

Example(1): Use Trapezoidal rule to evaluate the integral

$$\int_0^1 \frac{1}{1+x^2} dx$$

Considering $h=0.25$

Solution:

$$a=0, b=1, \quad f(x) = \frac{1}{1+x^2} \quad \text{and} \quad h=0.25$$

$$x_i = x_{i-1} + h \implies$$

$$x_0 = 0, \quad f(x_0) = f(0) = 1$$

$$x_1 = 0 + 0.25 = 0.25, \quad f(x_1) = f(0.25) = 0.9412$$

$$x_2 = 0.25 + 0.25 = 0.5, \quad f(x_2) = f(0.5) = 0.8$$

$$x_3 = 0.5 + 0.25 = 0.75, \quad f(x_3) = f(0.75) = 0.64$$

$$x_4 = 0.75 + 0.25 = 1, \quad f(x_4) = f(1) = 0.5$$

$$I[f] = \int_0^1 \frac{1}{1+x^2} dx = \frac{h}{2} [f_0 + 2(f_1 + f_2 + f_3) + f_4]$$

$$= \frac{0.25}{2} [1 + 2(0.9412 + 0.8 + 0.64) + 0.5]$$

$$= 0.7828$$

Example(2): Use Trapezoidal rule to evaluate the integral

$$\int_0^1 (x^3 + 1)dx$$

Considering $n=1$ and $n=4$.

Solution:

$$a=0, b=1, \quad f(x) = (x^3 + 1) \quad \text{and} \quad n=1 \implies h = \frac{b-a}{n} = 1$$

$$x_i = x_{i-1} + h \implies$$

$$x_0 = 0, \quad f(x_0) = f(0) = 1$$

$$x_1 = 0 + 1 = 1, \quad f(x_1) = f(1) = 2$$

$$\begin{aligned} I[f] &= \int_0^1 (x^3 + 1)dx = \frac{h}{2} [f_0 + f_1] \\ &= \frac{1}{2} [1 + 2] = 1.5 \end{aligned}$$

H.W.: $n=4$

Evaluation the number of subintervals in Trapezoidal rule:

From the error formula

$$E_T = -\frac{(b-a)^3}{12n^2} f''(\theta), \quad \theta \in (a, b)$$

Let $|f''(\theta)| \leq M$ then the error E_T for the composite Trapezoidal rule is less than accuracy ϵ

$$|E_T| = \left| -\frac{(b-a)^3}{12n^2} f''(\theta) \right| \leq \epsilon$$

$$\Rightarrow \frac{(b-a)^3}{12n^2} M \leq \epsilon$$

$$\Rightarrow n \geq \sqrt{\frac{(b-a)^3}{12\epsilon} M}$$

Example(3): Find the value of the integral with accuracy $\epsilon = 0.001$ by using Trapezoidal rule

$$I(f) = \int_{0.5}^1 \cos x \, dx$$

Solution:

$$a=0.5, b=1, \quad f(x) = \cos x \quad \text{and} \quad \epsilon = 0.001$$

$$f'(x) = -\sin x \quad \text{and} \quad f''(x) = -\cos x \quad \Rightarrow \quad |f''(x)| \leq 1 \quad \Rightarrow \quad M=1$$

$$n \geq \sqrt{\frac{(b-a)^3}{12\epsilon} M} = \sqrt{\frac{(1-0.5)^3}{12 * 0.001}} (1) \cong 3.2275$$

$$\Rightarrow n = 4$$

$$\Rightarrow h = \frac{b-a}{n} = \frac{1-0.5}{4} = 0.125$$

$$x_i = x_{i-1} + h$$

x	0.5	0.625	0.75	0.875	1
f(x)	0.8776	0.811	0.7317	0.641	0.5403

$$I[f] = \int_{0.5}^1 \cos x \, dx = \frac{h}{2} [f_0 + 2(f_1 + f_2 + f_3) + f_4]$$

$$\cong 0.3616$$

Algorithm of Trapezoidal rule:

Input: $a, b, n, f(x)$

Step(1): Evaluate $h=(b-a)/n$

Step(2): for $i=0,1,2,\dots,n$

 set $x_i=a+ih$ and $y_i=f(x_i)$

Step(3): set $sum=0$

Step(4): for $i=1,2,\dots,n-1$

 find $sum = sum + y_i$

Step(5): set $I=h[sum+1/2(y_0+y_n)]$

Step(6): print I and stop.