Lectures of Electrical Engineering Department

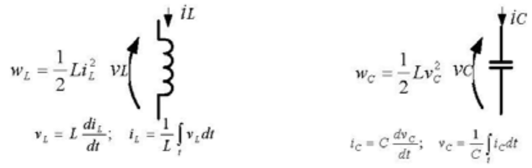
Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	Lecture sequences:	1st lecture	Instructor Name: Omer Sharaf Aldeen Ali Abbawi
	The major contents: 1- Transient Response of RL /RC Circuit		
	The detailed contents: 1- Natural Response of RL Circuit		

Chapter 1

Transient Response of RL /RC Circuit



- The inductor and capacitor are energy storage elements.
- The stored energy does not change promptly (in zero time) as this needs infinite power transfer (w=dp/dt)
- i_L and v_C are associated with the energy stored in the inductor and capacitor respectively; and therefore, they also do not change instantly.
- This chapter studies the time function of i_L and v_C as they change in RL and RC circuits.

<u>This chapter is divided into three parts:</u>

<u>Natural Response</u> (Week#1): i_L and v_C result by discharging energy stored in L or C in a resistive network.
<u>Step Response</u> (Week#2): i_L and v_C when the energy is being stored in an initially uncharged L and C.
<u>General Response</u> (Week #3): analysis or RL and RC circuits that applied to find the response of a circuit with any initial charge and subjected to any change(s) in DC voltage (or current) source.

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1. Natural Response of RL Circuit

Consider the circuit shown in Fig. 1 where the switch has been closed for a long time and opens at *t=0.*

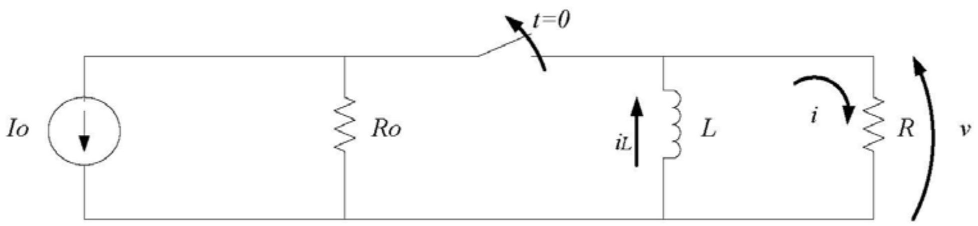


Fig.1

Before opening the switch (say at $t = 0^-$), the circuit at steady state, and therefore $v_L = L\frac{di}{dt} = 0$, so the inductor is acting as a short circuit and $i_L(t = 0^-) = I_o$.

After opening the switch, the current source and R_o are isolated and the inductor circuit becomes as in Fig. 2

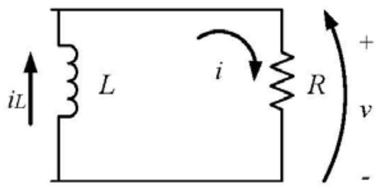


Fig. 2

Just after opening the switch (at $t = 0^+$), the inductor current is still I_o, but this makes $v(t = 0^+) = I_o R$ and this voltage will appear across the inductors and cause the inductor current to change. The following analysis describes the variation of the circuit current and voltage after (t=0),

For $t > 0$

The loop voltage equation...

$$L\frac{di}{dt} + iR = 0 \qquad (1)$$

Eq. (1) is a 1st order Differential Equation (DE) usually solved by integration; However we will apply alternative method here:

Assume:

$$i = Ae^{st} \qquad (2)$$

$$\frac{di}{dt} = Ase^{st} \qquad (3)$$

Substitute (2) and (3) into (1):

$$LAse^{st} + RAe^{st} = 0$$

$$Ae^{st}(Ls + R) = 0 \qquad (4)$$

Eq. (4) implies that **either:** $Ae^{st} = 0$, (i.e. $i = 0$), this solution has no significance and usually referred to as trivial solution **or:** $(\mathrm{Ls} + \mathrm{R} = 0)$ which is the interesting case, so:

$$s = -\frac{R}{L} \qquad (5)$$

In order to fully define the current (i), we must determine A in Eq. (1). Consider the **initial condition** $i(0) = I_0$

$$i = Ae^{-\frac{R}{L}t} \qquad (6)$$

At $(t = 0), i = I_o$

$$I_0 = Ae^{-\frac{R}{L}\times 0} = A \qquad (7)$$

Therefore:

$$i = I_o e^{-\frac{R}{L}t} \qquad (8)$$

It can be shown that L/R has the unit of time (sec), and (8) can be re-written as:

$$i(t) = I_o e^{-\frac{t}{L/R}} \qquad ,(t \geq 0) \qquad (8)$$

A plot of $i(t)$ according to Eq. (8) is shown in Fig. (3) for three values of L/R.

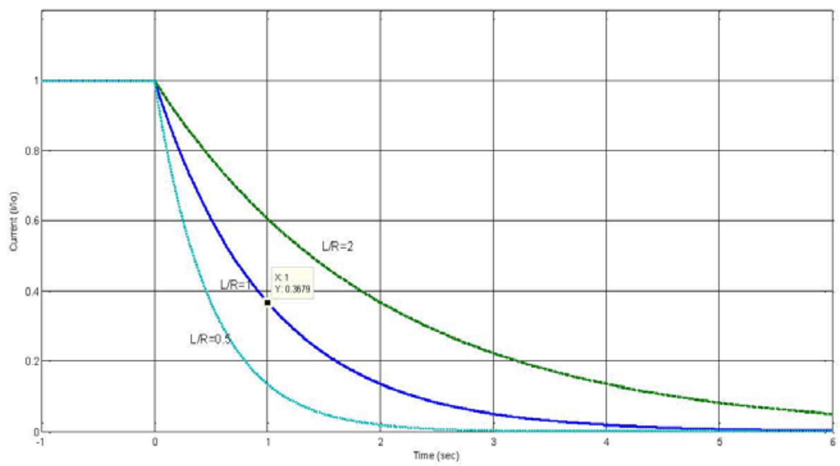


Fig. (3)

It can be noticed that L/R determines how fast the inductor current discharges and this parameter is defined as the RL circuit time constant, and denoted by τ (taw).

$$\text{Time constant, } \tau \triangleq \frac{L}{R} \qquad (9)$$

Table 1 mitigates the discharge rate seen in Fig. 3

Table 1						
Time	0	τ	2τ	3τ	4τ	5τ
$\frac{i}{I_o}$	1.00	0.3679	0.1353	0.0498	0.0183	0.0067
Approximation of $\frac{i}{I_o}$	1.00	≈0.37		<5%		<1%

Most of the time transient is considered over for any time >5τ; and any time >10τ can be described as long time.

<u>The inductor voltage:</u>

$$v_L = L\frac{di}{dt} = L\frac{d\left(I_o e^{-\frac{t}{\tau}}\right)}{dt} = -\frac{L}{\tau} I_o e^{-\frac{t}{\tau}} = -\frac{L}{L/R} I_o e^{-\frac{t}{\tau}}$$

$$v_L = -RI_o e^{-\frac{t}{\tau}} \qquad t \geq 0^+ \qquad (10)$$

Example 1: The switch in the circuit shown in Fig. 4 has been open for long time. The switch is closed at t=0, determine $i(t), (t \geq 0)$and $v(t), t > 0$

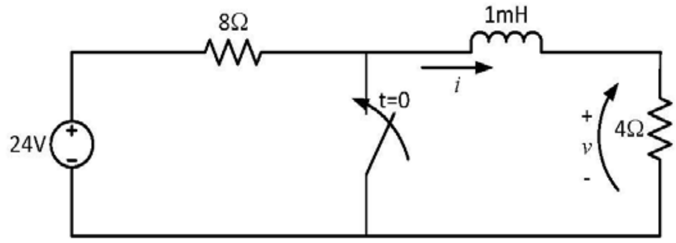
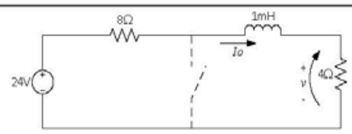
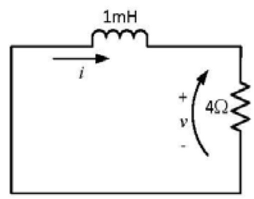


Fig. 4

Answer:

//The three solution steps are:		
(1) Determine I_o ($i_L(0^-)$) (by studying the circuit before transient starts)	For t=0$^-$: $I_0 = \frac{24V}{(8+4)\Omega} = 2A$	
(2) Determine τ by determining Req for (t>0)	For t>0, $R_{eq} = 4\Omega$ $\tau = \frac{L}{R} = \frac{1mH}{4\Omega} = 0.25 \times 10^{-3}sec$	
(3) write i_L according to Eq. (8)	$i(t) = I_o e^{-\frac{t}{\tau}}$ $i(t) = 2e^{-\frac{t}{0.25\times10^{-3}}} = 2e^{-4000t}(A)$, for t≥0	
(4) Other requirements://	$v(t) = i(t) \times R$ $v(t) = 8e^{-4000t}(V)$	

Example 2: The switch in the circuit shown in Fig. 5 has been closed for a long time before it is opened at t=0. Find:

a) $i_L(t)$ for t≥0;
b) $i_o(t)$ for t≥0$^+$;
c) $v_o(t)$ for t≥0, and:
d) The percentage of total energy dissipated in the 10Ω resistor

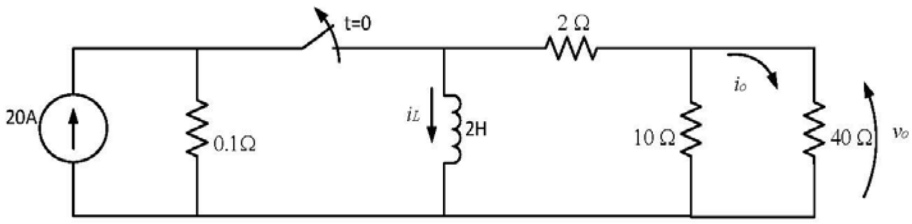


Fig. 5

Answer:

a)

- Initial value of inductor current, $i_L(t=0) = 20A$ (usually denoted by (I_o))
- R_{eq} =2+(40//10)=10Ω
- $\tau = \frac{2H}{10\Omega} = 0.2sec$
- $i_L(t) = 20e^{-5t}(A), \quad (t \geq 0)$

b)

$$i_0 = -i_L \frac{10\Omega}{(10+40)\Omega} = -0.2 i_L$$

$$i_o = -4e^{-5t}(A) \quad (t \geq 0^+)$$

c)

$$v_o = 40 i_o = -160e^{-5t}(V) \quad (t \geq 0^+)$$

d)

- $W_L = \frac{1}{2} L I_o^2 = \frac{1}{2} \times 2 \times 20^2 = 400J$
- $p_{10\Omega}(t) = \frac{v_o^2}{10} = 2560e^{-10t}(W) \quad (t \geq 0^+)$

$$W_{10\Omega} = \int_{t=0}^{\infty} p_{10\Omega}(t)dt = \int_{t=0}^{\infty} 2560e^{-10t}dt = \frac{2560}{-10}[e^{-10t}]_0^{\infty} = 256[1-0]$$

$$W_{10\Omega} = 256J$$

- Percentage energy dissipated in 10Ω resistor=$\frac{256}{400} \times 100\% = 64\%$

-Practice: show that the energy dissipated in the three resistors for (t>0) in Example 2 is equivalent to the energy initially stored in the inductor)

-Related book examples: 8.2, 8.3, 8.4

A graphical method to find (τ) can be applied if the circuit parameters are unknown.

The line tangent to i_L at t=0 (the switching instant) intersects the t axis at t=τ.

Proof:

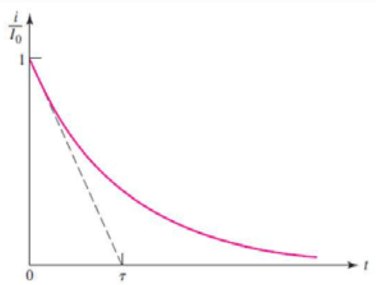
$\frac{di}{dt} = -\frac{1}{\tau} I_o e^{-\frac{t}{\tau}}; \frac{di}{dt}|_{t=0} = -\frac{1}{\tau} I_o$ (slope)

The line equation…

$$y = I_0 - \frac{I_o}{\tau} t = I_o\left(1 - \frac{t}{\tau}\right)$$

At t=τ, y=0#.

//(This method used in the lab to determine τ)//

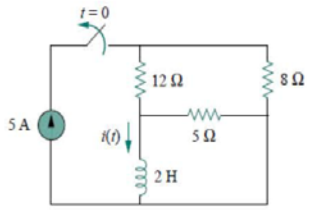


Related Book Problems:

Chapter 8 problems (1-13)

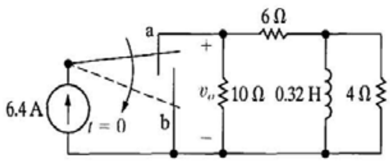
Homework (to be handed in the tutorial session This week)

1- For the circuit in Fig. , find $i(t)$ for $t > 0$.



2- At t=0, the make-before-break switch moves from position a to position b.

- Calculate $v_o(t), t \geq 0^+$
- What percentage of the initial energy stored in the inductor is dissipated in the 4Ω resistor.



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2. Natural Response of RC Circuit

Consider the circuit shown in Fig. 6, the switch has been in position (a) for long time and at t=0, the switch is moved to position (b). The following analysis shows the variation of the capacitor voltage after the switch is moved.

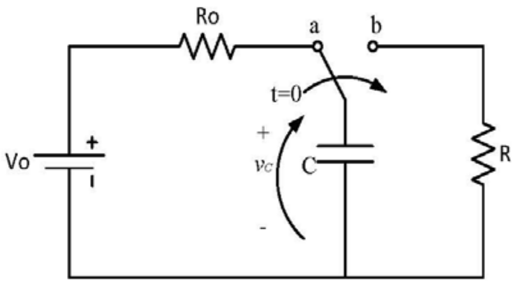


Fig. 6

- Before the switch moves to position (b), (at $t=0^-$), the capacitor is charged by the DC source to V_o.
- For t>0, the supply is removed and the capacitor equivalent circuit is as shown in Fig. 7

Applying the "node current" principle:

$i_C + i_R = 0$

$C\frac{dv_c}{dt} + \frac{v_c}{R} = 0$ (11)

(Eq. (11) is similar to Eq. (1)), Assume:

$v_C = Ae^{st}$ and $\frac{dv_C}{dt} = Ase^{st}$ (12a and 12b)

Substitute (12) into (11)

$Ae^{st}\left(Cs + \frac{1}{R}\right) = 0$ (13)

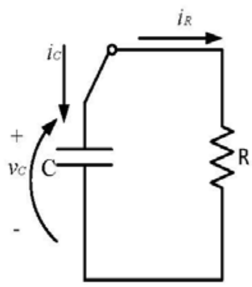


Fig. 7

Either: $Ae^{st} = 0$, (i.e. $v_{c=0}$) trivial solution (ignore).

Or: $\left(Cs + \frac{1}{R}\right) = 0 \Rightarrow s = -\frac{1}{RC}$ (14)

At t=0, $v_C(t=0) = V_o$ (15)

Substitute (15) into (12a)

$v_c(t=0) = V_o = Ae^{s\times 0} = A,$

$A = V_o$ (16)

Substitute (14) and (16) into (12a)

$$v_C(t) = V_o e^{-\frac{t}{RC}} \quad (V), \text{ for } (t \geq 0) \qquad (17)$$

It can be shown that RC has the unit of sec. defining the circuit time constant, τ as:

$$\tau \triangleq RC \qquad (18)$$

Substitute (18) into (17)

$$v_C(t) = V_o e^{-\frac{t}{\tau}} \quad (V), \text{ for } (t \geq 0) \qquad (19)$$

Eq. (19) describes the capacitor discharge in resistive circuit, and Fig. 8 shows the *natural response* of an RC circuit.

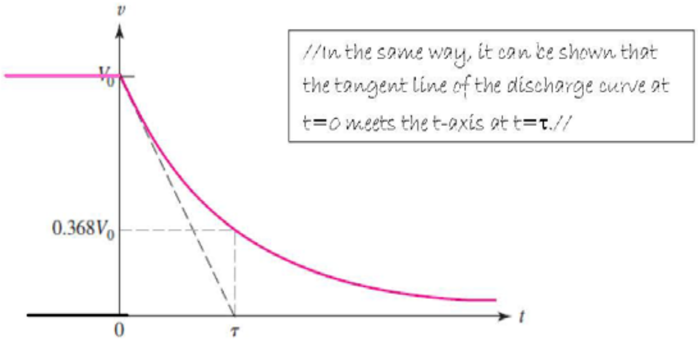


Fig. 8

Example 3: The switch in the circuit shown in Fig. 9 has been in position (x) for a long time. At t=0 the switch moves to position (y). Find

- $v_c(t), t \geq 0$
- $v_o(t), t \geq 0^+$
- $i_o(t), t > 0$
- The total energy dissipated in the 60kΩ resistor

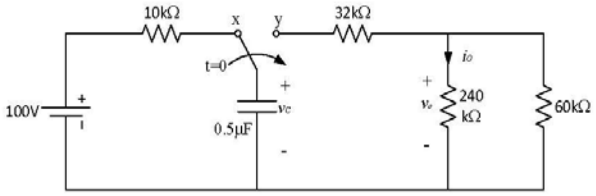
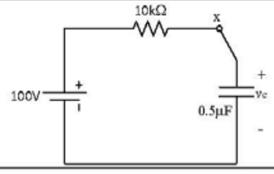
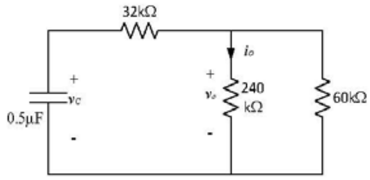


Fig. 9

Answer:

(a)

From the circuit condition for t<0. • The initial value of the capacitor voltage, $V_o = v_C(t=0) = 100V$	
From the circuit condition for t<0. • The equivalent resistance: $R_{eq} = \left(32 + (\frac{240 \times 60}{240 + 60})\right) k\Omega$ $R_{eq} = 80k\Omega$ • The time constant: $\tau = R_{eq}C = 80 \times 10^3 \times 0.5 \times 10^{-6}$ $\tau = 40msec.$ • The capacitor voltage (from Eq.19) $v_c(t) = 100e^{-25t}\,(V),\ \ t \geq 0$	

(b)

//We can find v_o by: (1) dividing the voltage v_c between the 32k and the parallel combination. OR (2) by determining i_o as follows://

$$i_C = C\frac{dv_c}{dt} = 0.5 \times 10^{-6}(-25 \times 100e^{-25t})$$

$$i_C = -1.25 \times 10^{-3}e^{-25t}(A), t > 0$$

$v_o = -i_C \times (240\backslash\backslash 60)k\Omega$ // the parallel sign is reversed as the correct one '//' turns fraction in Ms equation 😠 ☹ 😤//

$$v_o = -(-1.25 \times 10^{-3} \times e^{-25t}) \times \left(\frac{240 \times 60}{240 + 60}\right) \times 10^3$$

$$v_o = 60e^{-25t}\,(V)\ ,\quad t > 0$$

(c)

$$i_o = \frac{v_o}{60k}$$

$$i_o = e^{-25t}(mA),\quad (t > 0)$$

(d) $W_{60k} = \int_{t=0}^{\infty} {i_o}^2 R dt = \int_{t=0}^{\infty} (e^{-25t} \times 10^{-3})^2 R dt = \int_{t=0}^{\infty} e^{-50t} \times 10^{-6} \times 60 \times 10^3 dt$

$W_{60k} = 1.2mJ$

Example 4: The initial voltage of the capacitors C_1 and C_2 are as indicated. The switch is closed at t=0.

(a) Find $v_1(t), v_2(t)$ and $v(t)$ for t ≥ 0, and $i(t)$for t ≥ 0^+.

(b) Calculate the initial energy stored in C_1 and C_2

(c) Determine how much energy stored in the capacitors as t→ ∞.

(d) Show that the total energy delivered to the resistor is the difference between the results of (b) and (c).

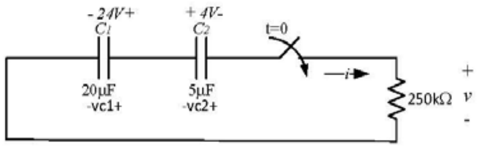


Fig. 10

Answer:

(a)

- Find C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \left(\frac{1}{20} + \frac{1}{5}\right)\frac{1}{\mu F} \rightarrow C_{eq} = 4\mu F$$

- Find τ

  $\tau = RC_{eq} = 250 \times 10^3 \times 4 \times 10^{-6}$=1sec.
- Find initial voltage of C_{eq}

  $v(t=0) = 24 - 4 = 20V$
- Find *v(t):*

  $v(t) = 20e^{-t}$(V), t>0

$$i(t) = \frac{v(t)}{R} = 80e^{-t}\mu A, (t > 0)$$

$$v_{C1}(t) = \frac{1}{C}\int_0^t -i(x)dx + v_{C1}(0)$$

$$v_{C1}(t) = -\frac{10^6}{5}\int_0^t 80 \times 10^{-6}e^{-x}dx - 4$$

$$v_{C1}(t) = 16e^{-t} - 20\ (V), t \geq 0$$

Similarly,

$$v_{C2}(t) = \frac{1}{C_2}\int_0^t -i(x)dx + v_{C2}(0)$$

$$v_{C2}(t) = -\frac{10^6}{20}\int_0^t 80 \times 10^{-6}e^{-x}dx + 24$$

$$v_{C2}(t) = 4e^{-t} + 20\ (V), t \geq 0$$

(b)

The initial energy stored in C_1n and C_2

$$W_{1,0} = \frac{1}{2}(5\mu)(4^2) = 40\mu J$$

$$W_{2,0} = \frac{1}{2}(25\mu)(24^2) = 5760\mu J$$

The total initial energy:

$$W_0 = W_{1,0} + W_{2,0} = 5800\mu J$$

(c)

As t→∞ : v_1→−20V and v_2→20V

And the energy in the two capacitors as t→∞

$$W_{1,\infty} = \frac{1}{2}(5\mu)(-20^2) = 1000\mu J$$

$$W_{2,\infty} = \frac{1}{2}(20\mu)(20^2) = 4000\mu J$$

The total final energy=

$$W_\infty = W_{1,\infty} + W_{2,\infty} = 5000\mu J$$

(d)The total energy dissipated in the resistor:

$$W_{250} = \int_{t=0}^{\infty} i^2 R dt = \int_{t=0}^{\infty} (80e^{-t} \times 10^{-6})^2 \times 250 \times 10^3 dt = 800\mu J$$

The power dissipated in the resistor is the difference between the initial power and the final value.

800μJ=5800μJ –5000μJ

Practice

8.6 Find values of v_C and v_o in the circuit of Fig. 8.23 at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) 1.3 ms.

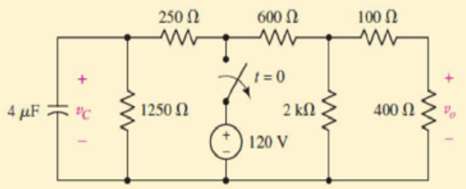


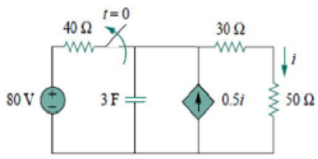
FIGURE 8.23

Ans: 100 V, 38.4 V; 100 V, 25.6 V; 59.5 V, 15.22 V.

Related book Problems 16-40

Homework (to be handed in the tutorial session this week)

3- For the circuit shown. Find *i(t)*for t<0 and t>0.



Lectures of Electrical Engineering Department

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Natural Response</td></tr>
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1- Natural Response of RC circuits

2- Step Response in RC circuits</td></tr>
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Chapter 1

Transient Response of RL /RC Circuit

This second lecture of chapter 1 deals with the step response. It presents the analysis of RL /RC circuit subjected to sudden application of dc voltage or current supply.

Definition:

The unit-step function $u(t)$: a function of time which is ZERO for all values of its argument less than zero and which is ONE for all positive values of its argument as shown in Fig. 11.:

$$u(t)=\begin{cases}0 & t<0\\ 1 & t>0\end{cases} \qquad (1)$$

More generally;

$$u(t-t_0)=\begin{cases}0 & t<t_o\\ 1 & t>t_o\end{cases} \qquad (2)$$

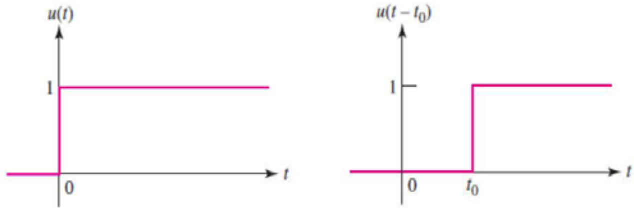


Fig. 11

Initially, we will use this function to activate some sources other applications will be indicated later. Fig. 12 showns two identical arrangements, the second uses unit step function to activate the current source.

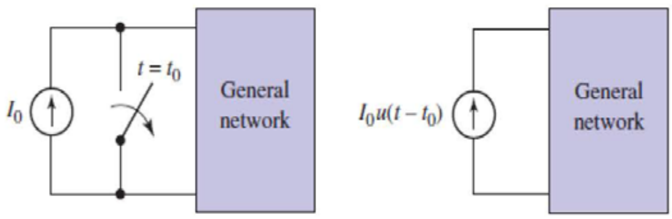


Fig. 12

3. Step Response of RL Circuit

Consider the circuit shown in Fig. 13, where the voltage supply is applied to the RL circuit at t=0. Assume that the initial value of the inductor current $i_L(t=0)=I_o$ (the source of I_o is not shown in Fig. 13)

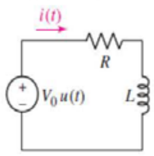


Fig.13

For t≥0, the loop equation:

$$V_s = R_i + L\frac{di}{dt} \qquad (3)$$

Assume the solution of Eq. (3) has the following form:

$$i(t) = Ae^{st} + K \qquad (4)$$

$$\frac{di}{dt} = Ase^{st} \quad (5)$$

Substitute (4) and (5) into (3):

$$V_s = R(Ae^{st} + K) + LAse^{st}$$

$$V_s = RAe^{st} + RK + LAse^{st} \quad (6)$$

By separating the constant and variable parts of Eq (6), we obtain two equations:

$$V_s = RK \Rightarrow \boxed{K = \frac{V_s}{R}} \quad (7)$$

$$0 = RAe^{st} + LAse^{st}$$

$$Ae^{st}(R + sL) = 0 \rightarrow s = -\frac{R}{L} \quad (8)$$

As in section 1, define $\tau \equiv \frac{L}{R}$,

Eq. (8) becomes $\boxed{s = -\frac{1}{\tau}}$ (8)

At t=0, $i = I_o$, substitute into (4)

$$i(t=0) = I_o = A + \frac{V_s}{R} \rightarrow \boxed{A = (I_o - \frac{V_s}{R})} \quad (9)$$

Substitute (7), (8) and (9) into (4)

$$\boldsymbol{i(t) = \frac{V_S}{R} + \left(I_o - \frac{V_S}{R}\right) e^{-\frac{t}{\tau}}}, t \geq 0 \quad (10)$$

Eq. (10) is a general equation, notice the following special cases:

-If we have no source (V_s=0), Eq. (10) becomes identical to the natural response equation (Eq.(8)_Lect.#1). $i(t) = I_o e^{-\frac{t}{\tau}}$.

- If the initial current is zero (I_o=0), Eq. (10) becomes $i(t) = \frac{V_S}{R} - \frac{V_S}{R} e^{-\frac{t}{\tau}}$ similar to Eq. [24] in the textbook.

Fig. 14 clarifies the inductor current variation with time, it is shown that:

- At t=τ, the inductor current changes by 63.2% of the total variation ($\frac{V_S}{R} - I_o$). Where 63.2% =$(1 - e^{-1})$.
- At t=3τ and 5τ the change of the inductor current is 95% and 99.3% of the total change respectively.

Note that $\frac{V_S}{R}$ represents the inductor current as the time → ∞, sometimes denoted by I_F

// No problem to have I_o> Vs/R. In that case the current will decrease during transient time //

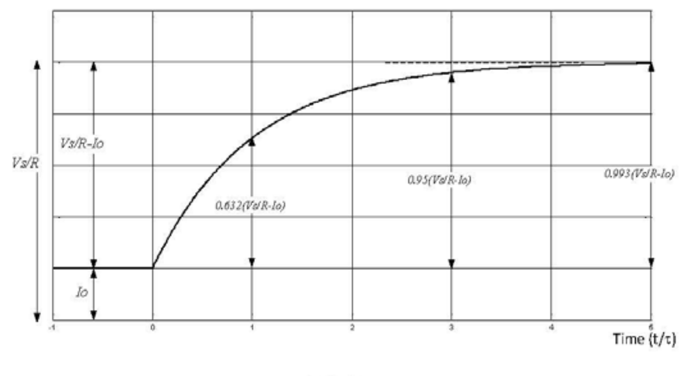


Fig. 14

Example 1: For the circuit shown in Fig. 15, determine $i(t), t \geq 0$.

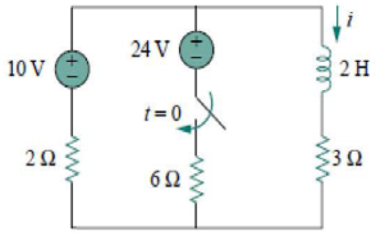
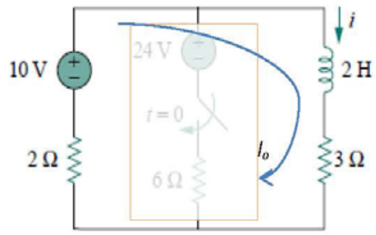
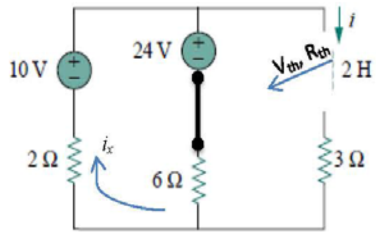
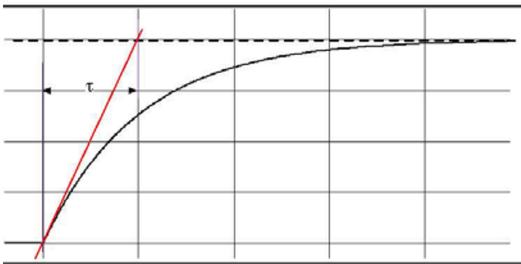


Fig. 15

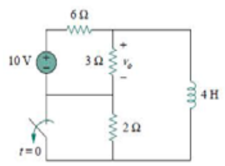
For t<0 $I_o = \frac{10V}{5\Omega} = 2A$	
For t>0 Vth: $24 + 8i_x - 10 = 0 \rightarrow i_x = -\frac{14}{8}$ $V_{th} = 24 + i_x \times 6 = 24 - \frac{14}{8} \times 6 = 13.5V$ $R_{th} = 3 + (2\backslash\backslash 6) = 4.5\Omega$ $\tau = \frac{L}{R_{th}} = \frac{2}{4.5} sec.$	
$i(t) = \frac{V_S}{R} + \left(I_o - \frac{V_S}{R}\right) e^{-\frac{t}{\tau}}$ $i(t) = \frac{13.5}{4.5} + \left(2 - \frac{13.5}{4.5}\right) e^{-\frac{t}{2/4.5}}$ $i(t) = 3 - e^{-2.25t} A, t \geq 0$	

	**<u>Exercise:</u>** show that the tangent line of the transient curve at t=0, reaches the final value of the transient at t=τ. As in the natural response case, this method can used to measure τ, if the circuit parameters are unknown.

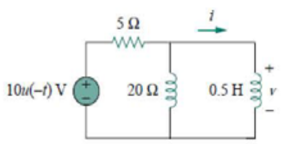
Related textbook Example 8.8, practice exercises 8.10, 8.11, and problems: 47-54.

H.W. submit in tutorial session

1. Find $v_o(t)$ for $t > 0$ in the circuit of Fig.



2. Obtain $v(t)$ and $i(t)$ in the circuit of Fig.



4. Step Response in RC Circuit

The capacitor shown in Fig. 16 is initially charged to a voltage, Vo ($v_c(t<0)=V_o$). At t=0 the switch is closed; we want to determine $v(t),(t\geq 0)$.

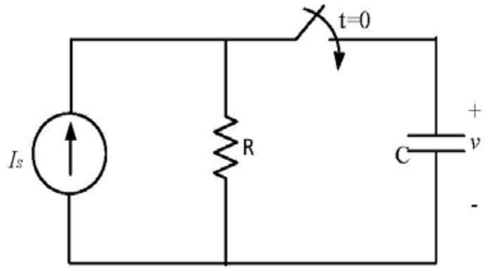


Fig. 16

For t≥0, the node currents equation:

$$I_s=\frac{v}{R}+C\frac{dv}{dt} \qquad (11)$$

Following steps similar to those written to derive equation (10), we can show that the solution of (11) is

$$\boldsymbol{v(t)=I_sR+(V_o-I_sR)e^{-\frac{t}{\tau}}},\ t\geq 0 \qquad (12)$$

Where $\tau=RC$

What has been indicated about Eq. (10), is also applicable to Eq. (12) for being:

- A general equation and zero forced voltage (I_SR), or zero initial voltage can be taken as special cases.
- The plot of v against time for t≥0 is similar to that shown in Fig. 14.

Example 2: The switch in the circuit shown in Fig. 17 has been in position (a) for a long time. At t=0 the switch is moved to position (b).

(a) What is the initial value of v_c?
(b) What is the final value of v_c?
(c) What is the time constant of the circuit when the switch is in position (b)?
(d) What is the expression of $v_c(t)$when t≥0?
(e) What is the expression of $i(t)$when t≥0$^+$?

(f) How long after the switch is in position (b) does the capacitor voltage equal zero?

(g) Plot $v_c(t)$ and $i(t)$ versus t.

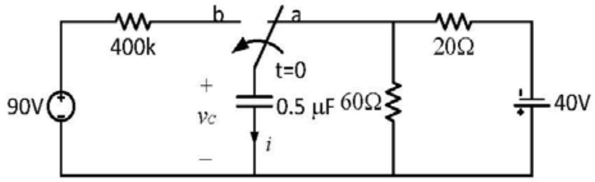
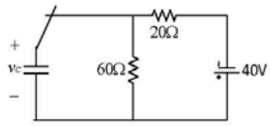


Fig. 17

Sol.

(a) t<0

$$v_C(t=0^-) = V_0 = \frac{-40}{20+60}60 = -30V$$



(b) $v_C(t=\infty) = V_F = 90V$

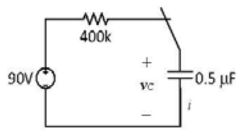
(c) $\tau = RC = (400 \times 10^3)(0.5 \times 10^{-6}) = 0.2sec$

(d) $\boldsymbol{v(t) = I_sR + (V_o - I_sR)e^{-\frac{t}{\tau}}}$

$$v(t) = 90 + (-30 - 90)e^{-5t}$$

$$v(t) = 90 - 120e^{-5t}, \text{ t}\geq 0$$

//We have used Thevenin's ←→Norton's transfer to replace I_sR; However we can always substitute I_sR; by the final value of the capacitor voltage //



(d) $0 = 90 - 120e^{-5t_z}$

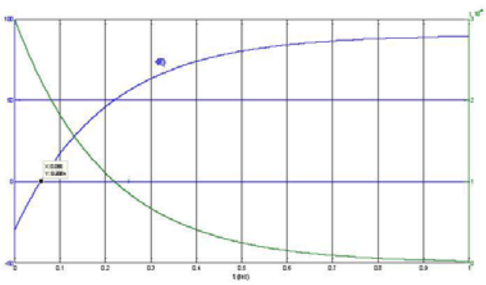
$$\ln\left(\frac{-90}{-120}\right) = -5t$$

$$t = 57.54ms$$

(e) $i_C = C\frac{dv_C}{dt} = 0.5 \times 10^{-6} \times \left((-5)(-120e^{-5t})\right)$

$$i_C = 0.3e^{-5t} \ (mA), t > 0$$

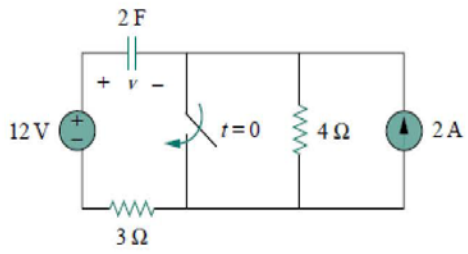
(f)



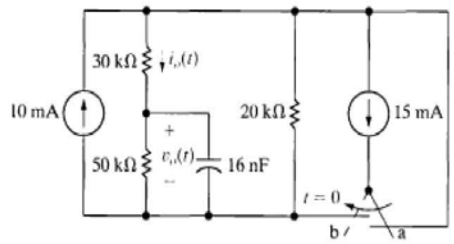
Related Book Examples 8.10 and 8.11; Book Practice 8.12 ; problems: 59-64

H.W. submit in tutorial session

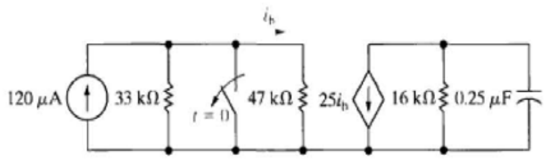
3. Calculate the capacitor voltage and current for t≥0 in the circuit shown:



4. The switch in the circuit shown has been in position (a) for a long time. At t=0 the switch moves to position (b), Find $v_o(t), t \geq 0$



5. The switch has been closed for a long time and it is opened at t=0>. How many milliseconds after the switch opens is the energy stored in the capacitor 25% of its final value?



Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	Lecture sequences:	3nd lecture	Instructor Name: Omer Sharaf Aldeen Ali Abbawi
	The major contents: Transient Response of RL /RC Circuit		
	The detailed contents: 1- General Response of RL/RC circuits 2- Sequential Switches		

Chapter 1

Transient Response of RL /RC Circuit

This third (and last) lecture of Chapter 1 deals with the general response which includes:
-Analysis of RL /RC circuit subjected to application of new dc supply at any instant (t_o), and
-Analysis of circuit subjected to sequence of application (or removal) of dc supplies.

---

1. General Response of RL/RC Circuit

To deal with the transient that start at **any time** (t_o) which is not necessarily zero, we can imagine a new shifted time axis (say t'-axis) with a zero at (t_o) as shown in Fig. 18.

Fig. 18

If we write the general response equation as a function of t', we will follow the method presented last week exactly. i.e.

$$x(t') = X_F + (X_o - X_F)e^{-\frac{t'}{\tau}} \qquad (1)$$

Where:

x is the inductor current for RL circuit or the capacitor voltage for RC circuit.

X_F is the steady state value of ($x(t' = \infty)$)

X_o is the initial condition ($x(t' = 0)$)

From Fig. 18 it can be noted that the relationship between t and t' is:

$$t' = t - t_o \qquad (2)$$

If (2) substituted in (1), we will get the circuit response as a function of the t, as follows:

$$x(t) = X_F + (X_o - X_F)e^{-\frac{(t-t_0)}{\tau}} \qquad (3)$$

Example 1: Refer to the circuit of Fig. 19, which contains a voltage-controlled dependent voltage source in addition to two resistors. (a) Compute the circuit time constant. (b) Obtain an expression for vx valid for all t. (c) Plot the power dissipated in the 4Ω resistor over the range of 6 time constants. (d) Repeat parts (a) to (c) if the dependent source is installed in the circuit upside down.
(e) Are both circuit configurations "stable"? Explain.

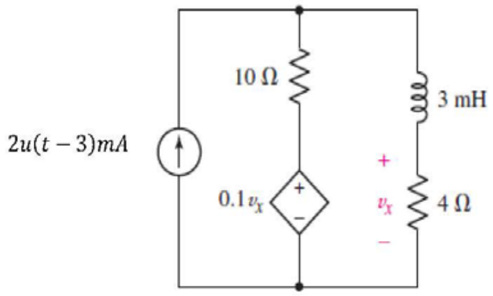


Fig. 19

// one last time I will recite the typical "solution steps":

1. *Identify x : (usually the inductor current or capacitor voltage)*
2. *Find the initial condition $x_o = x(t=t_o^-)$*
3. *Determine R_{th} and τ*
4. *Calculate x_F*
5. *Sub statue into Eq. (3)*

//

Sol.

(a) To find the time constant (τ), we need consider the circuit for $t>t_o$.

To find R_{th} looking from the inductor terminals, we will use the following $R_{th} = \frac{V_{OC}}{I_{SC}}$

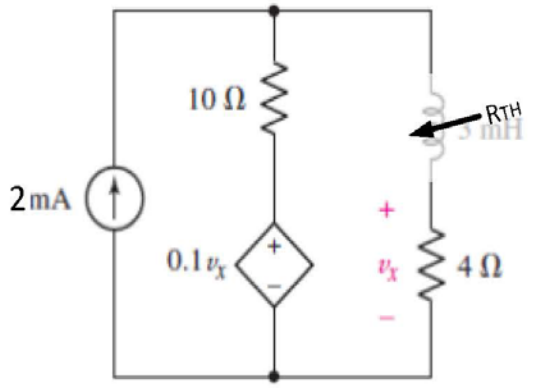
To find V_{OC}:

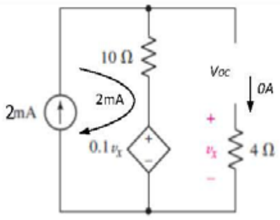
$$V_{OC} = 2\times10^{-3}\times 10 + 0.1v_x$$
$$v_x = 0\times(4\Omega) = 0V$$
$$V_{OC} = \mathbf{0.02V}$$

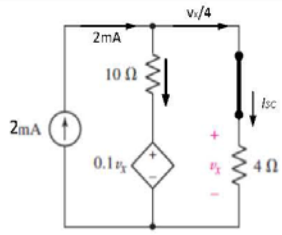
To find I_{SC}: (node current equation)

$$2\times10^{-3} = \frac{v_x - 0.1v_x}{10} + \frac{v_x}{4}$$
$$v_x = \frac{80}{13.6}\times10^{-3}$$
$$I_{SC} = \frac{v_x}{4} = \frac{\mathbf{0.02}}{\mathbf{13.6}}A$$
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{0.02}{0.02/13.6} = \mathbf{13.6\Omega}$$
$$\tau = \frac{L}{R} = \frac{3\times10^{-3}}{13.6}$$
$$\boxed{\tau = \mathbf{0.22msec}}$$

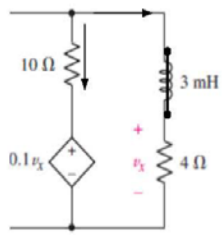
Type equation here.







(b) Find: (1)i_L before introducing the supply (t<3), and this represents the initial condition (I_o) and (2) i_L as (t→∞), and this represents the forced solution (I_F)

t<3	$\frac{v_x}{4} = \frac{v_x - 0.1v_x}{10}$ $10v_x = 3.6v_x$ $v_x = 0$ $I_o = 0A$	
t→∞	$I_F = \frac{\mathbf{0.02}}{\mathbf{13.6}}A$ *// Calculated in (a) as the short circuit current ...no need to repeat!//*	

Now we can write the expression of $i_L(t)$

$$i_L(t) = \begin{cases} 0 & t \le 3 \\ 1.47 \times 10^{-3}\left(1 - e^{-\frac{(t-3)}{0.22\times10^{-3}}}\right) & t \ge 3 \end{cases}$$

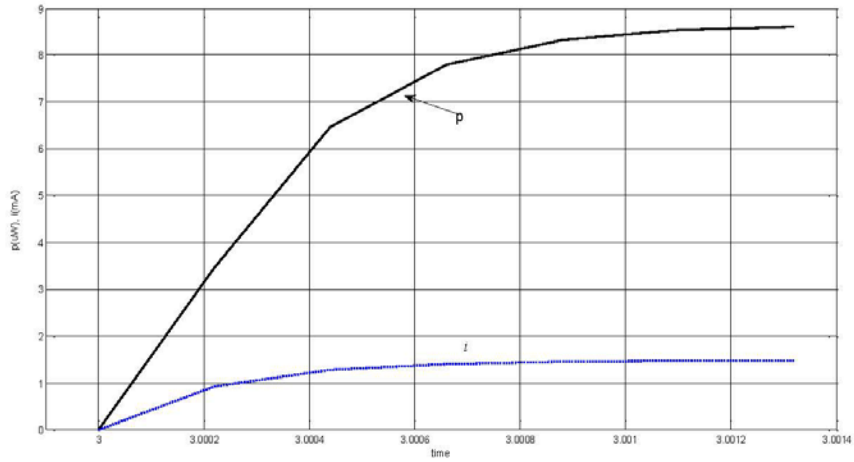
Since $v_x = 4 \times i_L$

$$v_x(t) = \begin{cases} 0 & t \le 3 \\ 5.88 \times 10^{-3}\left(1 - e^{-4545(t-3)}\right) & t \ge 3 \end{cases}$$

(c)

To plot obtain some points on p-curve

t	$i_L(t)$ (mA)	$p(t) = i_L^2 \times 4$ (μW)
3	0	0
3+τ	0.929	3.454
3+2τ	1.27	6.462
3+3τ	1.397	7.804
3+4τ	1.443	8.33
3+5τ	1.46	8.528
3+6τ	1.467	8.6



% the current saturates faster than the power!! (wow)

(d) and (e) are left for practice. (the circuit will be unstable if you have Rth≤0))

- Related book example: 8.7
- Book practice 8.9
-

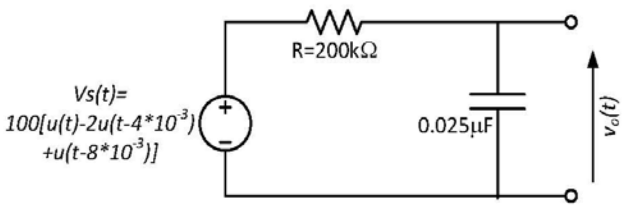
2-Sequential Switching

- We have sequential switching if the switching occurs in the circuit more than one time.
- In the analysis of circuits with sequential switching we consider that the circuit transient is not necessarily ended between the subsequent switching actions. (the time between the subsequent switching is possibly a short time (<3τ))
- We use the solution of an earlier state to determine the initial condition of the following state.

Example-2: The voltage source in the circuit shown in Fig.20 has "internal" sequential switching. Before t=0 there is no energy stored in the capacitor.

(a) Derive the expressions of $v_o(t)$ for the intervals (i) $t < 0$; (ii) $0 \le t \le 4ms$; (iii) $4ms \le t \le 8ms$; and $8ms \le t < \infty$.

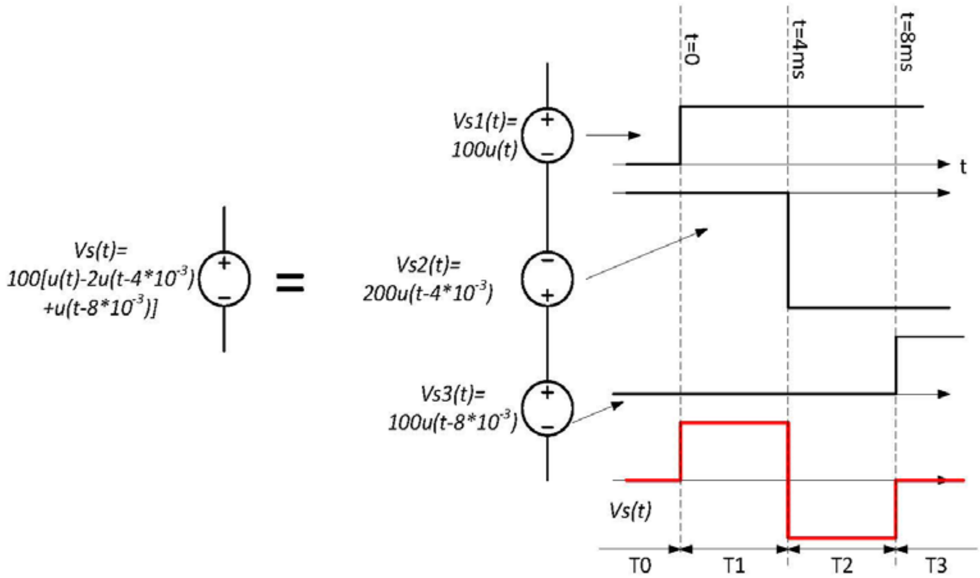
(b) Sketch $v_s(t)$ and $v_o(t)$ on the same coordinate axes.



Sol.

//simple circuit but complicated source! The following figure mitigates the supply and shows the shape of its voltage.

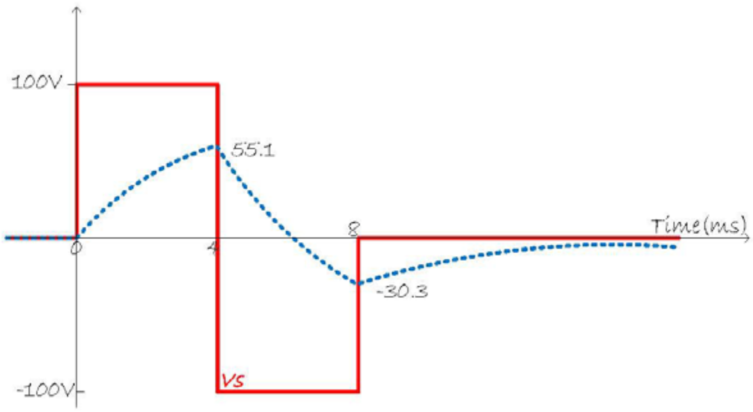
Some more, we have divided the time into 4 intervals T0-T3 //



The RC circuit time constant, $\tau = RC = 200 \times 10^3 \times 0.025 \times 10^{-6} = 5 \times 10^{-3} s$.

Interval	Initial condition V_0	Forced Sol. $V_F = v_s$	$v_c(t) = V_F + (V_o - V_F)e^{-\frac{(t-t_0)}{\tau}}$	Vc(end of interval)
T0 $(-\infty < t \le 0)$	0	0	0	0
T1 $(0 < t \le 4ms)$	0	100	$100 - 100e^{-200t}$	$100 - 100e^{-200\times4\times10^{-3}} = 55.1V$
T2 $(4ms < t \le 8ms)$	55.1	−100V	$-100 + 155.1e^{-200(t-0.004)}$	$-100 + 155.1e^{-200(0.008-0.004)} = -30.31V$
T3 $(8ms \le t < \infty)$	−30.31	0	$-30.13e^{-200(t-0.008)}$	0

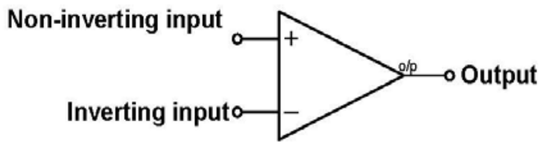
(b)

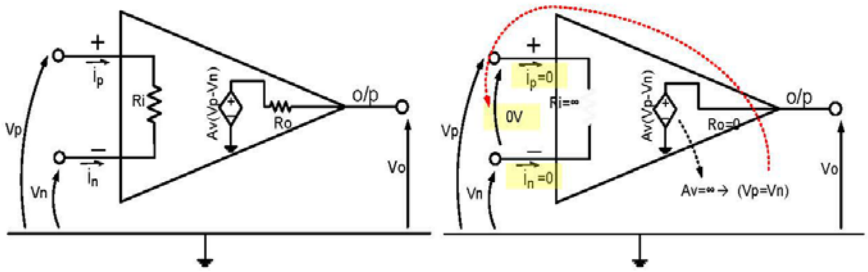


- Related Book Example: 8.9
- Related Book Practice 8.14
- Related Book Problems 41-45

Homework: Book Problem 35, 55

The Operational Amplifier

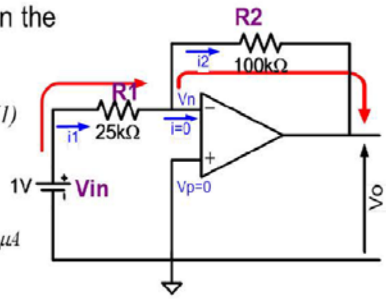




The ideal OPAMP:

- Ri=∞
- Av=∞
- Ro=0 I

- Calculate Vo in the circuit shown



*Vo=Vn-i2*100kW....(1)*

Vn=Vp=0

i2=i1

$$i1=\frac{1V-Vn}{25k\Omega}=\frac{1-0}{25k\Omega}=40\mu A$$

Substitute into (1)

*Vo=0-40μA*100kΩ*

Answer: ***<u>Vo=-4V</u>***

65. Obtain an expression for the voltage v_x as labeled in the op amp circuit of Fig. 8.94.

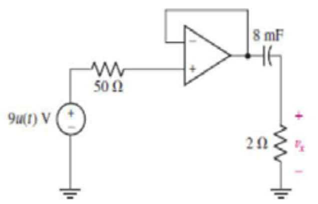
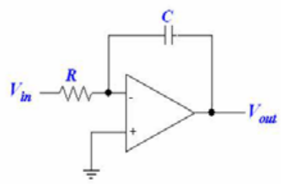


FIGURE 8.94

Homework: Assuming an ideal OPAMP, show that $v_{out}=-\frac{1}{RC}\int_{to}^{t}v_{in}dt+v_c(0)$



Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	Lecture sequences:	4th lecture	Instructor Name: Omer Sharaf Aldeen Ali Abbawi
	The major contents: Transient Response of RLC Circuit		
	The detailed contents: 1-Natural Response of Parallel of RLC Circuit 2- Analysis of Over-damped RLC Circuits 3- Analysis of Critically-damped RLC Circuits 4- Analysis of Under-damped RLC Circuits		

Chapter 2

Transient Response of RLC Circuit

- The response of the "second-order" RLC circuit is very different than that of the "first order" RL or RC system studied in Chapter 1.
- This chapter presents the natural and step response of parallel and series RLC circuits.

1. Natural response of Parallel RLC circuit

To find the natural response of a parallel RLC circuit, assume the circuit shown in Fig. 1 has been initiated at $t = 0$, with initial capacitor voltage $v_C(0^+) = V_o$ and initial inductor current, $i_L(0^+) = I_o$.

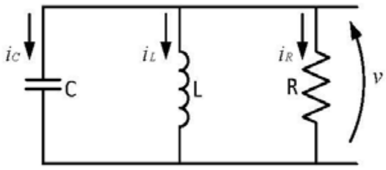


Fig. 1

The node current equation:

$$i_C + i_L + i_R = 0$$

Which can be written in terms of one variable, the parallel circuit voltage (v):

$$C\frac{dv}{dt} + \frac{1}{L}\int_0^t v dt + \frac{v}{R} = 0 \qquad (1)$$

To eliminate the integration in Eq. (1), we differentiate with respect to t:

$$C\frac{d2v}{dt^2} + \frac{1}{L}v + \frac{1}{R}\frac{dv}{dt} = 0 \qquad (2)$$

$$\frac{d2v}{dt^2} + \frac{1}{LC}v + \frac{1}{RC}\frac{dv}{dt} = 0 \qquad (3)$$

Eq. (3) is a second order DE, its solution has the form $v = f(t)$ and represents the circuit behavior for t>0.

In order to solve (3), assume:

$$v = Ae^t \qquad (4)$$

Substituting (4) into (3) and simplify gives:

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0 \quad (5)$$

For the reason indicated in Chapter 1, we ignore the case in which the first term is zero, giving:

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0 \quad (6)$$

- Eq. (6) determines the characters of $v(t)$and therefore it is known as the "**Characteristics Equation".**
- Eq (6) is a quadric equation has 2 roots, usually defined as follows:

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \quad (7)$$

Where:

$$\alpha = \frac{1}{2RC} \quad (8)$$

α is known as the **exponential damping coefficient** or **neper frequency**. And:

$$\omega_o = \frac{1}{\sqrt{LC}} \quad (9)$$

ω_o is known as the **resonant frequency**.

Both s_1 and s_2 given in Eq. (7) gives solutions of Eq. (3) if substituted in (4); however the general solution is formed as follows:

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (10)$$

The polarity of the term under the root of Eq. (7) determines the nature of *v*. This term $(\alpha^2 - \omega_o^2)$ gives three distinctive forms of v when being positive, zero or negative, as follows:

- if $\alpha > \omega_o$ gives two real roots and **Overdamped response**
- if $\alpha = \omega_o$ gives two real and equal roots and **Critical response**
- if $\alpha < \omega_o$ gives two complex conjugates roots and **Underdamped response**

The three forms will be discussed individually after the example.

Example 1: Consider a parallel RLC circuit having an inductance of 10 mH and a capacitance of 100 μF. Determine the resistor values that would lead to overdamped and underdamped responses.

Ans.:

From (9) $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2}x10^{-4}}} = 10^3\text{rad/sec}$

For overdamped response:

$$\alpha > \omega_o$$

$$\frac{1}{2RC} > 1000$$

$$R < \frac{1}{2 \times 1000 \times 10^{-4}}$$

$$\boldsymbol{R < 5\Omega}$$

For over damped response $\boldsymbol{R > 5\Omega}$

➔ Book Practice 9.1

2. Analysis of Overdamped RLC circuit

The voltage of the overdamped circuit is in the form of Eq. (10)

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (10)$$

Where, s_1 and s_2 are given in terms of circuit parameters in equations (7)-(9).

Now to determine A_1 and A_2, consider:

The initial capacitor voltage:

$$V_o = A_1 + A_2 \quad (11)$$

Take the differentiation of (10) at $t=0^+$

$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 \quad (12)$$

Given: $i_C(0^+) = C\frac{dv(0^+)}{dt}$

Or $$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} \quad (13)$$

$i_C(0^+)$ can be determined by applying KCL in terms of the inductor and resistor current:

$$i_C(0^+) = -i_L(0^+) - i_R(0^+)$$

$$i_C(0^+) = -I_o - \frac{V_o}{R} \quad (14)$$

Now we can substitute (13) and (14) into (12) to form the second equation to determine A_1 and A_2. However, closed form of this equation will not be given as it is difficult to memorize.

Usually the calculations procedure is given by the following sequence of applications:

- Given RLC, V_o, I_o
- Calculate α and ω_o
- Calculate s_1 and s_2
- Eq. (14)
- Eq. (13)
- Eq. (12)
- Eq. (11)   // alternatively this step can be done b4 Eq. (14)//
- Solve the simultaneous equations (11) and (12) obtain A_1 and A_2
- Write the expression of v Eq. (10)

Example 2: A source-free parallel RLC circuit is formed by 200Ω, 50mH, and 0.2μF. At t=0, the capacitor voltage $v_c(0) = 12V$ and the inductor current $i_L(0) = 30mA$.

- Find the expression of $v(t), t > 0$
- Plot $v(t), t > 0$

Ans.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 200 \times 0.2 \times 10^{-6}} = 12.5 \times 10^3 rad/sec$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 0.2 \times 10^{-6}}} = 10^4 rad/sec$$

Since $\alpha > \omega_o$ the response is overdamped

$$s_{1,2} = -12.5 \times 10^3 \pm \sqrt{1.5625 \times 10^8 - 10^8} = (-1.25 \pm .75) \times 10^4$$

$$s_1 = -5000 \; and \; s_2 = -20000$$

$\underline{V_o = A_1 + A_2 = 12}$ **(1)**

At t=0⁺,

$$i_L(0^+) = I_o = 30 \times 10^{-3} A$$

$$i_R(0^+) = \frac{v(0^+)}{R} = \frac{12}{200} = 60 \times 10^{-3} A$$

$$i_C(0^+) = -i_L(0^+) - i_R(0^+) = -90 \times 10^{-3} A$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-0.09}{0.2 \times 10^{-6}} = -45 \times 10^4$$

$$\frac{dv(0^+)}{dt} = s_1A_1 + s_2A_2$$

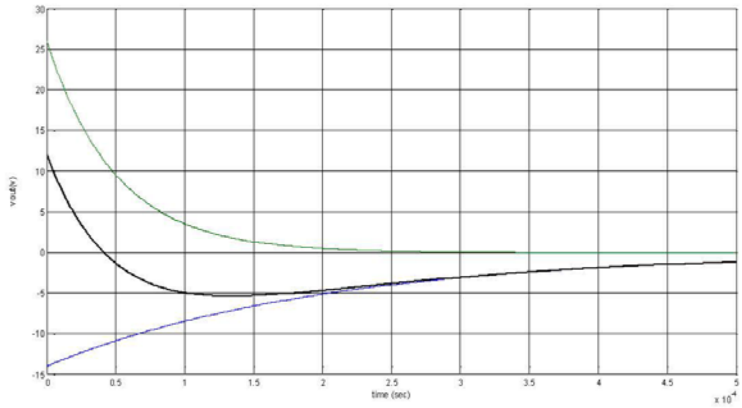
$$-45 \times 10^4 = -0.5 \times 10^4 A_1 - 2 \times 10^4 A_2$$

$\underline{90 = A_1 + 4A_2}$ **(2)**

(2)-(1)

$78 = 3A_2 \rightarrow$ A2=26 , A_1=-14

$$v(t) = -14e^{-5000t} + 26e^{-20000t}$$



Related Book Example 9.2, 9.3, 9.4

Practice 9.2 , 9.3, 9.4

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3. Analysis of Underdamped RLC circuit

If $\omega_0 > \alpha$, the roots of the characteristics equation described in Eq. (7) become complex.

$$s_{1,2} = -\propto \pm j\sqrt{\omega_o^2 - \alpha^2} \qquad (15)$$

The second term usually denoted by ω_d , where:

The damped radian frequency, $\omega_d \triangleq \sqrt{\omega_o^2 - \alpha^2}$

So

$$s_{1,2} = -\propto \pm j\omega_d \qquad (16)$$

// the reasons for this name will be explained in the lecture//

Substitute in Eq. (10)

$$v = A_1 e^{(-\propto + j\omega_d)t} + A_2 e^{(-\propto - j\omega_d)t}$$

$$v = A_1 e^{-\propto t} e^{j\omega_d t} + A_2 e^{-\propto t} e^{-j\omega_d t}$$

$$v = e^{-\propto t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t})$$

Use Eular's identity ($e^{jx} = cosx + jsin\,x$)

$$v = e^{-\propto t}(A_1(\cos(\omega_d t) + j\sin(\omega_d t)) + A_2(\cos(\omega_d t) - j\sin(\omega_d t)))$$

$$v = e^{-\propto t}((A_1 + A_2)\cos(\omega_d t) \quad + j(A_1 - A_2)\sin(\omega_d t))$$

Choose a different notation for the constants

$$\boldsymbol{v = e^{-\propto t}(B_1 \cos(\omega_d t) \quad + B_2 \sin(\omega_d t))} \qquad \textbf{(17)}$$

Where $B_1 = (A_1 + A_2)$ and $B_2 = j(A_1 - A_2)$

Now we use the initial conditions to determine the constant B_1 and B_2 as follows:

$$v(0^+) = V_0 = B_1$$

and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2$$

Example 3: A source-free parallel RLC circuit is formed by 20kΩ, 8H, and 0.125μF. At t=0, the capacitor voltage $v_c(0) = 0V$ and the inductor current $i_L(0) = -12.25mA$.

- Find the expression of $v(t), t > 0$
- Plot $v(t), t > 0$

Ans.

$$\alpha = \frac{1}{2RC} = 200\ rad/sec$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 10^3 rad/sec$$

Since $\propto < \omega_o$ the response is underdamped

$$\omega_d = \sqrt{10^6 - 4 \times 10^4} = 400\sqrt{6}$$

$$s_{1,2} = -200 \pm j400\sqrt{6}$$

Since V_o=0,

B_1=0

Also $i_R(0^+) = 0$

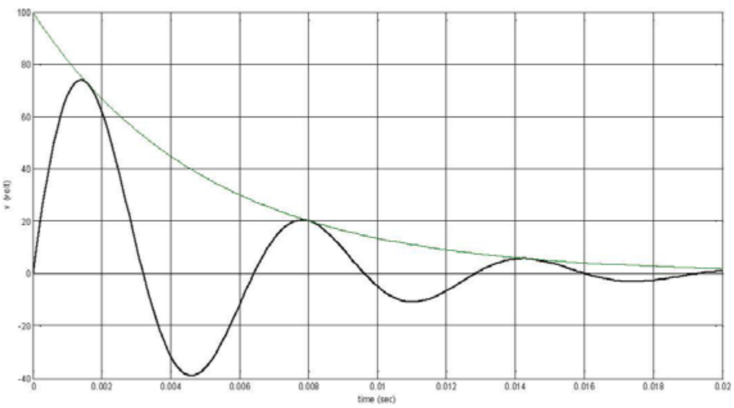
$$i_C(0^+) = -i_L(0^+) = 12.25mA$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = 98 \times 10^3 V/s$$

$$98 \times 10^3 = -\alpha B_1 + \omega_d B_2$$

$$B_2 = \frac{98 \times 10^3}{400\sqrt{6}} \cong 100V$$

Gives: $v(t) = 100e^{-200t} \sin 979.8t\ V$, for t≥0



Related Book Example 9.6

Book Practice 9.6

Home work : Book Problem 31 P366

4. Analysis of Critically-damped RLC circuit

If $\alpha = \omega_o$, the solution, the two roots of the DE will be identical

$$s_1 = s_2 = -\alpha \tag{18}$$

- If we carry on in the same way we will say that the solution will be similar to that of the 1st order DE given in chapter 1 ($Ae^{-\alpha t}$).
- This solution however cannot handle two different initial conditions. So there is a problem with this solution. Indeed the reason of that is due to the assumption proposed in Eq. 10. This form is not valid when the two root identical.
- In critical damping case the solution has the following form:

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \tag{19}$$

And the two arbitrary constant can be determined from initial conditions as follows:

$$V_o = D_2 \tag{20}$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2 \tag{21}$$

Example4: (book problem 23) A critically damped parallel RLC circuit is constructed from component values 40 Ω, 8 nF, and 51.2 μH, respectively.
(a) Verify that the circuit is indeed critically damped.
(b) Explain why, in practice, the circuit once fabricated is unlikely to be truly critically damped.
(c) The inductor initially stores 1 mJ of energy while the capacitor is initially discharged. Determine the magnitude of the capacitor voltage at t = 500 ns, the maximum absolute capacitor voltage, and the settling time.

Ans.

(a) $\alpha = \frac{1}{2RC} = \frac{10^9}{2*40*8} = 1.5625 \times 10^6$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{51.2 * 8 * 10^{-15}}} = 1.5625 \times 10^6$$

$\therefore \alpha = \omega_o$ critical damping

(b) In practice it is unusual to obtain components that are closer than 1 percent of their specified values. Thus, obtaining critical damping case not very likely. The model ignores the stray elements such as connection leads, the effect of temperature.

(c) determine *v(t)*

$$D_2 = V_o = 0$$

$$I_o = \sqrt{\frac{2W_L}{L}} = \sqrt{\frac{2 * 10^{-3}}{51.2 * 10^{-6}}} = 6.25A$$

$$i_C(0^+) = -i_L(0^+) - i_R(0^+) = -6.25A$$

$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-6.25}{8 * 10^{-9}} = -0.78125 \times 10^9 V/s$$

$$-0.78125 \times 10^9 = D_1 - \alpha D_2 = D_1$$

$$v(t) = -0.78125 \times 10^9 t e^{-1.5625 \times 10^6 t}$$

$$v(500ns) = -0.78125 \times 10^9 (500 \times 10^{-9}) e^{-1.5625 \times 10^6 (500 \times 10^{-9})}$$

$$v(500ns) = -178.8V$$

$$\frac{dv(t)}{dt} = -0.78125 \times 10^9 \left(t * -1.5625 \times 10^6 e^{-1.5625 \times 10^6 t} + e^{-1.5625 \times 10^6 t}\right)$$

Denote the peak voltage time by tp:

$$0 = -0.78125 \times 10^9 \left(tp * -1.5625 \times 10^6 e^{-1.5625 \times 10^6 tp} + e^{-1.5625 \times 10^6 tp}\right)$$

$$tp = \frac{-1}{-1.5625 \times 10^6} = 0.64 \mu sec$$

$$vp = -0.78125 \times 10^9 (0.64 * 10^{-6}) e^{-1.5625 \times 10^6 (0.64 * 10^{-6})}$$

$$vp = -184V$$

Settling time (ts) is the time at which the voltage drop to 5% of its peak

$$0.04(-184) = -0.78125 \times 10^9 ts e^{-1.5625 \times 10^6 ts} = -9.2V$$

$$ts e^{-1.5625 \times 10^6 ts} = 11.776 \times 10^{-9}$$

This nonlinear equation can be solved by iteration ts=3.676μsec.

Related Book Example 9.5

Practice 9.5

Homework

The natural voltage response of the circuit in Fig. 8.1 is

$$v(t) = 75e^{-8000t}(\cos 6000t - 4\sin 6000t)\text{V}, \quad t \geq 0,$$

when the inductor is 400 mH. Find (a) C; (b) R; (c) V_0; (d) I_0; and (e) $i_L(t)$.

Find the response $v_R(t)$ for $t > 0$ in the circuit in Fig. 8.102. Let $R = 3\ \Omega$, $L = 2$ H, and $C = 1/18$ F.

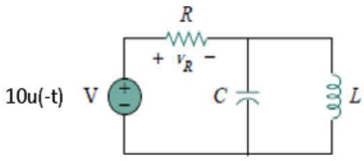


Figure 8.102 For Prob. 8.54.

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	**Lecture sequences:**	**5nd lecture**	**Instructor Name: Omer Sharaf Aldeen Ali Abbawi**
	The major contents: **Transient Response of RLC Circuit**		
	The detailed contents: 1- **The Complete Response pf Parallel RLC Circuits** **2-Derivation of Step Response Equations**		

Chapter 2

THE COMPLETE RESPONSE OF THE PARALLEL RLC CIRCUIT

In this lecture we will consider a parallel RLC with any initial conditions (inductor current and capacitor voltage) and we will determine the transient response after applying a DC current source at t=0. Unlike natural response case, the presence of the DC source causes non-discharged final conditions.

1. Derivation of Step Response Equations

Fig. 1 shows a parallel RLC circuit with initial capacitor voltage $v_C(0^+) = V_o$ and initial inductor current, $i_L(0^+) = I_o$; and a current source that is applied at t=0.

Notice: that the DC voltage source cannot be applied as the inductor will short circuit it!!

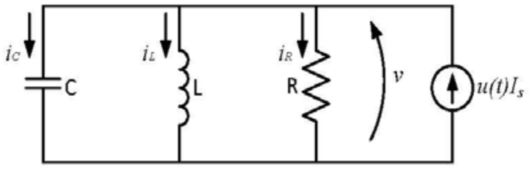


Fig. 1

The node current equation:

$$i_C + i_L + i_R = I \qquad (1)$$

By following steps similar to those used in last lecture, leads to:

$$\frac{d2v}{dt^2} + \frac{1}{LC}v + \frac{1}{RC}\frac{dv}{dt} = 0 \qquad (2)$$

Eq. (2) is identical to the natural response equation and leads to the same results obtained, which are:

- Over damping: $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (3)
- Under damping: $= e^{-\propto t}(B_1 \cos(\omega_d t) \;\; + B_2 \sin(\omega_d t))$ (4)
- Critical damping: $v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ (5)

The only difference in this case is that the effect of the supply must be taken into account (i.e. $i_C(t = 0^+) = \boldsymbol{I_s} - I_o - \frac{V_o}{R}$)

The evaluation of v of the parallel branches, however, is not sufficient to reveal full analysis as the expression of v, which approaches 0 at t→∞ does not reflect the end value of the inductor current. On the other hand if we have the inductor current we can find v easily! Therefore in the analysis of DC supplied parallel RLC circuit we will solve for $i_L(t)$ rather than v.

Before explaining the procedure by examples, let's look at the general expression of $i_L(t)$ as follows:

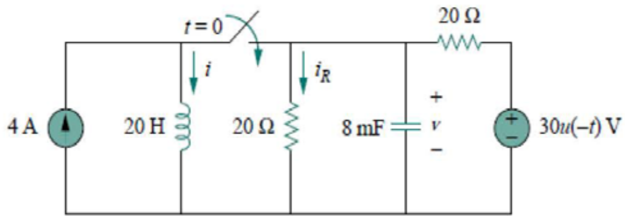
$$i_L(t) = I_f + i_n = I_s + (natural\ response\ function\ Eq.(3,4\ or\ 5)) \qquad (6)$$

The arbitrary constants of the natural response equation are determined from the initial conditions:

$$i_L(t = 0^+) = I_0 \ and \ \frac{di_L(t = 0^+)}{dt} = \frac{1}{L}V_o$$

Example 1:

In the circuit in Fig. 8.23, find $i(t)$ and $i_R(t)$ for $t > 0$.



Sol.

$I_o = 4A; V_o = 15V; v_L(0^+) = 15V; I_f = 4A$

$$\alpha = \frac{1}{2R_{eq}C} = \frac{1000}{2 * 10 * 8} = 6.25\frac{rad}{sec};\ \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 0.008}} = 2.5\ rad/sec$$

Notice that the parallel RLC circuit has two branches of resistors with 20Ω each. Therefore the equivalent resistor is 10Ω

$\alpha > \omega_o$ Overdamping response:

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -6.25 + 5.73 = -0.522;\ S_2 = -6.25 - 5.73 = -11.98$$

$$i = I_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i = 4 + A_1 e^{-0.522t} + A_2 e^{-11.98t}$$

At t=0, i=Io, gives

$$\mathbf{A_1 + A_2 = 0}$$

$$\frac{di(t=0^+)}{dt} = \frac{v_L(t=0^+)}{L} = A_1S_1 + A_2S_2$$

$$\frac{\mathbf{15}}{\mathbf{20}} = -\mathbf{0.522}A_1 - \mathbf{11.98}A_2$$

$\mathbf{0.75 = -0.522}A_1 + \mathbf{11.98}A_1;\ A_1 = \mathbf{0.0655};\ A_2 = \mathbf{0.0655}$

$$i = \mathbf{4 + 0.0655}e^{-0.522t} - \mathbf{0.0655}e^{-11.98t}$$

$$v_L = L\frac{di}{dt} = 20[-0.0342e^{-0.522t} + 0.7847e^{-11.98t}]$$

$$i_R = \frac{v_L}{\mathbf{20}} = \left[-\mathbf{0.0342}e^{-0.522t} + \mathbf{0.7847}e^{-11.98t}\right]$$

Practice: show that the energy dissipated in the resistors is equivalent to the energy supplied by the current source plus the energy initially stored in the capacitor.

Homework:

The switch in Fig. 8.4 was open for a long time but closed at $t = 0$. Determine: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.

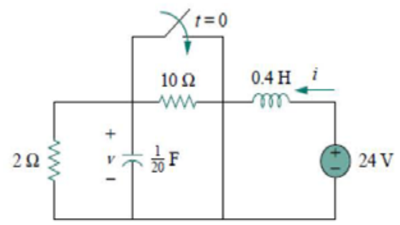
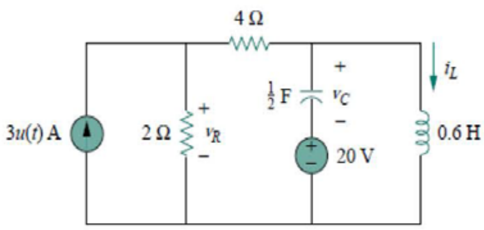


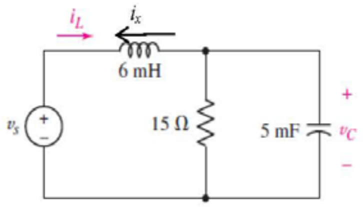
Figure 8.4 For Practice Prob. 8.1.

In the circuit of Fig. 8.5, calculate: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b) $di_L(0^+)dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$, (c) $i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$.



Example 2: (Book Problem 52)

52. Consider the circuit depicted in Fig. 9.52. If $v_s(t) = -8 + 2u(t)$ V, determine (*a*) $v_C(0^+)$; (*b*) $i_L(0^+)$; (*c*) $v_C(\infty)$; (*d*) $v_C(t = 150 \text{ ms})$.



Sol:

(a) $v_c(0^+) = V_o = -8V$

(b) $i_L(0^+) = I_o = -\frac{8}{15}A$

(c) $v_c(\infty) = -6V$

// the direction of i_L marked in the fig accompanying the question is opposite to that taken into consideration so just treat i_x as ($-i_L$) //

// solution plan: find i_x → find v_L → then v_c=v_L +v_s ☺ //

//1-) determination of the type of response://

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 15 \times 5 \times 10^{-3}} = 6.67\ rad/sec$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{10^3}{\sqrt{6 \times 5}} = 182.6 rad/sec$$

$\omega_o > \alpha$ Underdamping response $\omega_d = \sqrt{\frac{10^6}{30} - \frac{10^6}{22500}} \cong 182.45\ rad/sec$

//2-) Initial and forced conditions //

$I_o = \frac{8}{15} = 0.533A$, $V_o = -8V$, $v_L(t = 0^+) = v_c - v_s = -8 - (-6)$ =-2V, $I_f = \frac{6}{15} = 0.4\ A$

// write the equation and find the constants for iL//

$$i_X = I_f + e^{-\propto t}(B_1{}'\cos(\omega_d t) \quad + B_2{}'\sin(\omega_d t))$$

At t=0[+] $\quad 0.533 = 0.4 + B_1; B_1 = 0.133$

$$\frac{di_x(t=0^+)}{dt} = -\frac{2}{0.006} = \omega_d B_2' - \alpha B_1{}'$$

$$B_2 = \frac{-333.3 + 6.67 \times 0.133}{182.45} = -1.822$$

$$i_L = 0.4 + e^{-6.7t}(0.133\cos(182.45t) \quad - 1.822\sin(182.45t))$$

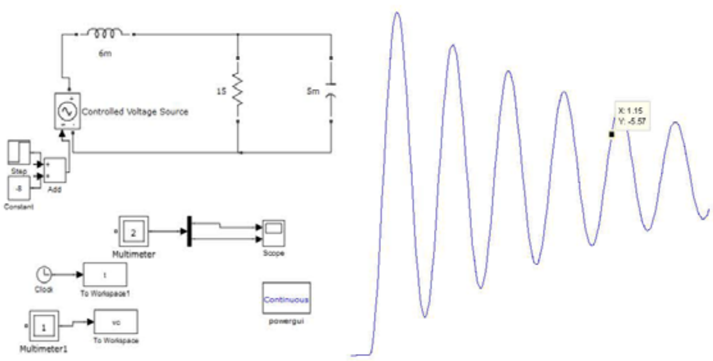
$$v_L = 0.006[e^{-6.67t}(-0.133 \times \omega_d \sin(\omega_d t) - 1.822\omega_d \cos(\omega_d t)) + (0.133\cos(\omega_d t) - 1.822\sin(\omega_d t))(-6.7e^{-6.7t})]$$

$$v_L = e^{-6.7t}(-2\cos(\omega_d t) - 0.072\sin(\omega_d t))$$

$$v_L(t = 150ms) = e^{-6.7*0.15}(-2\cos(182.45 * 0.15) - 0.072\sin(182.45 * 0.15))$$

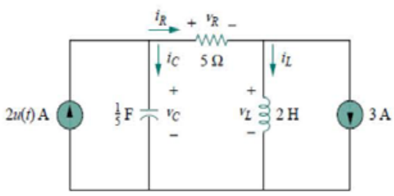
$$v_L(t = 150ms) = 0.433V$$

$$v_C = v_S + v_L = -6 + 0.4325 = \mathbf{-5.567V}$$



Practice:

For the circuit in Fig. 8.7, find: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$, (c) $i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$.



Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	**Lecture sequences:**	**6nd lecture**	**Instructor Name: Omer Sharaf Aldeen Ali Abbawi**
	The major contents: **Transient Response of RLC Circuit**		
	The detailed contents: 1- **Natural Response of Series RLC Circuits** **2- Step Response of Series RLC Circuits**		

Chapter 2

Transient Response of Series RLC Circuit

The procedure of natural and step response analysis for the series RLC circuit is similar to that of the parallel circuit. The only difference is in the definition of the damping ratio, α.

Your selection of the most appropriate variable for solution will also differ as you will see shortly.

1. Natural response of series RLC circuit

Assume the circuit shown in Fig. 1 is initiated at $t = 0$, with initial capacitor voltage $v_C(0^+) = V_o$ and initial inductor current, $i_L(0^+) = I_o$.

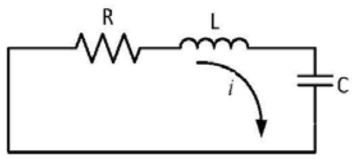


Fig. 1

The loop voltage equation:

$$v_R + v_L + v_C = 0$$

write the equation in terms of one variable, the current (i):

$$iR + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \qquad (1)$$

Differentiate with respect to *t*, rearrange and divide by L:

$$\frac{d2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \qquad (2)$$

Eq. (2) is similar to the parallel voltage equation of the parallel RLC circuit [Eq. (3) week#4]. Therefore, without repeating the derivation, we are going to state the characteristics equation and the current equations for the three cases. (The **bold** marks the difference compared to parallel voltage case):

$$s^2 + \frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{s} + \frac{1}{LC} = 0 \qquad (3)$$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad (4)$$

Where: $$\alpha = \frac{\boldsymbol{R}}{\boldsymbol{2L}} \qquad \textbf{(Rad/sec)} \qquad (5)$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad \text{(Rad/sec)} \qquad (6)$$

The current equation depends on the value under the root square in Eq. (4)

- if $\alpha > \omega_o$ Overdamping : $i = A_1e^{s_1t} + A_2e^{s_2t}$ (7)
- if $\alpha < \omega_o$ Underdamping $i = e^{-\propto t}(B_1\cos(\omega_d t) + B_2\sin(\omega_d t))$ (8)

  where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$ (9)
- if $\alpha = \omega_o$ Critical damping : $i = D_1te^{st} + D_2e^{st}$ (10)

In case of natural response it is a good choice to solve for the current, (*i*) especially if need to calculate more than one quantity (for example *i and* v_L) because the current of the three elements is equal.

Example 1 The 0.1 μF capacitor in the circuit shown in Fig 2 is charged to 100 V. At t = 0 the capacitor is discharged through a series combination of a 560 Ω resistor and 100 mH inductor.
a) Find i(t) for t > 0.
b) Find vc(t) for t > 0.

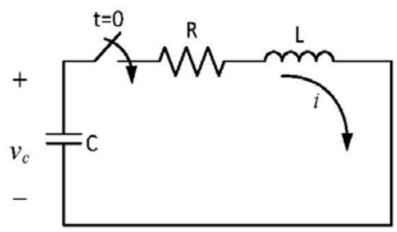


Fig. 2

Sol:

$\alpha = \frac{R}{2L} = \frac{560}{2\times0.1}$=2800 (rad/sec)

$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\times10^{-7}}} = 10^4$ rad/sec

((Underdamping))

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{10^8 - \alpha^2} = 9600 \text{ rad/sec}$$

$$i = e^{-\propto t}(B_1\cos(\omega_d t) + B_2\sin(\omega_d t))$$

$$i(0^+) = B_1 = 0$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

$-v_c(0^+) + i(0^+)R + v_L(0^+) = 0$ gives $\quad v_L(0^+) = 100V$

$\frac{di(0^+)}{dt} = 1000A/sec = \omega_d B_2 - \propto B_1$ gives $\quad B_2 = \frac{1000}{9600} \approx 0.1042$A

$\boldsymbol{i = 0.1042e^{-2800t}\sin(9600t)}$ t>0

$v_c = \frac{1}{C}\int idt$ // can be solved as ∫udv…but lets look for alternative//

OR $v_c(t) = iR + v_L = iR + L\frac{di}{dt}$

$$v_c = 560 \times \frac{10}{96}e^{-2800t}\sin(9600t) + 0.1\left(\frac{10}{96}\right)[e^{-2800t}(9600cos9600t) + sin(9600t)(-2800e^{-2800t})]$$

$$\boldsymbol{v_c = e^{-2800t}[100cos9600t - 29.17sin9600t]}$$

// what will be the case if we determined vc first ...??? ....try//

Practice: For the circuit in example 1,

(a) draw the variation of the capacitor voltage against time,
(b) determine the settling time, and
(c) show the all the energy initially stored in the inductors is dissipated in R.

2. Step response of series RLC circuit

In this case a DC source will be present in the circuit after the switching, this leads to $v_C(t = \infty) = V_F$ as seen in Fig. 3.

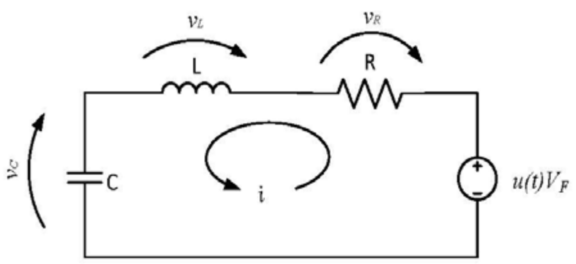


Fig. 3

In case of step response, the capacitor voltage is usually a good choice because it is the only variable which has a non-zero forced part (Solving for other variables is also possible). The forced response expressions for i_C for the three types of response are as follows:

- Overdamping : $v_C = V_F + A_1e^{s_1t} + A_2e^{s_2t}$ (11)
- Underdamping $v_C = V_F + e^{-\alpha t}(B_1\cos(\omega_d t) + B_2\sin(\omega_d t))$ (12)
- Critical damping : $v_C = V_F + D_1te^{st} + D_2e^{st}$ (13)

Example 2: In the circuit shown in Fig. 4, switch 1 has been in position (a) and switch two has been closed for a long time. At t=0 switch 1 moves to position (b) and switch 2 opens simultaneously. Find $v_C(t)$ for t>0.

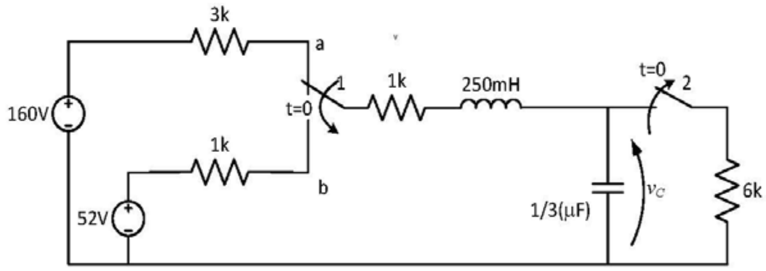


Fig. 4

Sol:

Initial and Forced Quantities:

$$V_{o=}v_C(0^-) = \frac{160}{3k + 1k + 6k}6k = 96V$$

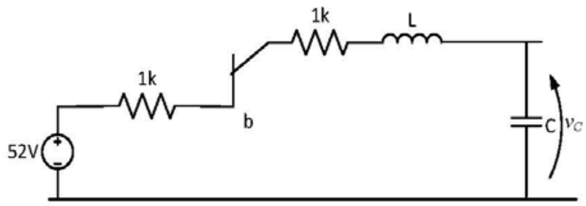
$$V_{F=}v_C(\infty) = 52V$$

$$I_o = i_L(0^-) = \frac{160}{10k} = 16mA$$

$$I_F = i_L(\infty) = 0$$

Type of response:

The circuit for t>0



$$\alpha = \frac{R}{2L} = \frac{2000}{2 \times 0.25} = 4000\ rad/sec$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\times\frac{1}{3}\times 10^{-6}}} = 2000\sqrt{3}\text{ rad/sec}$$

$\alpha > \omega_o$ overdamping response

$$S_{1,2} = -4000 \pm 1000\sqrt{16-12} = -4000 \pm 2000$$

$S_1 = -2000;\ S_1 = -6000$ (rad/sec)

$$v_C = V_F + A_1 e^{-2000t} + A_2 e^{-6000t}$$

At $t=0^+$

$$96 = 52 + A_1 + A_2$$

$A_1 + A_2 = 44V$------(1)

$$\frac{dv_C(0^+)}{dt} = \frac{I_o}{C} = \frac{16\times 10^{-3}}{\frac{1}{3}\times 10^{-6}} = 48\times 10^3 V/.sec$$

$$48\times 10^3 = -2000A_1 - 6000A_2$$

$24 = -A_1 - 3A_2$-----(2)

Add (1) and (2)

$68=-2A_2$.....gives $A_2=-34$

$A_1=78$

$$v_C = 52 + 78e^{-2000t} - 34e^{-6000t}\text{V},\qquad t>0$$

Example 3 (with dependent source)

49. Obtain an expression for i_1 as labeled in Fig. 9.49 which is valid for all $t > 0$.

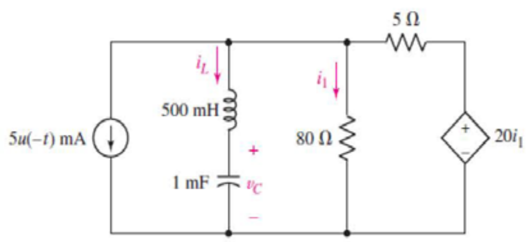
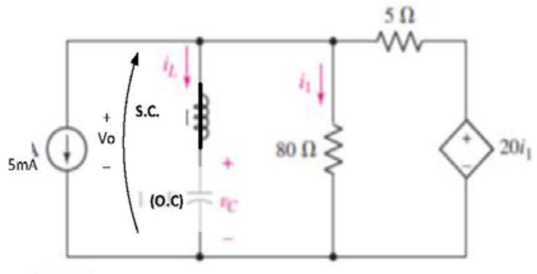


FIGURE 9.49

Sol.

Determination of initial conditions

Consider the circuit with t<0



Write the node currents equation:

$5\times10^{-3}+\frac{V_o}{80}+\frac{V_o-20i_1}{5}=0$

$$5\times10^{-3}+\frac{V_o}{80}+\frac{V_o-20\left(\frac{V_o}{80}\right)}{5}=0$$

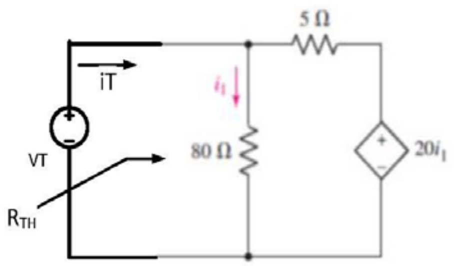
$$5\times10^{-3}+\frac{V_o}{80}+\frac{V_o}{5}-\frac{V_o}{20}=0$$

$$0.4+V_o+16V_o-4V_o=0$$

$$V_o=-\frac{0.4}{13};\ and\ I_o=0$$

For t>0

Determine the equivalent resistor (R_{TH}).



By applying a test voltage source (V_T), the resultant current is (I_T) and $R_{TH} = \frac{V_T}{I_T}$

$$I_T = \frac{V_T}{80} + \frac{V_T - 20\left(\frac{V_T}{80}\right)}{5} = \frac{V_T}{80} + \frac{V_T}{5} - \frac{V_T}{20}$$

$80I_T = V_T + 16V_T - 4V_T$ gives $\boldsymbol{R_{TH} = \frac{V_T}{I_T} = \frac{80}{13}\Omega}$

<u>Determination of type of response</u>

$$\alpha = \frac{R}{2L} = \frac{80}{13} = 6.153\ (rad/sec)$$

$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5*10^{-3}}} = \frac{100}{\sqrt{5}} \approx 44.72$ (Rad/sec)

Under damping response:

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} \approx 44.3\ rad/sec$$

$$i = e^{-\propto t}(B_1\cos(\omega_d t) + B_2\sin(\omega_d t))$$

At $t=0^+$, $i(0+)=0$ gives $B_1=0$

$$i = e^{-\propto t}(B_2\sin(\omega_d t))$$

$$\frac{di}{dt} = \frac{v_L(0^+)}{L} = \frac{-V_o - I_o R}{0.5} = -2\left(-\frac{0.4}{13} + 0\right)$$

$$\frac{di(t=0^+)}{dt} = \frac{0.8}{13} = \omega_d B_2$$

$$B_2 \approx 1.39mA$$

$$i = 1.39e^{-6.153t}\sin(44.3t)$$

$$i_1 = -i = -1.39e^{-6.153t}\sin(44.3t)$$

H.W.

Book Problem (Chapter 9) **56 Parts (a) and (b) only**. And Book Problem **57**

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	**Lecture sequences:**	**7nd lecture**	**Instructor Name: Omer Sharaf Aldeen Ali Abbawi**
	The major contents: **Balanced Three-Phase Circuits**		
	The detailed contents: 1- **The Three Phase Y- connected Source** **2- Balanced Three-Phase System** **3- The Three Phase Delta - connected Source**		

Chapter 3

Balanced Three-Phase Circuits

The electric energy is generated, transmitted and distributed using three-phase systems for economical reason. Large loads, such as industrial apparatus, are three-phase loads. This chapter introduces the three-phase systems and its basic analysis[1].

This lecture defines the basic elements, connections and circuits of the three-phase system.

1. The three-phase Y-connected source

Consider the three ac supplies shown in Fig. 1. The three sources represent a balanced three-phase supply if the voltages of the three supplies have the following form (in various representations)

1-Instanteos representation:

$$v_{an} = V_{ph}\sin(2\pi f t + \emptyset)$$
$$v_{bn} = V_{ph}\sin(2\pi f t + \emptyset - 120°)$$
$$v_{cn} = V_{ph}\sin(2\pi f t + \emptyset - 240°)$$
....(1)

OR

$$v_{an} = V_{ph}\sin(2\pi f t + \emptyset)$$
$$v_{bn} = V_{ph}\sin(2\pi f t + \emptyset + 120°)$$
$$v_{cn} = V_{ph}\sin(2\pi f t + \emptyset + 240°)$$
...(2)

2- Vector representation:

$$V_{an} = V_{ph}\angle\emptyset$$
$$V_{bn} = V_{ph}\angle\emptyset - 120°$$
$$V_{cn} = V_{ph}\angle\emptyset - 240°$$
...(3)

OR

$$V_{bn} = V_{ph}\angle\emptyset$$
$$V_{bn} = V_{ph}\angle\emptyset + 120°$$
$$V_{cn} = V_{ph}\angle\emptyset + 120°$$
...(4)

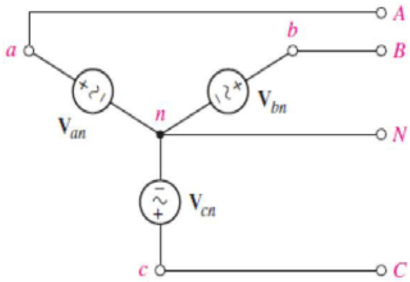


Fig. 1

Positive and negative sequence:

In equations (1) and (3), we can think of the voltage v_{an} as "leading" v_{bn} by 120°; and the voltage v_{bn} as leading v_{cn} by 120°. So the voltages are arranges as v_{an} - v_{bn} - v_{cn} as the time progresses. This sequence is known as the **positive sequence** of voltages.

[1] The material of this chapter is covered in chapter 12 of the textbook

In contrast, for the voltages represented in equations (2) and (4), the voltage v_{an} as "lagging" v_{bn} by 120°; and v_{bn} lags v_{cn} by 120°. So the voltages are arranges as $v_{an} - v_{cn} - v_{bn}$ as the time progresses. This sequence is known as the **negative sequence** of voltages.

To emphasize the above Fig. 2 shows the three voltages represented in Eq. (1) and Fig. 3 shows the vectors represented in Eq. (3). In Fig. 2 and Fig. 3 the, ϕ is considered to be zero and this is common practice.

In summary, we can say that the three-phase voltage system is composed of three ac voltages of:

a. Identical frequency,
b. Equal amplitudes, and
c. Phase shift of 120° between the three sources

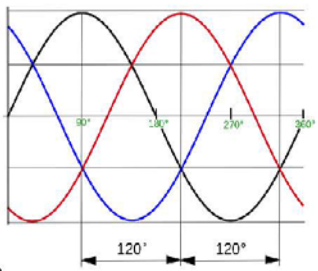


Fig. 2

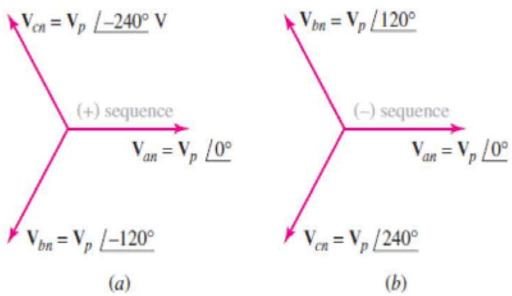


Fig. 3

To explain the practical implementation of three-phase source; The electricity is generated in bulk by a three-phase generates with rotating magnet that induces the voltage in three identical stator windings which are displaced by 120° in space as shown in Fig. 4. This maintains identical frequency and phase constant 120°-phase displacement of the three sources representing the voltages induced in the three windings.

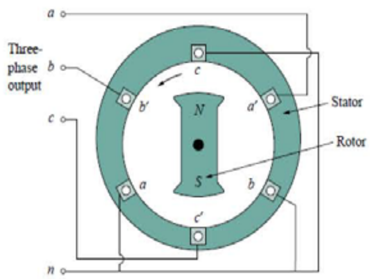


Fig. 4

The "a" operator

We can write the three phase quantities in more brief form by using the "a" operator. Where:

$$a \triangleq 1\angle 120^\circ \; or \; a \triangleq 1\angle -240^\circ \qquad ...(5)$$

The effect of multiplying a vector by a is that the vector rotates by 120°. Consequently:

$$a^2 = 1\angle -120^\circ \; or \; a^2 = 1\angle 240^\circ \qquad ...(6)$$

In positive sequence voltages:

$$V_{bn} = a^2 V_{an} \; and \; V_{cn} = aV_{an} \qquad ...(7)$$

And in negative sequence:

$$V_{bn} = aV_{an} \; and \; V_{cn} = a^2 V_{an} \qquad ...(8)$$

Zero-Sum property

One important property of the three-phase voltages is that it sums zero, we can show that analytically or graphically as shown below:

Based on Eq. (1)

$$v_{an} + v_{bn} + v_{cn} = V_{ph}[\sin(2\pi ft + \emptyset) + \sin(2\pi ft + \emptyset - 120) + \sin(2\pi ft + \emptyset + 120)]$$

Using the trigonometric identity: $\sin\,(A+B) = \sin A \cos B + \cos A \sin B$ and denoting $2\pi ft + \emptyset$ by x

$$v_{an} + v_{bn} + v_{cn} = V_{ph}[\sin x + \sin(x - 120) + \sin(x + 120)]$$

$$= \sin x + \sin x \cos(-120) + \mathbf{\cos x \sin(-120)} + \sin x \cos(120) + \mathbf{\cos x \sin(120)}$$

$$= \sin x\,[1 + \cos(-120) + \cos(120)] = \sin x\,[1 - 0.5 - 0.5] = \mathbf{0}$$

The graphical proof is based on adding the three vectors as shown in Fig. 5

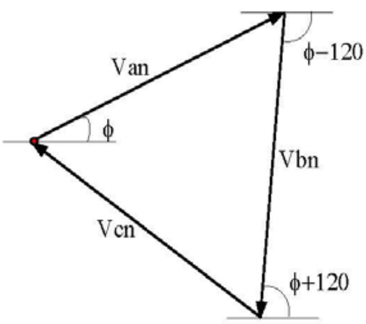


Fig. 5

<u>The Phase Voltages and the Line Voltages</u>

Referring to Fig. 1, in the configuration shown the sources is said to be Y-connected. The supply common point is known as the neutral point denoted by N and the other individual lines (A,B, and C) are known as the supply lines. The voltage between any line and the common neutral line is the ***phase voltage***. The three phase voltages are (V_{AN}, V_{BN} and V_{CN}).

The line-to-line voltages are the three voltages between the three lines; defined as:

$$V_{AB} = V_{AN} - V_{BN} = V_{ph} - \left(-aV_{ph}\right) = V_{ph}(1+a) = V_{ph}[(1+\cos(120)) + j(\sin(120))] = V_{ph}\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$$

$$= V_{ph}\left(\sqrt{\left(\frac{3}{2}\right)^2 + \frac{3}{4}}\right)\angle \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right) = V_{ph}\left(\sqrt{3}\right)\angle \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \sqrt{3}\boldsymbol{V_{ph}}\angle\boldsymbol{30°}$$

$$V_{AB} = \sqrt{3}\boldsymbol{V_{ph}}\angle\boldsymbol{30°} \qquad \text{...(9)}$$

In the same way it can be shown that the line-to-line voltages (V_{AB}, V_{BC} and V_{CA}) forms a three phase of voltages system; related to the phase voltages by the following:

- $\sqrt{3}$ amplitude ratio: $|V_{LL}| = \sqrt{3}|V_{LN}|$
- 30° phase shift:
    - For positive sequence systems, the line-to-line voltages lead the phase voltages by 30°.
    - For negative sequence systems, the line-to-line voltages lags the phase voltages by 30°.

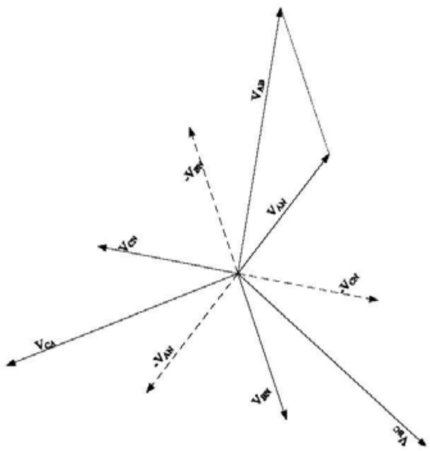


Fig. 6

2. The balanced three-phase system

In this section we will consider a three phase circuit analysis. The balanced system has a three identical parts connected to the terminals of the three phase supply. In the circuit shown in Fig. 7. The phase circuit is composed of the supply and a load impedance denoted by Z_p. The common point of the three load impedances (N) is also connected to the supply neutral point (n). This configuration is called Y-Y 4-wire system. Let's determine the current I_{Nn}:

$$I_{Nn} = I_A + I_B + I_C = \frac{V_{an}}{Z_p} + \frac{V_{bn}}{Z_p} + \frac{V_{cn}}{Z_p} = \frac{0}{Z_p} = 0$$

From the above, it is shown that the neutral line current is zero (in balanced systems) and therefore if we remove the wire connecting N-to-n, it will have no effect on the circuit operation, i.e. the points "n" and "N" will still have the same voltage and we can say that $V_{AN}=V_{an}$.

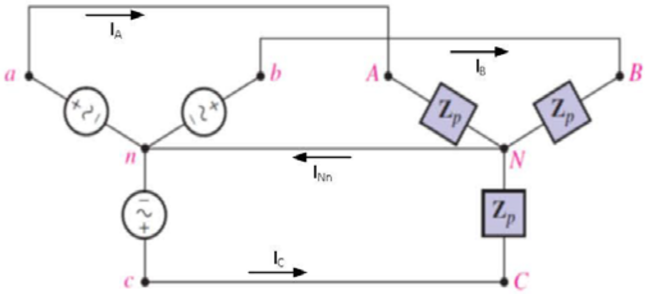


Fig. 7

Example 1: For the circuit of Fig. 8, find both the phase and line currents, and the phase and line voltages throughout the circuit; then calculate the total power dissipated in the load.

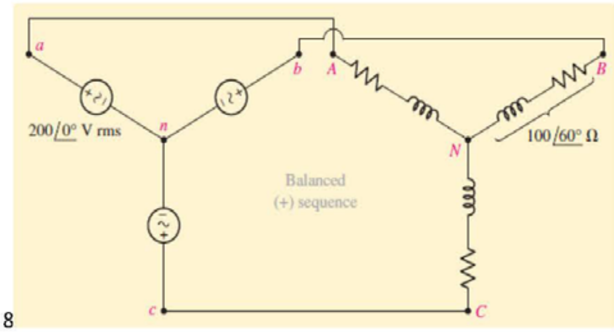


Fig. 8

Sol.

The phase voltages:

$$V_{AN} = V_{an} = 200\angle 0^\circ\ V;\ V_{BN} = V_{bn} = 200\angle -120^\circ\ V;\ V_{CN} = V_{cn} = 200\angle 120^\circ\ V$$

The line Voltages:

$$V_{AB} = \sqrt{3}V_{an}\angle 30^\circ = 346.4\angle 30^\circ\ V;\ V_{BC} = 346.4\angle -90^\circ\ V\ ;\ V_{CA} = 346.4\angle 150^\circ\ V$$

The phase (and line) currents

$$I_A = \frac{200\angle 0}{100\angle 60} = 2\angle -60^\circ\ A;$$

$$I_B = 2\angle(-60-120) = 2\angle 180^\circ\ A$$

$$I_C = 2\angle(-60+120) = 2\angle 60^\circ\ A$$

From Example 1, it is shown that symmetry between the phases can be utilized to extend the analysis of one phase to the other phases simply by shifting the phase angle. For more complex circuits the technique of "*single-line diagram*" is normally employed as shown in Example 2.

Before the example recall the Y-Δ conversion techniques used in circuit simplification. If we have equal impedances; the conversion becomes as follows:

$$Z_Y = \frac{Z_\Delta}{3} \qquad \text{...(9)}$$

Example 2: Determine I_{aA} *and* v_{Bn} for the circuit in Fig. 9

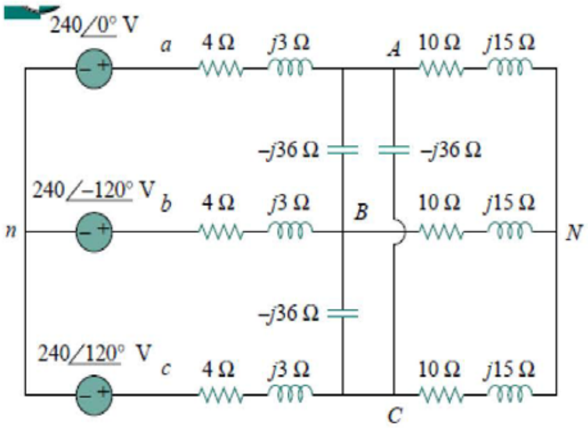


Fig. 9

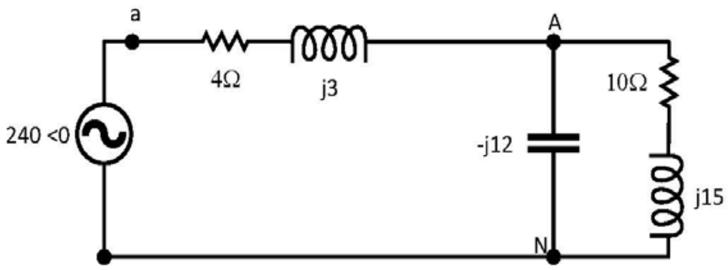
// notice that the Δ-connected element cannot be used in the single line diagram; (SLD) as the this diagram shows the connection to neutral;

Therefore, B4 drawing the SLD, we replace the Δ-connected elements by the Y-equivalents//

1- We replace the Δ-connected capacitor load by its Y-equivalent. The per-phase equivalent impedance:

$$Z_Y = -\frac{j36}{3} = -j12$$

2- We draw the per-phase equivalent circuit to determine, i_{aA} :



$$Z_{eq} = 4 + j3 + (-j12\backslash\backslash(10 + j15))$$

$$Z_{eq} = 4 + j3 + \frac{180 - j120}{10 + j3} = 4 + j3 + \frac{216.33\angle - 33.69°}{10.44\angle 16.7°}$$

$$Z_{eq} = 4 + j3 + 20.72\angle - 50.4° = 4 + j3 + 13.2 - j15.97$$

$$`Z_{eq} = 17.2 - j12.97 = 21.54\angle - 37°$$

$$I_{aA} = \frac{240\angle 0}{21.54\angle - 37} = 11.142\angle 37\ A$$

To determine v_{Bn}, we determine, v_{An}

$$v_{An} = 240\angle 0 - 11.142\angle 37 * (4 + j3)$$

$$v_{An} = 240\angle 0 - 55.71\angle 73.87 = 224.5 - j53.52$$

$$v_{An} = 230.8\angle - 13.4°\text{A}$$

$$v_{Bn} = v_{An}\angle - 120 = 230.8\angle - 133.4°$$

3- The Δ-connection

Phase and line quantitates for Y-connected element:

The relationship between the phase and line voltage of a Y-connected supply is derived [Eq. (9)] and defined in the past section. This relationship is re-stated and extended to Y-connected in Table 1 (Reference Table 12.1 textbook).

Phase and line quantitates for Δ-connected element:

The source may also be connected in Δ configuration, as shown in Fig. 10.

// The Δ-connection of the supply is not so common because any unbalance in the supply causes a considerable amount of "circulating current" in the Δ-circuit and causes losses; however unbalanced systems are beyond our field of study//

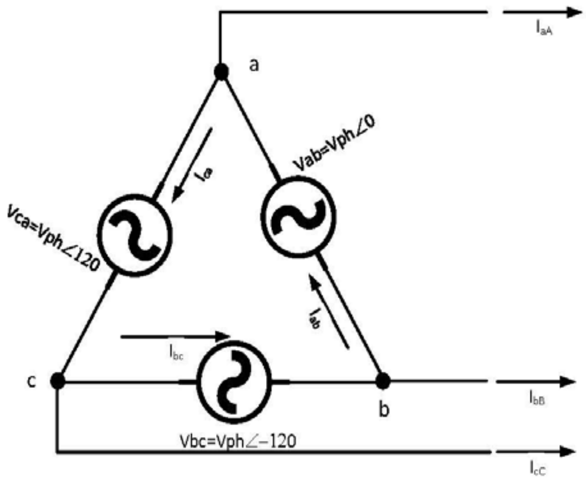


Fig. 10

In Δ-connected source the line voltages are equal to the phase voltages; while the amplitudes of phase currents (I_{ab} , I_{bc} and I_{ca} in Fig. 10) and the line current ((I_{aA} , I_{bB} and I_{cC}) are related by:

$$|I_{LL}| = \sqrt{3}\,|I_{ph}| \qquad ...(10)$$

The line current lags the phase currents by 30° for positive sequence systems as indicated in table 1. In Fig. 10, the line and phase current are related by:

$$I_{aA} = I_{ab} - I_{ca};\; I_{bB} = I_{bc} - I_{ab};\, I_{cC} = I_{ca} - I_{bc};$$

Table 1

Load	Phase Voltage	Line Voltage	Phase Current	Line Current	Power per Phase
Y	$\mathbf{V}_{AN} = V_p\angle 0^\circ$ $\mathbf{V}_{BN} = V_p\angle -120^\circ$ $\mathbf{V}_{CN} = V_p\angle -240^\circ$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{AN} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{BN} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{CN} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta =$ power factor of the load
Δ	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta =$ power factor of the load

Example 3

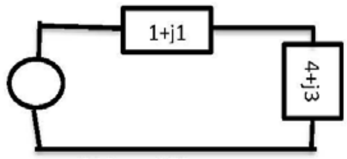
A negative-sequence balanced three-phase Y-connected source supplies power to a balanced, three-phase Δ- connected load with an impedance of 12 + j9 Ω/phase. The source voltage in the b-phase is 240∠-50° V. The line impedance is 1 + j1 Ω/phase. Draw the single phase equivalent circuit for the a-phase and use it to find the current in the a-phase of the load.

Sol.

$$Z_\Delta = \frac{Z_Y}{3} = 4 + j3\Omega$$

For negative-sequence supply $v_{bn} = a v_{an}$

$$v_{an} = \frac{v_{bn}}{a} = 240\angle -50 - 120 = 240\angle -170V$$



$$I_{aA} = \frac{240\angle -170}{(1+j1)+(4+j3)} = 37.48\angle 151.34$$

$$I_{AB} = \frac{37.48\angle 151.34}{\sqrt{3}}\angle -30 = 21.64\angle -121.34°A$$

Related Book Problems 15-35

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	Lecture sequences:	8nd lecture	Instructor Name: Omer Sharaf Aldeen Ali Abbawi
	The major contents: Analysis and Power Calculation of Three-Phase Circuits		
	The detailed contents: 1- Analysis of three-phase circuits with Y-connected supply 2- Analysis of systems with Delta-connected supply 3- Power Calculations in Three-Phase Systems		

Chapter 3

Analysis and Power Calculation of Three-Phase Circuits

This lecture presents the analysis of balanced three-phase circuits using single line diagram. A special emphasis is given for the Δ-connected supply. Power calculation in three-phase circuits in terms of phase and line quantities is also presented.

a. Analysis of three-phase circuits with Y-connected supply.

We have shown in the first lecture that the three-phase system can be seen as similar single phase systems (except the 120°-phase shift). We can use this symmetry to simplify the analysis using the single line diagram approach providing that we have neutral pints at the supply and load. In this section we will consider Y-connected supply. We can provide an imaginary neutral point for the Δ-connected load using Δ-Y transformation. For equal impedances.

$$Z_Y = \frac{Z_\Delta}{3} \qquad \text{...(1)}$$

Example 2: Determine I_{aA} *and* v_{Bn} for the circuit in Fig. 1

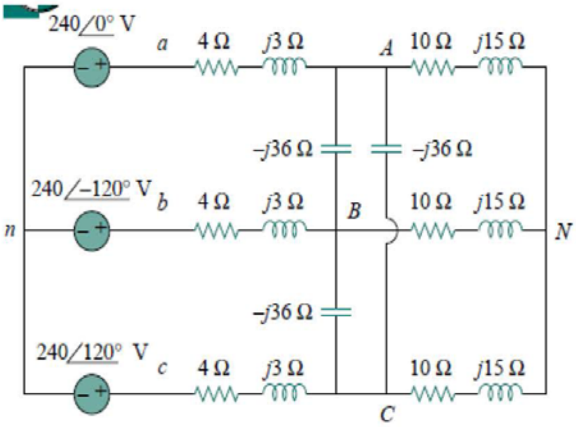


Fig. 1

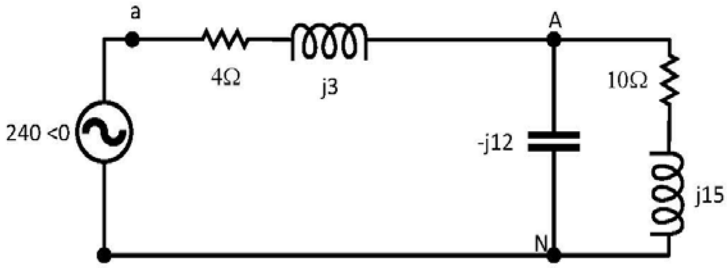
// notice that the Δ-connected element cannot be used in the single line diagram; (SLD) as the this diagram shows the connection to neutral;

Therefore, B4 drawing the SLD, we replace the Δ-connected elements by the Y-equivalents//

1- We replace the Δ-connected capacitor load by its Y-equivalent. The per-phase equivalent impedance:

$$Z_Y = -\frac{j36}{3} = -j12$$

2- We draw the per-phase equivalent circuit to determine, i_{aA} :



$$Z_{eq} = 4 + j3 + (-j12 \backslash\backslash (10 + j15))$$

$$Z_{eq} = 4 + j3 + \frac{180 - j120}{10 + j3} = 4 + j3 + \frac{216.33\angle -33.69°}{10.44\angle 16.7°}$$

$$Z_{eq} = 4 + j3 + 20.72\angle -50.4° = 4 + j3 + 13.2 - j15.97$$

$$`Z_{eq} = 17.2 - j12.97 = 21.54\angle -37°$$

$$I_{aA} = \frac{240\angle 0}{21.54\angle -37} = 11.142\angle 37\ A$$

To determine v_{Bn}, we determine, v_{An}

$$v_{An} = 240\angle 0 - 11.142\angle 37 * (4 + j3)$$

$$v_{An} = 240\angle 0 - 55.71\angle 73.87 = 224.5 - j53.52$$

$$v_{An} = 230.8\angle -13.4°\text{A}$$

$$v_{Bn} = v_{An}\angle -120 = 230.8\angle -133.4°$$

Table 1 presents the phase and line quantities for Y- and Δ- connected elements. This relationship has been defined earlier for Y-connected loads.

Table 1

Load	Phase Voltage	Line Voltage	Phase Current	Line Current	Power per Phase
Y	$\mathbf{V}_{AN} = V_p\angle 0°$ $\mathbf{V}_{BN} = V_p\angle -120°$ $\mathbf{V}_{CN} = V_p\angle -240°$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = (\sqrt{3}\angle 30°)\mathbf{V}_{AN} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = (\sqrt{3}\angle 30°)\mathbf{V}_{BN} = \sqrt{3}V_p\angle -90°$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = (\sqrt{3}\angle 30°)\mathbf{V}_{CN} = \sqrt{3}V_p\angle -210°$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta$ = power factor of the load
Δ	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90°$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210°$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90°$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210°$	$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = (\sqrt{3}\angle -30°)\frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = (\sqrt{3}\angle -30°)\frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = (\sqrt{3}\angle -30°)\frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta$ = power factor of the load

For Δ-connection: (1) Line voltage=Phase voltage and (2) Line current =$\sqrt{3}$ phase current with (−30°/+30°) phase shift for (positive sequence/negative sequence systems).

b. Analysis of systems with Δ-connected supply

The source may also be connected in Δ configuration, as shown in Fig. 2.

// The Δ-connection of the supply is not so common because any unbalance in the supply causes a considerable amount of "circulating current" in the Δ-circuit and causes losses; however unbalanced systems are beyond our field of study//

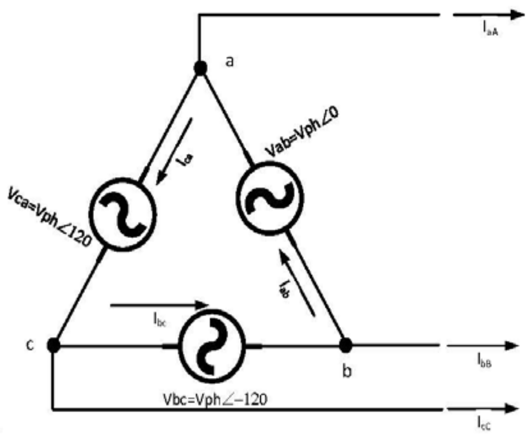


Fig. 2

In Δ-connected source the line voltages are equal to the phase voltages; while the amplitudes of phase currents (I_{ab} , I_{bc} and I_{ca} in Fig. 10) and the line current ((I_{aA} , I_{bB} and I_{cC}) are related by:

$$|I_{LL}| = \sqrt{3}\,|I_{ph}| \qquad ...(10)$$

The line current **lags** the phase currents by 30° for positive sequence systems as indicated in table 1. In Fig. 2, the line and phase current are related by:

$$I_{aA} = I_{ab} - I_{ca};\ I_{bB} = I_{bc} - I_{ab}; I_{cC} = I_{ca} - I_{bc};$$

The verification of the line- and phase- currents relationships s shown in Fig.3 using the phasor diagram.

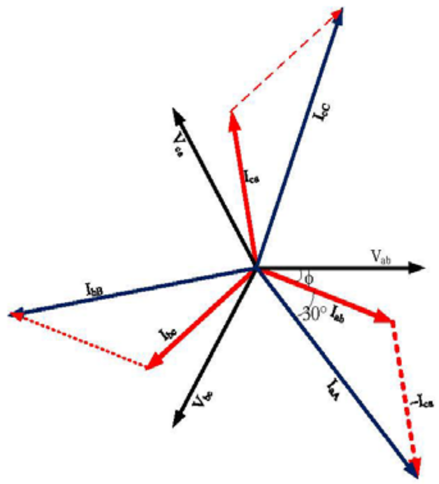


Fig.3

Example 2

A negative-sequence balanced three-phase source supplies power to a balanced, three-phase load with an impedance of 12 + j9 Ω/phase. The source voltage in the b-phase is 240∠-50° V. The line impedance is 1 + j1 Ω/phase. Draw the single phase equivalent circuit for the a-phase and use it to find the current in line current (I_{aA}) for each of the following combinations:

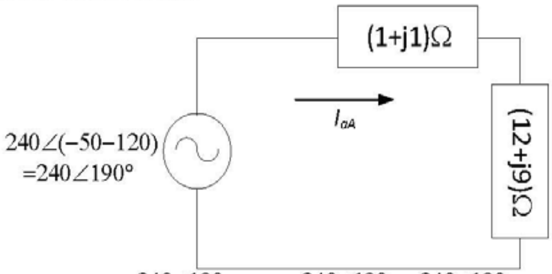
a. Y-connected source and Y-connected load
b. Y-connected source and Δ-connected load
c. Δ-connected source and Y-connected load
d. Δ-connected source and Δ-connected load.

Sol.

For negative-sequence supply $v_{bn} = av_{an}$

$$v_{an} = \frac{v_{bn}}{a} = 240\angle -50 - 120 = 240\angle -170V = 240\angle 190°$$

a. The per-phase equivalent circuit is shown:

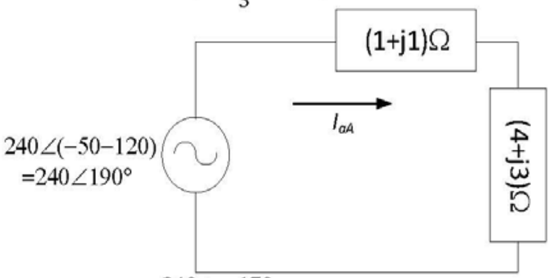


$$I_{aA} = \frac{240\angle 190}{(1+j1)+(12+j9)} = \frac{240\angle 190}{13+j10} = \frac{240\angle 190}{16.4\angle 37.6}$$

$I_{aA} = 14.6\angle 152.4°$A

b. B4 constructing the SLD we converter the load to it's Y-equivalent

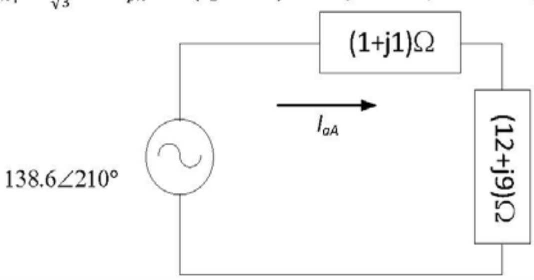
$$Z_Y = \frac{Z_\Delta}{3} = 4 + j3\Omega$$



$$I_{aA} = \frac{240\angle -170}{(1+j1)+(4+j3)} = 37.48\angle 151.34$$

c. The per-phase equivalent of a line voltage is determined by:

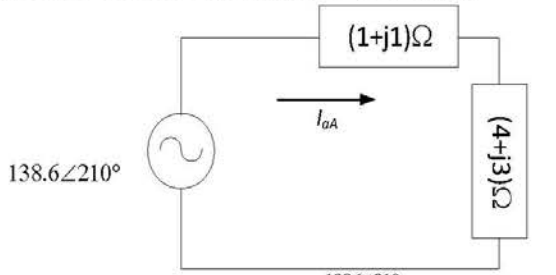
$|V_{ph}| = \frac{|V_L|}{\sqrt{3}}$; $\angle V_{ph} = \angle(V_L + 30°)$ for negative sequence voltages. The SLD is as shown:



$$I_{aA} = \frac{138.6\angle 210}{16.4\angle 37.6}$$
$$I_{aA} = 8.45\angle 172.4°\text{A}$$

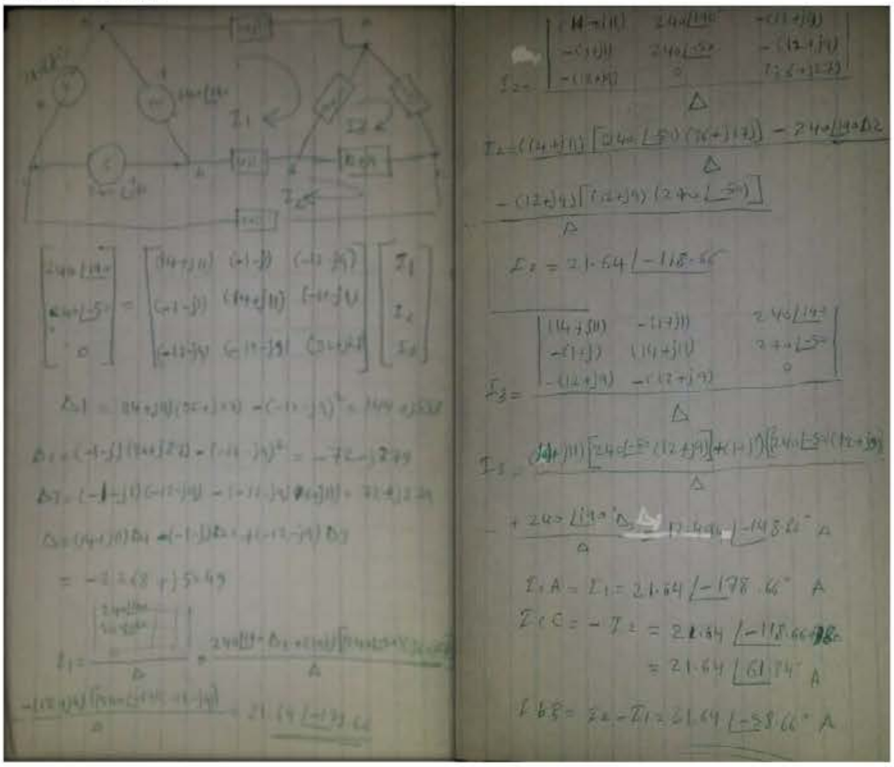
// what is the supply phase current I_{ab} ?//

d. *//We connect the source of (c) to load of (b)//*



$$I_{aA} = \frac{138.6\angle 210}{6.4\angle 38.7} = 21.65\angle 171.3\text{A}$$

// A great deal of simplification is achieved by the SLD approach. In order to appreciate this, try to analyze the circuit using the basic circuit methods; my attempt is shown below:



Related Book Problems 15-35

c. Power Calculations in Three-Phase Systems

Note : ALL Voltages and Curent indicated in the following material is in RMS which is equivalent (for sine wave to the peak /(√2); unless indicated otherwise.

The power asssocited to any element in the three-phase circuit (source, line or load) can be calculated using the SLD which represents one phase (or more preciesly one phase of the Y-Y connected equivalent circuit). As the three-phase system power is 3×(the power in the SLD). And since the quantities shown the in SLD are the per-phase quantities:

$$S_{3\emptyset} = 3 \times S_{1\emptyset} = 3 \times [V_{ph} I_{Ph}]$$

Similarly:

$$P_{3\emptyset} = 3 \times [V_{ph} I_{Ph} \cos\emptyset];\ and$$
$$Q_{3\emptyset} = 3 \times [V_{ph} I_{Ph} \sin\emptyset]$$

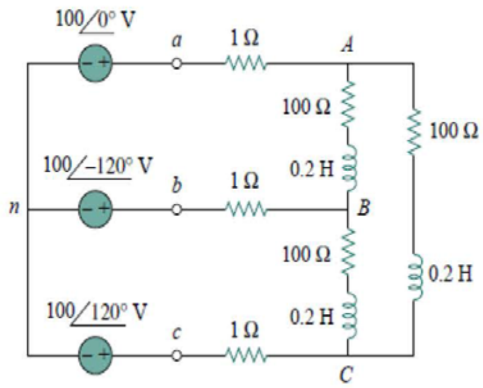
Recall that :

For Y-connection	For Δ-connection
$V_L = \sqrt{3} V_{ph} and\ I_L = I_{ph}$	$V_L = V_{ph} and\ I_L = \sqrt{3} I_{ph}$
For any connection : $S_{3\emptyset} = \sqrt{3} \times [V_L I_L]$ $P_{3\emptyset} = \sqrt{3} \times [V_L I_L \cos\emptyset]$ $Q_{3\emptyset} = \sqrt{3} \times [V_L I_L \sin\emptyset]$	

The three above equations are specially important when we have no information about the load and source and determining the power by line voltage and current information (or measurements).

Example3: The frequency of the supply in the circuit shown in below is 60Hz. Find for the three-phase system:

a) The load power, P.
b) The power dissipated in the line resistor
c) The supply apparent, average and rective power
d) The capacitace of a three delta connected capacitors to be introduced at the load side (points A,B, and C) to make the source power factor =1.



Sol.

-determine the inductive reatctance:

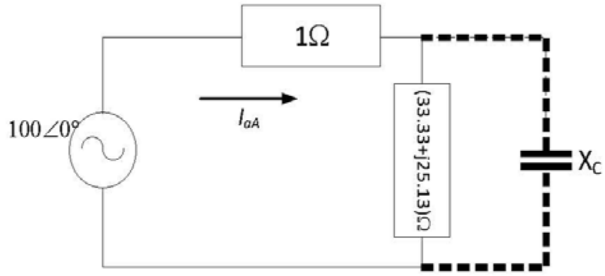
$$XL = 2\pi f L = 24\pi = 75.4$$

-determine the Y-equivalent impedance of the load

$$Z_Y = \frac{100 + j75.4}{3} = (33.3 + j25.13)\Omega$$

From the SLD:

$$I_{aA} = \frac{100\angle 0}{1 + (33.3 + j25.13)} = 1.8966 - j1.388\ A = 2.35\angle - 36.2°A$$



a) Now the load power can be determined using more than one method, as shown:

First method: $P_{Load} = 3(I_{ph})^2R = 3(|1.8966 - j1.388|)^2 33.33 = 552.4W$

Second Method: $P_{Load} = 3V_{ph}I_{ph}\cos\emptyset = 3(I_{ph}Z_L)I_{ph}\cos\emptyset$

$$P_{Load} = 3(1.8966 - j1.388)^2(33.3 + j25.13)\cos\tan^{-1}\left(\frac{25.23}{33.33}\right)$$

$$P_{Load} = 552.4W$$

//obviously the first method is simpler//

b) $P_{TL} = 3 * \left((2.35^2) * 1\right) = 16.572W$

c) $S_{supply} = 3V_{ph}I_{ph} = 3 * 100 * 2.35 = 705VA$

$P_{supply} = 3V_{ph}I_{ph}\cos\emptyset = 705\cos 36.2 = 568.9W$

$Q_{supply} = 3V_{ph}I_{ph}\sin\emptyset = 705\sin 36.2 = 416.38VAR$

d) First determine the capacitive reactace that appears in the SLD X_C

-Since the supply power factor is 1; the equivalent imedance seen by the supply has imajinary component impedance =0.

$$Im[1 + Z_{LC}] = Im[Z_{LC}] = 0$$

$$Im\left[\frac{(33.33 + j25.13) \times (-jX_C)}{(33.33 + jX)}\right] = 0$$

Where jX=j(25.13-Xc)

$$Im\left[\frac{(33.33 + j25.13) \times (-jX_C)}{(33.33 + jX)}\frac{(33.33 - jX)}{(33.33 - jX)}\right] = 0$$

$$Im[(25.13X_C - j33.33X_C) \times (33.33 - jX)] = 0$$

$$[(-25.13XX_C - 33.33^2X_C)] = 0$$

$$X = -\frac{33.33^2}{25.13} = -44.2$$

$$X_C = -44.2 - 25.13 = -69.33\Omega$$

As Xc is the Y equivalent of the actual Δ connected capacitors; we find the actual capacitive reactance:

$$Z_\Delta = 3Z_Y$$

$$X_{C\Delta} = 3(-j69.33) = -j208\Omega$$

$$C = \frac{1}{120\pi * 208} = 12.75\mu F$$

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	Lecture sequences:	9nd lecture	Instructor Name: Omer Sharaf Aldeen Ali Abbawi
	The major contents: Power Measurements in Three-Phase Circuits		
	The detailed contents: 1- The wattmeter 2- Power measurements in three phase systems 3- The two wattemeter method		

Chapter 3

Power Measurements in Three-Phase Circuits

The wattmeter is the basic power measurement instrument. In practical three-phase system it is required to consider unbalance. The famous power measurement method that uses two wattmeter and valid for balanced and unbalanced systems is presented in this lecture.

a. The wattmeter

In AC circuits, power measurement is accomplished through the use of a wattmeter that contains two separate coils. One of these coils is the current coil; the second coil is the potential (or voltage) coil. The value indicated by the wattmeter indicates the product of the current through its current coil and the voltage across its voltage coil. Fig. 1 shows the schematic diagram of the wattmeter.

In our discussion we will assume that the current coil has zero impedance and the voltage coil has infinite impedance.

In Fig. 1, one side of each coil is marked by (+); this indicates that the wattmeter is a polarity-sensitive device. If a positive current is entering that (+) side of the current coil and the voltage of the (+) side of the voltage coil is (+) with respect to the other side then the power is positive, or being transferred to the load side. Note the following:

- The wattmeter measures the average power (P). (Not the instantaneous power).
- Reversing the connection of one coil reverses the polarity of the power being measured. While revering both coils does not change the power polarity.

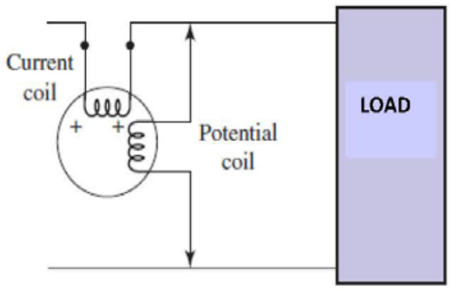


Fig.1

b. Power measurements in three phase systems.

To measure the power drawn by a three-phase load we will discuss the straightforward method first. We place one wattmeter in each of the three phases and add the results. The connections for a Y- and Δ-connected loads are shown in Fig.2.

The method is theoretically correct but **not practical**. One main reason is because the three phase loads have only three lines contacts and other **contacts are not accessible**.

The connection shown in fig. 3 uses the line currents and it establishes a common point (*x*) as a common voltage coil side for the three wattmeters. There are two possibilities:

- Either the system is balanced (the supply voltages and the load are balanced). In this case the point *x* has the same potential as the neutral and the voltage of each voltage coil is the phase voltage. The power indicated by the three wattmeters is equal.

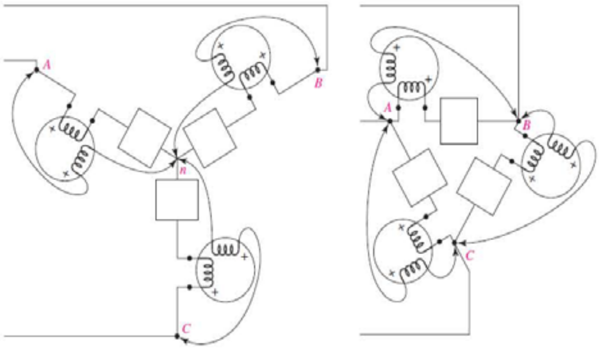


Fig. 2

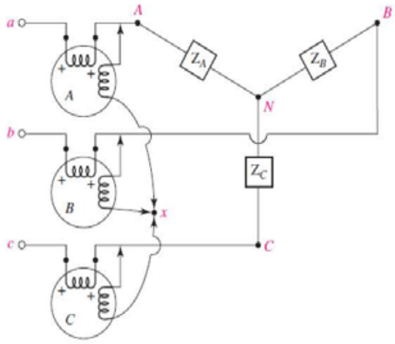


Fig. 3

- or there is a unbalance that makes a voltage difference between points (N and x). In this case the power measured:

$$P = P_A + P_B + P_C = \frac{1}{T}\int_0^T (v_{Ax}i_{aA} + v_{Bx}i_{bB} + v_{Cx}i_{cC})dt$$

The three voltages "seen" by the wattemeters are:

$$v_{A,x} = v_{A,N} + V_{Nx}$$
$$v_{B,x} = v_{B,N} + V_{Nx}$$
$$v_{C,x} = v_{C,N} + V_{Nx}$$

Substitutes to determine P

$$P = \frac{1}{T}\int_0^T (v_{AN}i_{aA} + v_{BN}i_{bB} + v_{CN}i_{cC})dt + \frac{1}{T}\int_0^T v_{Nx}(i_{aA} + i_{bB} + i_{cC})dt$$

Since this three-phase load has only three wires:

$$i_{aA} + i_{bB} + i_{cC} = 0$$

And

$$P = \frac{1}{T}\int_0^T (v_{AN}i_{aA} + v_{BN}i_{bB} + v_{CN}i_{cC})dt = P_{3\emptyset}$$

Example 1:

A three-phase supply has a positive sequence and a line voltage of 100V. The load is unbalanced impedances which are Δ-connected. The three impedances are as follows:

$$Z_{AB} = -j10\Omega; Z_{BC} = j10\Omega; and\ Z_{CA} = 10\Omega$$

Determine the average power dissipated by the load and the measurement of the three wattmeters which are connected as shown in Fig. 3, to show that the wattmeters measure the load in the unbalanced load.

Sol.

$$V_{AB} = 100\angle 0°;$$

$$V_{BC} = 100\angle -120° \; and$$

$$V_{CA} = 100\angle 120°.$$

$$I_{AB} = \frac{100\angle 0°}{10\angle -90°} = 10\angle 90°;$$

$$I_{BC} = \frac{100\angle -120°}{10\angle 90°} = 10\angle -210°;$$

$$I_{CA} = \frac{100\angle -240°}{10\angle 0°} = 10\angle -240°.$$

The average power dissipated in the load is the resistive part power (Z_C) only and it equals:

$$P = |I_{CA}|^2 R_C = 1000W$$

The line currents:

$$I_{aA} = I_{AB} - I_{CA} = 10\angle 90° - 10\angle -240° = 5.1764\angle 15$$

$$I_{bB} = I_{BC} - I_{AB} = 10\angle -210° - 10\angle 90° = 10\angle -150$$

$$I_{cC} = I_{CA} - I_{BC} = 10\angle -240° - 10\angle -210° = 5.1764\angle 45$$

Now determine the equivalent phase voltages

$$V_{AN} = \frac{100}{\sqrt{3}}\angle -30$$

$$V_{BN} = \frac{100}{\sqrt{3}}\angle -150$$

$$V_{CN} = \frac{100}{\sqrt{3}}\angle 90$$

The three measured powers:

$$P_A = 5.1764 \times \frac{100}{\sqrt{3}} \cos(-30 - 15) = 211.5W$$

$$P_B = 10 \times \frac{100}{\sqrt{3}} \cos(-150 - (-150)) = 577W$$

$$P_C = 5.1764 \times \frac{100}{\sqrt{3}} \cos(90 - 45) = 211.5W$$

$$\therefore P_{3\phi} = P_A + P_B + P_C$$

The example shows that the sum of the three wattmeters powers is equivalent to the actual power dissipated in this unbalanced load.

c. The two wattemeter method.

It is shown that point x, in Fig. 3; could be at any voltage and the three wattmeters will still measure the three phase power for balanced or unbalanced system. If we place point x at any one of the three lines (say line B as shown in Fig. 4), the wattmeter at that line sees zero voltage and measures zero power. Therefore, this wattmeter is removed and the remaining two wattmeters measure the voltage of the balanced or unbalanced three-phase load.

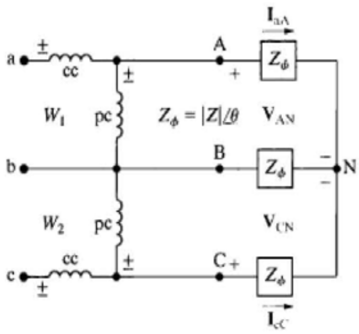


Fig. 4

In balanced systems let's look at the two wattmeter measurements, W1 and W2 according to Fig. 4

$$P_1 = |V_{AB}||I_{aA}| \cos\theta_1 = V_L I_L \cos\theta_1$$

$$P_2 = |V_{CB}||I_{cC}| \cos\theta_2 = V_L I_L \cos\theta_2$$

The load impedance: $Z_\emptyset = |Z_\emptyset|\angle\theta_\emptyset$; from the phasor diagram in Fig. 5:

$$\theta_1 = \theta_\emptyset + 30°$$

And

$$\theta_2 = \theta_\emptyset - 30°$$

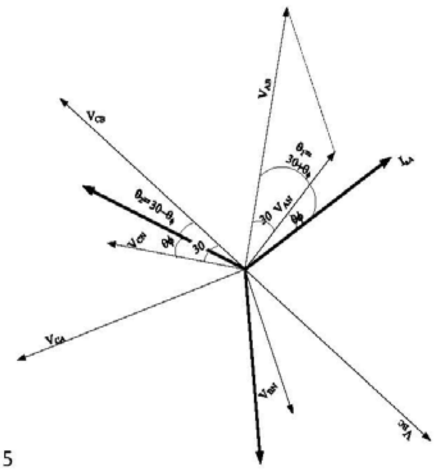


Fig. 5

Now consider the sum of the two measures:

$$P_1 + P_2 = V_L I_L (\cos(30 + \theta_\emptyset) + \cos(30 - \theta_\emptyset))$$

$$P_1 + P_2 = V_L I_L(\cos 30 \cos\theta_\emptyset - \sin 30 \sin\theta_\emptyset + \cos 30 \cos\theta_\emptyset + \sin 30 \sin\theta_\emptyset \qquad)$$

$$P_1 + P_2 = 2V_L I_L \cos 30 \cos\theta_\emptyset$$

$$P_1 + P_2 = \sqrt{3} V_L I_L \cos\theta_\emptyset = P_{3\emptyset}$$

Additionally we can obtain information about the phase angle and, hence, the power factor and reactive power as follows:

$$P_1 - P_2 = V_L I_L(\cos(30 + \theta_\emptyset) - \cos(30 - \theta_\emptyset))$$

$$P_1 - P_2 = V_L I_L(\sin\theta_\emptyset)$$

From the above, we can determine the impedance angle as:

$$\tan\theta_\emptyset = \frac{\sin\theta_\emptyset}{\cos\theta_\emptyset} = \sqrt{3}\frac{P_2 - P_1}{P_1 + P_2}$$

Example 2:

Refer to Fig.4; Calculate the reading of the two wattmeters and show that the sum of the two measures equals to the power delivered to the load if the load phase voltage is 120V and $Z_\emptyset = 8 + j6\Omega$. Determine also the impedance angle angle, the reactive power and the apparent power using the P_1 and P_2

Sol.

(a) $Z_\emptyset = 10\angle 36.87°, V_L = 120\sqrt{3}V$ and $I_L = \frac{120}{10} = 12A$

$$P_1 = 120\sqrt{3} \times 12 \times \cos(36.87 + 30) = 979.75W$$

$$P_2 = 120\sqrt{3} \times 12 \times \cos(36.87 - 30) = 2476.25W$$

$$P_{3\emptyset} = 3I^2R = 3 \times 12^2 \times 8 = 3456W = P_1 + P_2$$

Also...

$$\tan\theta = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1} = \sqrt{3}\frac{(1496.5)}{3456} = 0.75$$

$$\theta = \tan^{-1}(0.75) = 36.87°$$

The reactive power, $Q = S\sin\theta = \frac{P}{\cos\theta}\sin\theta = P\tan\theta = 3456 * 0.75 = 2592VAR$

The apparent power, $S = \frac{P}{\cos\theta} = \frac{3456}{0.8} = 4320VA$

Practice: Repeat Example 2 for the following values of Z:

(a) $Z_\emptyset = 8 - j6\Omega$.
(b) $Z_\emptyset = 5 + j5\sqrt{3}\Omega$.
(c) $Z_\emptyset = 10\angle -75\Omega$.

Related book problems: form 15 to 44

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

Lecture Contents	**Lecture sequences:**	**8nd lecture**	**Instructor Name: Omer Sharaf Aldeen Ali Abbawi**
	The major contents: **Magnetically-Coupled Circuits**		
	The detailed contents: 1- **Definition of Mutual Inductance** **2- Dot Convention** **3- Energy stored magnetically-coupled coils**		

Chapter 4

Magnetically-Coupled Circuits

The phenomenon described by mutual inductance is defined, and quantized. Circuits with magnetic coupling analysis is presented in this lecture. The energy stored in magnetically coupled coils is defined.

The following lectures present the most common device based on magnetic coupling: *the transformer*.

a. Definition of Mutual Inductance

Recall the inductance from its physical basis, refer to Fig.1

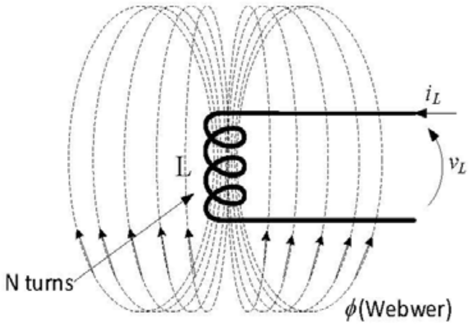


Fig. 1

Based of Amperes Law, the current carrying conductor (or coil) produces a magnetic flux proportional to the MMF (magneto-motive force) or the Ampere-turns.

$$\phi = \frac{Ni}{\Re} = \wp Ni$$

Where $\Re$ and $\wp$ represent the magnetic flux path reluctance and permanence respectively. This parameter is determined by the characteristics of magnetic flux path such as length, cross-section area and the medium permeability (μ).

According to Faraday's law, the voltage induced in a coil subjected to a time varying magnetic flux is given by:

$$v = \frac{d\lambda}{dt} = \frac{dN\phi}{dt} = \frac{dN \times \wp Ni}{dt} = \wp N^2 \frac{di}{dt}$$

Since: $v = L\frac{di}{dt}$, the circuit inductance can be related to the circuit magnetic characteristics by: $L = \wp N^2$.

Since L relates the voltage and current of the same coil, we will call it in this chapter "*the self- inductance*". In contrast, we will study the *mutual* inductance which (in a similar way) relates the voltage induced in one coil by the current of another coil.

Fig. 2 shows a current carrying coil with inductance L_1 and there is another coil L_2 within the magnetic field of L_1. The total flux of the coil L_1 is denoted by ϕ_1. This flux ϕ_1 is divided to two parts:

- ϕ_{21} is the part of ϕ_1 which is intersecting the windings of coil L_2, (both L_1 and L_2) and

- ϕ_{11} is the part of ϕ_1 which is intersecting the windings of coil L_1 only.

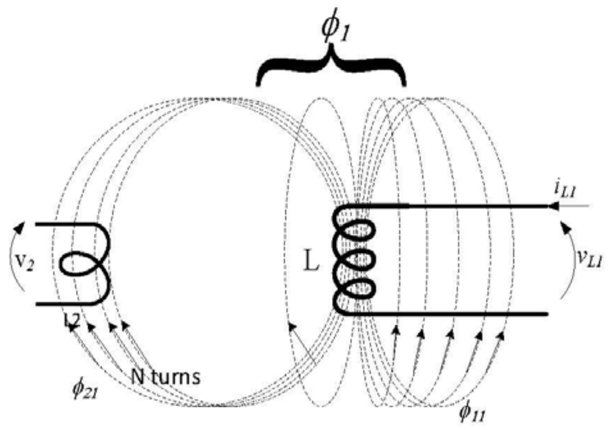


Fig. 2

Again apply Faraday's law for the two-coil case shown in Fig.2 (assuming a time-varying current)

$$v_1 = \frac{d\lambda_1}{dt} = \frac{dN_1\phi_1}{dt} = \wp_1 N_1{}^2 \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

And

$$v_2 = \frac{d\lambda_2}{dt} = \frac{dN_2\phi_{21}}{dt} = N_2 \frac{d\wp_{21}\phi_{21}}{dt}$$

$$v_2 = N_2 \frac{d\wp_{21}N_1 i_1}{dt} = \wp_{21} N_1 N_2 \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

Where M_{21} is the inductance that relates the voltage induced in coil 2 to the current of coil 1.

Since M_{21} depends on: the permanence of the magnetic flux path between the two coils and the product of the number of turns of the two coils, it must be true that:

If $v_1 = M_{12}\frac{di_2}{dt}$ $then\ M_{12} = M_{21} \triangleq M$ the mutual inductance between the two coils.

Dot Convention

It is important to relate the polarity of the induced voltage to the direction of the current in magnetically coupled coils. For this purpose, the dot (•) marking has been introduced.

In dot convention a • sign is placed at one end of each of the two coils which are mutually coupled. We determine the sign of the mutual voltage as follows:

> If di/dt of current entering the dotted terminal of one coil is positive it produces a positive voltage at the dotted terminal with respect to the undotted terminal of second coil.

Thus, in Fig. 3. i_1 enters the dotted terminal of L_1, v_2 is sensed positively at the dotted terminal of L_2, and v_2 = M di_1/dt. The voltage equations are also indicated in Fig. 3.
For AC circuits (sinusoidal supply) the two voltage equations can be expressed more easily in vector notation:

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$
$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt};$$
$$v_2 = M \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

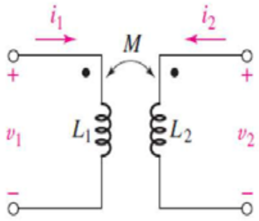
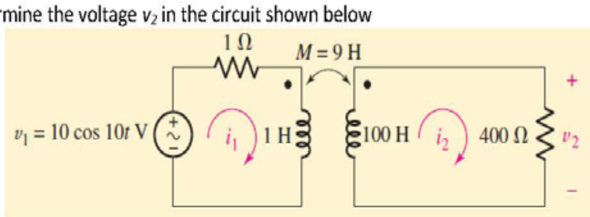


Fig. 3

Example 1: Determine the voltage v_2 in the circuit shown below



Sol.

- *// write the two loop equations to determine i_1 and i_2 //*

$V_1 = I_1(1 + j10 * 1) - I_2(j10 * 9)$ *//from loop 1; where w=10 and the current I_2 is leaving the dot therefore the second term has a negative sign//*

$$0 = -I_1(j10 * 9) + I_2(400 + j10 * 100)$$
$$I_2 = \frac{I_1(j90)}{(400 + j1000)}$$
$$V_1 = I_1(1 + j10) - \frac{I_1(j90)}{400 + j1000} j90$$
$$V_1 = I_1\left(1 + j10 + \frac{8100}{400 + j1000}\right)$$
$$I_1 = \frac{V_1}{(Z)} = 2.063\angle - 38.5°$$
$$I_2 = \frac{2.063\angle - 38.5° * (j90)}{(400 + j1000)} = 0.1724\angle - 16.7°$$
$$V_2 = 68.96\angle - 16.7°$$

---Book Practice 13.2 and 13.3

<u>The coupling coefficient: (Definition)</u>

$$L_1 = N_1{}^2 \wp_1$$
$$L_2 = N_2{}^2 \wp_2$$
$$L_1 L_2 = N_1{}^2 N_2{}^2 \wp_1 \wp_2$$
$$L_1 L_2 = N_1{}^2 N_2{}^2 (\wp_{11} + \wp_{21})(\wp_{22} + \wp_{12})$$

Since: $\wp_{12} = \wp_{21}$

$$L_1 L_2 = N_1{}^2 N_2{}^2 \wp_{12}{}^2 \left(\frac{\wp_{11}}{\wp_{12}} + 1\right)\left(\frac{\wp_{22}}{\wp_{12}} + 1\right)$$
$$L_1 L_2 = M^2 \left(\frac{\wp_{11}}{\wp_{21}} + 1\right)\left(\frac{\wp_{22}}{\wp_{12}} + 1\right)$$

$\wp_{11} and\ \wp_{22}$ will be zeros if the two coils are ideally tied magnetically in such a way that 100% of the flux of both coils is a mutual flux. In this case the ***coupling coefficient (k)*** said to be =1. i.e.

When k=1:

$$M(k=1)=\sqrt{L_1L_2}$$

Or in general

$$M=k\sqrt{L_1L_2} \text{ where } 0\le k\le 1$$

b. Energy stored magnetically-coupled coils

To determine the energy stored in two magnetically coupled coils, refer to Fig. 4. In this figure we consider the two coils (coil1 and coil 2) of self-inductances L1 and L2 and mutual inductance M. Initially the system carries no current and therefore stores no energy.

The synchronized graphs in Fig.4 show the voltages, currents powers and accumulative energy of the system drawn in a way consistent with the basic equations indicated in Fig. 3. Fig. 4 shows that:

- During t1: a constant voltage is applied to coil 1, while coil 2 is open-circuited. The current of coil 1 increases linearly. The power absorbed by coil 1 is the triangular shape and the energy transferred to the coils is given by:

$$W_{t1}=\int_0^{t1} p_1(t)+p_2(t)\,dt=\int_0^{t1} v_1(t)i_1(t)+v_2(t)i_2(t)\,dt$$

$$W_{t1}=\int_0^{t1} V_1\frac{V_1t}{L_1}+v_2(t)\times 0\,dt=\frac{V_1{}^2t_1{}^2}{2L_1}$$

Since $i_{L1}(t_1)=I_1=\frac{V_1t_1}{L_1}$

$$W_{t1}=\frac{1}{2}L_1I_1{}^2$$

Now assume that the current of coil 1 is marinated at I_1 after t1.

After the above described event:

- During t2: a constant voltage is applied to coil 2, while coil 1 current is held constant. The current of coil 2 increases linearly. The power absorbed by coil 2 is the triangular shape and the energy transferred to the coil is (by similarity to the above derivation)= $\frac{1}{2}L_2I_2{}^2$. During the same time another amount of energy is absorbed by the magnetically-coupled coils. This energy is equivalent to the area of the rectangular p_1 during t_2 and equivalent to MI_1I_2. This second amount of energy is supplied by the circuit of L_1 which is maintaining I_1 constant as suggested earlier.

So we can see that the total energy stored in the two magnetically-coupled coils is:

$$W=\frac{1}{2}L_1I_1{}^2+\frac{1}{2}L_2I_2{}^2+MI_1I_2$$

Example 2: Two magnetically-coupled coils with L_1 = 0.4 H, L_2 = 2.5 H, k = 0.6, and $i_1 = 4i_2$ =20 cos (500t − 20°) mA. Determine $v_1(0)$ and the total energy stored in the system at t = 0.

Sol.

In order to determine the value of v_1, we need to include the contributions from both the self-inductance of coil 1 and the mutual inductance:

$$M=k\sqrt{L_1L_2}=0.6\sqrt{0.4\times 2.5}=0.6H$$

$$v_1=L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$v_1(0)=0.4[20\times 500(-\sin(500(0)-20)]+0.6\tfrac{1}{4}[20\times 500(-\sin(500(0)-20)]\text{mV}$$

$$v_1(0)=5.5\times\sin(20)=1.8811V$$

$$i_1(0)=20\cos(-20)=18.794mA;\, i_2(0)=\frac{i_1(0)}{4}=4.698mA$$

$$W(0)=\frac{1}{2}0.4(18.794\times 10^{-3})^2+\frac{1}{2}2.5(4.698\times 10^{-3})^2+0.6\times 18.794\times 4.698\times 10^{-6}$$

$$W(0)=0.1512mJ$$

--- Book Practice 13.4 P504.
---covered Text Book Exercises 1-to-29

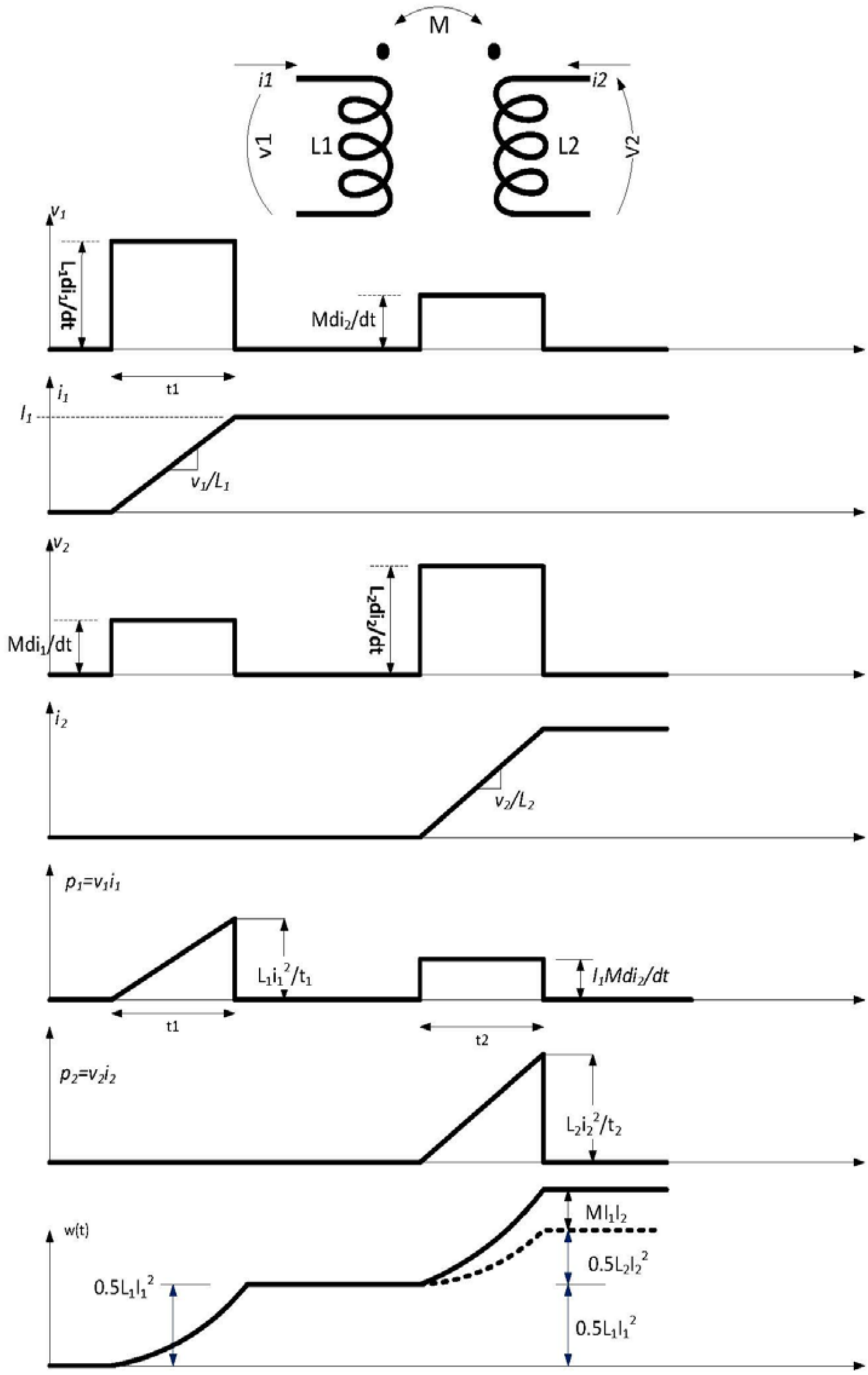


Fig.4

16. Consider the circuit of Fig. 13.46. The two sources are $i_{s1} = 2 \cos t$ mA and $i_{s2} = 1.5 \sin t$ mA. If $M_1 = 2$ H, $M_2 = 0$ H, and $M_3 = 10$ H, calculate $v_{AG}(t)$.

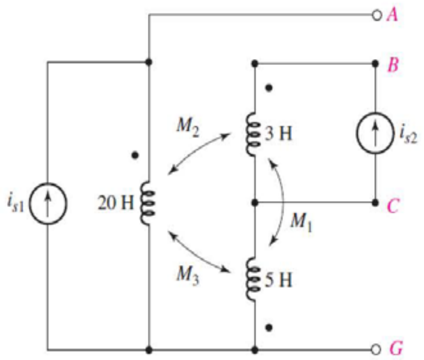


FIGURE 13.46

Sol.

$I_{s1} = 2\angle 0°$; $I_{s2} = 1.5\angle -90°$

$$\omega = 1 rad/sec$$

$$V_{AG} = I_1 jXL_1 + I_2 jXM_2 + I_3 jXM_3$$

$$V_{AG} = 2j20 + I_2 j0 + 0jXM_3$$

$$V_{AG} = j40 = 40\angle 0°$$

$$\boldsymbol{v_{AG} = 40 \cos(t + 90)\text{mV}}$$

21. Note that there is no mutual coupling between the 5 H and 6 H inductors in the circuit of Fig. 13.50. (*a*) Write a set of equations in terms of $\mathbf{I}_1(j\omega)$, $\mathbf{I}_2(j\omega)$, and $\mathbf{I}_3(j\omega)$. (*b*) Find $\mathbf{I}_3(j\omega)$ if $\omega = 2$ rad/s.

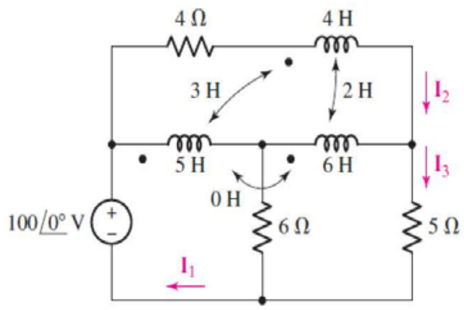


FIGURE 13.50

$$100\angle 0° = j10(I_1 - I_2) + j6I_2 + 6(I_1 - I_3)$$

$$100\angle 0° = I_1(6 + j10) - I_2(j4) - I_3(6) \qquad (1)$$

$$0 = 4I_2 + j8I_2 + j6(I_1 - I_2) + j4(I_3 - I_2) + j12(I_2 - I_3) - j4I_2 + j10(I_2 - I_1) - j6I_2$$

$$0 = I_1(-j4) + I_2(4 + j10) + I_3(-j8) \qquad (2)$$

$$0 = 6(I_3 - I_1) + j12(I_3 - I_2) + j4I_2 + 5I_3$$

$$0 = -6I_1 + I_2(-j8) + I_3(11 + j12) \qquad (3)$$

Rewrite in matrix form

$$\begin{bmatrix} 100\angle 0° \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (6 + j10) & -j4 & -6 \\ -j4 & (4 + j10) & -j8 \\ -6 & -j8 & (11 + j12) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6.067 - j9.54 \\ `6.115 - j4.178 \\ 2.521 - j3.51 \end{bmatrix} = \begin{bmatrix} 11.31\angle -57.55° \\ `\ 7.41\angle -34.34° \\ 4.32\angle -54.31° \end{bmatrix} (A)$$

Lectures of Electrical Engineering Department

Subject Title: Electrical Networks

Class: 2nd Stage

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2- The T-Model
3- The ∏ model</td></tr>
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Magnetically-Coupled Circuits: Linear Transformer

Two magnetically coupled coils form a transformer. The transformer circuits have two features: first they are analyzed as ac circuits and second the analysis focus on the effect of the transform on the load and the supply.

1. The Linear Transformer

The transform has two magnetically coupled coils one is connected to the supply and the other to the load. In the circuit shown in Fig. 1, the coil connected to the supply side (usually known as the primary side) has a resistance R1 and self-inductance L1. The second coil connected to the load is known as the secondary side has a resistance R2 and self-inductance L2. The impedance Zs represent the internal impedance of the supply and M is the mutual inductance between L_1 and L_2.

The following analysis aims to determine the effect of the transformer compared to the case of connecting the load directly to the supply. The analysis start by determining the currents I_1 and I_2 using the circuit parameters.

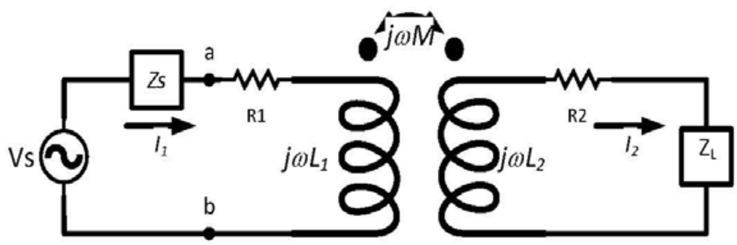


Fig. 1

Reflected Impedance:

To determine I_1 and I_2, write the loop equations:

$$V_s = I_1(Z_S + R_1 + j\omega L_1) - j\omega M I_2 \qquad ...(1)$$

$$0 = -j\omega M I_1 + I_2(R_2 + j\omega L_2 + Z_L) \qquad ...(2)$$

To simplify manipulation, denote: $(Z_S + R_1 + j\omega L_1)$ by Z_{11} and $(R_2 + j\omega L_2 + Z_L)$ by Z_{22}. Equation (2) gives:

$$I_2 = \frac{j\omega M}{Z_{22}} I_1$$

Substitute into (1):

$$V_s = I_1\left(Z_{11} + \frac{\omega^2 M^2}{Z_{22}}\right)$$

Giving:

$$I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$

And

$$I_2 = \frac{j\omega M}{Z_{22}} I_1 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2} V_s$$

Now the impedance seen by the voltage source is $\frac{V_s}{I_1}$

$$Z_t = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

The impedance of the circuit seen at the source terminals:

$$Z_{ab} = Z_t - Z_s$$

$$Z_{ab} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$$

The last term of the above equation is defined as the "reflated impedance" as it represents the image of the secondary circuit seen at the primary side.

$$Z_r \triangleq \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$$

Example 1: The parameters of a linear transformer are:

$R_1(\Omega)$	$R_2(\Omega)$	$L_1(\mathrm{H})$	$L_2(\mathrm{H})$	k
200	100	9	4	0.5

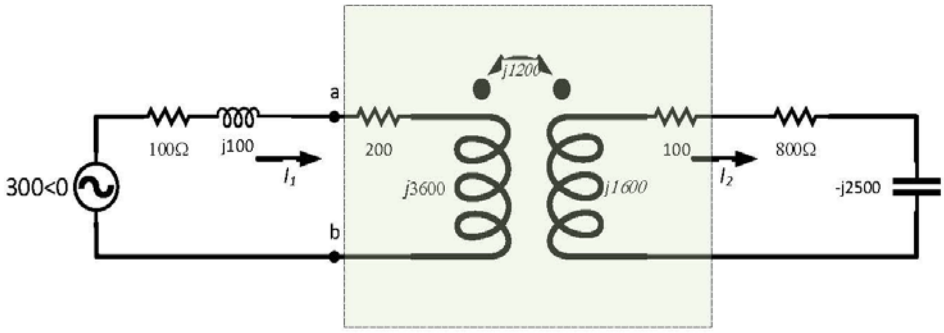
This transformer is used to connect a 300V, 400rad/sec source with internal impedance of (500+j100)Ω to a load consists of series 800Ω resistor and 1μF capacitor. *// the supply voltage should be taken as rms and at angle =zero; as it has not been indicated otherwise//*

(a) Draw the equivalent circuit in frequency domain *// in frequency domain (unlike time domain) implies to write the reactance as impedance and represent ac quantities as vectors// *
(b) Calculate the self-impedance of the primary side
(c) Calculate the self-impedance of the secondary side
(d) Calculate the impedance reflected into the primary winding.
(e) Calculate the impedance seen looking at the primary terminals of the transformer.
(f) Calculate the primary current.
(g) Calculate the secondary current.
(h) Calculate the voltage at the load terminals.
(i) Calculate the power delivered to load.
(j) Determine the percentage of the power delivered to the load to the power received by the transformer.

Sol.

L_1(H)	jωL₁ (Ω)	L_2(H)	jωL₂ (Ω)	k	M(H)	jωM (Ω)	C(F)	j1/ωC
9	j3600	4	j1600	0.5	3	j1200	1μ	−j2500

The equivalent circuit:



1. the self-impedance of the primary side :

$$Z_{11} = 100 + 200 + j(100 + 3600) = (300 + j3700)\Omega$$

2. the self-impedance of the secondary side

$$Z_{22} = 100 + 800 + j(1600 - 2500) = (900 - j900)\Omega$$

3. the impedance reflected into the primary winding.

$$Z_r = \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} = \frac{(1200)^2}{(900 - j900)} = (800 + j800)\Omega$$

4. the impedance seen looking at the primary terminals of the transformer.

$$Z_{ab} = 200 + j3600 + 800 + j800 = (1000 + j4400)\Omega$$

5. the primary current.

$$I_s = \frac{V_s}{Z_s + Z_{ab}} = \frac{300\angle 0}{1500 + j4500} = (20 - j60)mA$$

6. the secondary current.

$$I_2 = \frac{j\omega M}{Z_{22}} I_1 = \frac{j1200}{(900 - j900)}(20 - j60) = \frac{(80 + j160)}{3} mA$$

$$I_2 = 59.63\angle 63.43mA$$

7. the voltage at the load terminals.

$$V_L = I_2 Z_L = \frac{(80 + j160) * 10^{-3} * (800 - j2500)}{3}$$

$$V_L = 156.5\angle -8.82°V$$

8. the power delivered to load.

$$P_L = |I_2|^2 R_L = (0.05963)^2 \times 800 = 2.844W$$

9. the percentage of the power delivered to the load to the power received by the transformer.

$$P_{IN} = |I_1|^2 (R_1 + R_r) = 4W$$

$$\%\frac{Po}{Pin}=\frac{2.844}{4}\times 100\%=71.1\%$$

b. T and Π model

It is convenient to represent the magnetically-coupled coils by a basic inductor elements removing the mutual inductance. T and Π .models have been derived as equivalent circuit for the two mutual inductors in linear transformer. This section shows how to develop these model.

The T-Model:

Consider the linear transformer shown in Fig. 2; where the lower terminal of the two sides is connected to form a three-point arrangement. Refer to the T-equivalent model shown in Fig. 3. For both circuits the following equations are applicable:

$$v_1 = L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$
$$v_2 = M\frac{di_1}{dt}+L_2\frac{di_2}{dt}$$

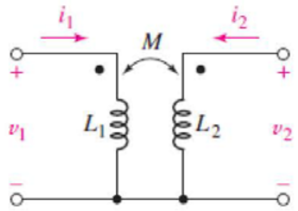


Fig. 2

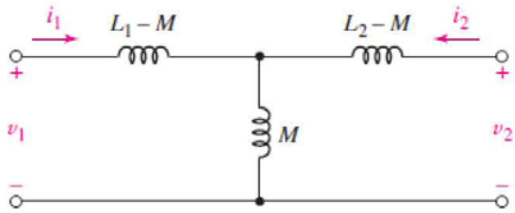
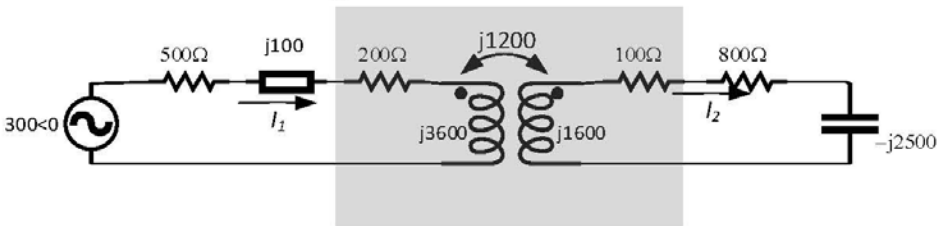


Fig. 3

If one of the dots of the coils in Fig. 2, is changed to the other coil side, the effect of this on the equation voltages can be seen as if M has been replaced by (−M) then the three elements in of the equivalent circuit in Fig. 3 become (L_1+M), (−M) and (L_2+M).

Example 2:

Find I1 and I2 in the circuit shown using T-model.



Sol.

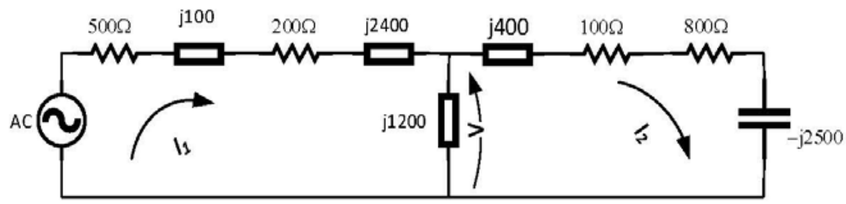
(a) Considering the dots positions:

$$XL_1 - XM = 3600 - 1200 = 2400$$

$$XL_2 - XM = 1600 - 1200 = 400$$

$$XM = 1200$$

Now redraw the circuit with the T-model



Now write the equation of the middle note currents in terms of V.

$$\frac{V-300}{700+j2500}+\frac{V}{j1200}+\frac{V}{900-j2100}=0$$

Gives

$$V = 136 - j8 = 136.24\angle - 3.37°$$

Then

$$I_1 = \frac{300-V}{700+j2500} = 63.25\angle - 71.57° mA$$

And

$$I_2 = \frac{V}{900-j2100} = 59.63\angle 63.43° mA$$

Practice: Repeat Example2 with the polarity dot on the secondary side moved to the lower terminal.

The Π-model:

We can derive the Π- model of the coils shown in Fig 2 using the voltage equations:

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

The following derivation is based on solving the equations for ${di_1}/{dt}$ and ${di_2}/{dt}$ and then regarding the resulting expressions as a pair of node-voltage equations; as follows:

$$\frac{di_1}{dt}=\frac{\begin{vmatrix} v_1 & M \\ v_2 & L_2 \end{vmatrix}}{\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}}=\frac{L_2}{L_1L_2-M^2}v_1-\frac{M}{L_1L_2-M^2}v_2;$$

$$\frac{di_2}{dt}=\frac{\begin{vmatrix} L_1 & v_1 \\ M & v_2 \end{vmatrix}}{\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}}=-\frac{M}{L_1L_2-M^2}v_1+\frac{L_1}{L_1L_2-M^2}v_2$$

By multiplying both sides of the above equations by dt the then integrating

$$i_1=i_1(0)+\frac{L_2}{L_1L_2-M^2}\int_0^t v_1\,dx\;-\frac{M}{L_1L_2-M^2}\int_0^t v_2\,dx$$

$$i_2=i_2(0)-\frac{M}{L_1L_2-M^2}\int_0^t v_1\,dx+\frac{L_1}{L_1L_2-M^2}\int_0^t v_2\,dx$$

Now regard $v_1 and\ v_2$ as two node equations of the circuit shown in Fig. 4

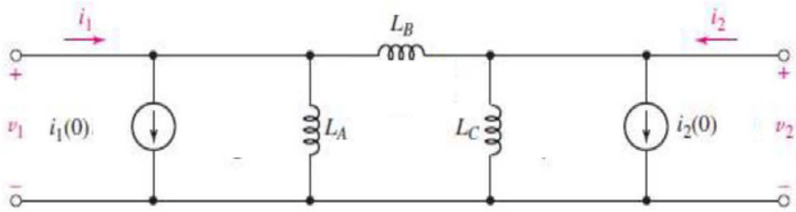


Fig. 4

$$i_1=i_1(0)+\frac{1}{L_A}\int_0^t v_1\,dx\;+\frac{1}{L_B}\int_0^t (v_1-v_2)\,dx$$

$$i_1=i_1(0)+\left(\frac{1}{L_A}+\frac{1}{L_B}\right)\int_0^t v_1\,dx-\frac{1}{L_B}\int_0^t v_2\,dx$$

And

$$i_2=i_2(0)+\frac{1}{L_C}\int_0^t v_2\,dx\;+\frac{1}{L_B}\int_0^t (v_2-v_1)\,dx$$

$$i_2=i_2(0)-\frac{1}{L_B}\int_0^t v_1\,dx+\left(\frac{1}{L_B}+\frac{1}{L_C}\right)\int_0^t v_2\,dx$$

By comparing to the above equations:

$$L_B=\frac{L_1L_2-M^2}{M}$$

$$L_A=\frac{L_1L_2-M^2}{L_2-M}$$

$$L_C=\frac{L_1L_2-M^2}{L_1-M}$$

Evidently in ac steady-state analysis, the initial current sources become zero; moreover we could have been used the Y-Δ equations to obtain the equivalent model //*without so much hustle* ☹//. But the Π-model that has been derived is more general indeed. It shows the effect of initial current which is necessary if the circuit transient model is required.

Example 3:

39. With respect to the network shown in Fig. 13.63, derive an expression for $\mathbf{Z}(j\omega)$ if M_1 and M_2 are set to their respective maximum values.

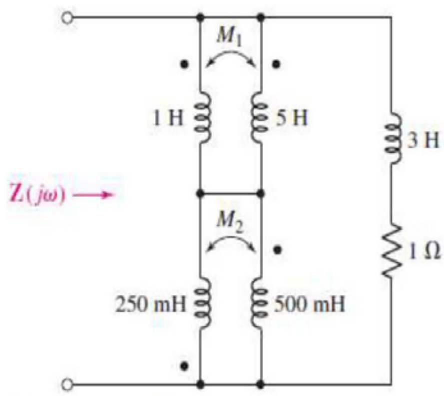


FIGURE 13.63

Sol.

$$M_{1,max} = \sqrt{1 * 5} = \sqrt{5}H$$

$$M_{2,max} = \sqrt{\frac{1}{4} * \frac{1}{3}} = \frac{1}{2\sqrt{2}}H$$

We replace the first transformer by the Π equivalent model and the second by T model

$$L_{A1} = \frac{L_1L_2 - M^2}{L_2 - M} = 0$$

$$L_{C1} = \frac{L_1L_2 - M^2}{L_1 - M} = 0$$

$$L_{x2} = L_1 + M = \frac{1}{4} + \frac{1}{\sqrt{8}}$$

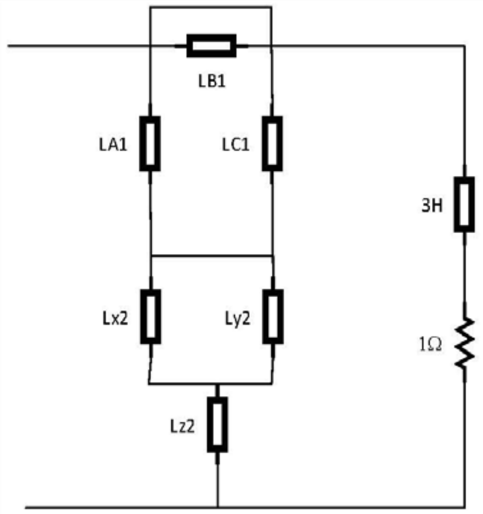
$$L_{y2} = L_2 + M = \frac{1}{2} + \frac{1}{\sqrt{8}}$$

$$L_{z2} = -M = -\frac{1}{\sqrt{8}}$$

The equivalent inductance of the two transformer branch =

$$L_{eq} = (L_{x2} \backslash\backslash L_{y2}) + L_{z2}$$

$$L_{eq} = \frac{\sqrt{8} - 1}{4\sqrt{8}} \cong 0.1616H$$



$$Z(j\omega) = (j\omega Leq)\backslash(1 + j3\omega)$$

$$Z(j\omega) = \frac{-3L_{eq}\omega^2 + j\omega Leq}{1 + j(3 + L_{eq})\omega}$$

$$Z(j\omega) = \frac{-0.4848\omega^2 + j\omega 0.1616}{1 + j(3.1616)\omega}\Omega$$