Example(2): Find the least square approximation of the form $y = c_1e^x + c_2e^{2x}$ for the points (0,1) , (1,-2) , (2,-40) .

Solution:

$f_1 = e^x$, $f_2 = e^{2x}$, n=3 , m=2

$x_1 = 0$, $y_1 = 1$, $x_2 = 1$, $y_2 = -2$, $x_3 = 2$, $y_3 = -40$

وبالتعويض في المنظومة

$$\sum_{i=1}^{m} c_i \sum_{j=1}^{n} f_i(x_j)f_k(x_j) = \sum_{j=1}^{n} y_i f_k(x_j)\,, \qquad k = 1,2,\ldots,m$$

نحصل على

$$a_{ik} = \sum_{j=1}^{3} f_i(x_j)f_k(x_j) \qquad , \qquad b_k = \sum_{j=1}^{3} y_i f_k(x_j)\,, \qquad k = 1,2$$

i=1 ⟶ k=1,2

$a_{11} = e^0e^0 + e^1e^1 + e^2e^2 \cong 63$

$a_{12} = e^0e^0 + e^1e^2 + e^2e^4 \cong 424$

i=2 ⟶ k=1,2

$a_{21} = e^0e^0 + e^2e^1 + e^4e^2 \cong 424$

$a_{22} = e^0e^0 + e^2e^2 + e^4e^4 \cong 3036$

$b_1 = 1.e^0 + (-2).e^1 + (-40).e^2 \cong -296$

$b_2 = 1.e^0 + (-2).e^2 + (-40).e^4 \cong -2198$

ويمكن كتابة منظومة المعادلات بشكل مبسط

$$\sum_{i=1}^{2} c_i\, a_{ik} = b_k \quad , \qquad k = 1,2$$

$36\, c_1 + 424\, c_2 = -296$

$424\, c_1 + 3036 c_2 = -2198$

$\longrightarrow \quad c_1 = 2 \quad , \quad c_2 = -1$

$\longrightarrow \quad y = 2e^x - e^{2x}$

Example(3): Find the engineering curve equation for the following data

x	1	2	4	5	6
y	3	11	50	72	110

Solution: The engineering curve equation is

$$y = ax^b$$

$$\ln y = \ln a + b \ln x$$

$$z = c_1 + c_2 t$$

Where $z = \ln y$, $c_1 = \ln a$, $c_2 = b$, $t = \ln x$

وباستخدام هذه التحويلات نحصل على

t= ln x	0	0.6931	1.3863	1.6094	1.7918
z=ln y	1.0986	2.3979	3.912	4.2767	4.7005

$\sum_{i=1}^{5} t_i = 5.4806 \quad , \quad \sum_{i=1}^{5} z_i = 16.3858$

$\sum_{i=1}^{5} {t_i}^2 = 8.2029 \quad , \quad \sum_{i=1}^{5} t_i z_i = 22.3905$

Now from eq.(6) we have

$$c_1 = \frac{\sum_{i=1}^{n} z_i \sum_{i=1}^{n} t_i^2 - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i z_i}{n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2} = 1.0656$$

$$c_2 = \frac{n\sum_{i=1}^{n} t_i z_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} z_i}{n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2} = 2.0176$$

$$\Longrightarrow \quad z = c_1 + c_2 t = 1.0656 + 2.0176\, t$$

$$a = e^{c_1} = 2.9026$$

$$b = c_2 = 2.0176$$

$$\Longrightarrow y = 2.9026x^{2.0176}$$

Example(3): Find the least squares method of the form $y = ae^{bx}$ for the following data

x	1	2	3	4	5	6	7	8
y	15.3	20.5	27.4	36.6	549.1	65.6	87.8	117.6

Solution: H.W.

The least squares algorithm of the form y=$\sum_{i=1}^{m} c_i f_i(x)$

let the data $\{(x_i, y_i)\}$ where $y_i = f_i(x)$ and $i=1,2,\dots,n$

Step(1): Compute the elements of the matrix A(m,m) from

$$a_{ik} = \sum_{j=1}^{n} f_i(x_j) f_k(x_j)$$

$i=1,2,\dots,m$, $k=1,2,\dots,m$

Step(2): Compute the elements of the vector b from

$$b_k = \sum_{j=1}^{n} y_i f_k(x_j), \quad k = 1,2,\dots,m$$

Step(3): Solve the linear system Ac=b to find the values c_j

$$c = [\, c_1 \;\; c_2 \;\; \dots \;\; c_m \,]^T$$

Step(4): Write the form

$$y = c_1 f_1(x) + c_2 f_2(x) + \dots + c_m f_m(x)$$