1.4 Simpson's Three-Eighth Method

The derivative of simple Simpson's Three-Eighth rule formula to approximate value of the integral in eq.(1) we approximate the function f(x) as Newton forward interpolation formula and then integration over the interval $[x_0,x_3]$, where n=3 we have

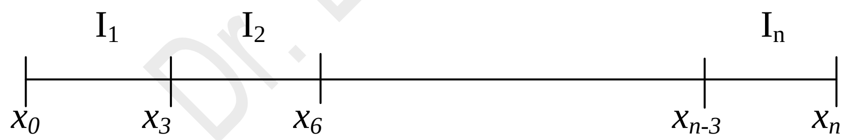
$$\int_{a=x_0}^{b=x_3} f(x)dx = h\int_0^3 f(x_m)dm$$

$$= \frac{3}{8}h[f_0 + 3f_1 + 3f_2 + f_3] \qquad (8)$$

Equation (8) is called **Simple Simpson's Three-Eighth Rule formula** ,the local truncation error in equation (8) is

$$\text{L.T.E.} = \frac{-3h^5}{80} f^4(\theta) \ , \ \ \theta\epsilon(x_0, x_3)$$

In general, we have subintervals $[x_{i-3}, x_i]$ and we apply eq.(8) at each subinterval we get:



$$\int_{x_i}^{x_{i+3}} f(x)dx \cong \ = \frac{3}{8}h[f_i + 3f_{i+1} + 3f_{i+2} + f_{i+3}]$$

then

$$\int_{a=x_0}^{b=x_n} f(x)dx = \int_{x_0}^{x_3} f(x)dx + \int_{x_3}^{x_6} f(x)dx + \cdots + \int_{x_{n-3}}^{x_n} f(x)dx$$

$$= \frac{3}{8}h[f_0 + 3f_1 + 3f_2 + f_3] + \frac{3}{8}h[f_3 + 3f_4 + 3f_5 + f_6] + \cdots + \frac{3}{8}h[f_{n-3} + 3f_{n-2} + 3f_{n-1} + f_n]$$

$$= \frac{3}{8}h[f_0 + 3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+1}) + 3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+2}) + 2\sum_{i=1}^{\frac{n}{3}-1} f(x_{3i}) + f_n] \qquad (9)$$

Equation (9) is called **Composite Simpson's Three-Eighth Rule formula**,

and the total truncation error in equation (9) is

$$E_{\frac{3}{8}S} = -\frac{3n}{80}h^5 f^4(\theta) = -\frac{3(b-a)}{80}h^4 f^4(\theta) \quad , \quad \theta\epsilon(a,b)$$

This is **Composite error term** in Composite Simpson's Three-Eighth Rule.

Example(4): Evaluate the following integral using Simpson's Three-Eighth Rule taking $h = \frac{1}{6}$.

$$\int_0^1 \frac{1}{1+x^2}dx$$

Solution:

a=0 , b=1 , $f(x) = \frac{1}{1+x^2}$ and $h = \frac{1}{6}$, $n = \frac{b-a}{h} = 6$

$x_i=x_{i-1}+h \Longrightarrow$

$x_0=0$, $f(x_0)=f(0)=1$

$x_1=0+1/6=1/6$, $f(x_1)=f(1/6)=36/37= 0.97297$

$x_2=1/6+1/6=1/3$, $f(x_2)=f(1/3)=9/10=0.9$

$x_3=1/3+1/6=1/2$, $f(x_3)=f(1/2)=4/5=0.8$

$x_4=1/2+1/6=2/3$, $f(x_4)=f(2/3)=9/13=0.6923$

x_5=2/3+1/6=5/6 , $f(x_5)$=f(5/6)=36/61=0.59016

x_6=5/6+1/6=1 , $f(x_4)$=f(1)=1/2=0.5

$$I[f]=\int_0^1 \frac{1}{1+x^2}dx=\frac{3}{8}h[f_0+3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+1})+3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+2})+2\sum_{i=1}^{\frac{n}{3}-1} f(x_{3i})+f_n]$$

$$=\frac{3h}{8}[1+3f_1+3f_2+2f_3+3f_4+3f_5+f_6]$$

$$=\frac{3h}{8}\ [1+3(\ f_1+f_2+f_4+f_5)+2f_3+f_6]$$

$$=0.7854$$

Example(5): Use Simpson's Three-Eighth Rule to evaluate the integral

$$\int_0^1 (x^3+1)dx$$

Considering h=1/6.

Solution: H.W.

Algorithm of Simpson's Three-Eighth rule:

Input: a , b , n , f(x)

Step(1): evaluate h=(b-a)/n

Step(2): for i=0,1,2,…,n

set x_i=a+ih

Step(3): evaluate the value of the integral

$$I=\frac{3}{8}h[f_0+3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+1})+3\sum_{i=0}^{\frac{n}{3}-1} f(x_{3i+2})+2\sum_{i=1}^{\frac{n}{3}-1} f(x_{3i})+f_n]$$

Step(4): print I and stop.