

Euler's Algorithm

Input: $a, b, n, y_0, f(x, y)$

Step(1): Set $x_0 = a$ and compute $h = \frac{b-a}{n}$

Step(2): For $k=1, 2, \dots, n-1$

Step(3): Evaluate $y_{k+1} = y_k + h f(x_k, y_k)$
and $x_{k+1} = x_k + h$

Step(4): print x_k and y_k then stop.

2- Modified Euler's Method (Heun's Method)

To solve the I.V.P. $y' = f(x, y)$ with $y(x_0) = y_0$ by this method

we use the formula:

$$x_{k+1} = x_k + h$$

$$y_{k+1} = y_k + \frac{h}{2} [f(x_k, y_k) + f(x_{k+1}, y_{k+1})] \quad \text{for } k = 0, 1, \dots, n-1$$

(4)

$$\text{where } y_{k+1} = y_k + h f(x_k, y_k)$$

This method is predictor – corrector method, and the local truncation error

of modified Euler's method is $O(h^3)$ (L.T.E. = $y''(c_1) \frac{-h^3}{12}$)

Example (3): Solve the I.V.P.

$$\frac{dy(x)}{dx} = x - y, \quad y(0) = 1$$

By using modified Euler's method at the points $x=0.1, 0.2, 0.3, 0.4$

Solution:

$$f(x,y)=x-y, \quad x_0=0, \quad y_0=1, \quad h=0.1, \quad n=4$$

$$y_{k+1} = y_k + \frac{h}{2} [f(x_k, y_k) + f(x_{k+1}, y_{k+1})] \quad \text{for } k = 0, 1, 2, 3$$

$$k=0 \longrightarrow y(x_1)=y(0.1)=y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)]$$

$$y_1 = y_0 + h f(x_0, y_0) = 1 + (0.1)(x_0 - y_0) = 1 + (0.1)(0 - 1) = 0.9$$

$$y(x_1) = 1 + \frac{0.1}{2} [f(0, 1) + f(0.1, 0.9)] = 1 + \frac{0.1}{2} [1 + (0.1 - 0.9)] = 0.91$$

$$k=1 \longrightarrow y(x_2)=y(0.2)=y_2 = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_2, y_2)]$$

$$y_2 = y_1 + h f(x_1, y_1) = 0.9 + (0.1)(0.1 - 0.9) = 0.82$$

$$y(x_2) = 0.91 + \frac{0.1}{2} [f(0.1, 0.91) + f(0.2, 0.82)] = 0.8385$$

$$k=2 \longrightarrow y(x_3)=y(0.3)=y_3 = y_2 + \frac{h}{2} [f(x_2, y_2) + f(x_3, y_3)]$$

$$= 0.782435$$

$$k=3 \longrightarrow y(x_4)=y(0.4)=y_4 = 0.741604$$

Example (4): Use modified Euler's method to approximate the solution for the initial value problem

$$y' = x^2 + 4x - \frac{1}{2}y, \quad y(0) = 4, \quad 0 \leq x \leq 0.25$$

with $h=0.05$.

Solution: H.W.

Modified Euler's Algorithm (Heun's Algorithm)

Input: $a, b, n, y_0, f(x, y)$

Step(1): Set $x_0 = a$ and compute $h = \frac{b-a}{n}$

Step(2): For $k=0, 1, 2, \dots, n-1$

Step(3): Set $x_{k+1} = x_k + h$

Step(4): Find $k_1 = h f(x_k, y_k)$

$$K_2 = h f(x_{k+1}, y_k + k_1)$$

Step(5): Evaluate $y_{k+1} = y_k + \frac{1}{2}(k_1 + k_2)$

Step(6): Print x_{k+1} and y_{k+1} then stop.