1.5 Midpoint Method

In this method one point is used, in particular

$$x_0 = \frac{a+b}{2} \quad , \quad p_0(x) = f(\frac{a+b}{2})$$

$$\int_a^b f(x)dx = \int_a^b p_0(x)dx = (b-a)\, f(\tfrac{a+b}{2}) \qquad (10)$$

The error term corresponding to midpoint rule is

$$E \quad = (b-a)^3(\frac{f''(\theta)}{24})$$

In general, we apply eq.(10) at each subintervals $[x_{i-1}, x_i]$ and we get

$$\int_a^b f(x)dx = \sum_{i=1}^n f(x)dx = h\sum_{i=1}^n f(\tfrac{x_{i-1}+x_i}{2}) \qquad (11)$$

Where h=(b-a)/n and the composite error of equation (11) is

$$E_M = (b-a)(\frac{h^2}{24})f''(\theta)$$

Example(6): Use Midpoint method to evaluate the integral

$$\int_0^2 e^x dx$$

Considering h=0.25.

Solution:

Clearly, a=0 , b=2 , $f(x)=e^x$ and $h=0.25$

, $n=\frac{b-a}{h}=8$

$x_i=x_{i-1}+h$ $\Longrightarrow$

$x_0=0$, $x_3=0.5+0.25=0.75$, $x_6=1.25+0.25=1.5$

$x_1=0+0.25=0.25$, $x_4=0.75+0.25=1$, $x_7=1.5+0.25=1.75$

$x_2=0.25+0.25=0.5$, $x_5=1+0.25=1.25$, $x_8=1.75+0.25=2$

$$\int_0^2 e^x dx = h\sum_{i=1}^{8} e^{\frac{x_{i-1}+x_i}{2}}$$

$$= h(e^{\frac{x_0+x_1}{2}} + e^{\frac{x_1+x_2}{2}} + e^{\frac{x_2+x_3}{2}} + e^{\frac{x_3+x_4}{2}} + e^{\frac{x_4+x_5}{2}} + e^{\frac{x_5+x_6}{2}} + e^{\frac{x_6+x_7}{2}} + e^{\frac{x_7+x_8}{2}})$$

$$= 0.25(e^{0.125} + e^{0.375} + e^{0.625} + e^{0.875} + e^{1.125} + e^{1.375} + e^{1.625} + e^{1.875})$$

=0.25(1.1331+1.4550+1.8682+2.3989+3.0802+3.9551+5.0784+6.5208)

=0.25(25.4897)

=6.3724

Example(7): Evaluate the integration $\int_0^1 (x^3+1)dx$ using Midpoint method with h=0.25.

Solution: H.W.

Midpoint Algorithm:

Input: a , b , n , f(x)

Step(1): evaluate h=(b-a)/n

Step(2): for i=0,1,2,…,n

set $x_i=a+ih$

Step(3): set sum=0

Step(4): for i=1,2,…,n set

sum =sum + $f(\frac{x_{i-1}+x_i}{2})$

Step(5): evaluate I=h*sum

Step(6): print I and stop.