University of Kirkuk/College of Engineering/Petroleum Department

Production Engineering II

[Inflow Performance Relationship (IPR)]

2017-2018

Syllabus:

1. Types of reservoirs and radial flow in the reservoirs.
2. Productivity index.
3. Inflow performance relationship (IPR).
4. Effect of stratification and water cut on IPR productivity index test.
5. IPR methods, Vogel method, Standing method, Couto method, Fetkovich method, Al-Saadoon method.
6. Mathematical and physical principles for pressure drop calculations.
7. Flow pattern and its relation with pressure drop.
8. Poettman and Carpenter method, Dukler method.
9. Working charts.
10. Analysis of choke performance.
11. Prediction of restricted and unrestricted production.
12. Effect of other parameters on well performance.
13. Derivation and solutions of diffusivity equation.
14. Application of Horner solution.
15. Multi-rates test.
16. Build-up test.
17. Draw-down test.
18. Effect of skin factor on well testing analysis.

الكتاب المنهجي (هندسة إنتاج النفط II)، حازم حسن العطار،رشيد هليل العاني.

References:

1. Production optimization using nodal analysis, Dale Beggs.
2. Petroleum production engineering, Boyun Guo, William C. Lyons, Ali Ghalambor.

3. Heriot watt University, production engineering technology II.
4. The technology of artificial methods, volume 1, Kermit E. Brown, H. Dale Beggs.
5. Pressure transient test, John Lee, John B. Rollins, John P. Spivey.
6. Reservoir engineering handbook , Tarek Ahmed.
7. Advance in well testing analysis, Robert, C. Earlougher,Jr.
8. Papers.
9. Other resources: internet.

Division mark

1. Final average (40%).
2. Final exam (60%).

Examinations

1. Open part.
2. Close part.

Important Terms
Flow Regimes

There are basically three types of flow regimes that must be recognized in order to describe the fluid flow behavior and reservoir pressure distribution as a function of time. There are three flow regimes:

- Steady-state flow
- Unsteady-state flow
- Pseudosteady-state flow

1) Steady-State Flow

The flow regime is identified as a steady-state flow if the pressure at every location in the reservoir remains constant, i.e., does not change with time. Mathematically, this condition is expressed as:

$$\left(\frac{\partial p}{\partial t}\right)_i = 0$$

The above equation states that the rate of change of pressure **p** with respect to time **t** at any location **i** is **zero**. In reservoirs, the steady-state flow condition can only occur when the reservoir is completely recharged and supported by strong aquifer or pressure maintenance operations.

2) Unsteady-State Flow

The unsteady-state flow (frequently called ***transient flow***) is defined as the fluid flowing condition at which the rate of change of pressure with respect to time at any position in the reservoir is not zero or constant.

This definition suggests that the pressure derivative with respect to time is essentially a function of both position **i** and time **t**, thus

$$\left(\frac{\partial p}{\partial t}\right) = f(i, t)$$

3) Pseudosteady-State Flow

When the pressure at different locations in the reservoir is declining linearly as a function of time, i.e., at a constant declining rate, the flowing condition is characterized as the pseudosteady-state flow. Mathematically, this definition states that the rate of change of pressure with respect to time at every position is constant, or

$$\left(\frac{\partial p}{\partial t}\right)_i = \text{constant}$$

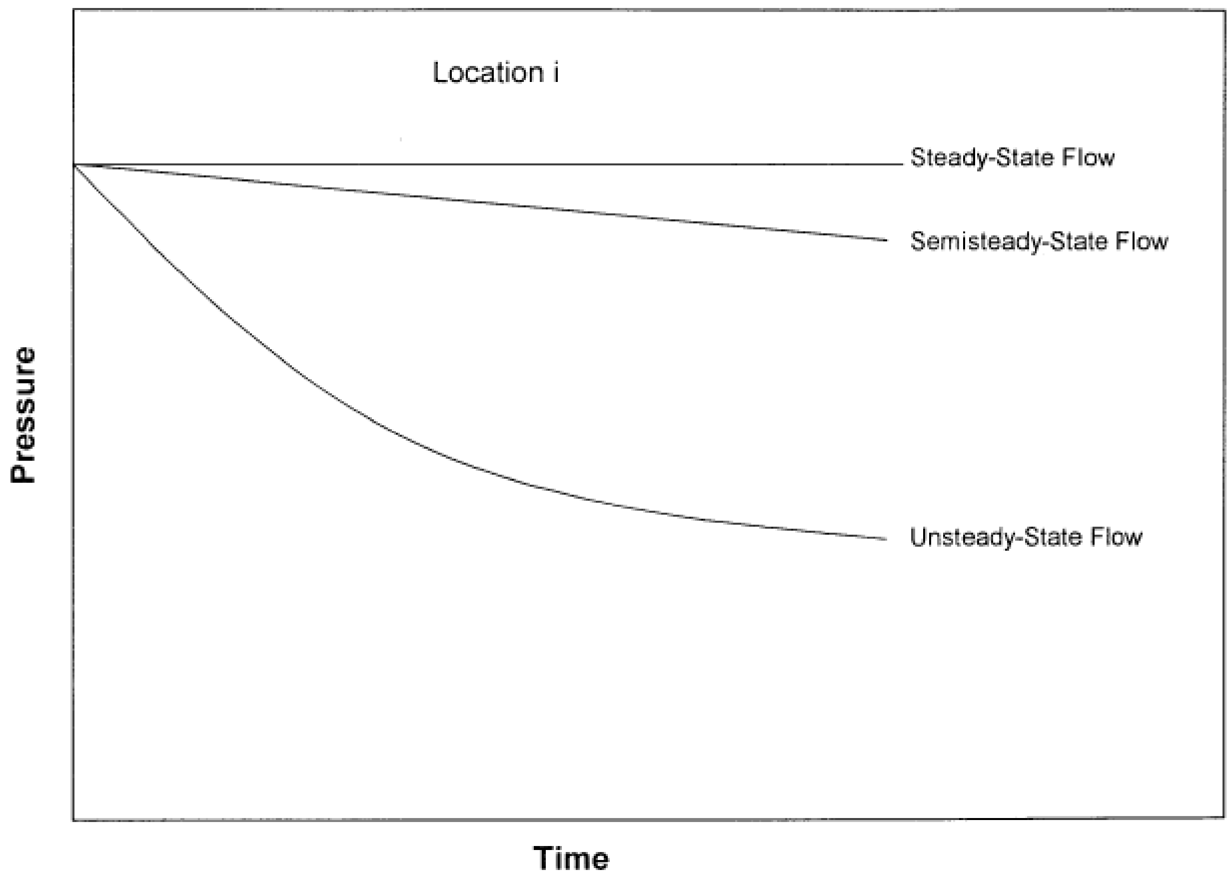


Fig. (1-1): Flow regimes.

Types of Reservoirs and Radial Flow in the Reservoirs

In general, reservoirs are conveniently classified on the basis of the location of the point representing the initial reservoir pressure **pi** and temperature **T** with respect to the pressure-temperature diagram of the reservoir fluid. Accordingly, reservoirs can be classified into basically two types. These are:

1. **Oil reservoirs:** If the reservoir temperature **T** is **less than** the critical temperature $\mathbf{T_c}$ of the reservoir fluid, the reservoir is classified as an oil reservoir.
2. **Gas reservoirs:** If the reservoir temperature is **greater than** the critical temperature of the hydrocarbon fluid, the reservoir is considered a gas reservoir.

Oil Reservoirs

Depending upon initial reservoir pressure **pi**, oil reservoirs can be sub-classified into the following categories:

- **Under-saturated oil reservoir:** If the initial reservoir pressure **pi** (as represented by **point 1** on Figure (1-2), is **greater than** the bubble-point pressure $\mathbf{p_b}$ of the reservoir fluid, the reservoir is labeled an under-saturated oil reservoir.
- **Saturated oil reservoir:** When the initial reservoir pressure is **equal** to the bubble-point pressure of the reservoir fluid, as shown on Figure (1-2) by **point 2**, the reservoir is called a saturated oil reservoir.
- **Gas-cap reservoir:** If the initial reservoir pressure is **below** the bubble point pressure of the reservoir fluid, as indicated by **point 3** on Figure (1-2), the reservoir is termed a gas-cap or two-phase reservoir, in which the gas or vapor phase is underlain by an oil phase. The appropriate quality line gives the ratio of the gas-cap volume to reservoir oil volume.

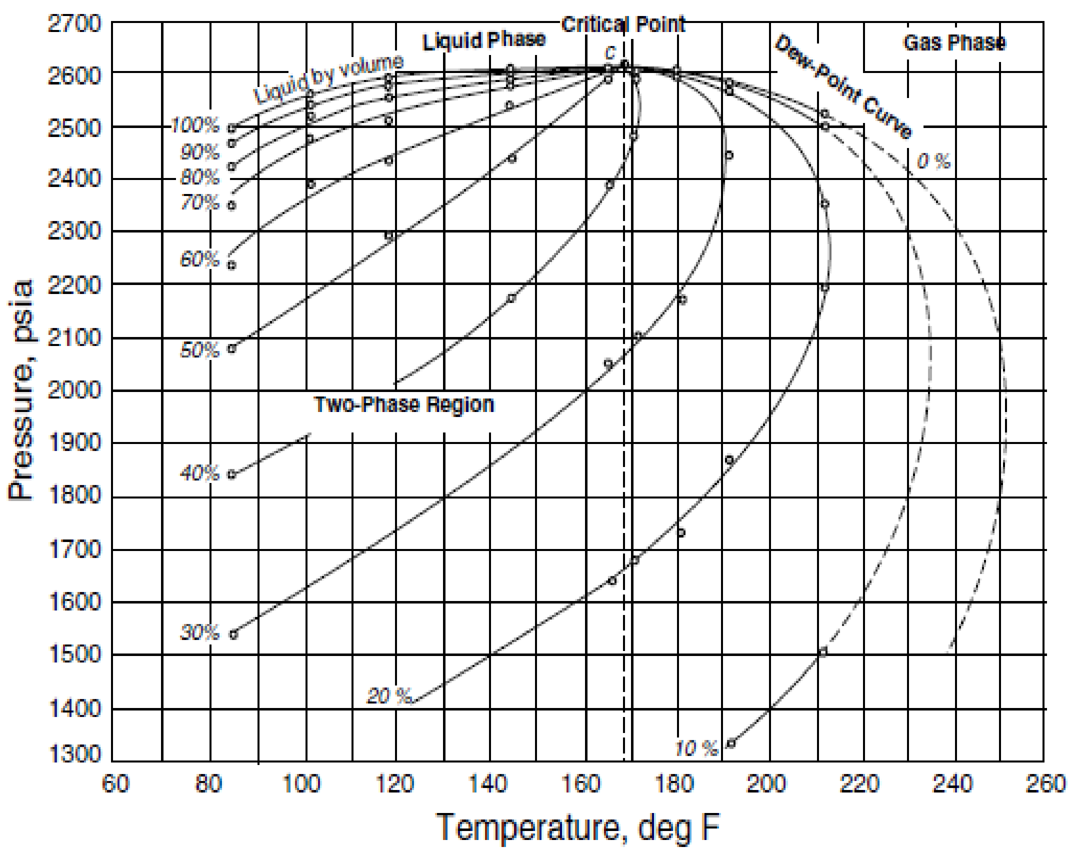


Fig.(1-2): Typical P-T diagram for a multicomponent system.

Reservoir Drive Mechanisms

Ideally the hydrocarbons are recovered from the reservoir pore spaces by exploiting a drive mechanism, precluding the need for artificial method. Drive mechanisms have two classifications:

1. **Internal drive:** Using the internal energy of the reservoir configuration.
2. **External drive:** Which involves the invasion of the pore spaces by a replacement fluid, this type of drive called "**Secondary recovery or Enhanced oil recovery"**.

1) Internal drive

This is known as primary recovery, which includes three drive mechanisms see Figure (1-3):

1. Depletion or internal gas drive
2. External gas cap drive
3. Water drive

1) Depletion or internal gas drive

The compressibility of oil and water is relatively small. As soon as production commences, it is accompanied by a rapid drop of pressure in the producing zone which soon reaches the bubble point of entrained gas. Initially, this gas is dispersed, but it rapidly expands and assists in dispelling the oil. Eventually, however, the gas will start to form a gas front, which, having more mobility than the oil, will increase the production gas to oil ratios. This depletion or dissolved gas drive is characterized by a rapid decline in reservoir pressure and by the recovery of only a small percentage of the oil in situation, e.g. 5 to 20% maximum see Figure (1-4).

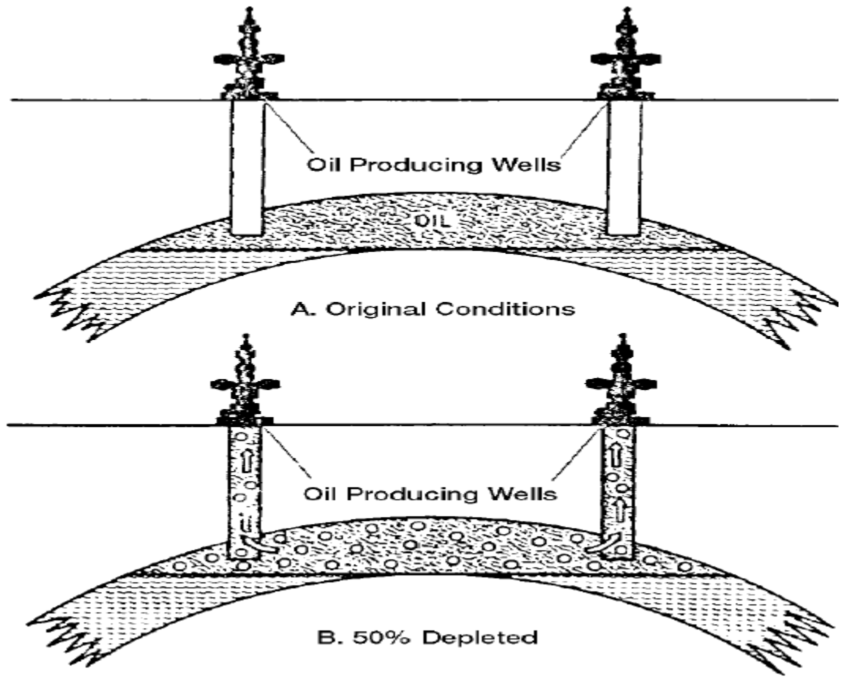


Fig. (1-4): Depletion or internal gas drive.

2) External gas cap drive

Where the oil has a gas cap, the gas cap pressure together with the pressure of gas in solution tends to maintain pressure in the reservoir much longer than depletion drive. Therefore, gas cap reservoirs have higher recovery rates e.g. 20 to 40% see Figure (1-5).

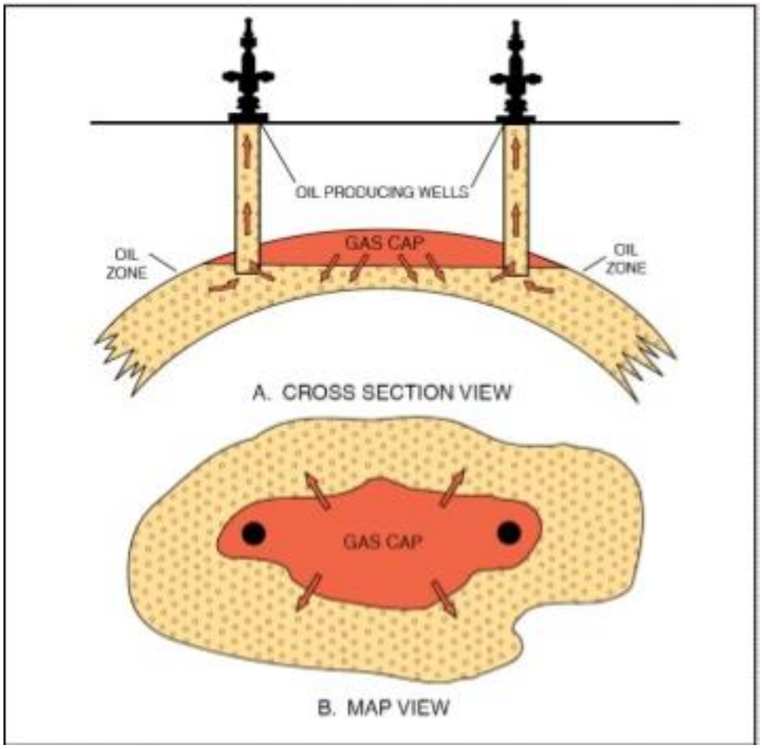


Fig. (1-5): External gas cap drive.

3) Water drive

Water drive is characterized by large local deposits of water which expand as pressure is reduced in the reservoir. Eventually, recovery will decrease due to the greater mobility of the water front which eventually breaks through to the wellbore with increased water to oil ratios. Nonetheless, water drive is the most efficient of all the drive mechanisms and can produce recovery rates as high as 60% see Figure (1-6).

All three-drive mechanisms may be present to varying degrees at the same time although one will predominate see Figure (1-7).

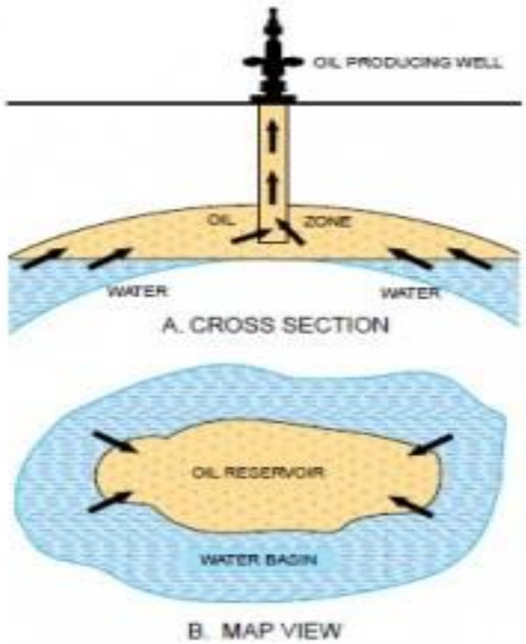


Fig. (1-6): Water Drive Mechanisms.

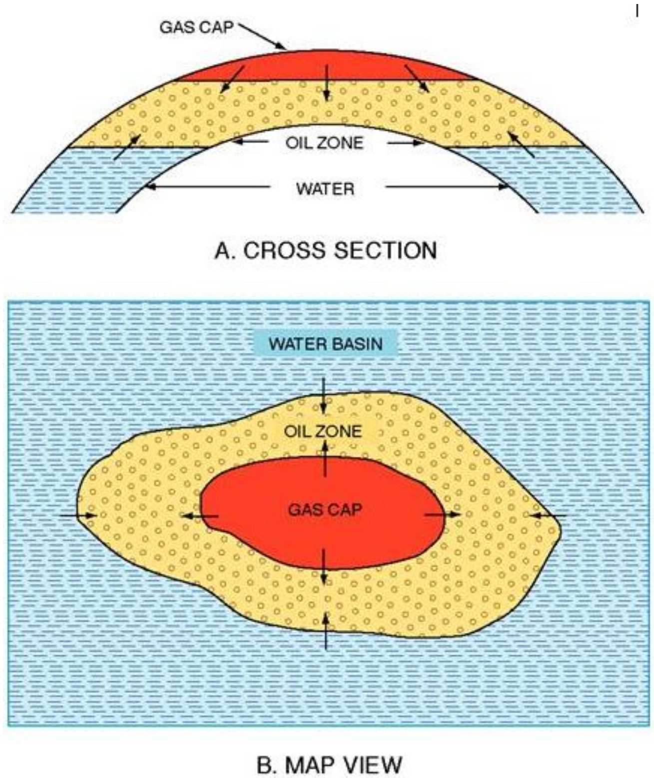


Fig. (1-7): Combined Drive Mechanisms.

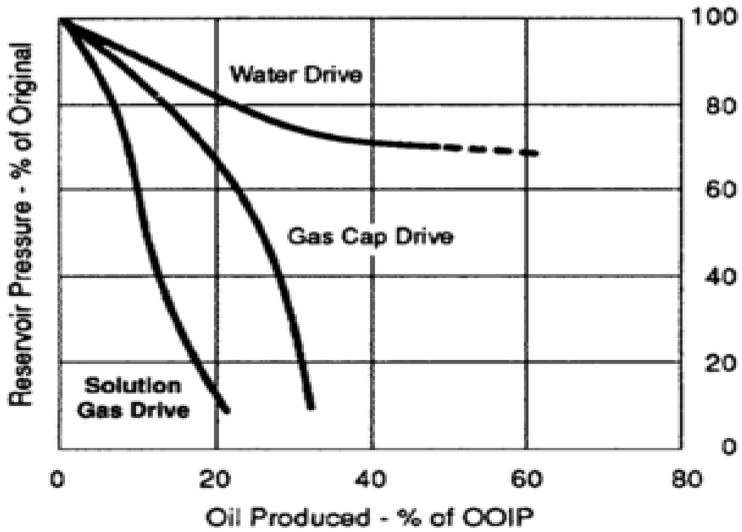


Fig. (1-8): Reservoir – Pressure Trends for Various Drive Mechanisms.

1) External drive

If fluid is injected into a well so that the volumetric rate of fluid replacement is equal to the volumetric rate of fluid extraction, then the average reservoir pressure will tend to remain constant. Injection stimulates secondary recovery.

Depending on the **type and configuration of the reservoir**, pressure can be maintained therefore by:

1. Gas injection
2. Water injection
3. Miscible and immiscible fluid injection.

In general, gas is injected into the crest, and water injection into the base or periphery of the reservoir. Particular consideration must be given to the quality of the injection fluid. They must be compatible with existing reservoir fluids, filtered to prevent formation plugging, possess a viscose significantly higher than formation water. Variation in reservoir permeability, and injection rate should also be considered. If the injection rate is excessive, the water front may advance unevenly, thus giving rise to early water breakthrough, or to unstable coning round the borehole.

Well Performance

A simple producing system is illustrated in Figure (1-9).

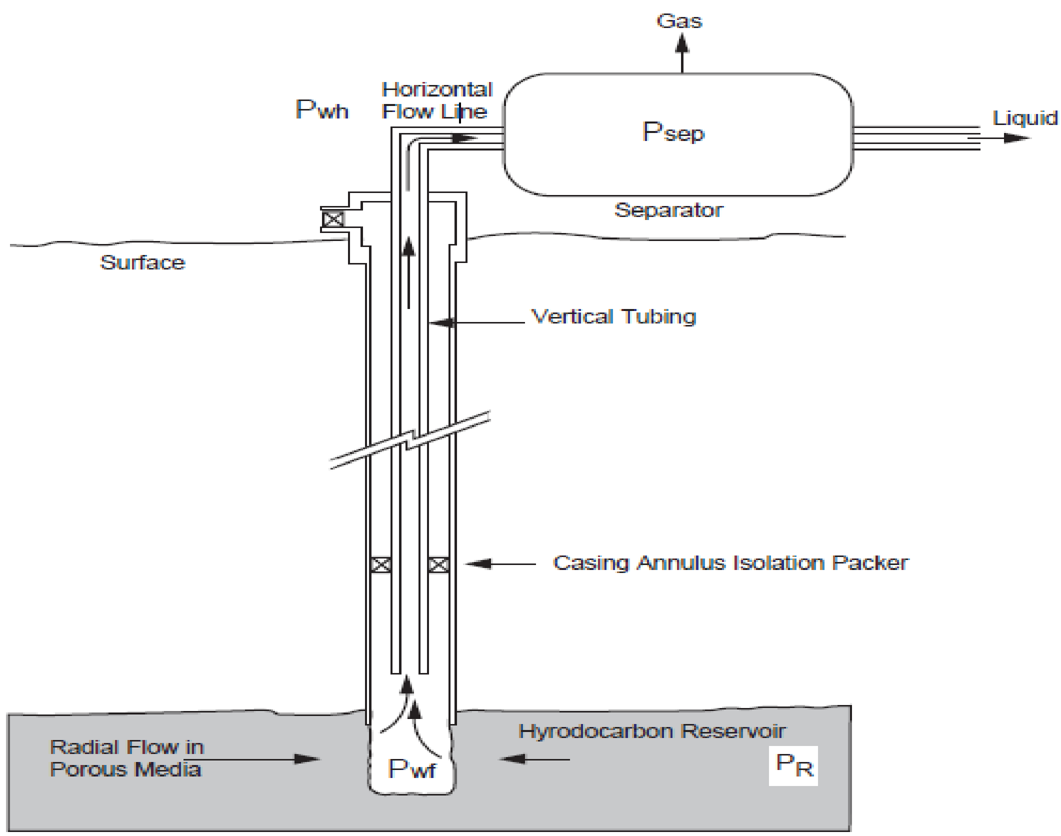


Fig. (1-9): Simplified hydrocarbon production system.

The hydrocarbon fluid flows from the reservoir into the well, up the tubing, along the horizontal flow line and into the oil storage tank. During this process the fluid's pressure is reduced from the reservoir pressure to atmosphere pressure in a series of pressure loss processes Figure (1-10):

1) Across the reservoir
2) Across the completion (perforation/gravel pack etc.)
3) Across the tubing and any restrictions
4) Across the sub surface safety valve
5) Across the surface choke
6) Across flowline

These pressure losses can be grouped into three main components:

1) Summarizes the total pressure losses in the reservoir and completion
2) Summarizes the total pressure losses in the tubing
3) Summarizes the total pressure losses at the surface

A pump or compressor is often used to aid evacuation of fluids (gas/water/oil) from the separator. The separator is operated under gas pressure control and liquid (oil and water) level control. Hence it normally acts as the end point of the flowing system since a pump is necessary to aid evacuation of the liquids from the separator.

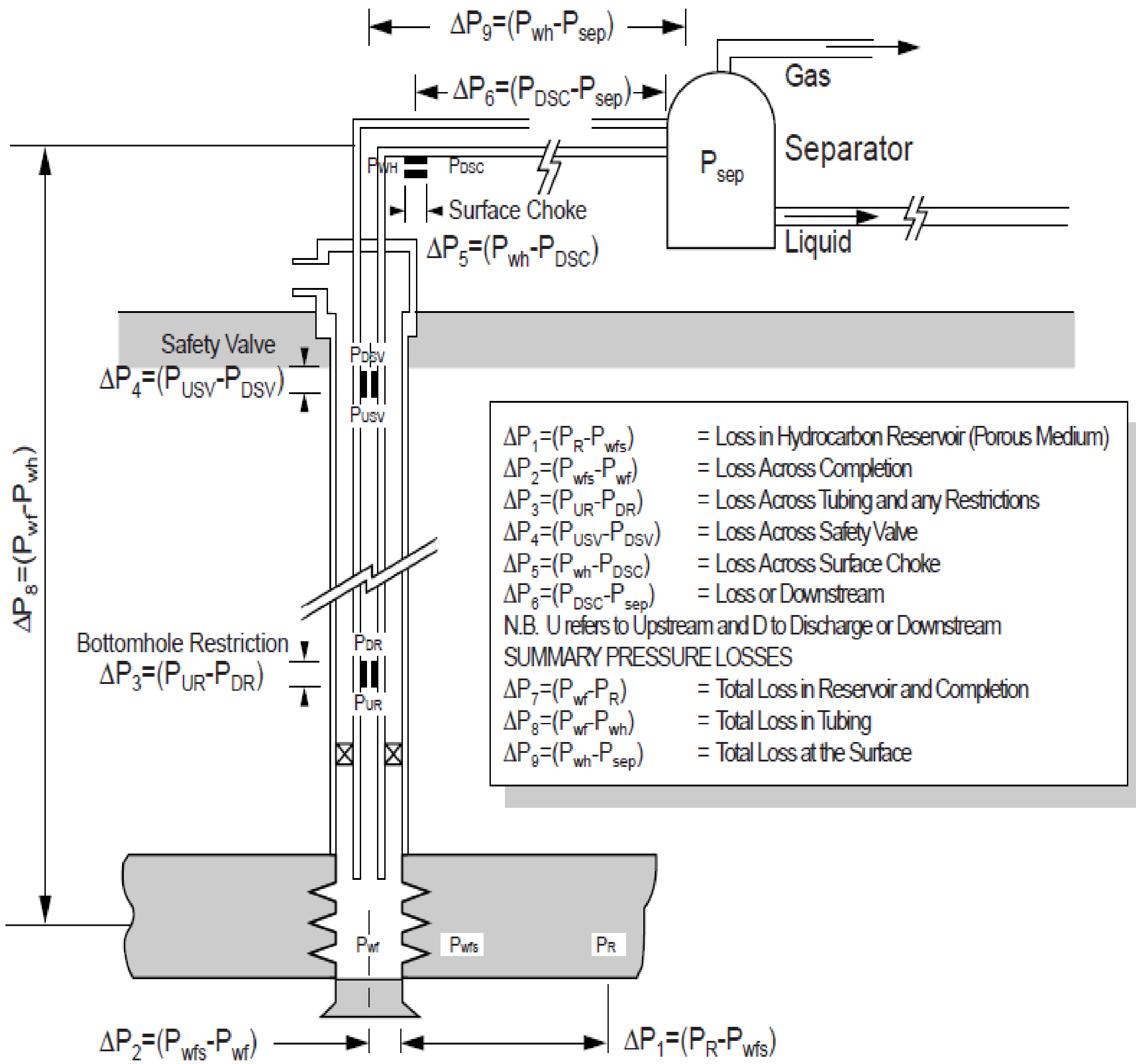


Fig. (1-10): Pressure Losses during Production.

P_R: Reservoir Pressure

P_{wfs}: Flowing sand face Pressure

P_{wf}: Flowing Bottom Hole Pressure

P_{UR}: Upstream Restriction Pressure

P_{DR}: Downstream Restriction Pressure

P_{USV}: Upstream Safety Valve Pressure

P_{DSV}: Downstream Safety Valve Pressure

P_{WH}: Well Head Pressure

P_{DSC}: Downstream surface Choke Pressure

P_{sep}: Separator Pressure

The magnitude of these individual pressure losses depend on **the reservoir properties and pressures**; fluid being produced and the well design. Production Technologists/ Engineers need to understand the interplay of these various factors so as to design completions which maximize profitability from the oil or gas production. There are no standard "rules of thumb" which can be used. Figure (1-11) schematically represents the pressure distribution across the production system shown in Figure (1-10). It identifies the most significant components, flowline, tubing and the reservoir and completion where pressure losses occur.

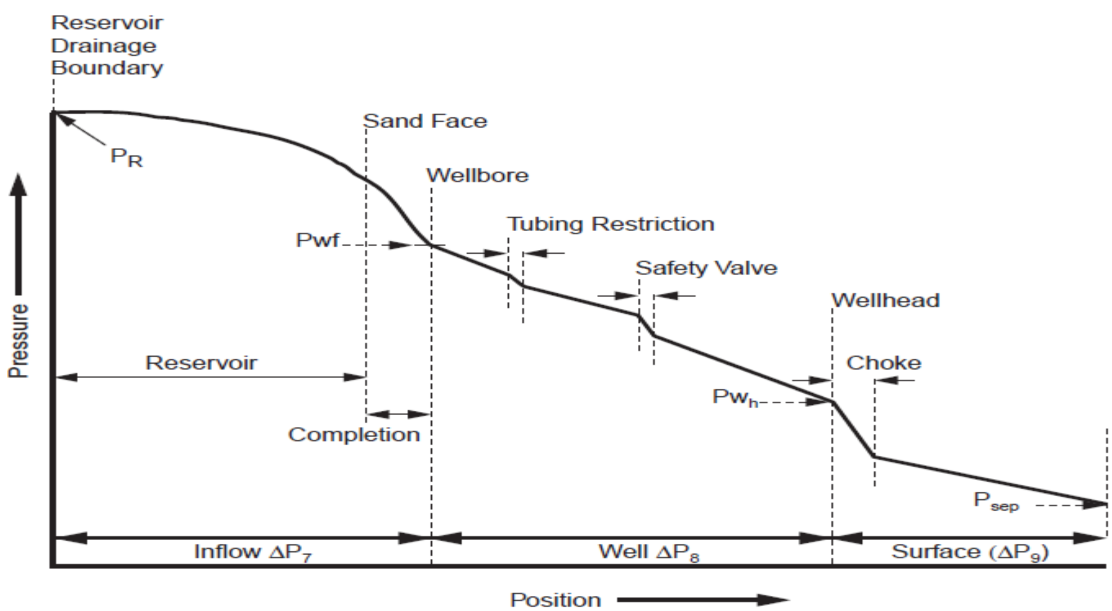


Fig.(1-5): Pressure across production system.

Inflow Well Performance

The flow of oil, water and gas from the formation into the bottom of the well (Well bore), is typified, as far as gross liquid production is concerned, by PI (Productivity Index) of the well or, more generally, by the IPR (Inflow Performance Relationship). The analysis of the production performance is essentially based on the following fluid and well characteristic;

- Fluid PVT Properties.
- Relative permeability data.
- Inflow performance relationship (IPR) & productivity index (PI).

Productivity Index (PI) & Inflow Performance Relationship (IPR)

A commonly used to measure of the ability of the well to produce (give fluids) is the **Productivity Index** (PI), denoted by **J**. **Productivity Index** is the ratio of the total liquid flow rate to the pressure drawdown and could present as a plot of $\mathbf{P_{wf}}$ **versus q**. as shown in Figure (1-12).

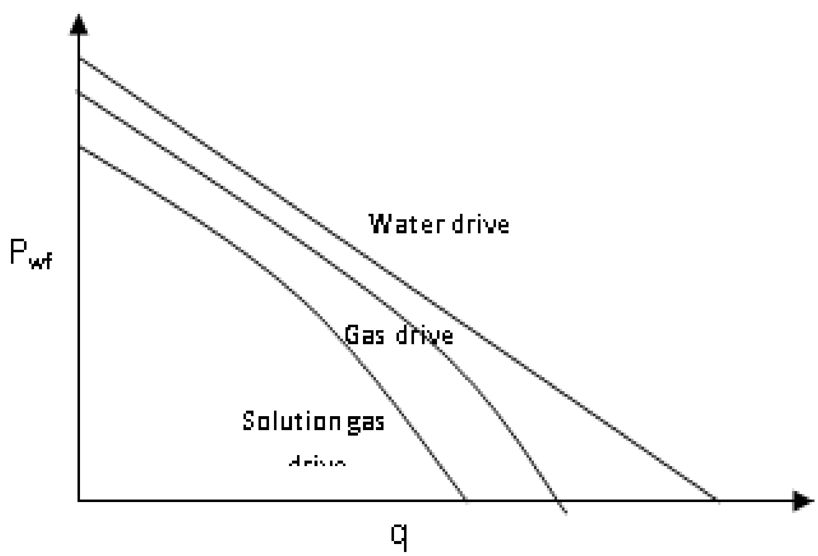


Fig. (1-12): P_{wf} vs Q.

The producing pressure $\mathbf{P_{wf}}$ at the bottom of the well is known as the **flowing BHP**, and the difference between this and the well's static pressure $\mathbf{P_s}$ is the **drawdown**;

Drawdown = $\mathbf{P_s - P_{wf}}$

The productivity index is given by;

$$\mathbf{PI = J = \frac{Q_o}{P_s - P_{wf}} = \frac{Q_o}{\Delta p}} \quad \text{-------------------- } \mathbf{(1.1)}$$

Where:

q = Oil flow rate, STB/day

J = Productivity index, STB/day/psi

P_s = Static pressure (volumetric average drainage area pressure, p_r), psi

P_{wf} = Bottom-hole flowing pressure, psi

The productivity index is generally measured during a production test on the well. The well is shut-in until the static reservoir pressure is reached. The well is then allowed to produce at a constant flow rate and a stabilized bottom-hole flow pressure.

It is important to note that the productivity index is a valid measure of the well productivity potential only if the well is flowing at pseudosteady state conditions. Therefore, in order to accurately measure the productivity index to a well, it is essential that the well is allowed to flow at a constant flow rate for a sufficient amount of time to reach the pseudostead state as illustrated in Figure (1-13). The Figure indicates that during the transient flow period, the calculated values of the productivity index will vary depending upon the time at which the measurements of $\mathbf{P_{wf}}$ are made.

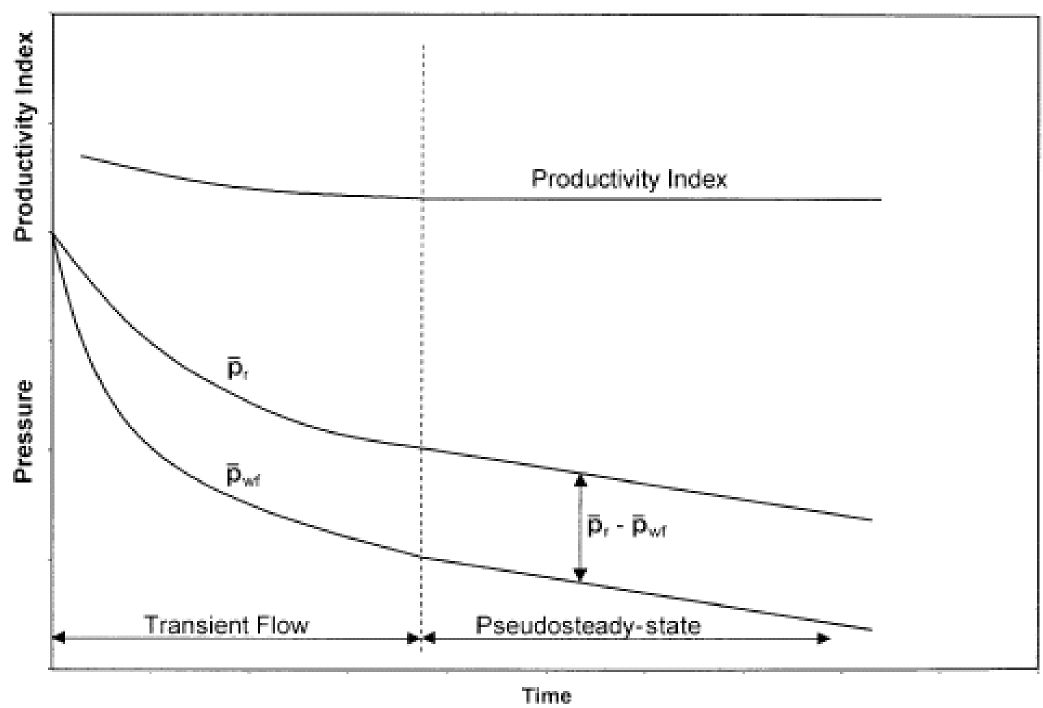


Fig. (1-13): Productivity Index during Flow Regimes.

The productivity index can be numerically calculated by recognizing that ***J*** must be defined in terms of semisteady-state flow conditions;

$$\mathbf{q_o = \frac{0.000708\ k_o h (P_r - P_{wf})}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}} \quad \text{---} \quad \mathbf{(1.2)}$$

Combine Eq. (1.1) with Eq. (1.2);

$$\mathbf{J = \frac{0.000708\ k_o h}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}} \quad \text{---} \quad \mathbf{(1.3)}$$

Since most of the well life is spent at a flow regime that is approximating the pseudosteady-state, the productivity index is a valuable methodology for predicting the future performance of wells. Further, by monitoring the productivity index during the life of the a well, it is possible to determine if the well has become damage due to completion , workover, production, injection operations, or mechanical problems. If a measured ***J*** has unexpected decline, one of the indicated problems should be investigated.

The productivity index may vary from well to well because of the variation in thickness of the reservoir; it is helpful to normalize the indices by dividing each by the thickness of the well. This is defined as the specific productivity index **(*J_s*)**.

$$\mathbf{J_s = \frac{J}{h} = \frac{Q_o}{h(P_s - P_{wf})}} \quad \text{---} \quad \mathbf{(1.3)}$$

Assuming that the well's productivity index is constant, Eq. (1.1) can be rewritten as:

$$\mathbf{Q_o = J\,(P_s - P_{wf}) = J\Delta p} \quad \text{---} \quad \mathbf{(1.4)}$$

Eq. (1.4) indicates that the relationship between q and Δp is a straight line passing through the origin with a slope of ***J*** as in figure (1-14).

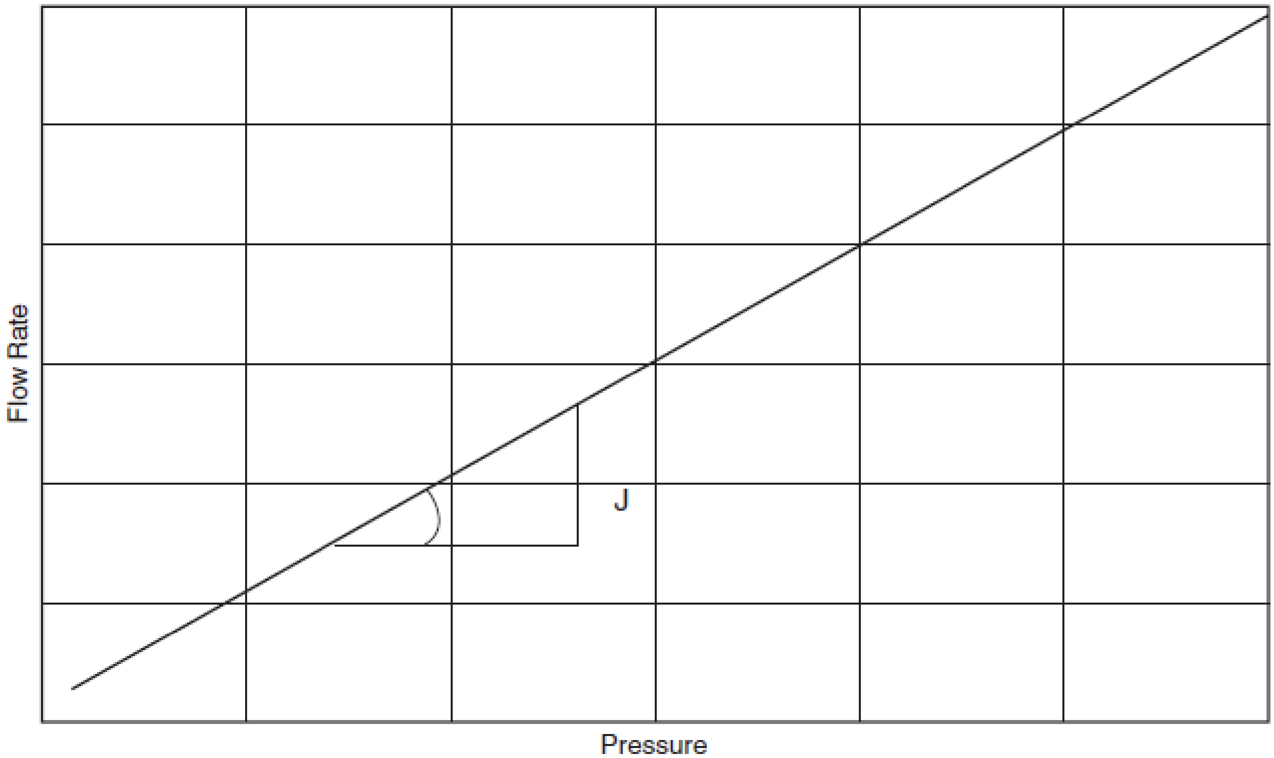


Fig. (1-14): q vs. Δp relationship.

Alternatively, Eq. (1.1) can be written as:

$$\mathbf{P_{wf} = P_s - \frac{q}{J}} \quad \text{-----} \quad \mathbf{(1.5)}$$

The above expression shows that the plot P_{wf} against q is a straight line with a slope of **(- 1/*J*)** as shown schematically in Figure (1-15). This graphical representation of the relationship that exists between the oil flow rate and bottom-hole flowing pressure is called the **Inflow Performance Relationship** and referred to as **IPR**.

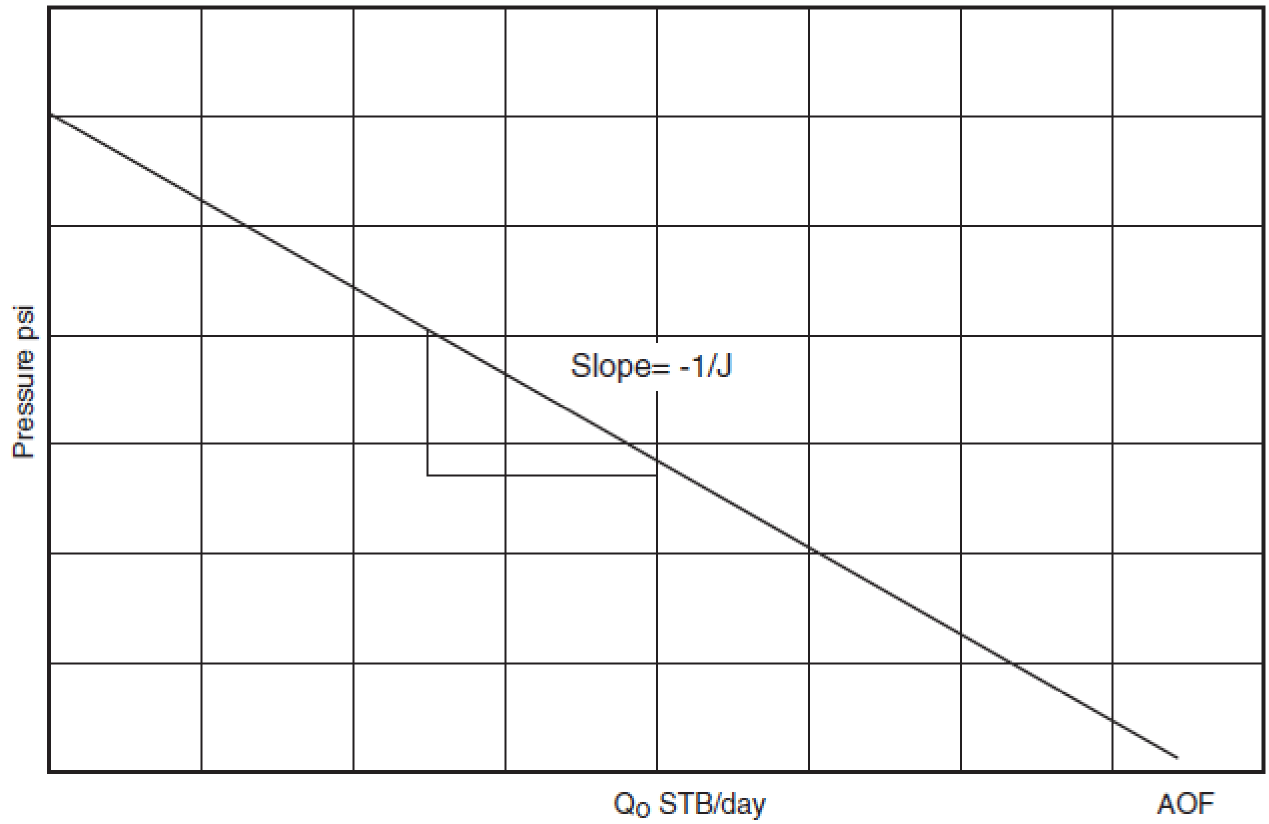


Fig. (1-15): IPR.

Several important features of the straight-line **IPR** can be seen in Figure (1-15);

- When $\mathbf{P_{wf}}$ equals average reservoir pressure, the flow rate is zero due to the absence of any pressure drawdown.
- Maximum rate of flow occurs when $\mathbf{P_{wf}}$ is zero. This maximum rate is called **Absolute Open Flow** and referred to as **AOF**. Although in practice this may not be a condition at which the well can produce, it is a useful definition that has widespread applications in the petroleum industry, (comparing flow potential of different wells in the field). The **AOF** is then calculated by;

  $$\mathbf{AOF} = J\,P_S \qquad (1.6)$$

- The slope of the straight line equals the reciprocal of the productivity index.

Inflow Performance Test

The following stepwise procedure is offered for obtaining data in an inflow performance test:

1) Closed the well (shut in) for (24-72 hrs) to obtained pressure build up test.
2) With the recording pressure gauge on bottom, place the well on its lowest production rate and obtain a flowing pressure recording. Ample time must be allowed for the production rate and flowing BHP to stabilized. If possible at least 48 hours should be allowed for each rate. (The well open to flow for 48 hrs in order to reach the stabilized condition, pseudo steady state regime).
3) After that change the well production rate for different values of rates and recorded the BHP for each rate, allowing approximately 48 hrs to production (at least three values).
4) Plot IPR curve **(q vs. p_{wf})** on Cartesian paper.
5) Again shut the well in and obtain a BHP buildup survey.

Importance of Knowing the IPR of a Well

An allowable production from a certain well = 50 bbl/day (only oil, zero water cut), in order to make the allowable rate, a pump has been installed in the well. For the first few years of its life, the well has produced 50 bbl/day. However, recently production has been less than the allowable.

One of two things has happen:

1. The reservoir (formation) is no longer capable of producing from the well 50 bbl/day.
2. There is some mechanical defect in the well's equipment resulting in a low lifting efficiency (from the bottom of the well to the surface). To know the exact problem that cause the reduction in production is to determine the well's IPR. The result might be either as shown in Figure (1-16) curve.

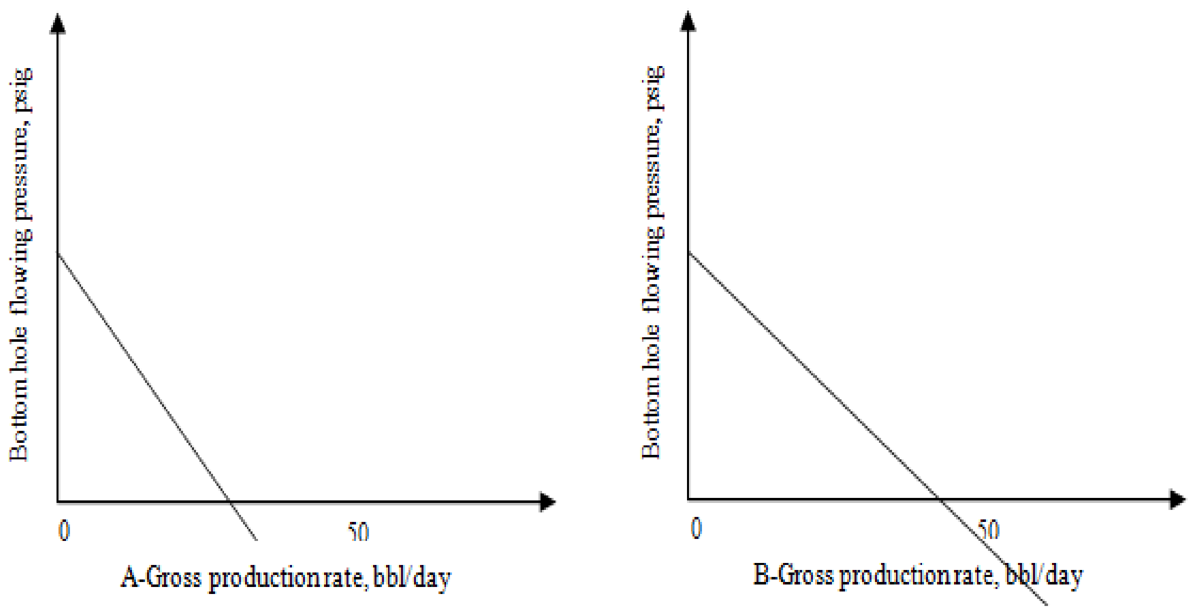


Fig. (1- 16): IPR showing formation incapable of desired production rate.

If the IPR were as illustrated in Figure (1-16A), the well's owner could be certain that no amount of pump changing would result in a production rate of 50 bbl/day and would either have to become reconciled to a below-allowable rate or else undertake a formation-stimulation workover such as a fracturing or an acidizing job. If, on the other hand, the IPR were as illustrated in Figure (1-16B), the owner would be reasonably sure that a mechanical workover of the equipment in the well would restore production to its allowable rate.

As a second example of the importance of knowing the IPR, suppose that a company has been carrying out a formation-stimulation program on some of its wells and that to gauge the success of this program, "before" and "after" *production*-rate, figures are used. Let the results on two wells (both cutting zero water) be as follows:

Well	Before treatment	1 week after treatment
A	60, flowing	100, flowing
B	35, pumping	36, pumping

The treatment would probably be accounted successful on well A and unsuccessful on well B. but while this may in fact be true, insufficient evidence has been presented to warrant such a conclusion; the before and after IPR`s of the well's might be as illustrated in figures (1-17) and (1-18).

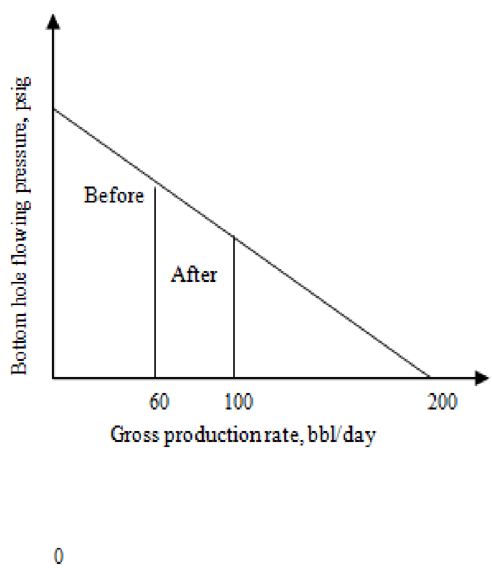


Fig. (1-17) formation stimulation a failure despite increased production rate

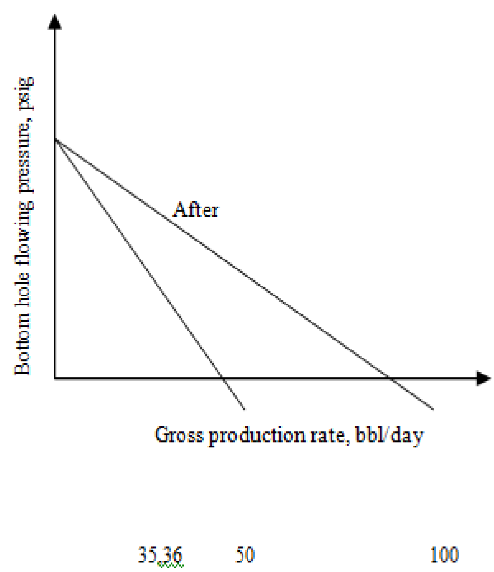


Fig. (1-18) formation stimulation a success despite unaltered production rate

The treatment has had no effect at all on the IPR of well A; that is, the formation inflow performance has not been improved in any way, so the treatment was completely unsuccessful. The production increase from 60 to 100 bbl/day was fortuitous and might have been caused by the treatment dislodging some tubing obstruction, by different-sized tubing having been run into the hole after the job or by a different choke having been inserted in the flow line at the surface.

On the other hand, the treatment on well B has increased the formation's potential considerably and was an undoubted success. Why then were the before and after rates almost identical? There are several possible reasons: the pump might not have been properly seated after the treatment; the pump might have been damaged in some way when it was pulled for the treatment to be undertaken; the producing GOR of the formation might have been increased by the treatment, resulting in reduced pump efficiencies; or the truth of the matter may lie with one or more of various other possible explanation.

Factors Influencing Shape of IPR

The discussion that follows will concentrate on effects resulting from the pressure of free gas in the formation and, consequently, will lead to some conclusions relating to the dependence of producing GLRs on drawdown. In oil reservoir, gas does not be free until BHP of formation reaches value less than bubble point pressure value. So at pressure below bubble point pressure gas being free and the free gas could moving when saturation of free gas (S_g) be greater than critical gas saturation (S_{gc}, at this value gas be able to moving).

It is evident from the form of the radial- flow equation that the greater part of the pressure drop (from static pressure to flowing BHP) in a producing formation occurs in the neighborhood of the well bore (pressure drop is occurring within 20 ft of the well bore).

Suppose the flowing BHP at the well is below the bubble point of the oil. As oil moves in toward the well, the pressure on it drops steadily, allowing gas to come out of solution. The free gas saturation in the vicinity of the oil body steadily increases, and so the relative permeability to gas steadily increases at the expense of the relative permeability to oil. The greater the drawdown, that is, the lower the sand-face pressure at the well , the more market this effect will de , so that it would be reasonable to expect the PI (which depends on the effective oil permeability) to decrease and the producing GOR (which depends on the effective gas permeability) to increase as the drawdown is increased. Such an argument leads to the conclusion that a curved IPR, as shown in Figure (1-19), is to be expected whenever the flowing BHP is below the bubble-point pressure.

Last as long as the value of the flowing BHP remains above the saturation pressure, no free gas will be evolved in the formation and the PI will remain constant; that is, the portion of the IPR applicable to values of the flowing BHP higher than the saturation pressure will be a straight line, as shown in Figure (1-19).

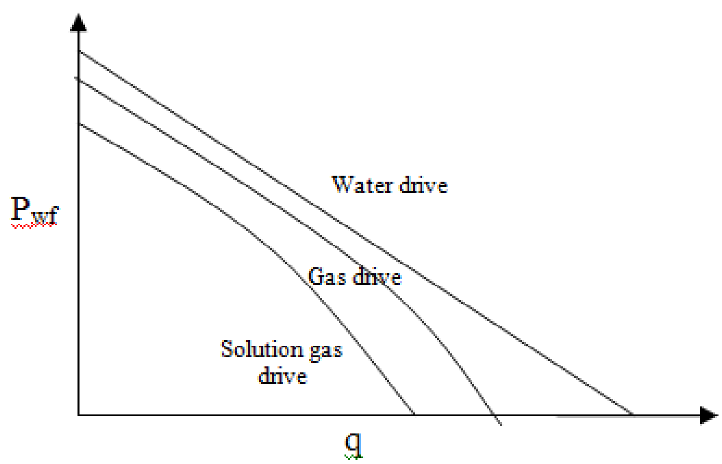


Fig. (1-19): P_{wf} vs. q.

1. Stratified Formation

Practically every production formation is stratified to some extent; that is to say, it contains layers of differing permeability. To illustrate the type of effects that such stratification may have upon the shape of the IPR and upon the dependence of GOR on production rate, consider an example in which there are three different zones having permeabilities of 10,100, and 1 md, respectively. It will be assumed that there is no vertical communication between the zones, except through the well bore itself as shown in Figure (1-20). Production from this formation will evidently be drawn chiefly from the

100 md zone, with the result that the static pressure in this zone will drop below those in the other two, the 1 md zone exhibiting the highest static pressure.

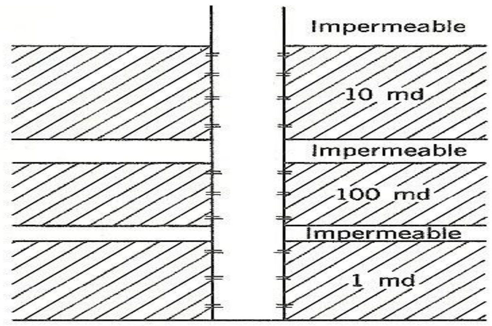


Fig. (1-20) Idealized stratified formation.

Suppose that a stage has been reached in which the pressure in the 100 md zone is 1000 psig, that in the 10 md zone is 1200 psig, and that in the 1 md zone is 1500psig, the well is now tested at various production rates to establish the IPR. If the individual IPRs of the three zones are as illustrated in Figure (1-21), the composite IPR, which will be the sum of these three curves, will have the shape shown. It follows as a generalization that many wells will, because of stratification and subsequent differential depletion of the zones on production, exhibit a composite IPR curve of the type illustrated in Figure (1-22) that is the say, an improving PI with increasing production rate at lower rates, but a deteriorating PI at the higher rates.

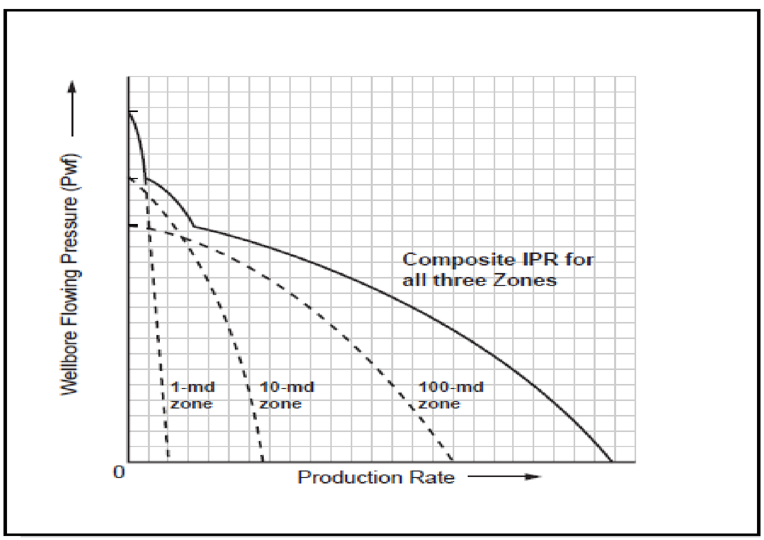


Fig. (1-21): Composite IPR for heterogeneous formation.

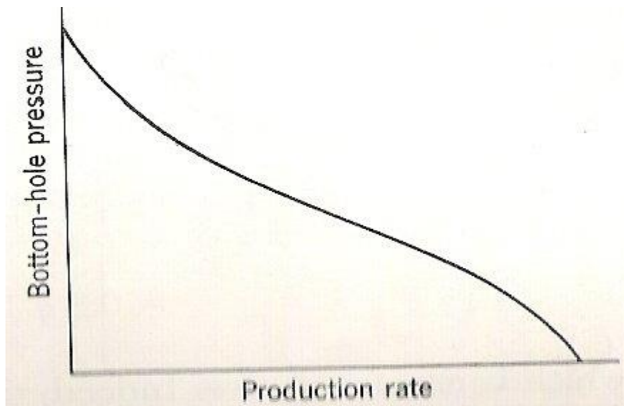


Fig. (1-22): Typical IPR Curve.

At the low rates the flowing BHP will be high and only the higher-pressured layers will contribute to the production. These layers will be those with the greater degree of consolidation and cementing, that is, with the higher values of the gas/oil permeability ratio. In other words, the producing layers at the low rates of flow are those which produce with a high GOR.

As the well's rate of production is gradually increased, the less consolidated layers will begin to produce one by one (at progressively lower GORs) and so the overall ratio of the production will fall as the rate is increased. If, however, the most highly depleted layers themselves produce at high ratios owing to high free gas saturation.

2. Effect of water cut on IPR

If water is moving from the water source to the well stringers in the formation, it is possible to determine whether, at the well bore, the pressure in the water is greater than or less than the pressure in the oil sands (that is, whether it is high-pressure or low-pressure water) from an analysis of the gross IPR and three or four water-cut values taken at different gross rates. The method of approach may be illustrated by means of an example.

Problem (1-1): A series of tests is made on a certain well with the following results;

Gross rate, bbl/day	water cut, water/gross%	flowing BHP,psig
47	85	1300
90	60	920
125	48	630
162	45	310

Determine the static pressure and the productivity index of the oil and water zones, respectively. Based on the results, at what rate could water be expected to flow into the oil sand if the well left shut down?

Referring to Figure (1-23).

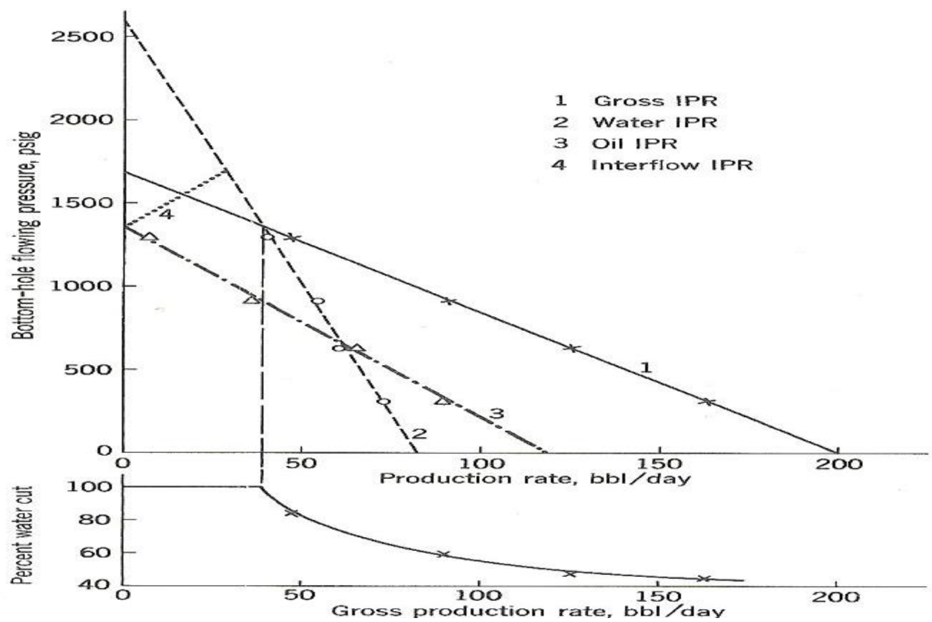


Fig. (1-23): IPR and water-cut curves: high-pressure water.

The first step is to plot the gross IPR (line 1).

From the gross rate and the measured water cuts the water and oil IPRs are calculated as follows (line 2 and 3):

$q_o = q_t - q_w$ -- **(1.7)**

$q_o = q_t\ (1- q_w/q_t)$ -- **(1.8)**

$q_o = q_t\ (1- wc)$ -- **(1.9)**

Gross rate, bbl/day	water cut, water/gross%	water rate, bbl/day	oil rate, bbl/day	flowing BHP,psi
47	85	40	7	1300
90	60	54	36	920
125	48	60	65	630
162	45	73	89	310

Then plot water cut ratio versus gross rate.

Evidently, from the figure,

Static pressure of oil zone = 1350 psig

PI of oil zone = 120/1350 = 0.089 bbl/(day)(psi)

Static pressure of water zone = 2600 psig

PI of water zone = 82/2600 = 0.0315 bbl/(day)(psi)

When the well is shut in, it might be expected (from the gross IPR) that the BHP would stabilized at about 1700 psig and that water would flow into the oil zone at some 28 bbl/day.

It is of interest to note the shape of the water cut versus rate curve (also shown on Figure (1-23), which is typical of high-pressure water, namely, a 100 percent cut (pure water) is obtained at low rates, the oil content gradually increasing with the offtake rate. In Figure (1-24) the case of low-pressure water is similarly illustrated, and the typical water cut versus rate curve is shown; namely, the cut starts at or near zero and increases with rate.

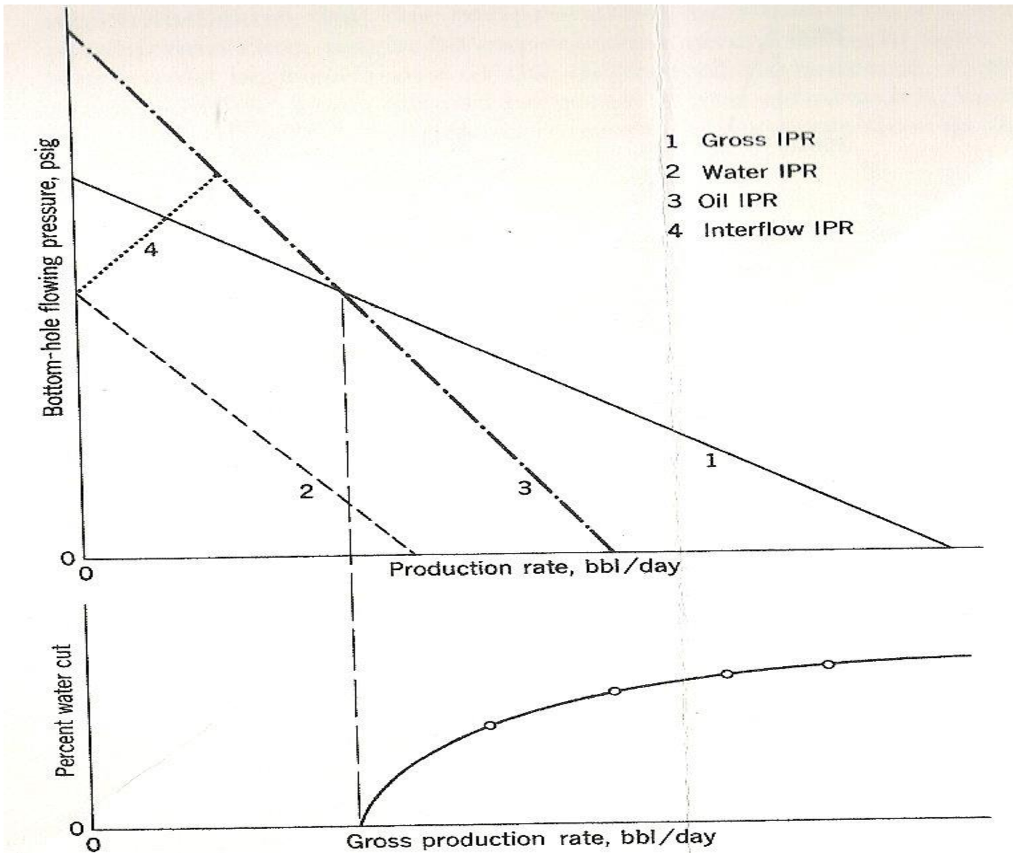


Fig. (1-14): IPR and water-cut curves: low-pressure water.

Problem (1-2): A productivity test was conducted on a well. The test results indicate that the well is capable of producing at a stabilized flow rate of **110** STB/day and a bottom-hole flowing pressure of **900** psi. After shutting the well for **24** hours, the bottom-hole pressure reached a static value of **1300** psi.

Calculate:

- Productivity index
- AOF
- Oil flow rate at a bottom-hole flowing pressure of 600 psi
- Wellbore flowing pressure required to produce 250 STB/day

Solution:

1- $J = \frac{q}{P_r - P_{wf}}$

$$J = \frac{110}{1300-900} = 0.275 \text{ STB /psi}$$

2- $AOF = J\,(P_r - 0)$

$$AOF = 0.275(1300 - 0) = 375.5 \text{ STB /day}$$

3- $Q_o = J(P_r - P_{wf})$

$$Q_o = 0.257(1300 - 600) = 192.5 \text{ STB / day}$$

4- $P_{wf} = P_r - \left(\frac{1}{J}\right) Q_o$

$$P_{wf} = 1300 - \left(\frac{1}{0.275}\right) 250 = 390.9 \text{ psi}$$

Equation (1.4) suggests that the inflow into a well is directly proportional to the pressure drawdown and the constant of proportionality is the productivity index. Muskat and Evinger (1942) and Vogel (1968) observed that when the pressure drops below the bubble-point pressure, the IPR deviates from that of the simple straight-line relationship as shown in Figure (1-25).
Recalling Equation (1.2):

$$\mathbf{J} = \left[\frac{0.00708\,hk}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s}\right]\left(\frac{k_{ro}}{\mu_o B_o}\right) \text{ ---------- } \mathbf{(1.10)}$$

Treating the term between the two brackets as a constant c, the above equation can be written in the following form:

$$\mathbf{J} = \mathbf{C}\left(\frac{k_{ro}}{\mu_o B_o}\right) \text{ ---------- } \mathbf{(1.11)}$$

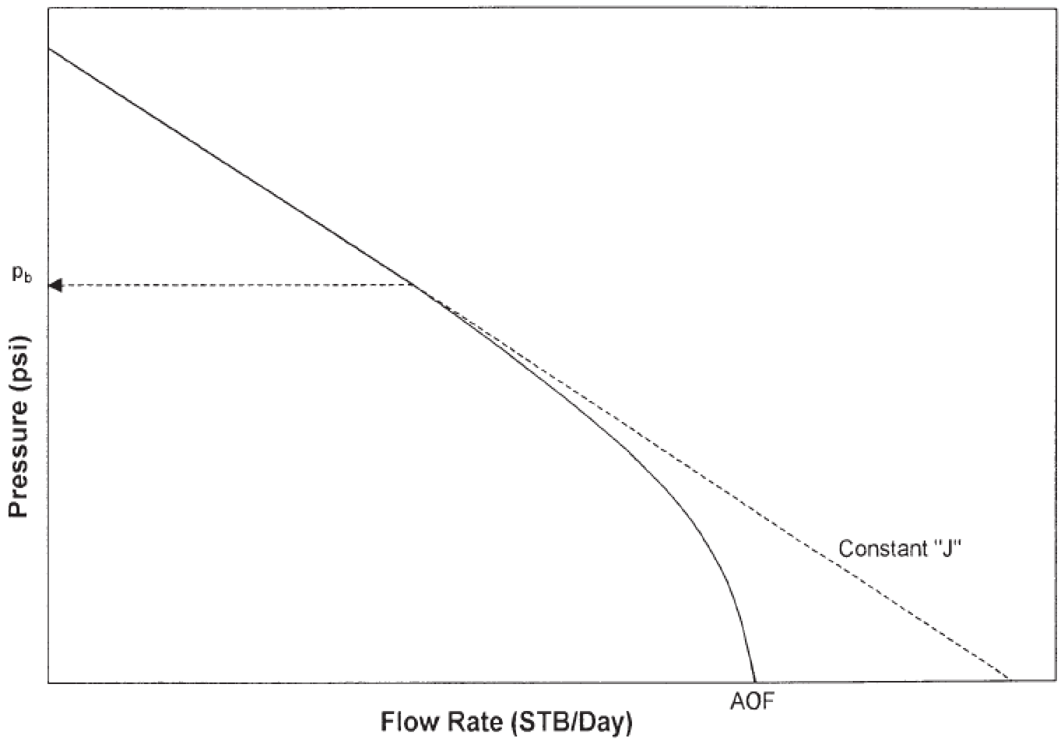


Fig. (1-25): IPR below P_b.

With the coefficient c as defined by:

$$\mathbf{C} = \frac{0.00708\,kh}{\ln\left(\frac{r_e}{r_w}\right)+0.75+s} \quad \text{------------------------} \quad \mathbf{(1.12)}$$

Equation (1.11) reveals that the variables affecting the productivity index are essentially those that are pressure dependent, i.e.:

- Oil viscosity μ_o
- Oil formation volume factor B_o
- Relative permeability to oil k_{ro}

Figure (1-26) schematically illustrates the behavior of those variables as a function of pressure. Figure (1-27) shows the overall effect of changing the pressure on the term $(k_{ro}/\mu_o B_o)$. Above the bubble-point pressure $\mathbf{P_b}$, the relative oil permeability $\mathbf{k_{ro}}$ equals unity ($\mathbf{k_{ro}}$ =1) and the term $(k_{ro}/\mu_o B_o)$ is almost constant. As the pressure declines below $\mathbf{P_b}$, the gas is released

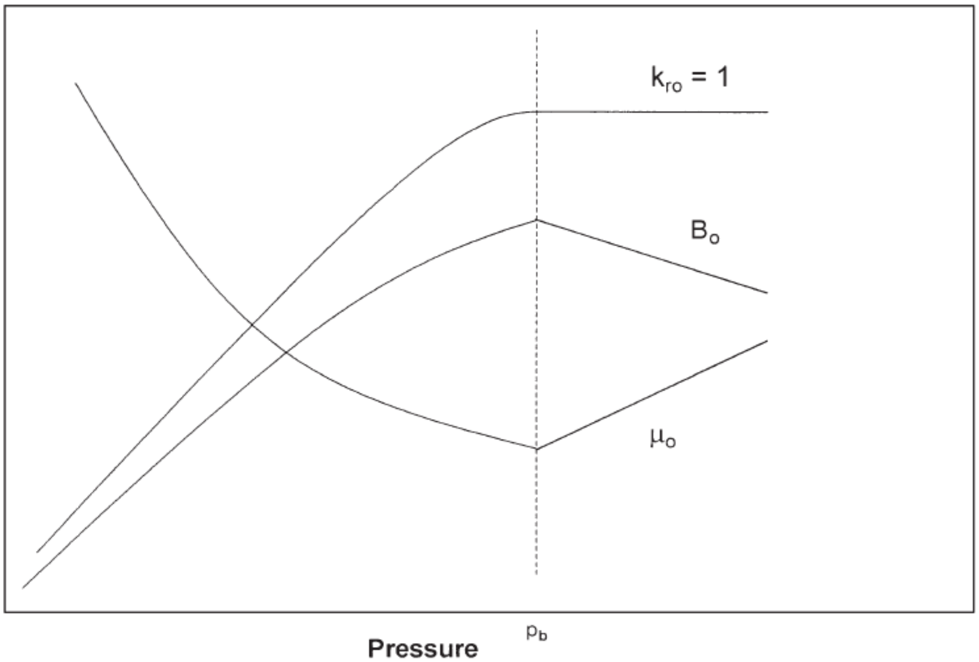


Fig. (1-26): Effect of pressure on B_o, μ_o, and k_{ro}

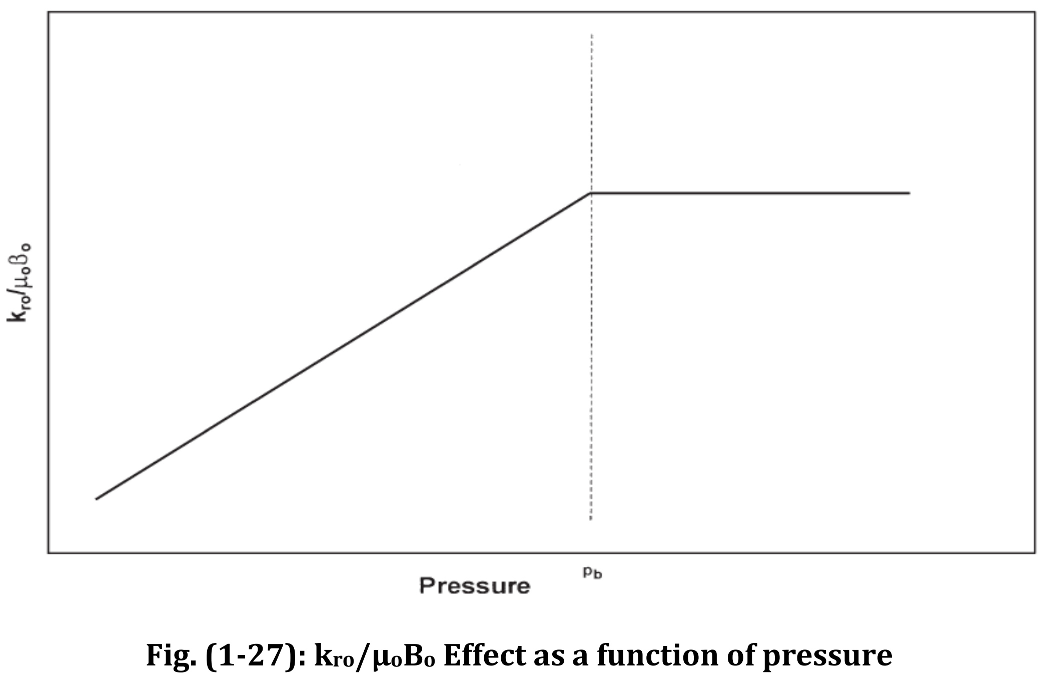


Fig. (1-27): k_{ro}/μ_oB_o Effect as a function of pressure

From solution, which can cause a large decrease in both k_{ro} and (k_{ro}/μ_oB_o). Figure (1-28) shows qualitatively the effect of reservoir depletion on the IPR.

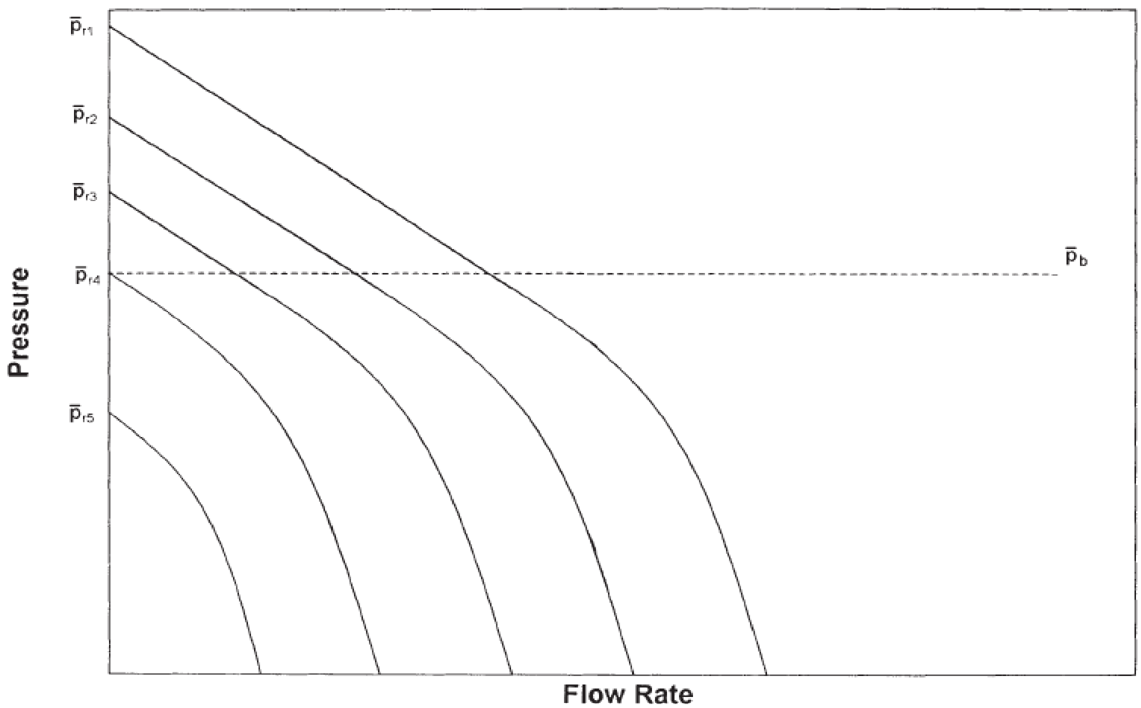


Fig. (1-28): Effect of reservoir pressure on IPR

Methods of Calculation

There are several empirical methods that are designed to predict the non-linearity behavior of the IPR for solution gas drive reservoirs. Most of these methods require at least one stabilized flow test in which Q_o and P_{wf} are measured. All the methods include the following two computational steps:

- Using the stabilized flow test data, construct the IPR curve at the current average reservoir pressure P_r.
- Predict future inflow performance relationships as to the function of average reservoir pressures.

The following empirical methods that are designed to generate the current and future inflow performance relationships:

1. Vogel's method
2. Standing's method
3. Couto`s Method
4. Al saadoon`s Method
5. Fetkovich's method
6. Wiggins' method
7. The Klins-Clark method

1) Vogel's Method

Vogel (1968) based on a computer simulation of dissolved gas drive reservoirs, where in his calculated IPRs using a wide range of reservoir and fluid parameters, proposed the general IPR curve of Figure (1-29). Often this same Vogel relation is successfully applied to other types of reservoir drive systems.

Vogel normalized the calculated IPRs and expressed the relationships in a dimensionless form. He normalized the IPRs by introducing the following dimensionless parameters:

- Dimensionless pressure = $\frac{P_{wf}}{P_r}$
- Dimensionless flow rate = $\frac{Q_o}{(Q_o)_{max}}$

Where $(Q_o)_{max}$ is the flow rate at zero wellbore pressure (100% drawdown), i.e., AOF.

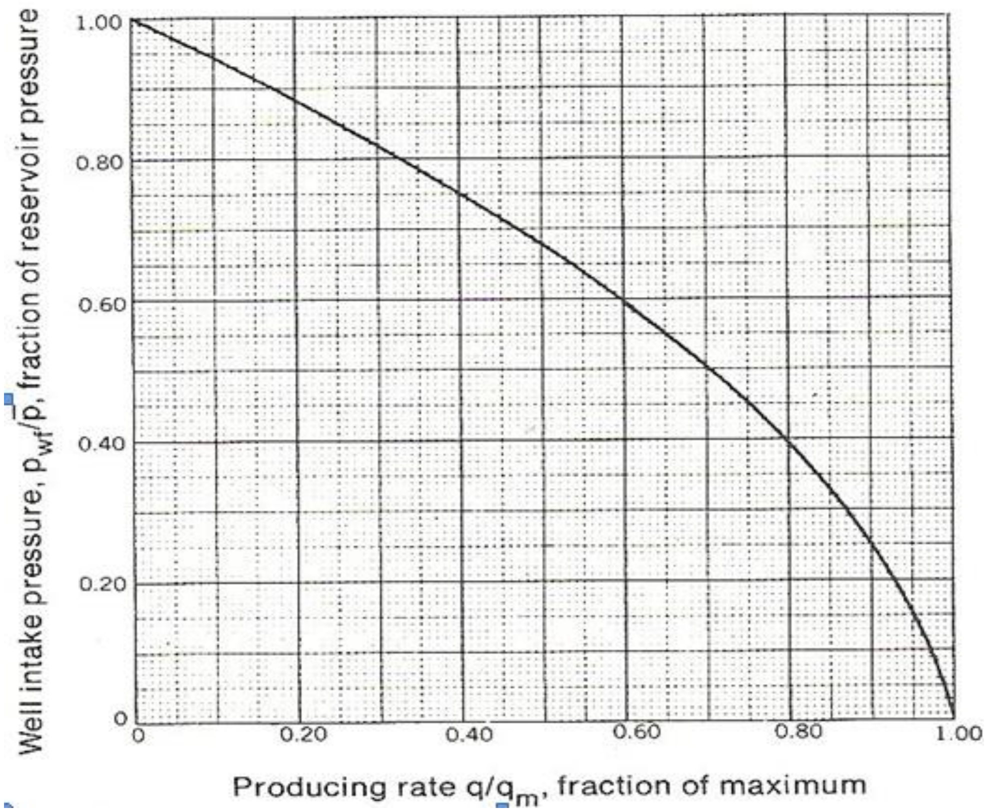


Fig. (1-29): Inflow performance relation (Vogel).

Vogel plotted the dimensionless IPR curves for all the reservoir cases as shown in Figure (1-29) and arrived at the following relationship between the above dimensionless parameter:

$$\frac{Q_o}{(Q_o)_{max}} = 1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2 \text{ ------------------------------------ } (1.13)$$

Where:

Q_o = oil rate at P_{wf}

$(Q_o)_{max}$ = maximum oil flow rate at zero wellbore pressure, i.e., AOF

P_r = current average reservoir pressure, psig

P_{wf} = wellbore pressure, psig

Vogel's method can be extended to account for water production by replacing the dimensionless rate with $Q_L/(Q_L)max$ where $Q_L = Q_o + Q_w$.

This has proved to be valid for wells producing at water cuts as high as 97%.

The method requires the following data:

- Current average reservoir pressure $\mathbf{P_r}$
- Bubble-point pressure $\mathbf{P_b}$
- Stabilized flow test data that include $\mathbf{Q_o}$ at $\mathbf{P_{wf}}$

Vogel's methodology can be used to predict the IPR curve for the following two types of reservoirs:

- Saturated oil reservoirs $\mathbf{P_r \le P_b}$
- Undersaturated oil reservoirs $\mathbf{P_r > P_b}$

➢ Saturated Oil Reservoirs

When the reservoir pressure equals the bubble-point pressure, the oil reservoir is referred to as a ***saturated oil reservoir***. The computational procedure of applying Vogel's method in a saturated oil reservoir to generate the IPR curve for a well with a stabilized flow data point, i.e., a recorded $\mathbf{Q_o}$ value at $\mathbf{P_{wf}}$, is summarized below:

Step 1: Using the stabilized flow data, i.e., $\mathbf{Q_o}$ and $\mathbf{P_{wf}}$, calculate: $\mathbf{(Q_o)_{max}}$ from Equation

$$(Q_o)_{max} = \frac{Q_o}{1-0.2\left(\frac{P_{wf}}{P_r}\right)-0.8\left(\frac{P_{wf}}{P_r}\right)^2} \quad \text{------} \quad \mathbf{(1.14)}$$

Step 2: Construct the IPR curve by assuming various values for $\mathbf{P_{wf}}$ and calculating the corresponding $\mathbf{Q_o}$ from:

$$\mathbf{Q_o = (Q_o)_{max}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]} \quad \text{------} \quad \mathbf{(1.15)}$$

Problem (1-3): A well is producing from a saturated reservoir with an average reservoir pressure of **2500** psig. Stabilized production test data indicated that the stabilized rate and wellbore pressure are **350** STB/day and **2000** psig, respectively. Calculate:

1. Oil flow rate at P_{wf} = **1850** psig
2. Calculate oil flow rate assuming constant J
3. Construct the IPR by using Vogel's method and the constant productivity index approach.

Solution:

Part A.

Step 1: Calculate $(Q_o)_{max}$:

$$(Q_o)_{max} = \frac{350}{1 - 0.2\left(\frac{2000}{2500}\right) - 0.8\left(\frac{2000}{2500}\right)^2} = \mathbf{1076.1} \text{ STB /day}$$

Step 2: Calculate Q_o at p_{wf} = 1850 psig by using Vogel's equation

$$Q_o = (Q_o)_{max}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]$$

$$Q_o = 1076.1\left[1 - 0.2\left(\frac{1850}{2500}\right) - 0.8\left(\frac{1850}{2500}\right)^2\right] = \mathbf{441.7} \text{ STB/day}$$

Part B.

Calculating oil flow rate by using the constant **J** approach
Step 1: Apply Equation (1.1) to determine **J**

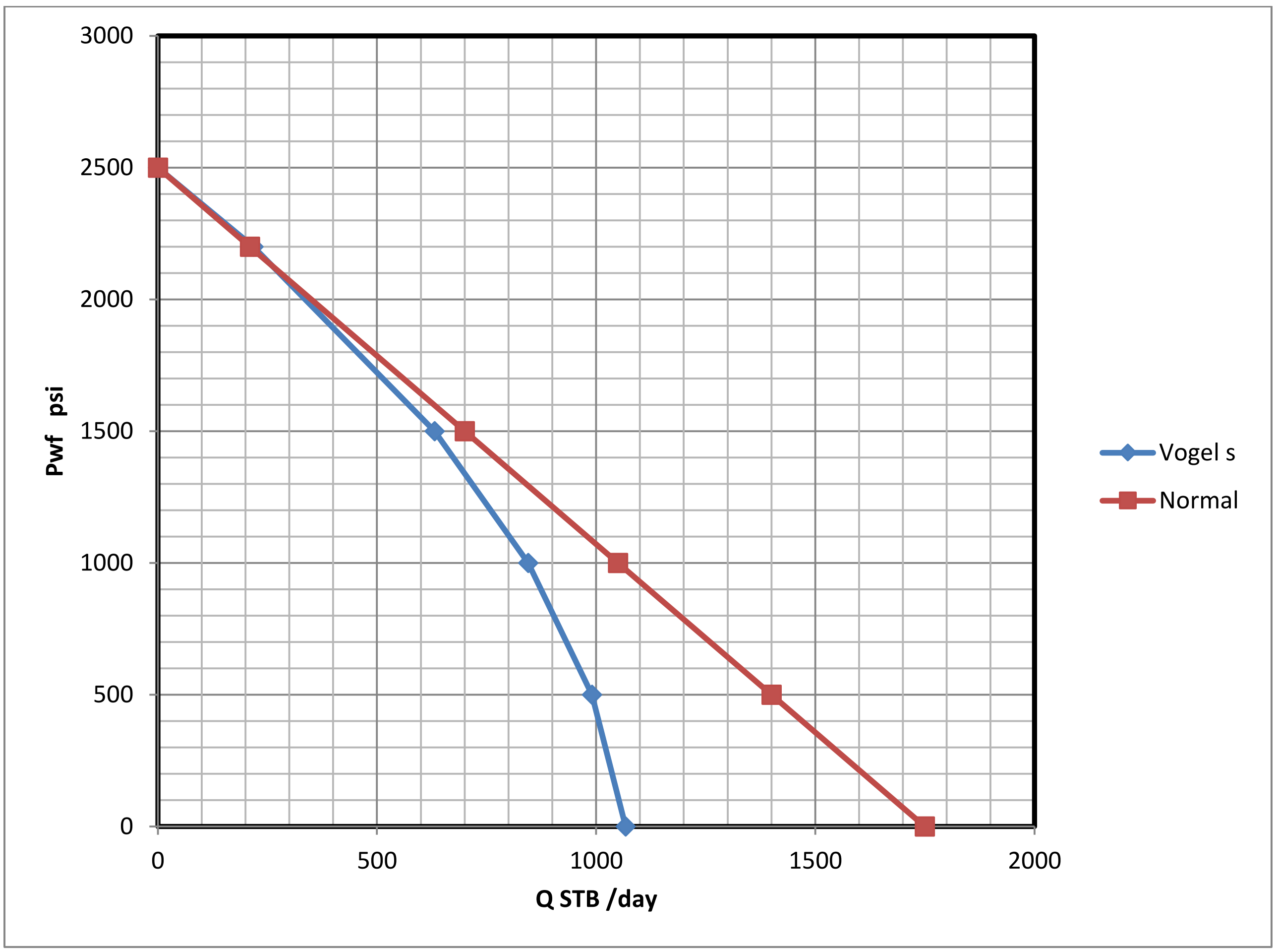
$$J = \frac{Q_o}{P_r - P_{wf}}$$

$$J = \frac{350}{2500 - 2000} = 0.7 \text{ STB /day / psi}$$

Step 2: Calculate Q_o
$Q_o = J\,(Pr - P_{wf}) = 0.7\,(2500 - 1850) = 455$ STB/day

Part C.
Generating the IPR by using the constant **J** approach and Vogel's method:
Assume several values for $\mathbf{P_{wf}}$ and calculate the corresponding $\mathbf{Q_o}$.

p_{wf}	Vogel`s	$Q_o = J(p_r - p_{wf})$
2500	0	0
2200	218.2	210
1500	631.7	700
1000	845.1	1050
500	990.3	1400
0	1067.1	1750



➢ Under-saturated Oil Reservoirs

Beggs (1991) pointed out that in applying Vogel's method for under-saturated reservoirs, there are **two possible outcomes to the recorded stabilized flow test data** that must be considered, as shown schematically in Figure (1-30):

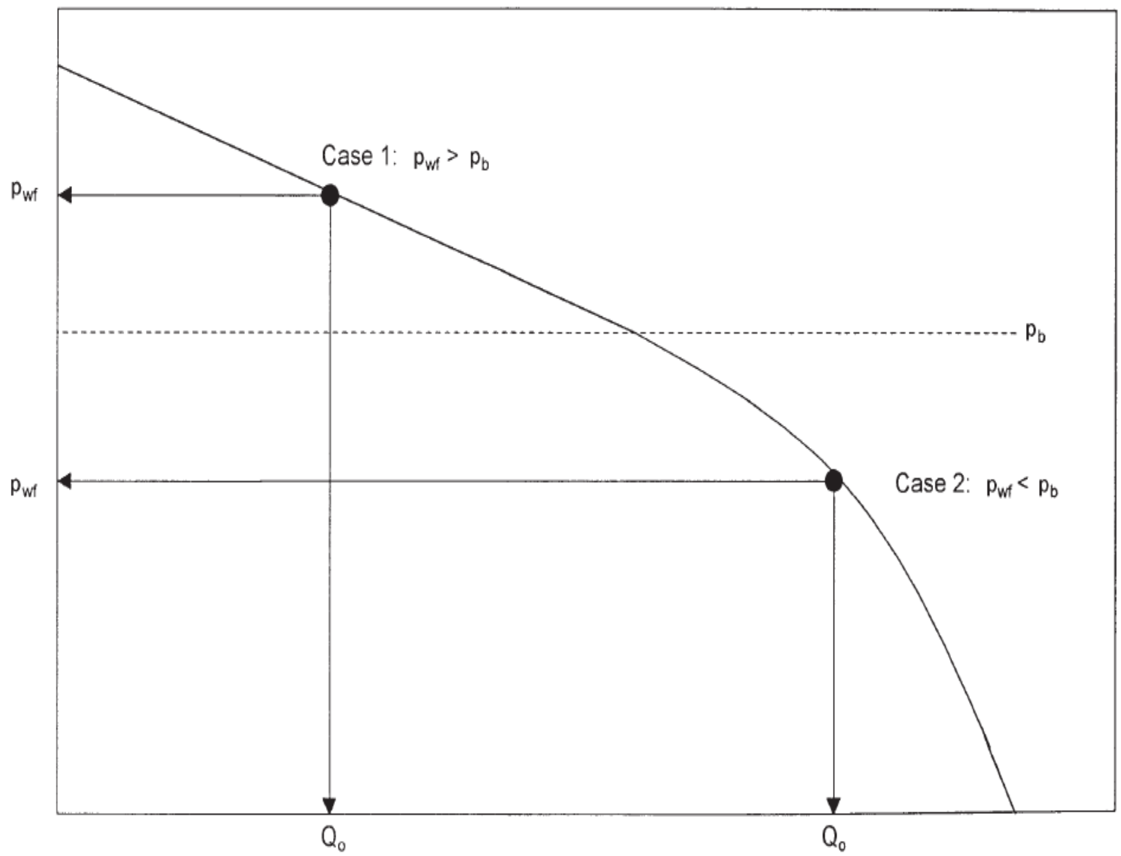


Fig. (1-30): Stabilized flow test data.

- The recorded stabilized bottom-hole flowing pressure is greater than or equal to the bubble-point pressure, i.e. $\mathbf{P_{wf} \geq P_b}$
- The recorded stabilized bottom-hole flowing pressure is less than the bubble-point pressure $\mathbf{P_{wf} < P_b}$

Case 1: The Value of the Recorded Stabilized $\mathbf{P_{wf} \geq P_b}$

Beggs outlined the following procedure for determining the IPR when the stabilized bottom-hole pressure is greater than or equal to the bubble point pressure Figure (1-30):

Step 1: Using the stabilized test data point ($\mathbf{Q_o}$ and $\mathbf{P_{wf}}$) calculate the productivity index **J**:

$$\mathbf{J = \frac{Q_o}{P_r - P_{wf}}}$$

Step 2: Calculate the oil flow rate at the bubble-point pressure:

$\mathbf{Q_{ob} = J\ (P_r - P_b)}$ -- **(1.16)**

Where:

Q_{ob}: is the oil flow rate at $\mathbf{P_b}$

Step 3: Generate the IPR values below the bubble-point pressure by assuming different values of $\mathbf{P_{wf}} < \mathbf{P_b}$ and calculating the corresponding oil flow rates by applying the following relationship:

$$\mathbf{Q_o = Q_{ob} + \frac{JP_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]} \quad \textbf{(1.17)}$$

The maximum oil flow rate (Q_{omax} or AOF) occurs when the bottomhole flowing pressure is zero, i.e. $P_{wf} = 0$, which can be determined from the above expression as:

$$\mathbf{Q_o = Q_{ob} + \frac{JP_b}{1.8}} \quad \textbf{(1.18)}$$

It should be pointed out that when $\mathbf{P_{wf} \geq P_b}$, the IPR is linear and is described by:

$\mathbf{Q_o = J(P_r - P_{wf})}$

Problem (1-4): An oil well is producing from an under-saturated reservoir that is characterized by a bubble-point pressure of **2130** psig. The current average reservoir pressure is **3000** psig. Available flow test data show that the well produced **250** STB/day at a stabilized P_{wf} of **2500** psig. Construct the IPR data.

Solution:

The problem indicates that the flow test data were recorded above the bubble-point pressure; therefore, the Case 1 procedure for under-saturated reservoirs as outlined previously must be used.

Step 1: Calculate J using the flow test data.

$$J = \frac{Q_o}{P_r - P_{wf}}$$

$$J = \frac{250}{3000 - 2500} = 0.5\ \text{STB/day/psi}$$

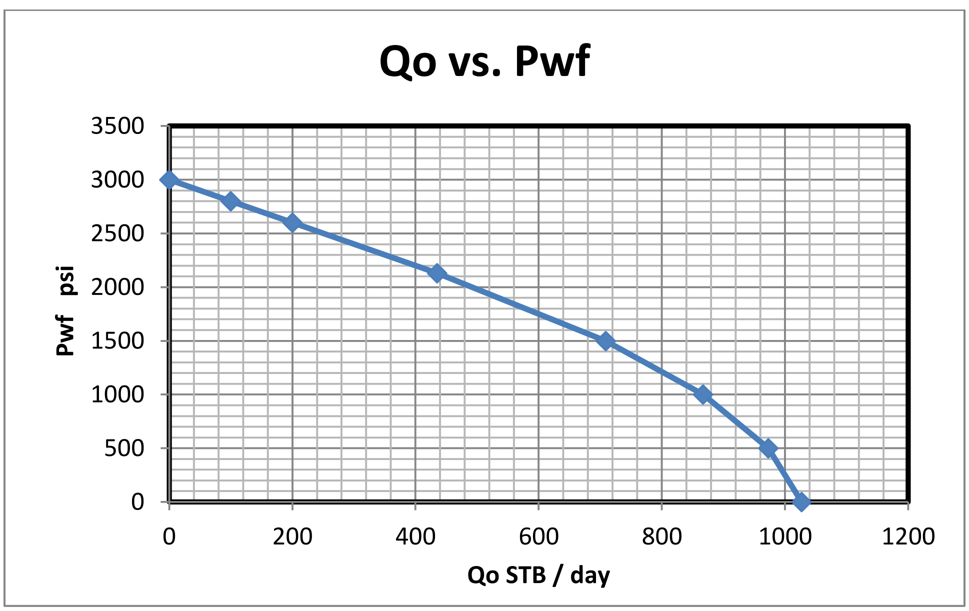
Step 2: Calculate the oil flow rate at the bubble-point pressure by applying

$Q_{ob} = J\,(P_r - P_b)$

$Q_{ob} = 0.5\,(3000 - 2130) = 435$ STB/day

Step 3: Generate the IPR data by applying the constant J approach for all pressures above $\mathbf{P_b}$ and equation (1.17) for all pressures below $\mathbf{P_b}$.

P_{wf}	Equation	Q_o
3000	(1.4)	0
2800	(1.4)	100
2600	(1.4)	200
2130	(1.4)	435
1500	(1.17)	709
1000	(1.17)	867
500	(1.17)	973
0	(1.17)	1027



Case 2: The Value of the Recorded Stabilized $P_{wf} < P_b$

When the recorded $\mathbf{P_{wf}}$ from the stabilized flow test is below the bubble- point pressure, as shown in Figure (1-30), the following procedure for generating the IPR data is proposed:

Step 1: Using the stabilized well flow test data and combining Equation (1.16) with (1.17), solve for the productivity index **J** to give:

$$J = \frac{Q_o}{(P_r - P_b) + \frac{P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]} \qquad (1.19)$$

Step 2: Calculate Q_{ob} by using Equation (1.16), or:

$$Q_{ob} = J\,(P_r - P_b)$$

Step 3: Generate the IPR for $\mathbf{P_{wf} \geq P_b}$ by assuming several values for $\mathbf{P_{wf}}$ above the bubble point pressure and calculating the corresponding $\mathbf{Q_o}$ from:

$$Q_o = J\,(P_r - P_{wf})$$

Step 4: Use equation (1.17) to calculate $\mathbf{Q_o}$ at various values of $\mathbf{P_{wf}}$ below $\mathbf{P_b}$, or:

$$Q_o = Q_{ob} + \frac{JP_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]$$

Problem (1-5): The well described in problem (1-4) was retested and the following results obtained:

P_{wf} = **1700** psig, Q_o = **630.7** STB/day

Generate the IPR data using the new test data.

Solution:

Notice that the stabilized P_{wf} is less than P_b

Step 1: Solve for J by applying equation (1.19).

$$J = \frac{Q_o}{(P_r - P_b) + \frac{P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2\right]}$$

$$J = \frac{630.7}{(3000 - 2130) + \frac{2130}{1.8}\left[1 - 0.2\left(\frac{1700}{3000}\right) - 0.8\left(\frac{1700}{3000}\right)^2\right]} = \mathbf{0.5}\ \text{STB/day/psi}$$

Step 2: Q_{ob} = 0.5 (3000 − 2130) = **435** STB/day

Step 3: Generate the IPR data.

P_{wf}	Equation	Q_o
3000	(1.4)	0
2800	(1.4)	100
2600	(1.4)	200
2130	(1.4)	435
1500	(1.17)	709
1000	(1.17)	867
500	(1.17)	973
0	(1.17)	1027

2) Standing's method

The initial work of Vogel assumed a flow efficiency of 1.00 and did not account for wells that were damaged or improved. Standing (1970) essentially extended the application of Vogel's (Vogel did not consider formation damage) proposed a companion chart to account for conditions where the flow efficiency was not equal to 1.00, as shown in Figure (1-31).

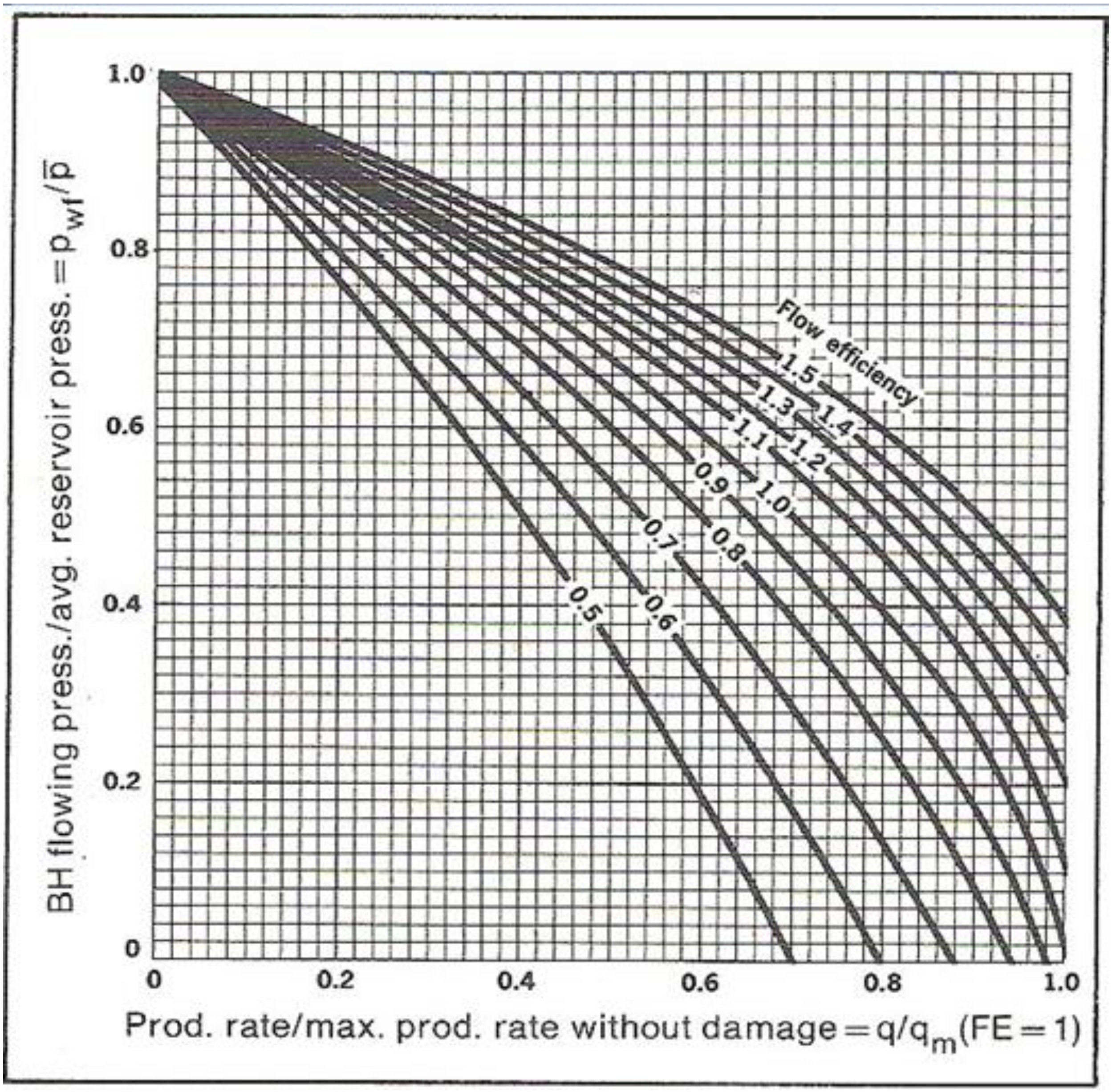


Fig. (1-31): Inflow performance relation, modified by standing.

Flow efficiency is defined as:

$$\mathbf{FE = \frac{Ideal\ drawdown}{Actual\ drawdown} = \frac{P_r - P'_{wf}}{P_r - P_{wf}}} \text{ ---------- } \mathbf{(1.19)}$$

Where:

$$\mathbf{P'_{wf} = P_{wf} + \Delta P_{skin}} \text{ ---------- } \mathbf{(1.20)}$$

Substituting:

$$\mathbf{FE = \frac{P_r - (P_{wf} + \Delta P_{skin})}{P_r - P_{wf}} = \frac{P_r - P_{wf} - \Delta P_{skin}}{P_r - P_{wf}}} \text{ ---------- } \mathbf{(1.21)}$$

Which of the ratio of useful pressure drop a cross the system to total pressure drop. For a well drianing a cylinderical volume:

$$\mathbf{FE = \frac{\ln \frac{0.47\ r_e}{r_w}}{\left[\ln \frac{0.47 r_e}{r_w} + s\right]}} \text{ ---------- } \mathbf{(1.22)}$$

Where:
S is the dimensionless skin factor.
The $\mathbf{\Delta P_{skin}}$ is thus seen to be the difference between $\mathbf{P`_{wf}}$ and $\mathbf{P_{wf}}$. There may be many factors which cause or control this added resistance to flow near the well-bore, **including invasion of the zone by mud or "Kill-fluids", swelling of shale, and others**. This may also represent a region of improvement after a stimulation treatment.
The determination of $\mathbf{\Delta P_{skin}}$ is made by first determining **S**, skin factor from a standard pressure build up test on a well. $\mathbf{\Delta P_{skin}}$ was defined by Van Everding as:

$$\mathbf{\Delta P_{skin} = 141.2 \frac{Q_o \mu_o B_o}{k_o h}} \text{ ---------- } \mathbf{(1.23)}$$

The standard equation for determining skin is:

$$\mathbf{S = 1.151 \left[\frac{P_{1hr} - P_{wf}}{m} - \log \frac{k}{\varphi \mu c r_w^2} + 3.23\right]} \text{ ---------- } \mathbf{(1.24)}$$

We may recall that:

S = 0 indicate no alteration.

S = + indicate damage.

S = - indicate improvement and that values of -3 to -5 are common for fractured reservoir.

The value of ΔP_{skin} is then calculated from:

$$\Delta P_{skin} = 0.87\ S\ m \quad \text{------------------} \quad (1.25)$$

m = slope from straight line portion of the pressure build up curve, determined from the following equation:

$$m = \frac{162.5 q_o \mu_o B_o}{k_o h} \quad \text{------------------} \quad (1.26)$$

Standing constructed Figure (1-31), which shows IPR curves for flow efficiencies between 0.5 and 1.5. Several things can be obtained from this plot:

1. The maximum rate possible for a well with damage.
2. The maximum rate possible if the damage is removed and FE = 1.0
3. The rate possible if the well is stimulated and improved.
4. The determination of the flow rate possible for any flowing pressure for different values of FE.
5. The construction of IPR curves to show rate versus flowing pressure for damaged and improved wells.

Figure (1-31) can be slightly confusing if not studied carefully. The abscissa is the ratio of the producing rate divided by the producing rate with no damage that is, each value that is read from the curves is a value to calculate Q_{omax} with FE corrected to 1.

Equation (1.13) may be simplified to the following form:

$$\frac{Q_{o\ FE=j}}{(Q_o)_{max\ FE=1}} = j\,(1-R)[1.8 - 0.8j(1-R)] \quad \text{------------------} \quad (1.27)$$

Where:

$$R = \frac{P_{wf}}{P_r}$$

Equation (1.27) can replaced Standing`s` chart for IPR of damaged/stimulation wells. plotting equation (1.27) for j = 0.6, 0.8, ..., 1.6 reproduced Standing`s chart as shown in Figure (1-31).

Problem (1-6): Given the following information:

Q_o =**70** bbl/day, P_r = **2400** psi, P_{wf} = **1800** psi, FE= **0.7**.

Our first requirement is to find the maximum flow rate possible assuming the well has no damage (FE=1).

$Q_{o\,(FE=0.7)} / Q_{omax\,(FE=1)} = 0.281$

$Q_{omax\,(FE=1)} = 70/0.281 = 249$ bbl/day.

Our next requirement is to find the maximum flow rate from the damaged well.

The maximum rate occurs when $P_{wf} = 0$, then $P_{wf} / P_r = 0$, and from Figure (1.31) curve we find $Q_o / Q_{omax} = 0.87$, then $Q_o = 0.87 * 249 = 216$ bb/day.

$Q_{o\,(FE=0.7)} \neq (0.7) Q_{omax\,(FE=1)}$ because of the non-linear IPR relationship for solution gas drive reservoir system.

Our next requirement is to find the maximum flow rate if the well is improved

Assume that a stimulation job is performed on the above well and that FE is increased to 1.3. What is the maximum rate possible?

$Q_{omax\,(FE=1)} = 249$ bbl/day (from curve) for $P_{wf} = 0$, then $(P_{wf} / P_r) = 0$, and from Figure (1-31) on the FE = 1.3 curve (by extrapolation) $Q_o / Q_{omax} = 1.1$, then $Q_{omax\,(FE=1.3)} = 1.1 * 249 = 274$ bb/day.

The extrapolation of the curves is not recommended since they appear to give erroneous results for values on the abscissa greater than 1. The solution by equation (1.27) does not appear correct either. Fortunately in practices we normally do not need values of Q_o / Q_{omax} greater than 1 and the curves and equation handle these problems in a satisfactory manner.

➢ Predict Future Inflow Performance Relationship

To predict future inflow performance relationship of a well as a function of reservoir pressure, Standing noted that Vogel's equation (1.13) could be rearranged as:

$$\frac{Q_o}{(Q_o)_{max}} = 1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2$$

$$\frac{\mathbf{Q_o}}{\mathbf{(Q_o)_{max}}} = \left(\mathbf{1} - \frac{\mathbf{P_{wf}}}{\mathbf{P_r}}\right)\left[\mathbf{1} + \mathbf{0.8}\left(\frac{\mathbf{P_{wf}}}{\mathbf{P_r}}\right)\right] \text{ ----------- } \mathbf{(1.28)}$$

Standing introduced the productivity index ***J*** as defined by equation (1.1) into equation (1.28) to yield:

$$\mathbf{J} = \frac{\mathbf{(Q_o)_{max}}}{\mathbf{P_r}}\left[\mathbf{1} + \mathbf{0.8}\left(\frac{\mathbf{P_{wf}}}{\mathbf{P_r}}\right)\right] \text{ ----------- } \mathbf{(1.29)}$$

Standing then defined the present (current) zero drawdown productivity index as:

$$\mathbf{J_p^*} = \mathbf{1.8}\left[\frac{\mathbf{(Q_o)_{max}}}{\mathbf{P_r}}\right] \text{ ----------- } \mathbf{(1.30)}$$

Where J^*_p is Standing's zero-drawdown productivity index. The J^*_p is related to the productivity index ***J*** by:

$$\frac{\mathbf{J}}{\mathbf{J_p^*}} = \frac{\mathbf{1}}{\mathbf{1.8}}\left[\mathbf{1} + \mathbf{0.8}\left(\frac{\mathbf{P_{wf}}}{\mathbf{P_r}}\right)\right] \text{ ----------- } \mathbf{(1.31)}$$

Equation (1.1) permits the calculation of **J^*p** from a measured value of **J**.

To arrive to the final expression for predicting the desired IPR expression, Standing combines equation (1.31) with equation (1.28) to eliminate **$(Q_o)_{max}$** to give:

$$\mathbf{Q_o} = \left[\frac{\mathbf{J_f^*(P_r)_f}}{\mathbf{1.8}}\right]\left\{\mathbf{1} - \mathbf{0.2}\left[\frac{\mathbf{P_{wf}}}{\mathbf{(P_r)_f}}\right] - \mathbf{0.8}\left[\frac{\mathbf{P_{wf}}}{\mathbf{(P_r)_f}}\right]^2\right\} \text{ ----------- } \mathbf{(1.32)}$$

Where the subscript **f** refer to future condition.

Standing suggested that J^*_f can be estimated from the present value of J^*_p by the following expression:

$$\mathbf{J_f^*} = \mathbf{J_p^*}\frac{\mathbf{(k_{ro}/\mu_o B_o)_f}}{\mathbf{(k_{ro}/\mu_o B_o)_p}} \text{ ----------- } \mathbf{(1.33)}$$

Where the subscript **p** refer to present condition.

If the relative permeability data is not available, J^*_f can be roughly estimated from:

$$\mathbf{J_f^*} = \mathbf{J_p^*}\left[\frac{\mathbf{(P_r)_f}}{\mathbf{(P_r)_p}}\right]^2 \text{ ----------- } \mathbf{(1.34)}$$

Standing's methodology from predicting a future IPR is summarized in the following steps:

1) Using the current time condition and the available flow test data, calculate $(Q_o)_{max}$ from equation (1.13) or (1.28).
2) Calculate J^* at the present condition, i.e., J^*_p, by using equation (1.30). Notice that other combinations of equations (1.28) through (1.31) can be used to estimate J^*_p.
3) Using fluid property, saturation and relative permeability data, calculate both $[k_{ro}/ \mu_o B_o]_f$ and $[k_{ro}/ \mu_o B_o]_p$.
4) Calculate J^*_f by using equation (1.33). Use equation (1.34) if the oil relative permeability data is not available.
5) Generate the future IPR by applying equation (1.32).

Problem (1-7): A well is producing from a saturated oil reservoir that exists at its saturation pressure of **4000** psig. The well is flowing at a stabilized rate **600** bbl/day and a P_{wf} = **3200** psig. Material balance calculations provide the following current and future predictions for oil saturation and PVT properties.

Parameter	Present	Future
p_r	4000	3000
μ_o, cp	2.4	2.2
B_o, bbl/STB	1.2	1.13
k_{ro}	1	0.66

Generate the future IPR for the well at **3000** psig by using Standing's method.

Solution:

Calculate the current $(Q_o)_{max}$ from equation (1.28).

$$(Q_o)_{max} = \frac{Q_o}{\left(1-\frac{P_{wf}}{P_r}\right)\left[1+0.8\left(\frac{P_{wf}}{P_r}\right)\right]}$$

$$(Q_o)_{max} = \frac{600}{\left(1-\frac{3200}{4000}\right)\left[1+0.8\left(\frac{3200}{4000}\right)\right]} = \mathbf{1829}\text{ STB / day}$$

Step 2: Calculate $\mathbf{J^*_p}$ by using equation (1.30).

$$J^*_p = 1.8\left[\frac{(Q_o)_{max}}{P_r}\right]$$

$J_p^* = 1.8 \left[\frac{1829}{4000}\right] = 0.823$

Calculate the following pressure-function:

$[k_{ro}/ \mu_o B_o]_p = [1 / 2.4^* 1.2]_p = 0.3472$

$[k_{ro}/ \mu_o B_o]_f = [0.66 / 2.2^* 1.15]_f = 0.2609$

Calculate J^*_f by applying equation (1.33)

$J^*_f = 0.832 [0.2609] / [0.3472] = 0.618$

Generate the IPR by using equation (1.32)

P_{wf}	(Q_o) STB / day
3000	0
2000	527
1500	721
1000	870
500	973
0	1030

It should be noted that one of the main disadvantages of Standing's methodology is that it requires reliable permeability information; in addition, it also requires material balance calculations to predict oil saturations at future average reservoir pressures.

3) Couto`s method

He suggested a procedure to solve for flow efficiency (FE) from two flow tests on the well. His procedure makes use of Vogel's equation and dose requires that we known $\mathbf{P_r}$.

From Standing's` work;

$$\mathbf{FE} = \frac{P_r - P'_{wf}}{P_r - P_{wf}}$$

$$\mathbf{FE} = \frac{P_r - P_{wf} - P_{skin}}{P_r - P_{wf}}$$

Since Standing assumed a constant skin value (s, independent of rate and time). Then it should obtain the same FE value from each flow test. Therefore, in general, this solution

(Couto`s method) is **trial and error solution**, in that a value of FE is assumed and a value of $(Q_o)_{max}$ is solved for each flow test, others values are assumed until the same $(Q_o)_{max}$ value are obtained from each flow test.

Recalling Vogel`s equation;

$$\frac{Q_o}{(Q_o)_{max}} = 1 - 0.2\left(\frac{P_{wf}}{P_r}\right) - 0.8\left(\frac{P_{wf}}{P_r}\right)^2$$

In the form used by Standing we can write the equation;

$$\frac{\mathbf{Q_{o\ FE=j}}}{\mathbf{(Q_o)_{max\ FE=1}}} = \mathbf{1 - 0.2\left(\frac{P'_{wf}}{P_r}\right) - 0.8\left(\frac{P'_{wf}}{P_r}\right)^2} \text{ -- } \mathbf{(1.35)}$$

Where j = value of FE and $P`_{wf}$ is the ideal flowing pressure.

From equation (1.35) we can write;

$$\frac{\mathbf{P'_{wf}}}{\mathbf{P_r}} = \mathbf{1 - FE + FE\left(\frac{P_{wf}}{P_r}\right)} \text{ -- } \mathbf{(1.36)}$$

By substituting equation (1.35) in equation (1.36);

$$\frac{\mathbf{Q_{o\ FE=j}}}{\mathbf{Q_{omax\ FE=1}}} = \mathbf{1 - 0.2\left(1 - FE + FE\left(\frac{P_{wf}}{P_r}\right)\right) - 0.8\left(1 - FE + FE\left(\frac{P_{wf}}{P_r}\right)\right)^2} \text{ ---------------- } \mathbf{(1.37)}$$

Problem (1-8)

P_r = 2000 psi

Test	Q_o	P_{wf}
1	165	1500
2	298	1000

Find the FE for the well.

Solution:

$$\frac{P_{wf1}}{P_r} = \frac{1500}{2000} = 0.75$$

$$\frac{P_{wf2}}{P_r} = \frac{1000}{2000} = 0.5$$

$$\frac{P'_{wf}}{P_r} = 1 - FE + FE(0.75) = 1 - 0.25\ FE$$

$$\frac{P'_{wf}}{P_r} = 1 - FE + FE(0.5) = 1 - 0.5\ FE$$

Assume a value of FE = 0.6

Then: $\frac{P'_{wf}}{P_r} = 1 - 0.25\ \text{FE} = 0.85$

$\frac{P'_{wf}}{P_r} = 1 - 0.5\ \text{FE} = 0.7$

Now use equation (1.35) for test No. 1 and test No. 2:

$\frac{Q_{o\ FE=0.6}}{(Q_o)_{max\ FE=1}} = 1 - 0.2(0.85) - (0.85)^2 = 0.252$

$\frac{Q_{o\ FE=0.6}}{(Q_o)_{max\ FE=1}} = 1 - 0.2(0.7) - (0.7)^2 = 0.468$

In the same manner values are calculated for assumed values of FE = 0.7, 0.8, 0.9, 1, 1.1, 1.2. These are noted in the following table:

FE values	$Q_o / Q_{o\ max}$ (test no.1	$Q_o / Q_{o\ max}$ (test no.2)	Ratio
0.6	0.252	0.468	0.539
0.7	0.29	0.532	0.545
0.8	0.328	0.592	0.554
0.9	0.364	0.648	0.562
1	0.4	0.7	0.571
1.1	0.434	0.748	0.58
1.2	0.468	0.792	0.591

From our table the ratio of Q_{o1} / Q_{o2} $^{(FE = j)}$ = 165/298 = 0.554
And this occurs at a value of j = 0.8 as noted in the table.

4) Al Saadoon`s method

This method is used to predict the present and future IPR based on one value of flow test data, where:

PI = *J* = - $\partial Q / \partial p_{wf}$ -- (1.38)

The **PI** is not constant in a two-phase reservoir liquid and gas flow, therefore, it conclude that a plot of $\mathbf{P_{wf}}$ versus $\mathbf{Q_o}$ will not yield a straight line but a curved line concave to the origin. It appears, therefore, that the **PI** is subject to change at any specific time under differing drawdown conditions. It also appears that the PI changes throughout the life of a well for a particular drawdown condition.

If J cannot be assumed to be constant (as for solution-gas-drive reservoirs operating at or below the saturation pressure), then J can be obtained, by derivative Vogel's equation curve (1.13) with respect to $\mathbf{P_{wf}}$, as shown;

$$\mathbf{J = \frac{(Q_o)_{max}}{5P_r}\left(1 + 8\frac{P_{wf}}{P_r}\right)} \quad \text{------} \quad \mathbf{(1.39)}$$

For present IPR, $J = J^*_p$ when $p_{wf} \rightarrow p_r$;

$$\mathbf{J^*_P = \frac{9}{5}\left(\frac{(Q_o)_{max}}{P_r}\right)} \quad \text{------} \quad \mathbf{(1.0)}$$

$$\mathbf{\frac{J}{J^*_P} = \frac{1}{9}\left(1 + 8\frac{P_{wf}}{P_r}\right)} \quad \text{------} \quad \mathbf{(1.41)}$$

A single generalized dimensionless graph is constructed with (Q_o / Q_{omax}) and (J / J^*) on the ordinate and (p_{wf} / p_r) on the abscissa, as shown in Figure (1-32).

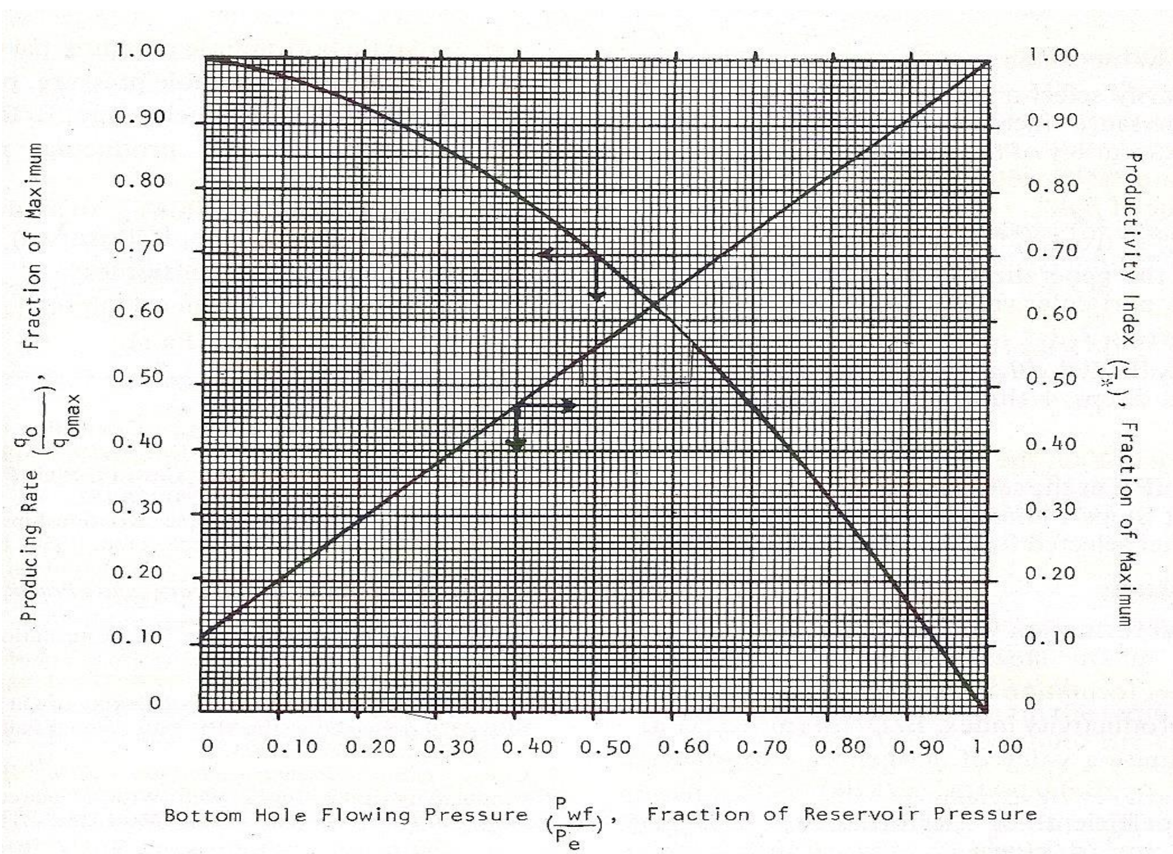


Fig. (1-32): Inflow performance relationship and productivity index relationship, (Al Saadoon)

Thus, the calculation procedure to be followed in developing present and future IPR from a single data point is as follows.

To predict present IPR curve it should be follow this steps;

1) Use Figure (1-32) to find (Q_o / Q_{omax}) and (J / J^*_p) value at any particular value of (p_{wf} / p_r).
2) Calculate (Q_{omax}) from (Q_o / Q_{omax}).
3) Find J^*_p from (Q_{omax}) by use equation (1.40).
4) Find J from (J / J^*_p).
5) Again use Figure (1-32) to find (Q_o/Q_{omax}) and (J/J^*_p) value at any other values of (p_{wf} / p_r).
6) Find (Q_o) from (Q_o / Q_{omax}).
7) Find J from (J / J^*_p).
8) Repeat steps 5 through 7 at other values of (p_{wf} / p_r).
9) Plot (Q_o) and (J) on Y-axis versus (p_{wf}) on X-axis, this plot is the present IPR curve.

To predict future IPR curve it should be follow this steps;

1) Arbitrarily select a future value of static reservoir pressure (average reservoir pressure, p_r). The use of small pressure increment is recommended to increase the accuracy of the calculations.
2) Compute J^*_f from J^*_p by using equation (1.33) or (1.34) at the selected value of (p_r) in step 1.
3) Find (Q_{omax}) from J^*_f by using equation (1.40).
4) Using Figure (1-32) to find (Q_o / Q_{omax}) and (J / J^*_f) value at any particular value of (p_{wf} / p_r).
5) Find J from (J / J^*_f).
6) Find (Q_o) from (Q_o / Q_{omax}).
7) Repeat steps 4 through 6 at other values of (p_{wf} / p_r).
8) Plot (Q_o) and (J) on the Y-axis versus (p_{wf}) on X-axis, this plot is the well`s IPR curve at the selected future value (p_r) in step 1.
9) Repeat steps 1 through 8 to plot the well`s IPR`s curve at other selected future values of (p_r), for different time periods in future.

Problem (1-8): given the following information:

Q_o =**70** bbl/day, P_r = **2400** psi, P_{wf} = **1800** psi. Construct present and future IPR.

Solution:

1) Present IPR

$$(Q_o)_{max} = \frac{70}{0.47} = 148.94 \text{ bbL / day}$$

$$J_P^* = \frac{9}{5}\frac{(Q_o)_{max}}{P_r} = \frac{148.94}{2400} = 0.11$$

P_{wf}	P_r	P_{wf}/P_r	Q_o/Q_{omax}	J / J_P	Q_o	Q_{omax}	J_P	J
2400	2400	1.00	0	1	0.00	148.94	0.11	0.11
2000	2400	0.83	0.28	0.85	41.70	148.94	0.11	0.09
1680	2400	0.70	0.47	0.73	70.00	148.94	0.11	0.08
1440	2400	0.60	0.595	0.645	88.62	148.94	0.11	0.07
960	2400	0.40	0.795	0.47	118.40	148.94	0.11	0.05
480	2400	0.20	0.93	0.29	138.51	148.94	0.11	0.03
0	2400	0.00	1	0.11	148.94	148.94	0.11	0.01

2) Future IPR

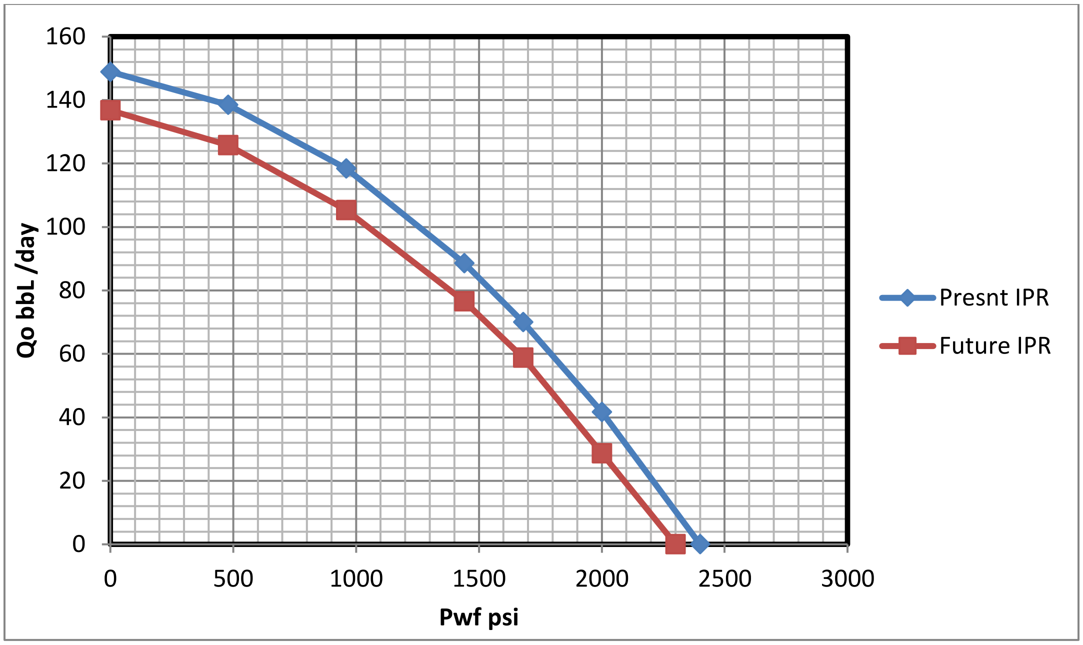
P_r = **2300** psi

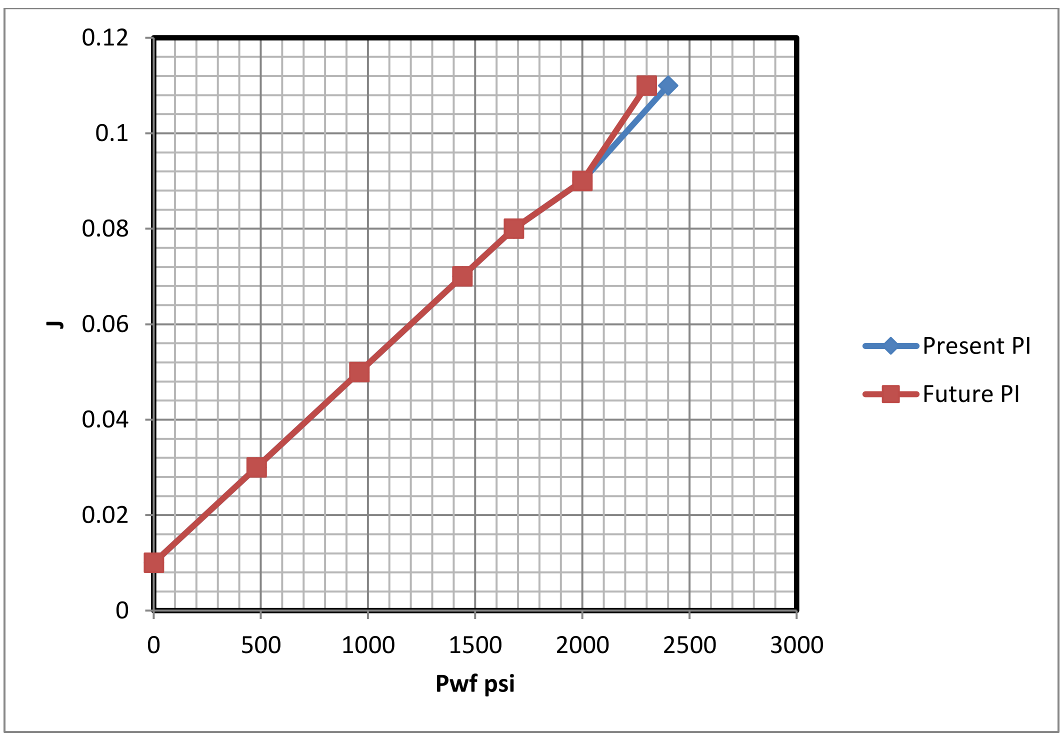
$$J_f^* = J_p^*\left[\frac{(P_r)_f}{(P_r)_p}\right]^2 = 0.11$$

$$(Q_o)_{max} = \frac{5}{9}(P_r \times J_f^*) = 136.78 \text{ bbL/day}$$

P_{wf}	P_r	P_{wf}/P_r	Q_o/Q_{omax}	J / J_F	Q_o	Q_{omax}	J_P	J_F	J
2300	2300	1.00	0.00	1.00	0.00	136.78	0.12	0.11	0.11
2000	2300	0.87	0.21	0.88	28.72	136.78	0.12	0.11	0.09
1680	2300	0.73	0.43	0.76	58.82	136.78	0.12	0.11	0.08
1440	2300	0.63	0.56	0.67	76.60	136.78	0.12	0.11	0.07
960	2300	0.42	0.77	0.49	105.32	136.78	0.12	0.11	0.05
480	2300	0.21	0.92	0.30	125.84	136.78	0.12	0.11	0.03
0	2300	0.00	1.00	0.11	136.78	136.78	0.12	0.11	0.01

Figures below shows the relationship between $\mathbf{Q_o}$ and J versus $\mathbf{P_{wf}}$.





5) Fetkovich method

Fetkovich Proposed a method for calculating the inflow performance for oil wells using the same type of equation that has been used for analyzing gas wells for many years. The procedure was verified by analyzing isochronal and flow-after-flow tests conducted in reservoirs with permeabilities ranging from **6** md to greater than **1000** md. Pressure conditions in the reservoirs ranged from **highly undersaturated to saturated at initial pressure and to a partially depleted field with a gas saturation above the critical**.

In all cases, oil-well back-pressure curves were found to follow the same general form as that used to express the inflow relationship for a gas well. That is:

$$\mathbf{q_o = C\left(P_r^2 - P_{wf}^2\right)^n} \text{ --- } \mathbf{(1.42)}$$

Where:

q_o: producing rate

P_r: average reservoir pressure

P_{wf}: flowing wellbore pressure

C: flow coefficient

n: exponent depending on well characteristic

The value of **n** ranged from **0.568 to 1.000** for the 40 field tests analyzed by Fetkovich. The applicability of eq. (1.42) to oil well analysis was justified by writing Darcy's equation as:

$$\mathbf{q = \frac{0.00708\,kh}{\ln(0.472 r_e/r_w)s} \int_{P_{wf}}^{P_r} f(p)dp} \text{ --- } \mathbf{(1.43)}$$

$$f(p) = \frac{k_{ro}}{\mu_o B_o}$$

For an undersaturated reservoir, the integral is evaluated over two regions as:

$$\mathbf{q = C\int_{P_{wf}}^{P_b} f_1(p)dp + \int_{P_b}^{P_r} f_2(p)dp} \text{ --- } \mathbf{(1.44)}$$

Where:

$$C = \frac{0.00708\,kh}{\ln(0.472 r_e/r_w)s}$$

It was assumed that for **P > P_b**, **k_{ro}** is equal to **one** and that **μ_o** and **B_o** could be considered constant at $\acute{P} = \frac{P_r + P}{2}$

It was also assumed that for **P < P_b, f(p)** could be expressed as a linear function of pressure, that is:

f(p)= ap+b -- (1.45)

Making these substitutions into eq. (1.43) and integrating gives:

$$\mathbf{q_o = C_1(P_b^2 - P_{wf}^2) + C_2(P_r - P_b)} \text{ ------------------------- } \mathbf{(1.46)}$$

Fetkovich then stated that the composite effect results in an equation of the form:

$$\mathbf{q_o = C(P_r^2 - P_{wf}^2)^n} \text{ ------------------------- } \mathbf{(1.42)}$$

Once values for **C** and **n** are determined from test data eq. (1.42) can be used to generate a complete IPR. As there are **two** unknowns in eq. (1.42), at least **two** tests are required to evaluate **C** and **n**, assuming **P_r** is unknown. However, in testing gas wells it has been customary to use at least **four** flow tests to determine **C** and **n** because of the possibility of data errors. This is also recommended for oil well testing.

By taking the log of both sides of eq. (1.42) and solving for log $(P_r^2 - P_{wf}^2)$ the expression can be written as:

$$\mathbf{\log(P_r^2 - P_{wf}^2) = \frac{1}{n}\log q_o - \frac{1}{n}\log C}$$

A plot of $\mathbf{(P_r^2 - P_{wf}^2)}$ versus **q_o** on log-log scales will result in a straight line having a slope of **1/n** and an intercept of **q_o = C at** $(\mathbf{P_r^2 - P_{wf}^2}) = \mathbf{1}$. The value of **C** can also be calculated using any point on the linear plot once **n** has been determined. That is:

$$\mathbf{C = \frac{q_o}{(P_r^2 - P_{wf}^2)^n}}$$

Three types of tests are commonly used for gas-well testing to determine **C** and **n**. These tests can also be used for oil wells and will be described in this section. The type of test to choose depends on the **stabilization time** of the well, which is a function of the **reservoir permeability**. If a well stabilizes fairly rapidly, a conventional flow after-flow test can be conducted.

- For tight wells, an isochronal test may be preferred.

- For wells with very long stabilization times, a modified isochronal test may be more practical.

The stabilization time for a well in the center of a circular or square drainage area may be estimated from:

$$\mathbf{t_p = \frac{380\emptyset\mu_o C_t A}{k_o}}$$

t_p: stabilization time, hrs

φ = porosity

C_t = total fluid compressibility, psi^{-1}

A = drainage area, ft^2

k_o = permeability to oil, md

μ_o= oil viscosity, cp

1. **Flow-After-Flow, testing**

A flow-after-flow test begins with the well shut in so that the pressure in the entire drainage area is equal to $\mathbf{P_r}$. The well is placed on production at a constant rate until the flowing wellbore pressure becomes constant. The flowing pressure should be measured with a bottomhole pressure gage, especially for oil-well tests. Once $\mathbf{P_{wf}}$ has stabilized, the production rate is changed, and the procedure is repeated for several rates. The idealized behavior of production rate and wellbore pressure with time is shown in Figure (1-33). The test may also be conducted using a decreasing rate sequence.

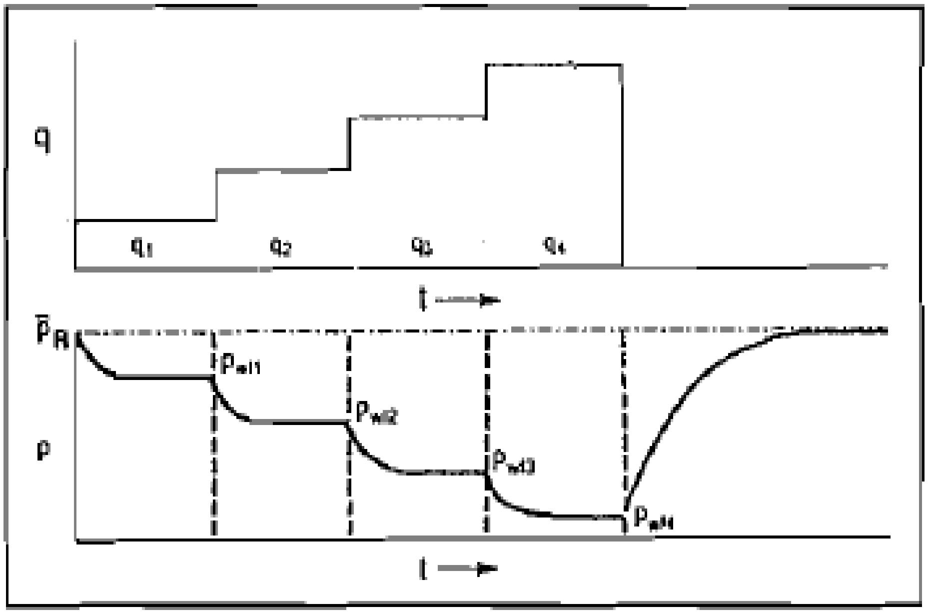


Fig. (1-33): Conventional test-producing rate and pressure diagrams.

The test is analyzed by $\left(\mathbf{P_r^2 - P_{wf}^2}\right)$ versus $\mathbf{q_o}$ on log-log coordinates and drawing the best straight line through the points. The exponent **n** is determined from the reciprocal of the slope of the line, That is:

$$n = \frac{\Delta \log q_o}{\Delta \log\left(P_r^2 - P_{wf}^2\right)}$$

It is common practice to read the change in $\mathbf{q_o}$ over one leg cycle of change in$\left(\mathbf{P_r^2 - P_{wf}^2}\right)$, since the difference in the log value over one cycle is equal to one.

2. Isochronal Testing

If the time required for the well to stabilize on each choke size or producing rate is excessive, an isochronal or equal time test is preferred. The procedure for conducting an isochronal test is;

1. Starting at a shut-in condition, open the well on a constant production rate and measure pat specific time periods. The total production period for each rate may be less than the stabilization time.
2. Shut the well in and allow the pressure to build up to $\mathbf{P_r}$.
3. Open the well on another producing rate and measure the pressure at the same time intervals.

4. Shut the well in again until $P_{ws} = P_r$.
5. Repeat this procedure for several rates,

The values $\left(P_r^2 - P_{wf}^2\right)$ determined at the specific time periods are plotted versus q_o and **n** is obtained from the slope of the line. To determine a value for **C**, one test must be a stabilized test. The idealized behavior of producing rate and pressure as a function of time is shown in Figure (1-34).

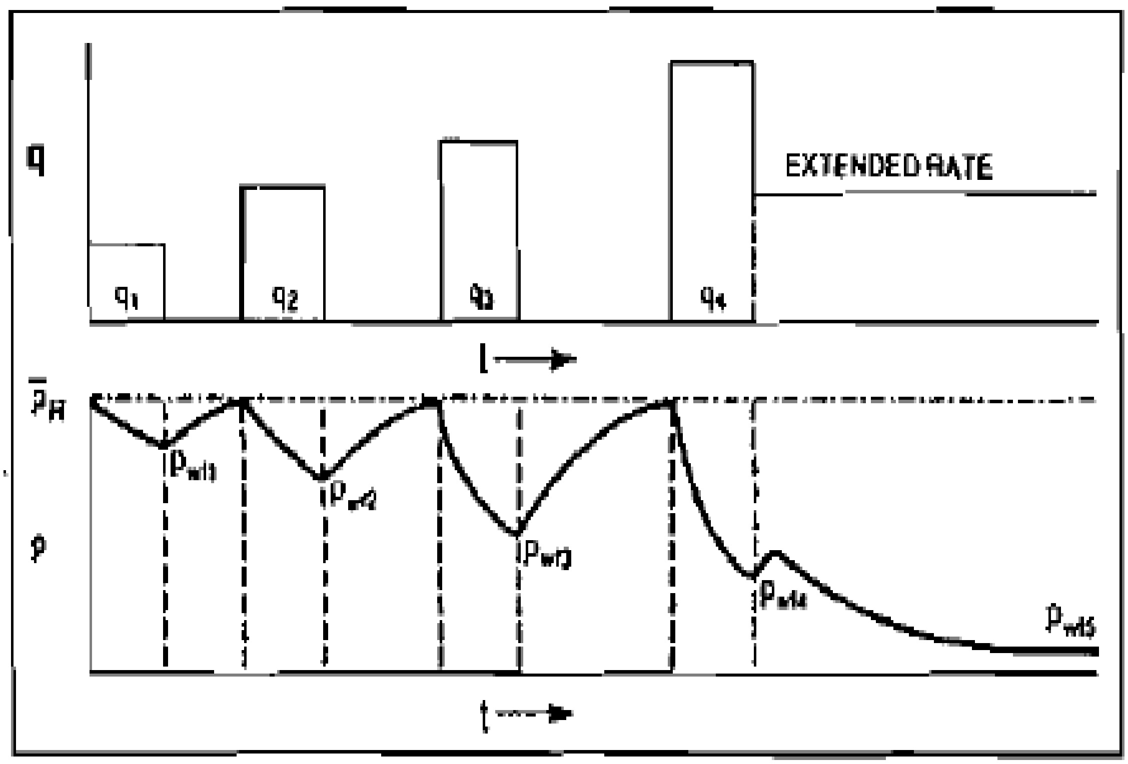


Fig. (1-34): Isochronal test-producing rate and pressure diagrams.

3. Modified Isochronal Testing

If the shut-in time required for the pressure to build back up to P_r between flow periods is excessive, the isochronal test may be modified. The modification consists of shutting the well in between each flow a period of time equal to this producing time. The static well bore pressure P_{ws}, may not reach P_r, but a plot of $\left(P_r^2 - P_{wf}^2\right)$ versus q_o, will usually produce a straight line, from which **n** may be obtained. A stabilized test is still required to calculate a value for **C**. The testing procedure is illustrated in Figure (1-35).

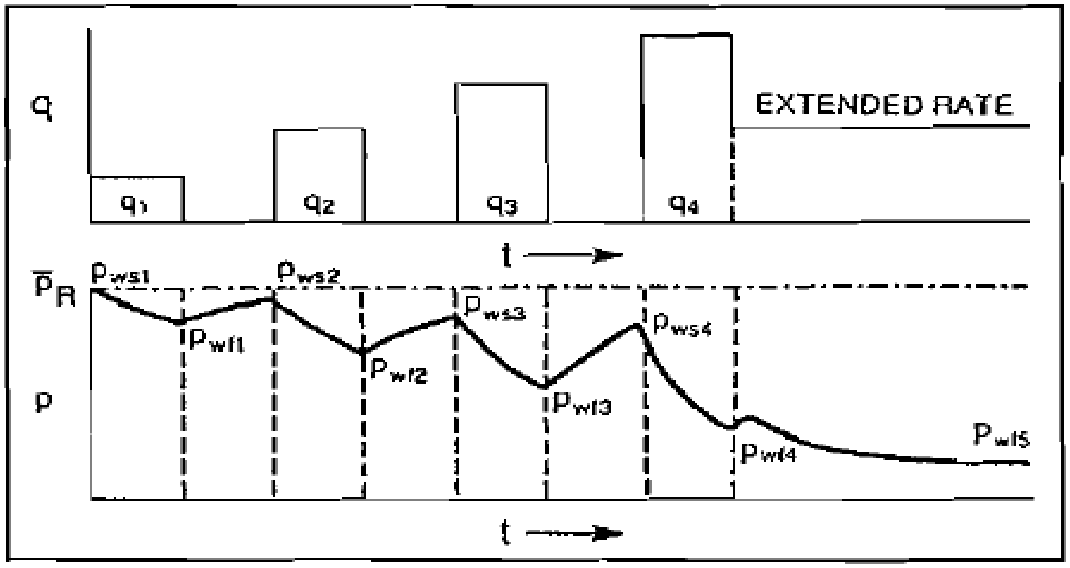


Fig. (1-35): Modified isochronal test-producing rate and pressure diagrams.

To construct the future IPR when the average reservoir pressure declines to $(p)_f$, Fetkovich assumes that the performance coefficient **C** is a linear function of the average reservoir pressure and, therefore, the value of **C** can be adjusted as:

$$(C)_f = (C)_p \left[(p_r)_f / (p_r)_p\right] \qquad (1.47)$$

Problem (1-9)

A four-point stabilized flow test was conducte on a well producing from reservoir in which P_r= **3600** psia. The test results were:

q_o STB / day	P_{wf} psi
263	3170
383	2897
497	2440
640	2150

1. Construct a complete IPR for this well and determine $q_{o\,(max)}$.
2. Construct the IPR when the reservoir pressure declines to **2000** psi.

Solution:

Part A

Step1: Construct the following table:

q_o STB / day	P_{wf} psi	$(P_r^2 - P_{wf}^2) \times 10^{-6}$, psi^2
263	3170	2.911
383	2890	4.567
497	2440	7.006
640	2150	8.338

Step 2: Plot $(P_r^2 - P_{wf}^2)$ versus **Q** on log-log paper as shown in Figure (1-36) and determine the exponent **n**, or:

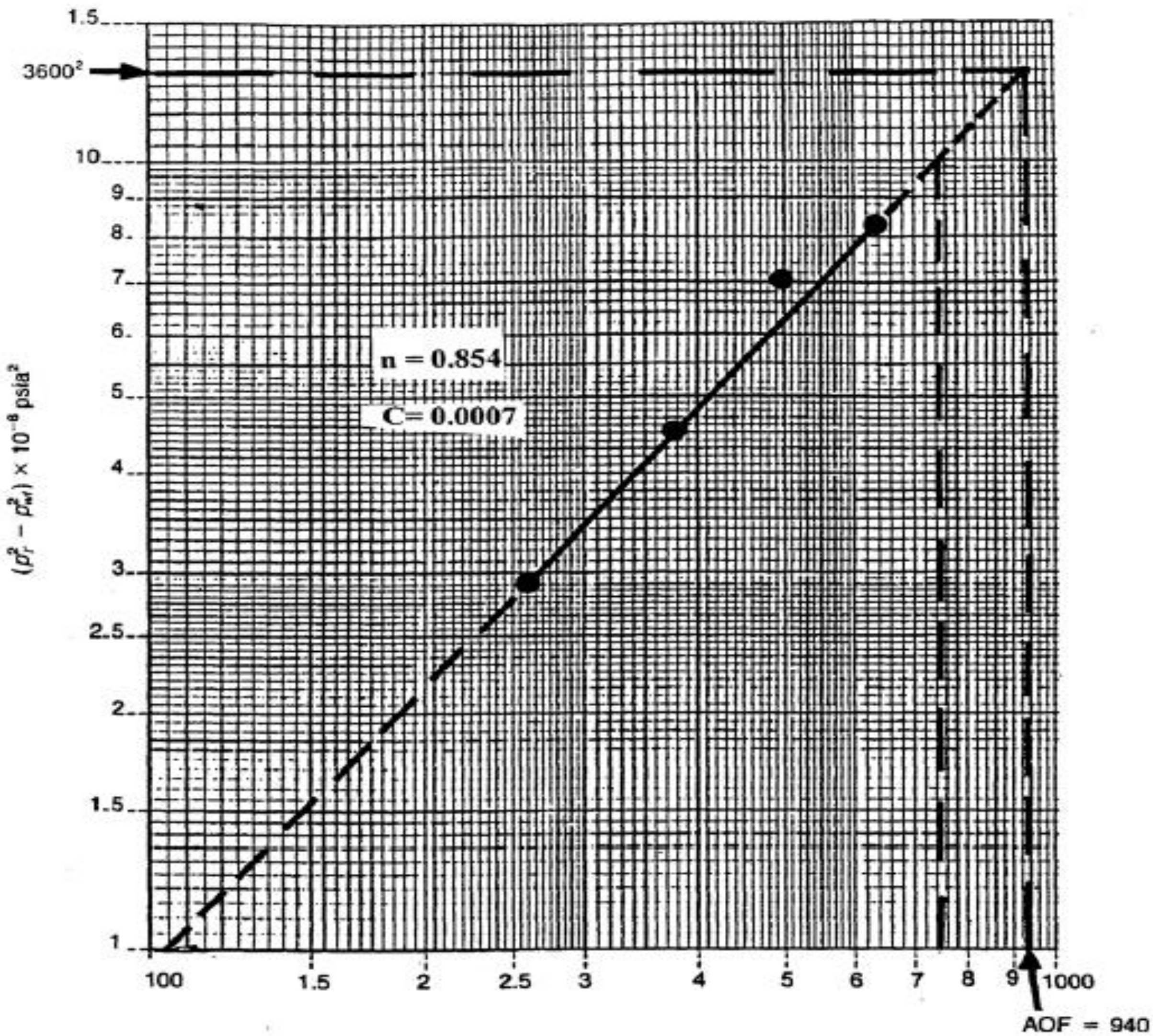


Fig. (1-36): flow after flow data, Fetkovich plot.

$$n = \frac{\Delta \log q_o}{\Delta \log\left(P_r^2 - P_{wf}^2\right)}$$

$$n = \frac{\log(750) - \log(105)}{\log(10^7) - \log(10^6)} = 0.854$$

Step 3: Solve for the performance coefficient C:

$$C = \frac{q_o}{\left(P_r^2 - P_{wf}^2\right)^n}$$

C= 0.00079 STB/day-psia$^{1.71}$

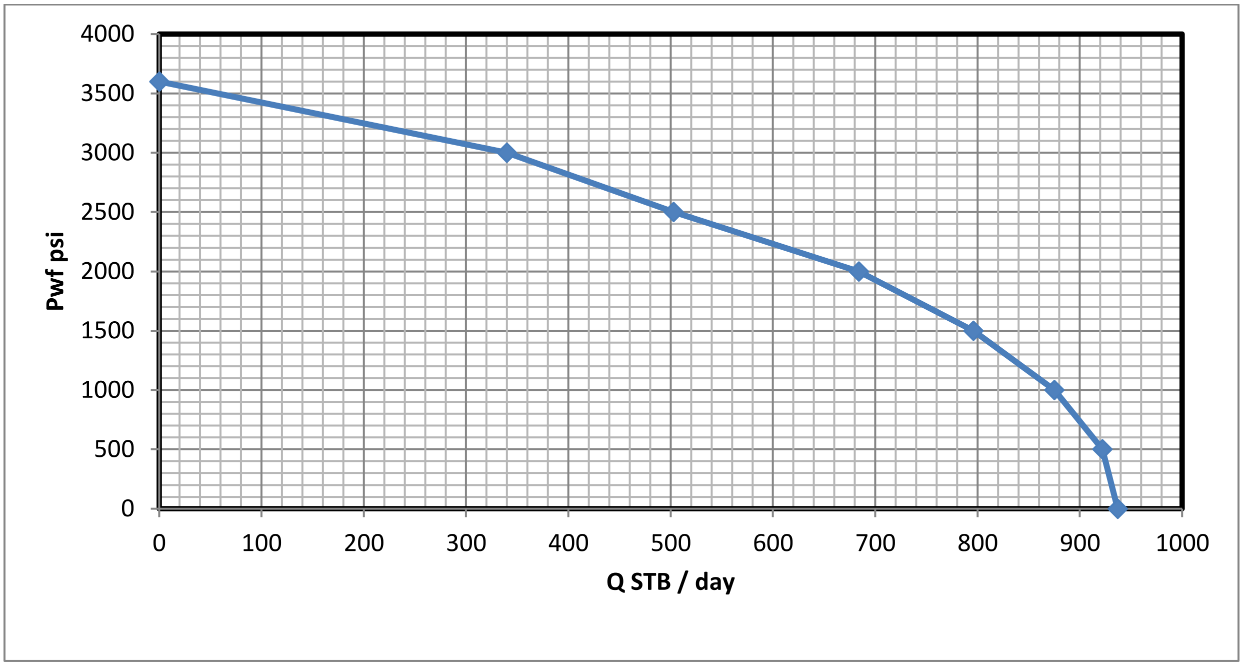
Step 4: Generate the IPR by assuming various values for p_{wf} and calculating the corresponding flow rate from equation (1.42):

$Q_o = 0.00079\ (3600^2 - p_{wf}^2\ )^{0.85}$

P_{wf} psi	Q_o STB / day
3600	0
3000	340
2500	503
2000	684
1500	796
1000	875
500	922
0	937

The IPR curve is shown in figure below. Notice that the AOF, i.e.,:

$(Q_o)_{max}$ = 937 STB/day.



Part B

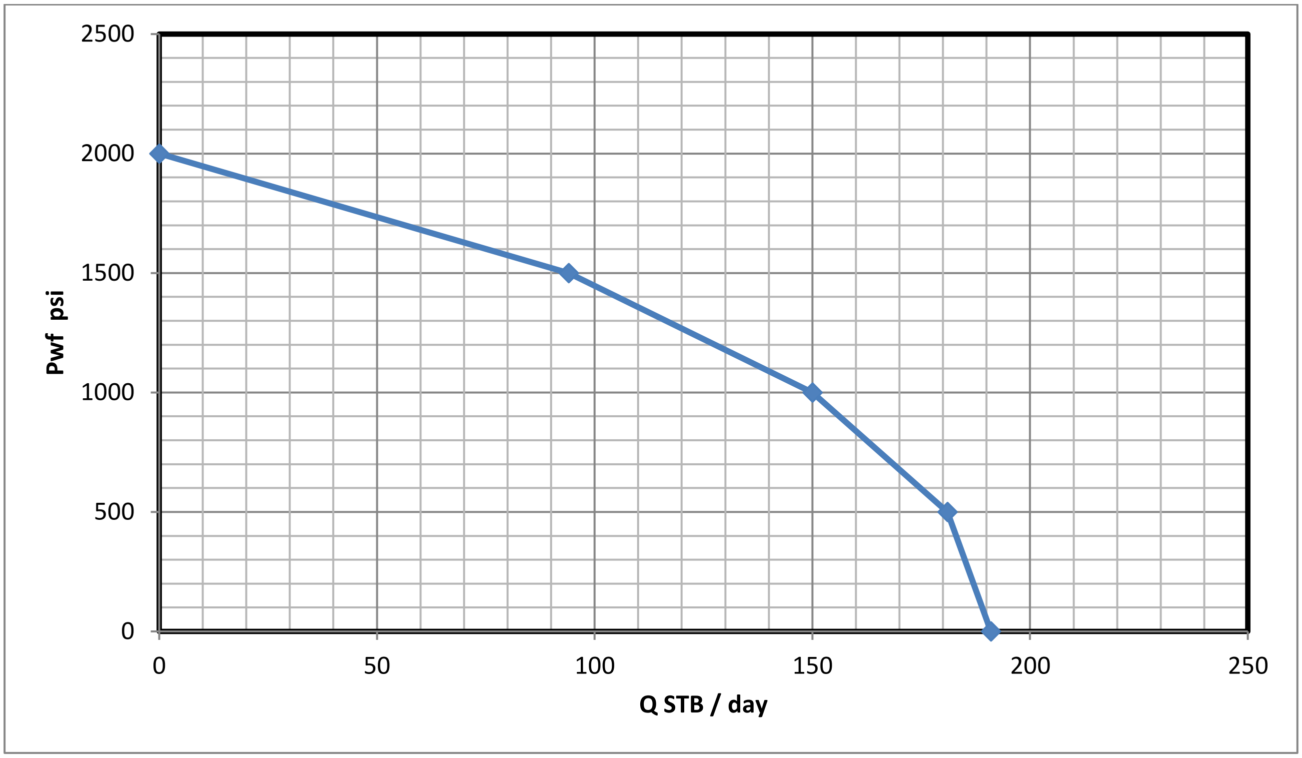
Step 1: Calculate future C by applying equation (1.47).

$$(C)_f = 0.00079\left(\frac{2000}{3600}\right) = 0.000439$$

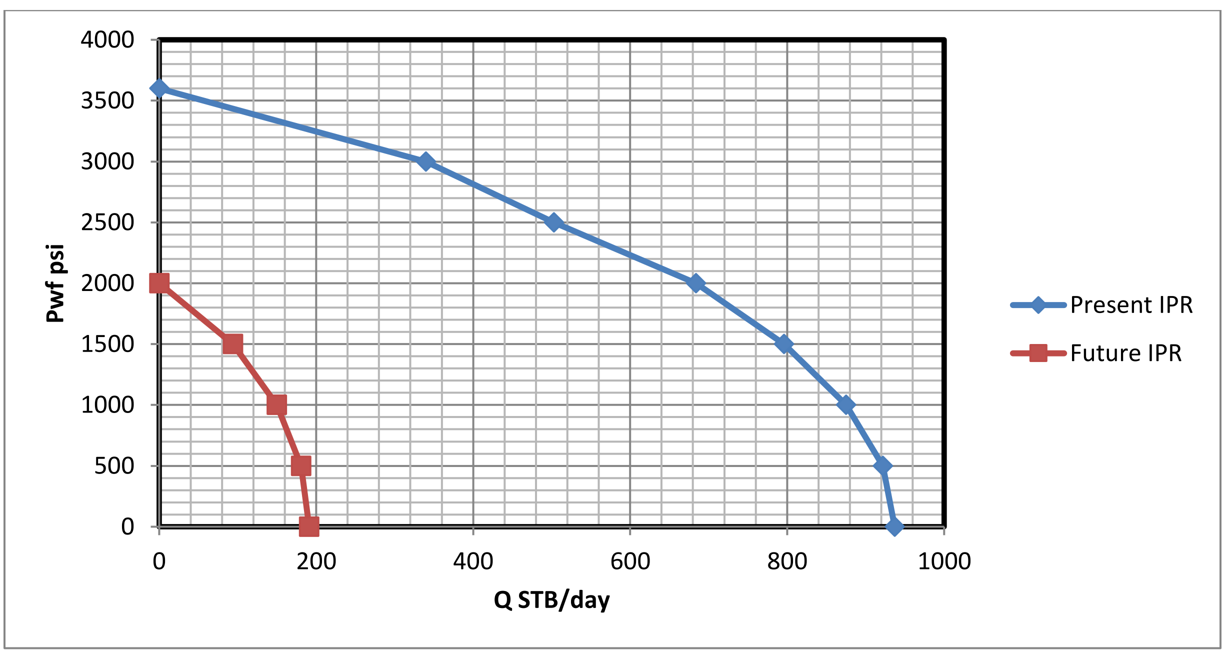
Step 2: Construct the new IPR curve at 2000 psi by using the new calculated C and applying the inflow equation.

$$Q_o = 0.000439\,(2000^2 - P_{wf}^2)^{0.854}$$

P_{wf} psi	Q_o STB / day
2000	0
1500	94
1000	150
500	181
0	191



Both the present time and future IPRs are plotted in Figure below.



The Fetkovich equation can be modified to a form similar to Vogel's equation and stated in terms of Productivity Index J or $q_{L(max)}$ (AOF).

$$q_L = C\left(P_r^2 - P_{wf}^2\right)^n$$

$$q_{L(max)} = C(P_r^2 - 0)^n$$

Eliminating the coefficient C gives.

$$\frac{q_L}{q_{L(max)}} = \frac{\left(P_r^2 - P_{wf}^2\right)^n}{P_r^2} = \left[1 - \left(\frac{P_{wf}}{P_r}\right)^2\right]^n$$

It can also be shown that as drawdown approaches Zero, that is as $\mathbf{P_{wf}}$, approaches $\mathbf{P_r}$,

$$\mathbf{q_{L(max)} = \frac{JP_r}{2}}$$

Therefore, the Fetkovich equation can be expressed as: -

$$\mathbf{q_L = \frac{JP_r}{2}\left[1 - \left(\frac{P_{wf}}{P_r}\right)^2\right]^n} \quad \text{--} \quad \mathbf{(1.48)}$$

Fetkovich also suggested that the analysis could be further broken down for undersaturated reservoirs as:

$$\mathbf{q_L = J(P_r - P_b) + \frac{JP_b}{2}\left[1 - \left(\frac{P_{wf}}{P_r}\right)^2\right]^n} \quad \text{------------------------------------} \quad \mathbf{(1.49)}$$

Application of either eq. (1.42) or eq. (1.49) except eq. (1.47) to analyze a flow-after-flow test requires at least two stabilized production tests. For isochronal testing at least two transient rates and one stabilized rate are required. This results from the fact that there are two unknowns in the equations, either **C** or **n** or **J** and **n**. It should be pointed out that if only one stabilized test is available, it is often assume to be one and either **C** or **J** can be calculated directly. This method of analysis usually gives more conservative results than those obtained using the Vogel method with **FE = 1**. Taking the log of both sides of eq. (1.48) gives:

$$\log q_L = \log\left(\frac{JP_r}{2}\right) + n\log\left[1 - \left(\frac{P_{wf}}{P_r}\right)^2\right]$$

A plot of $\left[\mathbf{1}-\left(\frac{\mathbf{P_{wf}}}{\mathbf{P_r}}\right)^{\mathbf{2}}\right]$versus $\mathbf{q_L}$ on log-log scales will result in a straight line having a slope equal to the exponent **n**. A value of **J** can then be calculated using any point on the linear plot from:

$$J=\frac{2q_L}{P_r\left[1-\left(\frac{P_{wf}}{P_r}\right)^2\right]^n}$$

Problem (1-10): The well described in problem (1-9) is to be analyzed using the Fetkovich equation with the assumption that n=**1**. One production test on the well resulted in a rate of 282 STB/day for P_{wf} = 1765 psig=1780 psia. The static reservoir pressure is 2085 psig = 2100 psia. Calculate:

Calculate:

1. Productivity Index J
2. The new producing rale if P_{wf} 1500 psia.
3. The value of P_{wf} required for q_L = 400 STB/day
4. $q_{L(max)}$ or AOF.

Solution:

1. $J=\frac{2q_L}{P_r\left[1-\left(\frac{P_{wf}}{P_r}\right)^2\right]^n}$

$$J=\frac{2\times282}{2100\times\left[1-\left(\frac{1780}{2100}\right)^2\right]^1}=\mathbf{0.95}\text{ STB/day}-\text{psi}$$

2. $q_L=\frac{JP_r}{2}\left[1-\left(\frac{P_{wf}}{P_r}\right)^2\right]^n$

$$q_L=\frac{0.95\times2100}{2}\left[1-\left(\frac{1500}{2100}\right)^2\right]^1=\mathbf{489}\text{ STB/day}$$

Solving eq. (1.48) for P_{wf} and assuming n = 1:

$$P_{wf}=P_r\left(1-\frac{2q_L}{JP_r}\right)^{0.5}$$

$$P_{wf} = 2100 \times \left(1 - \frac{2 \times 400}{0.95 \times 2100}\right)^{0.5} = \mathbf{1625}\ \text{psia}$$

4. $q_{L(max)} = AOF = \frac{JP_r}{2}$

$$q_{L(max)} = AOF = \frac{0.95(2100)}{2} = \mathbf{998}\ \text{STB/day}$$

The values for $q_{L(max)}$ obtained from the three methods used to analyze this well test may be compared:

Method	$q_{L(max)}$
Constant J	1835
Vogel	1097
Fetkovich (n = 1)	998

6) Wiggins (1996) Method

In 1996, Wiggins derived an equation for the prediction of oil well performance. His equation can be used as follows:

1. Saturated Oil Reservoirs $P_r \le P_b$

- Use the stabilized wellbore rate and pressure ($\mathbf{P_{wf}}$ **&** $\mathbf{Q_o}$) to calculate ($\mathbf{Q_{omax}}$) as follows:

$$(\mathbf{Q_o})_{\mathbf{max}} = \frac{\mathbf{Q_o}}{\left[1 - 0.0933\left(\frac{P_{wf}}{P_r}\right) - 1.6183\left(\frac{P_{wf}}{P_r}\right)^2 + 1.5579\left(\frac{P_{wf}}{P_r}\right)^3 - 0.8464\left(\frac{P_{wf}}{P_r}\right)^4\right]}$$

- Construct the IPR curve by assuming various values of $\mathbf{P_{wf}}$ and calculating the corresponding $\mathbf{Q_o}$ as follows:

$$\mathbf{Q_o} = (\mathbf{Q_o})_{\mathbf{max}} \left[1 - 0.0933\left(\frac{P_{wf}}{P_r}\right) - 1.6183\left(\frac{P_{wf}}{P_r}\right)^2 + 1.5579\left(\frac{P_{wf}}{P_r}\right)^3 - 0.8464\left(\frac{P_{wf}}{P_r}\right)^4\right]$$

2. Undersaturated Oil Reservoirs $P_r > P_b$

- Use the stabilized wellbore rate and pressure ($\mathbf{P_{wf}}$ **&** $\mathbf{Q_o}$) to calculate ($\mathbf{Q_{omax}}$) to calculate the productivity index ***J*** as follows:

$$J = \begin{cases} \dfrac{Q_o}{(P_r - P_{wf})} & P_{wf} \geq P_b \\ \dfrac{Q_o}{(P_r - P_b) + \dfrac{P_b}{1.8}\left[1 - 0.0933\left(\dfrac{P_{wf}}{P_b}\right) - 1.6183\left(\dfrac{P_{wf}}{P_b}\right)^2 + 1.5579\left(\dfrac{P_{wf}}{P_b}\right)^3 - 0.8464\left(\dfrac{P_{wf}}{P_b}\right)^4\right]} & P_{wf} < P_b \end{cases}$$

- Calculate the oil flow rate at the bubble-point pressure:

$$\mathbf{Q_{ob} = J\,(P_r - P_b)}$$

- Construct the IPR curve by assuming various values of $\mathbf{P_{wf}}$ and calculating the corresponding $\mathbf{Q_o}$ as follows:

$$Q = \begin{cases} J(P_r - P_{wf}) & P_{wf} \geq P_b \\ Q_{ob} + \dfrac{JP_b}{1.8}\left[1 - 0.0933\left(\dfrac{P_{wf}}{P_b}\right) - 1.6183\left(\dfrac{P_{wf}}{P_b}\right)^2 + 1.5579\left(\dfrac{P_{wf}}{P_b}\right)^3 - 0.8464\left(\dfrac{P_{wf}}{P_b}\right)^4\right] & P_{wf} < P_b \end{cases}$$

PI and IPR

Summary

The ability of a well to produce fluids.

The uses of the Productivity Index and IPR Curves

- Find the well's potential, q', the maximum production rate.
- Predict production rates for planning production schedules and sizing production equipment
- Reference point for the comparison of wells in a field.
- Find Flow Efficiency of the well to plan or verify completion techniques.
- During production monitoring to help diagnose production problems if any.
- Equipment or reservoir problem
- Selecting testing procedures to identify production problems.
- Comparing the PI of the field test to a calculated PI to verify reservoir properties or an indication of skin in the well.

Productivity Index J

Simplest of the methods, one production and pressure point and a straight line. But the least accurate for calculating the well's potential, greater error as $\mathbf{P_{wf}}$ is lower.

Very good for calculating flow efficiencies, **FE**.

IPR Curves

A very high accuracy if obtained by using a multipoint production test data.

For one point tests

- **Vogel Method**

Reasonable accuracy for the plot and the potential. Problem is that good mobility data is needed for a calculated ideal curve and for future average reservoir plots.

- **Fetkovich Method**

Not as accurate as Vogel. But because of the straight forward method easier for a quick calculation.

Flow Efficiency

The Flow Efficiency of the well which is the ratio of the actual PI to the ideal PI is used to check if the well is a candidate for a work over to remove damage. Also it can be sued to verify a stimulation job. If FE < 1 possible damage, FE > 1 a stimulated zone.

PROBLEMS

Q1: An oil well is producing under steady-state flow conditions at **300** STB/day. The bottom-hole flowing pressure is recorded at **2500** psi.

Given: h = **23** ft, k = **50** md, μ_o = **2.3** cp, r_w = **0.25** ft, B_o = **1.4** bbl/STB, r_e = **660** ft, S = **0.5**

Calculate:

a. Reservoir pressure

b. AOF

c. Productivity index

Q2: A well is producing from a saturated oil reservoir with an average reservoir pressure of **3000** psig. Stabilized flow test data indicate that the well is capable of producing **400** STB/day at a bottom-hole flowing pressure of **2580** psig.

a. Oil flow rate at P_{wf} = **1950** psig

b. Construct the IPR curve at the current average pressure.

c. Construct the IPR curve by assuming a constant J.

d. Plot the IPR curve when the reservoir pressure is **2700** psig.

Q3: An oil well is producing from an undersaturated reservoir that is characterized by a bubble-point pressure of **2230** psig. The current average reservoir pressure is **3500** psig. Available flow test data show that the well produced **350** STB/day at a stabilized P_{wf} of **2800** psig. Construct the current IPR data by using:

a. Vogel's correlation

b. Generate the future IPR curve when the reservoir pressure declines from **3500** psi to **2230** and **2000** psi.

Q4: A well is producing from a saturated oil reservoir that exists at its saturation pressure of **4500** psig. The well is flowing at a stabilized rate of **800** STB/day and a P_{wf} of **3700** psig. Material balance calculations provide the following current and future predictions for oil saturation and PVT properties.

Parameter	Present	Future
P_r	4500	3300
μ_o	1.45	1.25
B_o	1.23	1.18
K_{ro}	1	0.86

Generate the future IPR for the well at **3300** psig by using Standing's method.