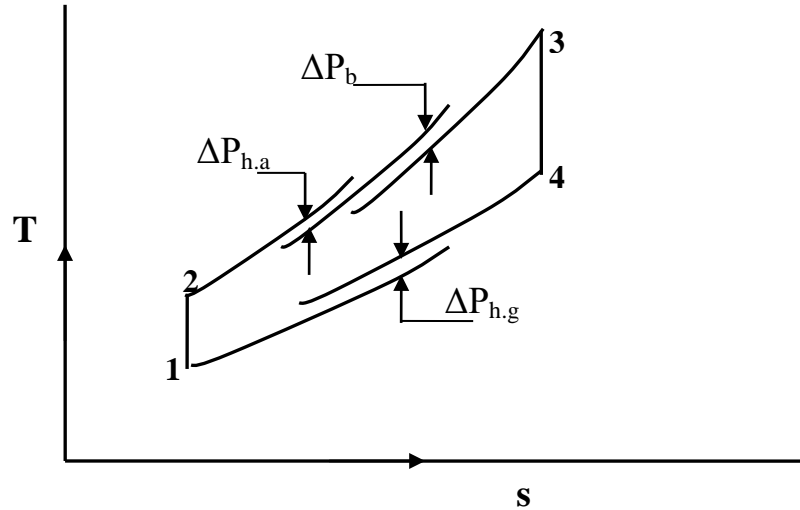


Figure (8) : T-S diagram showing pressure losses

Fixed values of the losses can be fed into the cycle calculation directly

$$P_3 = P_2 - \Delta P_{h,a} - \Delta P_b$$

$$P_4 = P_1 - \Delta P_{h,g}$$

And the compressor pressure ratio is $r_{p_c} = \frac{P_2}{P_1}$

And the turbine pressure ratio is $r_{p_t} = \frac{P_3}{P_4}$

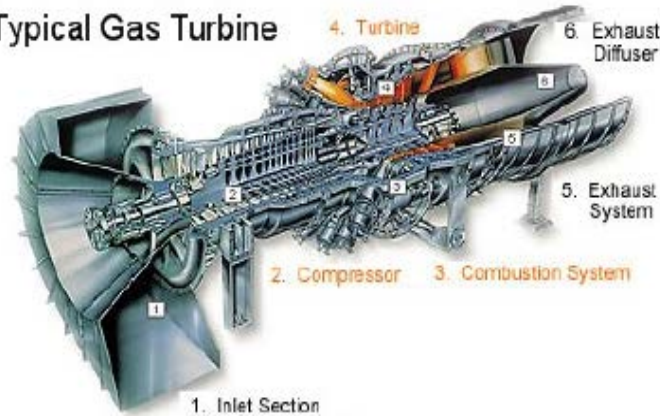
Example 4.

Determine the specific work output, specific fuel consumption and thermal efficiency of a gas turbine power plant which produce a power output of 20 MW. The pressure ratio across the compressor is 4. Hot gases enters the turbine at 1100 K, the isentropic efficiencies of the compressor and turbine are 85% and 87% respectively. Pressure losses across the combustion chamber, heat exchanger air-side and heat exchanger gas-side are 0.08bar, 0.12bar and 0.04 bar respectively. The used fuel of LCV= 44000 kJ/kg, the heat exchanger effectiveness of 80%, ambient conditions are 1.0 bar and 288 K.

Note for air $\gamma = 1.4$ and $C_p = 1.005$ kJ/kg.K for hot gases $\gamma = 1.33$ and $C_p = 1.135$

Figure 1

Typical Gas Turbine



Courtesy of Siemens Westinghouse



Figure 4. Typical simple cycle gas turbine power generator

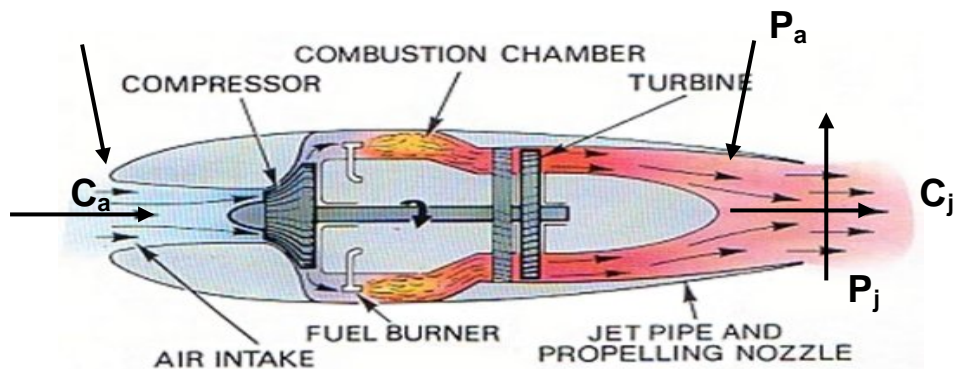
Gas Turbine Cycles for Aircraft Propulsion

Aircraft gas turbine differ from industrial gas turbine (power plant)cycles in the useful power output. The power output of aircraft propulsion is produced as a result of expansion in a propelling nozzle.

Also in gas turbine of aircraft engines, the effect of forward speed and altitude should be considered on the performance of aircraft. There are four basic types of aircraft engines (turbines): the turbojet, the turboprop, the turbofan, and the ramjet. Each has its advantages and disadvantages for specific cruise speeds. Engineers look for two things when designing a jet engine: thrust to weight ratio and fuel consumption. Most aircraft are designed for low fuel consumption, even though it means lower thrust capability. Some aircraft, such as fighter jets need a lot of thrust and are not as concerned about the amount of fuel used, if the mission requires it.

Criteria of Performance.

Consider the schematic diagram of a propulsive duct shown in figure(1) relative to the engine, the air enters the intake with a velocity C_a equal and opposite to the forward speed of the aircraft, and the power unit accelerates the air so that it leaves with the jet velocity C_j .



If the exhaust gases are not expanded completely to P_a in the propulsion duct, The thrust is given as follows:-

$$T = m (C_j - C_a) + A_j (P_j - P_a) \quad (1)$$

In present study, we shall assume there is complete expansion to P_a in

the nozzle, therefore equation will be

$$T = m (C_j - C_a) \quad (2)$$

Where:

T: thrust (N)

C_j : hot gases exit velocity (m/sec)

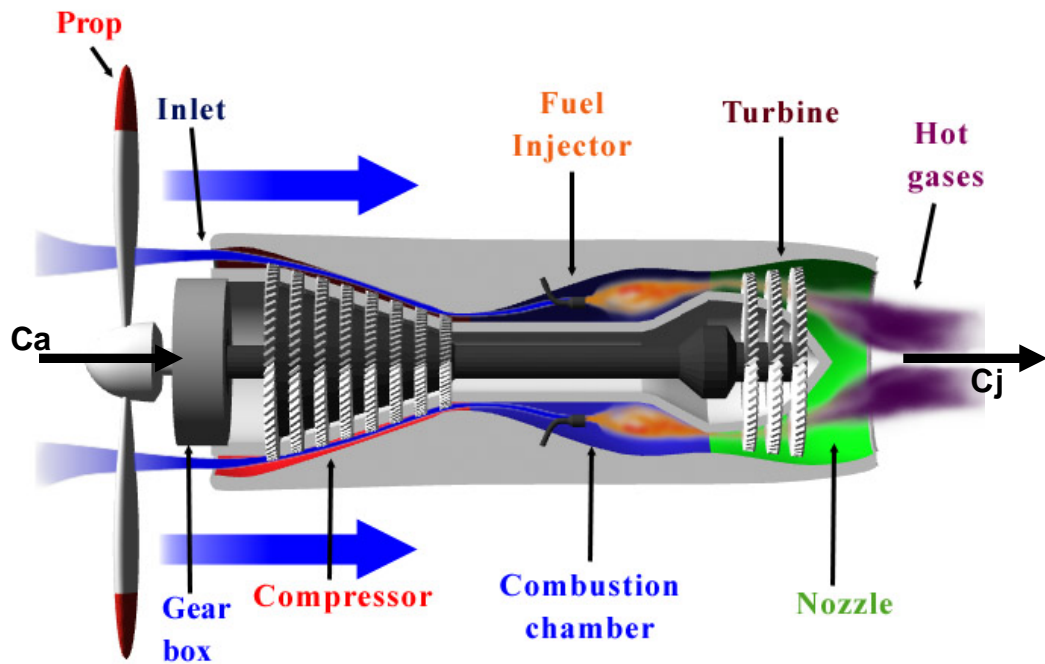
C_a : the air intake velocity which is equal to the speed of aircraft (m/sec)

It is clear that the required thrust can be obtained by designing the engine to produce either a high velocity jet of small mass flow or a low velocity jet of high mass flow.

1) Turboprop engine

The principle of a propeller is to accelerate a large quantity of air to a relatively slow velocity. This type of engine mainly is used when a low speed of air-craft is required about less than 800 km/hr. Expansion in turbine is completed to atmospheric pressure. However, the turbo propeller engines can be characterized as follows:-

- a. gas turbine drive the compressor and propeller
- b. most of the thrust is from the propeller
- c. works by accelerating large volumes of air to moderate velocities
- d. turbo propeller engine are best suited for low speed flight less than 800 km/h



a) Thrust $T = m (C_j - C_a)$

b) Propulsive efficiency:- which can be defined as the useful thrust power (useful propulsive energy) to the sum of that energy "thrust power" and the unused kinetic energy of the jet relative to the earth

$$\eta_{pr} = \frac{m \cdot C_a (C_j - C_a)}{m \cdot C_a (C_j - C_a) + \frac{1}{2} m \cdot (C_j - C_a)^2} \quad (3)$$

$$\eta_{pr} = \frac{m \cdot C_a (C_j - C_a)}{m [C_a (C_j - C_a) + \frac{1}{2} (C_j - C_a)^2]} \quad (4)$$

$$\eta_{pr} = \frac{m \cdot C_a (C_j - C_a)}{m (C_j - C_a) [C_a + \frac{1}{2} (C_j - C_a)]} \quad (5)$$

$$\eta_{pr} = \frac{C_a}{C_a + \frac{1}{2} (C_j - C_a)} \quad (6)$$

$$\eta_{pr} = \frac{2C_a}{C_a + C_j} = \frac{2}{1 + \frac{C_j}{C_a}} \quad (7)$$

Propulsive efficiency is often called Froud efficiency. From equations 2 and 7 it can be concluded the following:-

- 1- Thrust is maximum when $C_a = 0$, i.e under static condition
- 2- Propulsive efficiency (η_{pr}) is maximum when $C_j/C_a = 1.0$ as shown in equation 7, but thrust is zero

c) Thermal efficiency:- may be defined as the ratio of output (Kinetic energy) to the input (fuel energy), and can be expressed in the following mathematical form:-

$$\eta_{th} = \frac{\text{Kinetic energy}}{\text{fuel energy}}$$

$$\eta_{th} = \frac{\frac{1}{2} m \cdot (C_j^2 - C_a^2)}{m_f \cdot \text{LCV}} \quad (8)$$

d) Overall efficiency

$$\eta_{ov} = \frac{\text{Thrust power}}{\text{Fuel energy}}$$

Which can be written as follows:

$$\eta_{ov} = \frac{\text{Thrust power}}{\text{kinetic energy}} * \frac{\text{kinetic energy}}{\text{fuel energy}}$$

$$\text{Since } m \cdot C_a (C_j - C_a) + \frac{1}{2} m \cdot (C_j - C_a)^2 = \frac{1}{2} m \cdot (C_j^2 - C_a^2)$$

$$\eta_{ov} = \frac{m \cdot C_a (C_j - C_a)}{m \cdot C_a (C_j - C_a) + \frac{1}{2} m \cdot (C_j - C_a)^2} * \frac{\frac{1}{2} m \cdot (C_j^2 - C_a^2)}{m_f \cdot \text{LCV}}$$

$$\eta_{ov} = \eta_{pr} * \eta_{th} \quad (9)$$

Turbofan Engine

As engineers struggled to overcome the limitations of the

turboprop engine for airplanes at higher speeds, a new design emerged: the turbofan. It can be described as a compromise between the turboprop and the turbojet engines. It includes a large, internal propeller (sometimes called a ducted fan) and 2 streams of air flowing through the engine. The primary stream travels through all of the components like a turbojet engine, while the secondary stream is usually accelerated through a nozzle to mix with the primary exhaust stream. The figure below illustrates the design of a turbofan engine.

There are several advantages to the turbofan over the other two engines. The fan is not as large as a propeller, so the increase of speeds along the blades is less. Also, by enclosing the fan inside a duct or cowling, the aerodynamics are better controlled. There is less flow separation at the higher speeds and less trouble with shocks developing.

A turbofan engine can fly at transonic speeds up to Mach 0.9. While the fan is smaller than the propeller, it does suck in much more air flow than the turbojet engine, so it gets more thrust. Like the turboprop engine, the turbofan has low fuel consumption compared to a turbojet. The turbofan engine is the engine of choice for high-speed, subsonic commercial airplanes.

Turbofan engine has the following characteristic:

1. best choice for fuel economy and speed
2. high speed exhaust gases are mixed with lower speed air in the by-pass resulting in a considerable noise reduction
3. by-pass ratio can be adjusted, its typical value are 5-6
4. typically used for speed up to 1250 km/h

$$\text{By - pass ratio} = \frac{\text{mass flow passing the combustion chamber}}{\text{mass flow through the combustion chamber}}$$

$$B = \frac{m_c}{m_h} \quad (10)$$

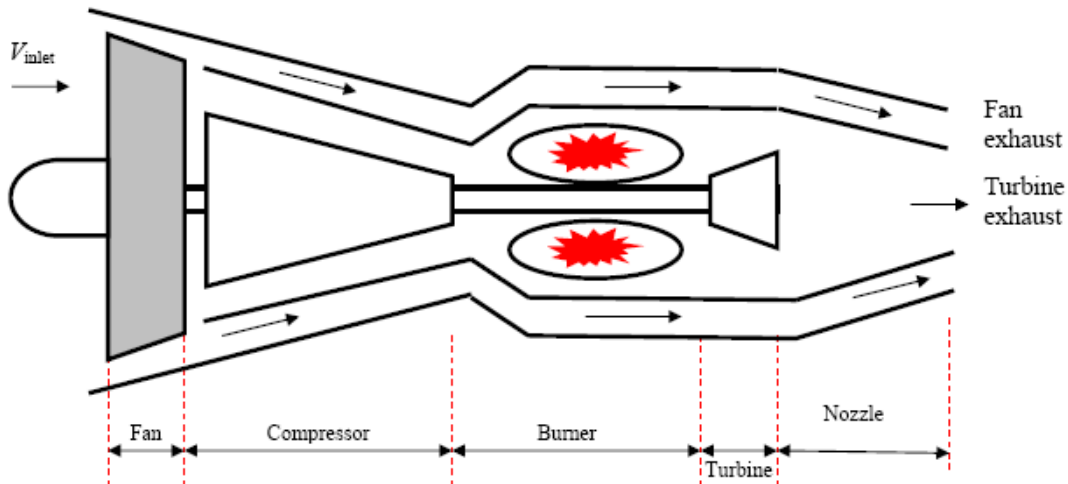
Where m_c ; mass flow rate of cold stream

m_h ; mass flow rate of hot stream

$$m = m_c + m_h$$

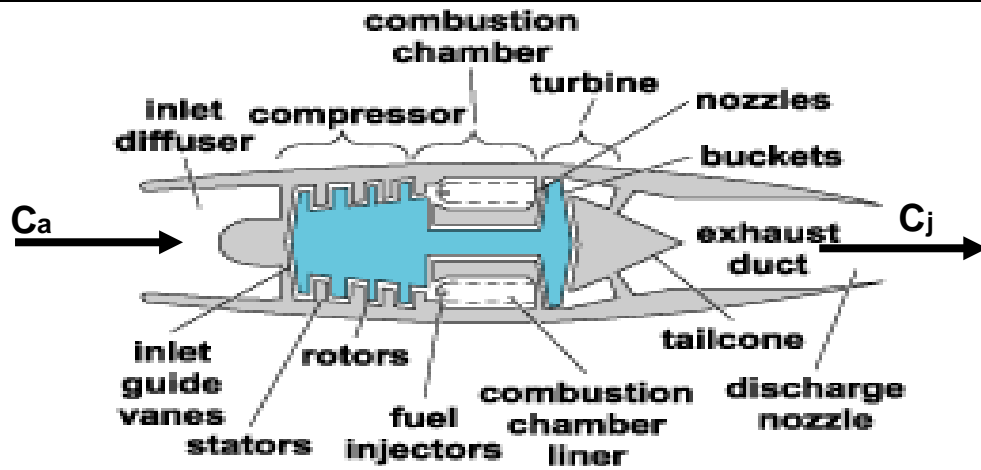
the thrust can be obtained as follows

$$\text{Thrust} = (m_c C_{j_c} + m_h C_{j_h}) - m C_a \quad (11)$$



2) Turbojet Engine

The typical turbojet engine has all five of the components: an inlet, a compressor, a combustor, a turbine, and a nozzle. The figure below shows a basic turbojet schematic with the 5 components clearly identified. The principle of a turbojet engine is to accelerate a small quantity of air to a relatively high velocities to obtain the thrust required. The expansion in turbine is just enough to drive the compressor and auxiliary power required. This type of engines are normally used for high speed aircraft with speed higher than 800 km/hr.



However the thrust is given as in equation 2

- a) Thrust $T = m (C_j - C_a)$
- b) Thrust power $= m C_a (C_j - C_a)$ (12)
- c) Propulsion efficiency:- which is the ratio of thrust power to the propulsion power

$$\eta_p = \frac{\text{thrust power}}{\text{Propulsion power}}$$

$$\eta_p = \frac{m \cdot C_a (C_j - C_a)}{\frac{1}{2} m (C_j^2 - C_a^2)}$$

$$\eta_p = \frac{2 C_a}{C_j + C_a} \Rightarrow \eta_p = \frac{2 \frac{C_a}{C_j}}{1 + \frac{C_a}{C_j}} \Rightarrow \eta_p = \frac{2 N}{1 + N} \quad (13)$$

Where $N = \frac{C_a}{C_j}$

- d) Thermal efficiency: it can be defined as the ratio of propulsion power to the fuel energy.

$$\eta_{th} = \frac{\text{Propulsion power}}{\text{Fuel energy}}$$

$$\eta_{th} = \frac{\frac{1}{2} m (C_j^2 - C_a^2)}{m_f \cdot \text{LCV}} \quad (14)$$

e) Overall efficiency :- may be defined as the ratio of thrust power to the fuel energy

$$\eta_{ov} = \frac{\text{thrust power}}{\text{fuel energy}} \Rightarrow \eta_{ov} = \frac{\text{thrust power}}{\text{propulsion power}} * \frac{\text{propulsion power}}{\text{fuel energy}}$$

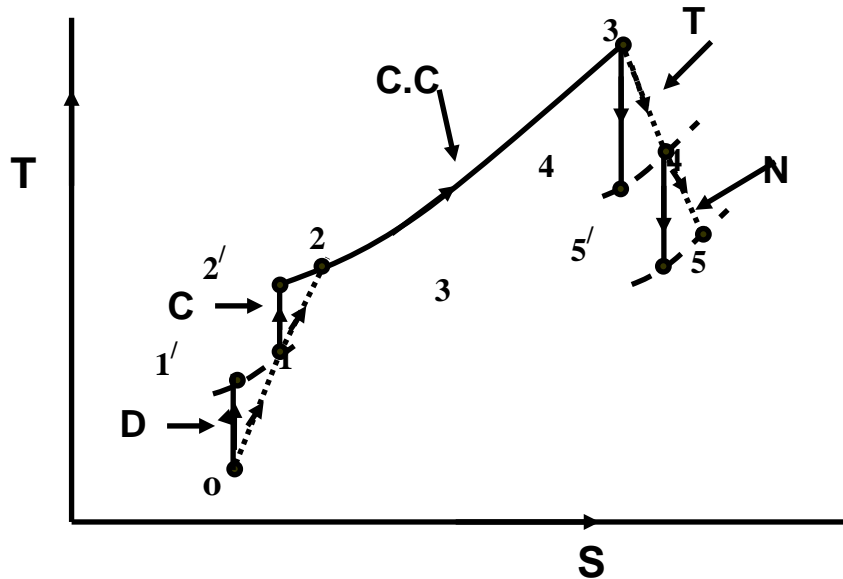
$$\eta_{ov} = \eta_p * \eta_{th} \quad (15)$$

f) specific fuel consumption (SFC) (kg/kW. hr)

$$\text{SFC} = \frac{\text{mass of fuel}}{\text{thrust power}}$$

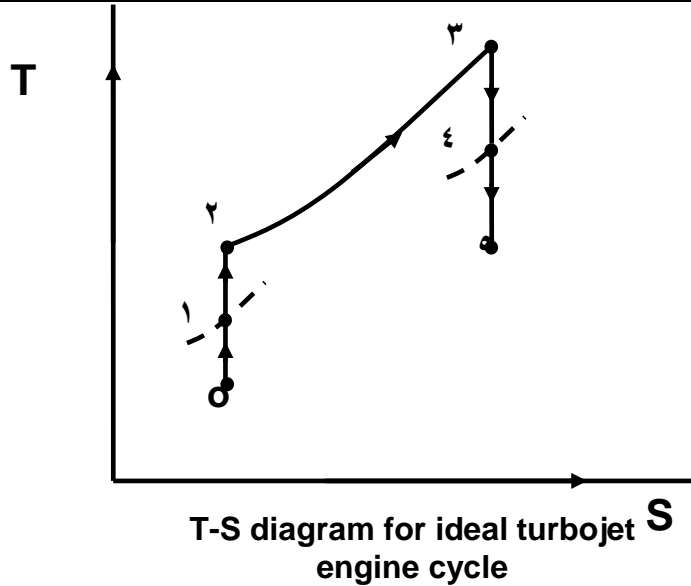
$$\text{SFC} = \frac{m_f}{m \cdot C_a (C_j - C_a)} \quad (16)$$

Representation of Turbojet Engine on T-S Diagram



T-S diagram for turbojet engine cycle

D: Diffuser C.C : combustion chamber
C: compressor T: Turbine N: Nozzle



For a turbojet engine operating on test-bed design or when the engine running at land on the airport, hence the diffuser effect will be ignored and the thermodynamic analysis of the cycle can be carried as follows:-

Process 1-2' : isentropic compression in the compressor

hence
$$T_{2'} = T_1 (r_{pc})^{\frac{\gamma-1}{\gamma}}$$

And
$$\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1}$$

The compressor work per unit mass can be obtained as:-

$$W_c = C_{p_a} (T_2 - T_1)$$

Process 2-3 : heat supplied as constant pressure

$$q_{add} = q_{23} = C_{p_g} (T_3 - T_2) \quad (\text{kJ/kg})$$

Process 1-4' : isentropic expansion in the turbine

$$w_t = w_c = C_{p_g} (T_3 - T_4) \quad (\text{kJ/kg})$$

$$T_{4'} = T_3 \frac{1}{(r_{pt})^{\frac{\gamma-1}{\gamma}}} \quad \text{and} \quad \eta_t = \frac{T_3 - T_2}{T_3 - T_{4'}}$$

Process 4-5' : isentropic expansion in nozzle which placed at the end of turbine to accelerate the velocity of hot gases leaving the turbine to high

velocity C_j

For the nozzle, apply the steady flow energy equation

$$Q - W = \Delta H + \Delta K.E + \Delta P.E$$

$$\text{Hence } m(h_{5'} - h_4) = -\frac{1}{2}(C_{5'}^2 - C_4^2) \quad \text{But } C_4 = 0$$

$$\text{i.e. } 2Cp_g(T_{5'} - T_4) = -C_{5'}^2$$

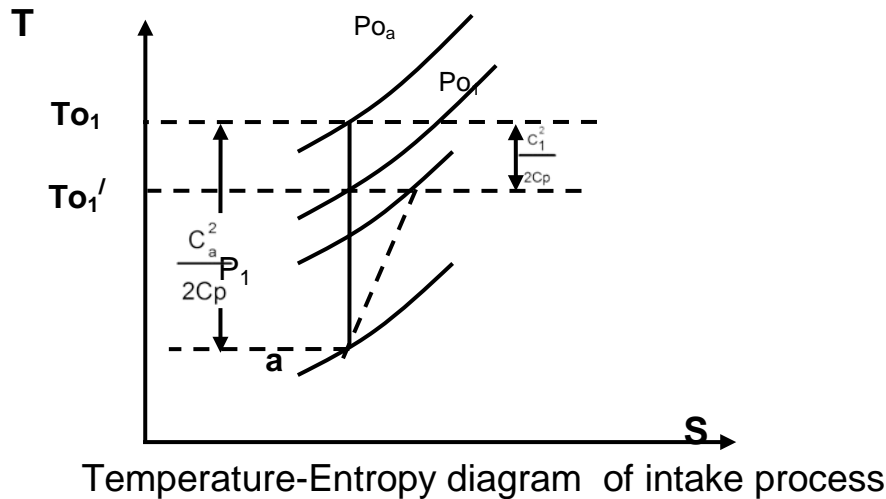
$$\text{or } 2Cp_g(T_4 - T_{5'}) = C_{5'}^2$$

$$\text{i.e. } C_{j'} = C_{5'} = \sqrt{2Cp_g(T_4 - T_{5'})}$$

$$\text{and } T_{5'} = T_4 \frac{1}{(r_{pN})^{\frac{\gamma-1}{\gamma}}} \quad \text{also } \eta_N = \frac{C_{5'}^2}{C_4^2}$$

$$\text{or } \eta_N = \frac{(T_4 - T_{5'})}{(T_4 - T_{5'})}$$

for a turbojet engine operating at high altitude (flying), a diffuser will be considered in this case, the diffuser will reduce the velocity of air



entering to the compressor and increase the compressor inlet pressure. The intake efficiency can be expressed in different ways, but the most commonly used are the isentropic efficiency (defined in term of

temperature rise) and ram efficiency η_r (defined in term of pressure rise)

.

Referring to T-S diagram of intake process , we have

$$T_{O1} = T_a + \frac{C_a^2}{2C_p}$$

And
$$\frac{P_{O1}}{P_a} = \left(\frac{T_{O1'}}{T_a} \right)^{\frac{\gamma}{\gamma-1}}$$

Also the diffuser efficiency can be written in term of temperature as:

$$\eta_D = \frac{(T_{O1'} - T_a)}{(T_{O1} - T_a)}$$

Hence $T_{O1'} - T_a = \eta_D (T_{O1} - T_a)$

i.e $T_{O1'} - T_a = \eta_D \left(T_a + \frac{C_a^2}{2C_p} - T_a \right)$

or $T_{O1'} - T_a = \eta_D \left(\frac{C_a^2}{2C_p} \right)$

i.e $T_{O1'} = T_a + \eta_D \left(\frac{C_a^2}{2C_p} \right)$

substitute the above equation in pressure ratio equation, we get

$$\frac{P_{O1}}{P_a} = \left(\frac{T_a + \eta_D \left(\frac{C_a^2}{2C_p} \right)}{T_a} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{O1}}{P_a} = \left(1 + \eta_D \left(\frac{C_a^2}{2C_p T_a} \right) \right)^{\frac{\gamma}{\gamma-1}}$$

However, the ram efficiency η_r may be defined as the ratio of the ram pressure rise to the inlet dynamic head.

$$\eta_r = \eta_D = \frac{(P_{O1} - P_a)}{(P_{Oa} - P_a)}$$

Example 1.

A turbojet engine operating at sea level condition of 15 °C and 1.0 bar. The pressure ratio of the compressor is 4:1 , compressor and turbine efficiencies is 80%. The gas inlet temperature to the turbine is 1055 K . Determine the pressure at turbine exit , $C_{p_a} = 1.005 \text{ kJ/kg.K}$, $\gamma_{\text{air}} = 1.4$, $C_{p_g} = 1.135 \text{ kJ/kg.K}$ and $\gamma_g = 1.33$

If the mass flow rate of air is 22.6 kg/s and nozzle efficiency is 94% , calculate the exit area of nozzle and thrust obtained assuming :-

1. when the unit stationary
2. when the unit is fitted on aircraft flying at 972 km/h, take $P_a = 0.45 \text{ bar}$ and $T_a = 240 \text{ K}$

Turbojet Engine With After Burner

Afterburner is popular in military aircrafts and it used whenever a need for extra thrust arises, such as for short takeoffs or combat conditions. After burner is similar to a reheat process; it is located after the turbine and before the nozzle. It produce a higher temperature and pressure at the nozzle inlet , result in an increase in velocity and ultimately in thrust. Stoichiometric combustion is desirable in this process for maximum thrust augmentation, however specific fuel consumption would be high.

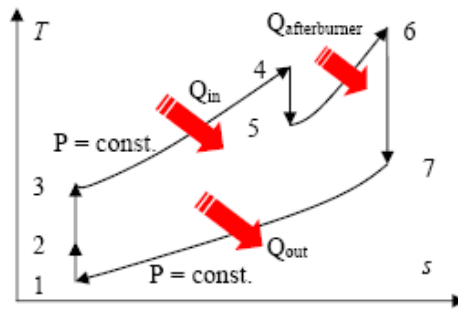
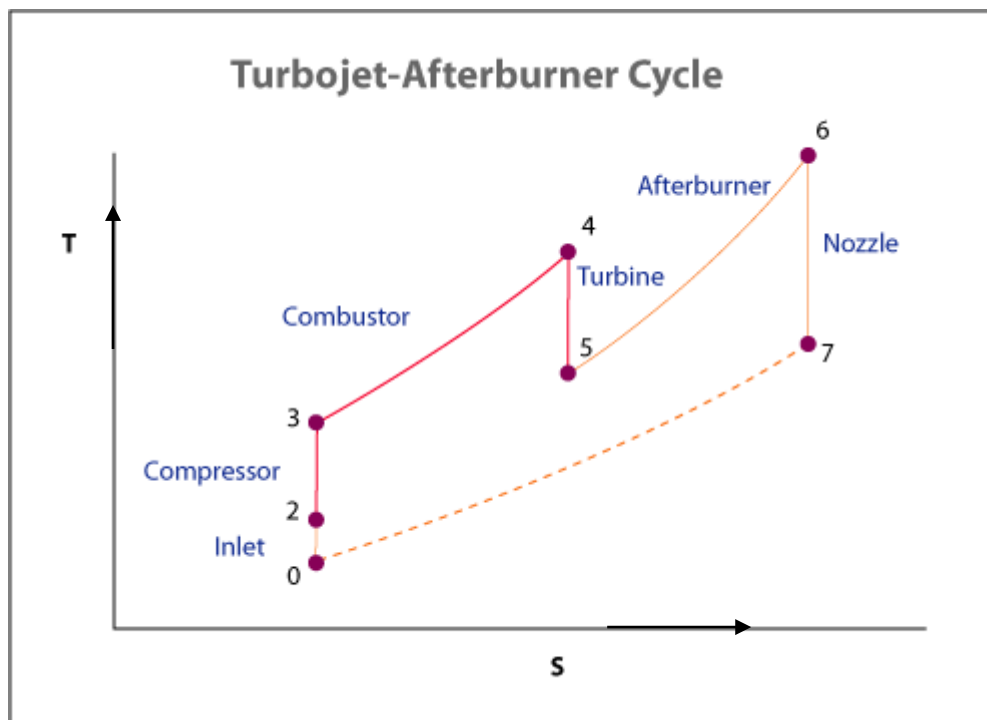
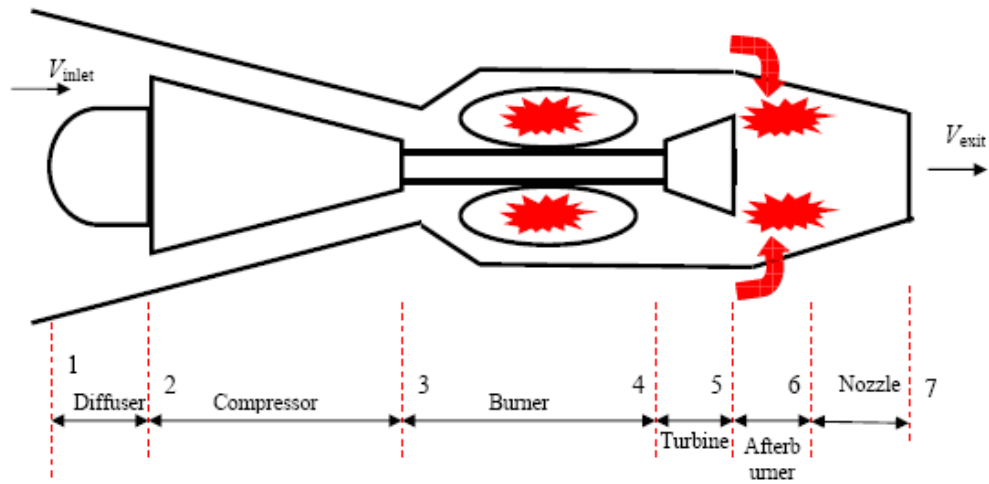


Fig. 3: T-s diagram for an ideal turbojet with afterburner cycle.



Thrust Augmentation

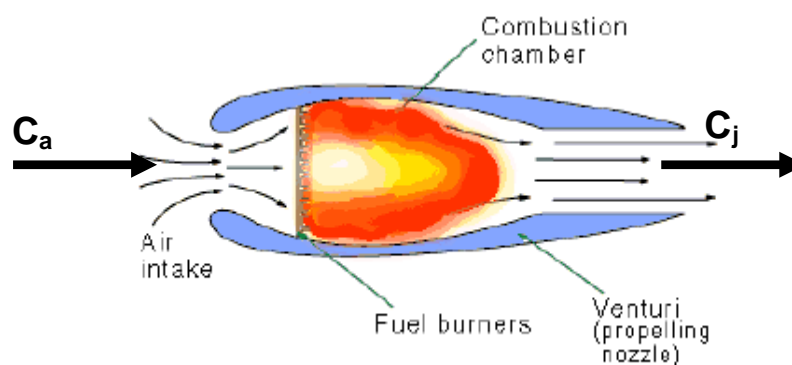
Two basic method can be used

1. increase turbine inlet temperature
2. increase the mass flow rate

both above methods simply are restricted by the designer and metallurgical limit of turbine blades.

Ramjets Engine

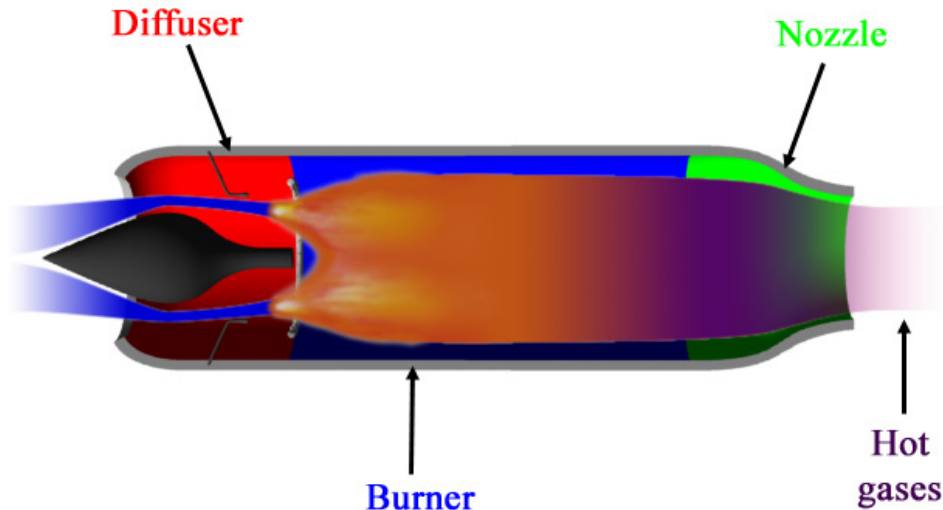
Below Mach 1.0 a compressor is very much needed as a component of an air-breathing engine. As an airplane increases its speed past Mach 1.0 the air pressure created from the speed of the air flow decreases the need for a compressor. As speeds approach Mach 3.5 - 4.0, a compressor isn't even needed. The ramjet is the most efficient engine because it has less components. The ramjet doesn't have a compressor or a turbine, and it has a much higher tolerance to high temperatures. A schematic of a ramjet engine is shown below. It has an inlet, a burner, and a nozzle.



A ramjet does have limitations. The first is that it will not work at less than supersonic speeds; another engine must first power the aircraft to supersonic speeds. Another limitation is the burning of the fuel and air mixture in the combustor. The ramjet inlet must slow the air flow from the

supersonic speeds to subsonic speed for ignition in the burner. As the ramjet approaches Mach 6.0 the air coming into the burner is too hot to burn. This is due to the friction created as the supersonic air is slowed at the inlet to subsonic speed. At this speed not enough thrust is being generated to continue performance.

There is a proposed solution to the ramjet's speed limitation (Mach 6.0). It is called supersonic combustion ramjet (SCRAMJET). Instead of slowing the air flow down to subsonic speeds for combustion, the SCRAMJET will ignite the air while still supersonic (thus avoiding the friction at the inlet). Fuel must still be injected into the air stream to be ignited. Unfortunately, today's fuels do not ignite quickly enough. The development of a workable fuel injection system for the SCRAMJET is still in its early stages.

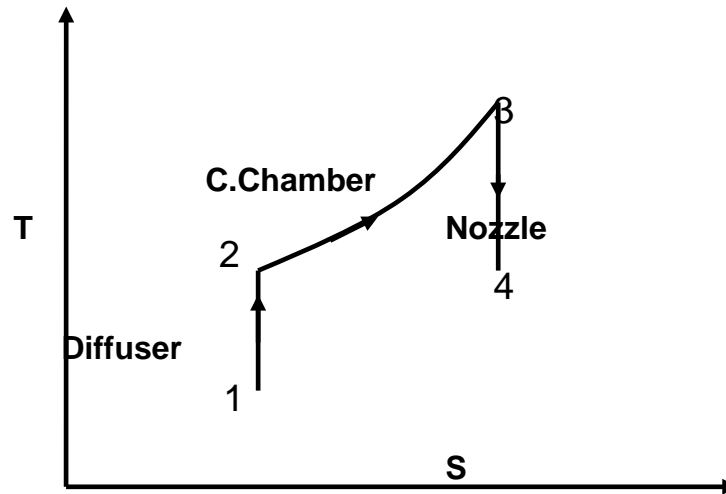


Analysis of Ramjet Engine

The following assumption will be considered;

1. isentropic processes through the diffuser and nozzle
2. heat addition at constant pressure

3. very low Mach number in combustion chamber
4. C_p and C_v are constant



T-S Diagram for Ramjet Engine

Thrust = mass* (exit velocity – inlet velocity)

$$T = m(C_j - C_a)$$

Specific thrust I

$$I = \frac{T}{m} = \frac{m(C_j - C_a)}{m} = (C_j - C_a)$$

However $\sqrt{\frac{T_3}{T_2}} = \frac{C_j}{C_a}$ and $M_a = \frac{C_a}{C_s}$

Hence
$$I = C_s M_a \left[\sqrt{\frac{T_3}{T_2}} - 1 \right]$$

Where C_s : speed of sound (m/s)

From gas dynamic relation, we have

$$T_2 = T_1 \left[1 + \frac{\gamma - 1}{2} M_a^2 \right]$$

Hence thrust can be written in the following form:

$$I = C_s M_a \left[\sqrt{\frac{T_3/T_1}{1 + \frac{\gamma-1}{2} M_a^2}} - 1 \right]$$

Example 2

Determine the specific fuel consumption for a ramjet engine which is being considered to be fly at Mach number of 1.5 and altitude of 15240 m ($P_a=0.116$ bar and $T_a= -56.5$ m) . The maximum temperature in the cycle is 2222 K , the fuel of lower calorific value of fuel is given to be 41860 kJ/kg.

Internal Combustion Engine Emissions

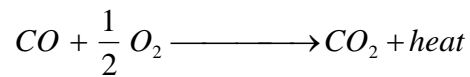
Most of the pollutions from internal combustion engines are emitted through the exhaust . However some hydrocarbons escape from the crankcase as a result of blow by and from the fuel tank and carburetor because of evaporation .

The primary pollutants emitted from internal combustion engines are :

- 1) Carbon monoxide CO
- 2) Hydrocarbons HC.
- 3) Nitrogen oxides NO_x .
- 4) Sulfur dioxides SO_x
- 5) Carbon dioxide CO_2

1- Carbon monoxide CO

Carbon monoxide a colorless , odorless ,poisonous gas , is generated in an engine when its operated with a fuel-rich equivalence ratio as shown in figure (1).When there is not enough oxygen to convert all carbon to CO_2 , some fuel does not get burned and some carbon ends up as CO. Typically the exhaust gasses of an SI engine will be about 0.2 to 5 % carbon monoxide .Not only is CO considered an undesirable emission , but it also represents lost chemical energy that was not fully utilized in the engine .



Maximum CO is generated when an engine runs rich , such as when starting or when accelerating under load . Even when the air-fuel mixture is stoichiometric or lean , some CO will be generated in the engine . Poor mixing , local rich region and incomplete combustion will create some CO.

2- Hydrocarbons HC.

Exhaust gasses leaving the combustion chamber of spark ignition engine contain up to 6000 ppm of hydrocarbon components , the equivalent of 1-1.5 % of the fuel . About 40 % of this is unburned gasoline fuel components. The other 60 % consists of partially reacted components that were not present in the original fuel .

Most unburned hydrocarbon emissions result from the droplets that where transported or injected into the "quench layer" during combustion . This is the region immediately adjacent to the combustion chamber surfaces , where heat transfer outward through the cylinder wall causes the mixture temperature to be too low to support combustion .

Partially unburned hydrocarbons can occur for a number of reasons :

- 1) Non-Stoichiometric air-fuel ratio .
- 2) Poor air and fuel homogeneity due to incomplete .
- 3) Excessive large fuel droplets (Diesel engine)
- 4) Low cylinder temperature due to excessive cooling through the wall or early cooling of these gases by expansion of the combustion volume caused by piston motion before combustion is completed.

All of these conditions can be caused by either poor maintenance or faulty design .

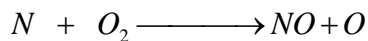
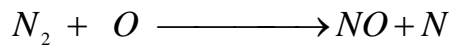
3- Oxides of Nitrogen NO_x.

Exhaust gases of an engine can have up to 2000 ppm of oxides of Nitrogen . Most of this will be nitrogen oxide NO, with a small amount of nitrogen dioxide NO₂ .

The maximum thermal NO_x production occurs at a slightly lean fuel mixture ratio because of the excess availability of oxygen for reaction as shown in figure (1). Clearly the formation of NO_x depends on pressure , temperature , air-fuel ratio and combustion time within the cylinder. However the maximum flame temperature will occur at stoichiometric air-fuel ratio . Figure (1) shows that maximum NO_x is formed at a slightly lean equivalence ratio of about $\Phi=0.95$. This can be explained that the flame temperature is still high at $\Phi=0.95$ and in addition , there is an excess of (O₂) that can combine with the nitrogen to form various oxides .

Figure (2) shows the relationship between the NO_x and combustion time at difference equivalence ratio . The amount of NO_x is reduced in modern engines with fast burn combustion chambers . The amount of NO_x generated also depends on the location with the combustion chamber . The highest concentration is formed a round the spark plug when the highest temperature occur .

At high temperatures both N_2 and O_2 molecules in the combustion air adsorb the heat energy up to the point when they are dissociated into their respective atomic states , N and O . The subsequent reaction of these atoms to create NO_x is described by the zeldovich mechanism as follows :



The rate of these reactions are highly dependent upon the air – fuel ratio , combustion temperature and residence time at the combustion temperature .

In compression engines the formation of NO_x is due to high temperature and pressure . A diesel fuel and natural gas are predominate fuels for this source .

In compression ignition engine Nitrogen Oxide NO_x emission are directly affected by the amount of premixing which , in turn , is a function of ignition delay . When ignition delay is large , there is more premixing and a grater energy release rate at the short of combustion . This generally leads to higher temperature and accordingly , higher NO_x emission .

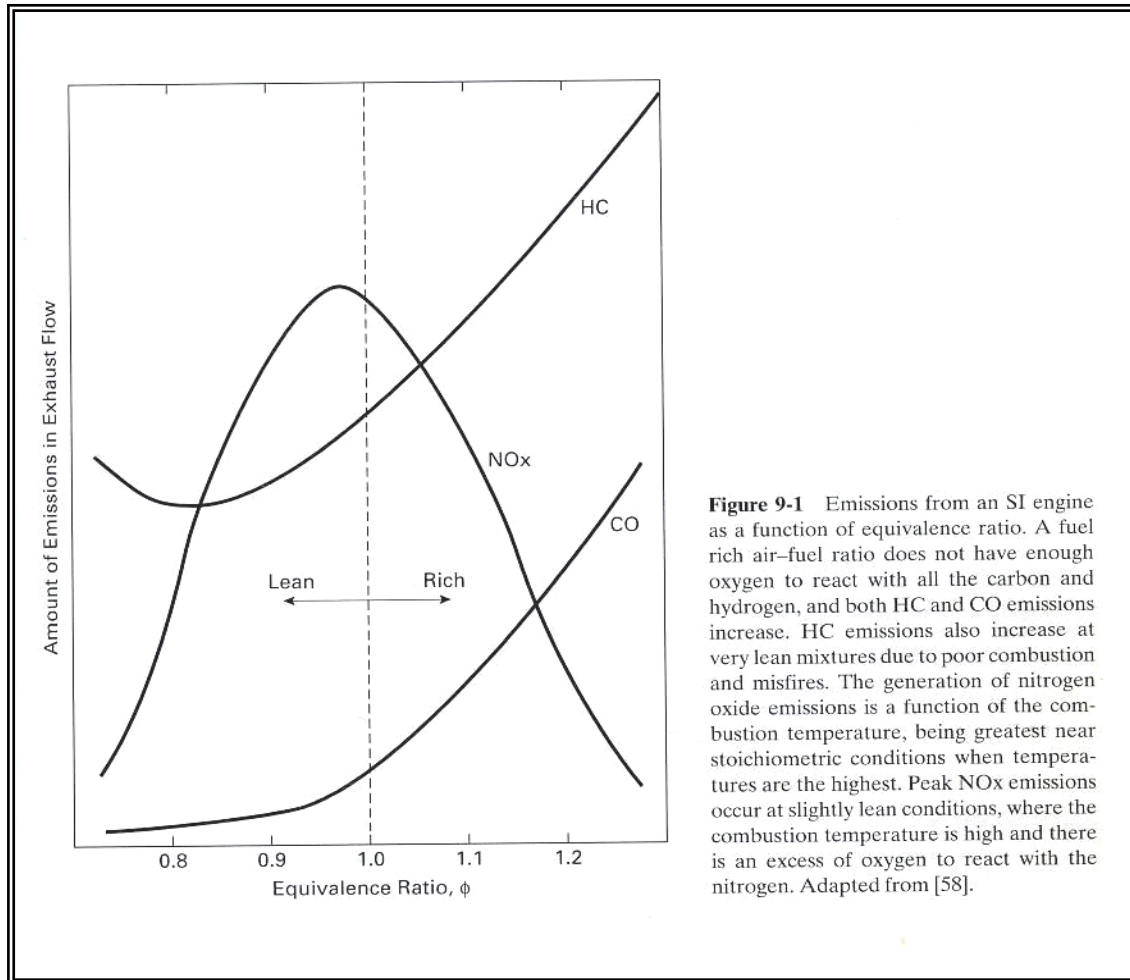
4- Sulfur Oxides SO_x .

Sulfur oxide emission are a function of the sulfur content in the fuel only , rather than of any combustion variables . In fact , during combustion process essentially all the sulfur in the fuel is oxidized to SO_2 . The oxidation of SO_2 gives sulfur trioxide SO_3 , which reacts with water to give sulfur acid H_2SO_4 , and contribute to acid precipitation sulfuric acid reacts with basic substance to give sulfates , which are fine particulates that contribute to visibility reduction . The reduction of SO_x would also minimize corrosion of the engine parts .

5- Carbon Dioxide CO_2 .

Concern about the increasing release of green gas such as CO_2 has grown out of research that documents the build up of gases in atmosphere and estimates the implication of continued accumulation .

Carbon dioxide is largely transported to incoming solar radiation , but can adsorb infrared radiation remitted by earth . Because of this energy "trapping "properly , such a gas is referred to as a green house gas .



The warming of the surface of earth and cooling of the stratosphere is achieved via molecular absorption and radiation in the infrared spectrum . The earth emits the energy it absorbs back to space , thus maintaining an energy balance . Because the earth is much colder than the sun , the bulk of earth's emission takes place at longer wave lengths than those of incoming radiation . Although CO_2 is a relatively inefficient absorber of solar radiation , it is a strong absorber of long wave (infrared) radiation .

Although some of the emitted energy is released to space , some of the radiation is transmitted downward , leading to a net trapping of long wave and warming of the surface .

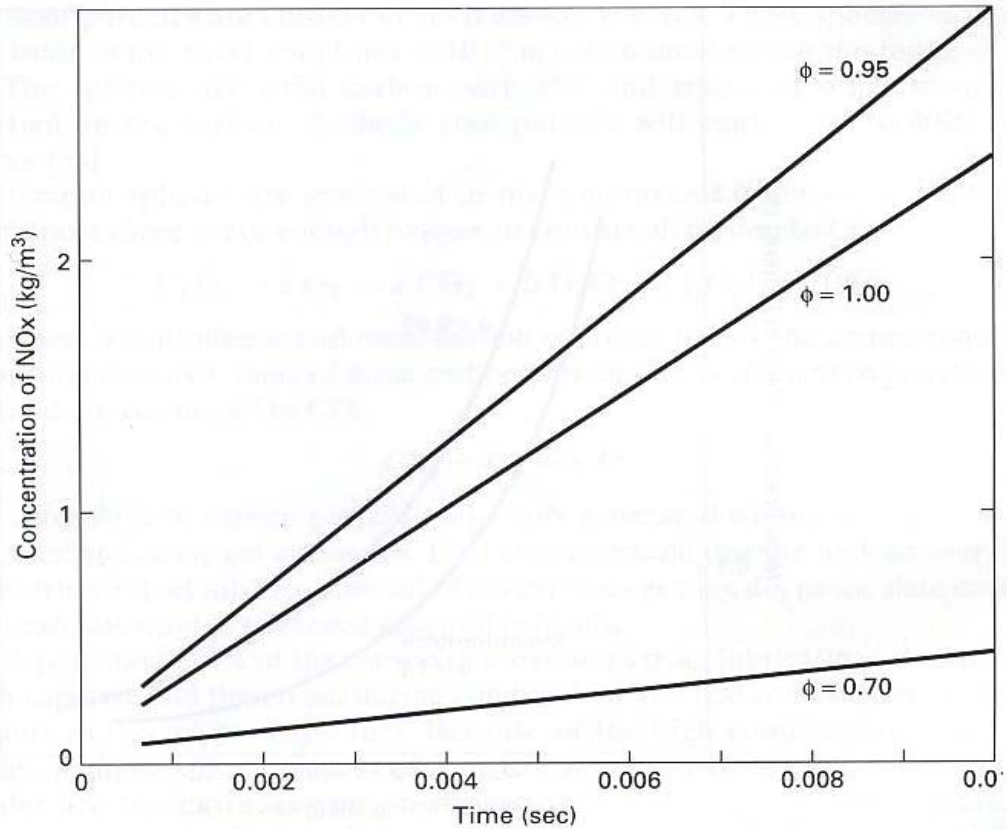


Figure 9-3 Generation of NO_x in an engine as a function of combustion time. Many modern engines produce lower NO_x emissions due to fast-burn combustion chamber design. Adapted from [92].

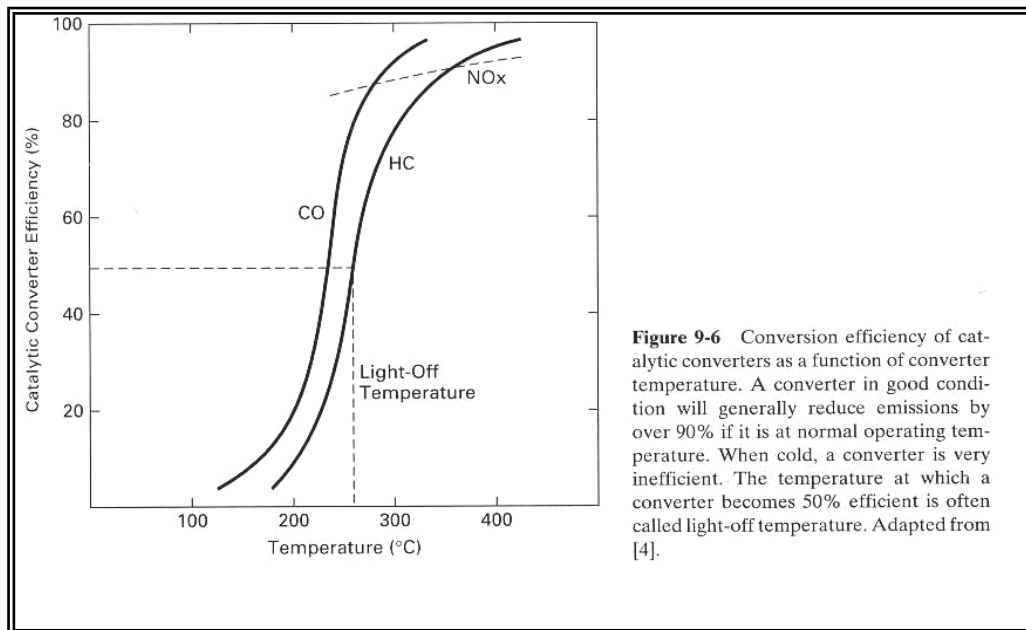


Figure 9-6 Conversion efficiency of catalytic converters as a function of converter temperature. A converter in good condition will generally reduce emissions by over 90% if it is at normal operating temperature. When cold, a converter is very inefficient. The temperature at which a converter becomes 50% efficient is often called light-off temperature. Adapted from [4].

THANK YOU