Example(5): Use Simpson's One-Third Rule to evaluate the integral

$$\int_{0}^{1} (x^3 + 1)dx$$

Considering n=2.

Solution:

a=0 , b=1 , $f(x) = (x^3 + 1)$ and

n=2 $\Longrightarrow$ $h = \frac{b-a}{n} = \frac{1}{2}$

$x_i = x_{i-1} + h$ $\Longrightarrow$

$x_0=0$, $f(x_0)=f(0)=1$

$x_1=0+1=1/2$, $f(x_1)=f(1/2)=9/4$

$x_2=1/2+1/2=1$, $f(x_2)=f(1)=2$

$$I[f] = \int_{0}^{1} (x^3 + 1)dx = \frac{h}{3}[f_0 + 4f_1 + f_2]$$

$$= \frac{1/2}{3}\left[1 + 4(\frac{9}{4}) + 2\right] = \frac{12}{15} = 1.25$$

Evaluation the number of subintervals in Simpson's One-Third rule :

From the error formula

$$E_T = -\frac{(b-a)^5}{180n^4} f^4(\theta) \quad , \quad \theta \epsilon (a,b)$$

Let $|f^4(\theta)| \leq M$ then the error E_T for the composite Simpson's One-Third rule is less than accuracy ϵ

$$|E_T| = \left| -\frac{(b-a)^5}{180n^4} f^4(\theta) \right| \leq \in$$

$$\Longrightarrow \quad \frac{(b-a)^5}{180n^4} M \leq \in$$

$$\Longrightarrow \quad n \geq \sqrt[4]{\frac{(b-a)^5}{180\in} M}$$

Example(6): Find the value of the integral with accuracy $\in = 0.001$ by using Simpson's One-Third rule

$$I(f) = \int_{0.5}^{1} \cos x \, dx$$

Solution:

a=0.5 , b=1 , $f(x) = \cos x$ and $\in = 0.001$

$$I[f] = \int_{0.5}^{1} \cos x \, dx \cong 0.36206$$

Algorithm of Simpson's One-Third rule:

Input: a , b , n , f(x)

Step(1): Evaluate h=(b-a)/n

Step(2): for i=0,1,2,…,n

set x_i=a+ih and y_i=f(x_i)

Step(3): set s1=0 and s2=0

Step(4): for i=1,2,…,n-1 set

s1 =s1 + y_i (for i is even)

s2 =s2 + y_i (for i is odd)

Step(5): evaluate I=(h/3)*[y_0+2*s1+4*s2+y_n]

Step(6): print I and stop.