Lectures of Electrical Engineering Department

Subject Title: Power Electronics

Class: 3rd Power and Machines

Lecture Contents	**Lecture sequences:**	**First lecture**	**Instructor Name: Mohamad N AbdulKadir**
	The major contents: **1- Rectifier** **2- Inverter** **3- Chopper** **4- AC Controller** **5- Cyclovonverter** **6- Matrix Converter**		
	The detailed contents: **1-** **2-** **3-** **4-**		

Chapter 1

Introduction

1- Definition and basics

Lecture aims:

1- To present the definition of “power electronics”
2- To explain the applications and types of power converters
3- To describe the main required characteristics of power converter.
4- To construct a hypothetical power converter and define its switching states
5- To use the genetic converter for basic conversion actions.

1- Definition of power electronics

Power electronics can be defined as a branch of electrical engineering devoted to ***conversion and control*** *of electric power, using electronic* ***converters*** *based on* ***semiconductor power switches***.

2- Application and types of power converters

Power ***Converter*** is the general name of the power electronics circuit.

Converters are used to match the available power source to a load of special demand, the matching may include:

-Changing the type: AC←→ DC
-Adjusting voltage
-Adjusting frequency
….etc.

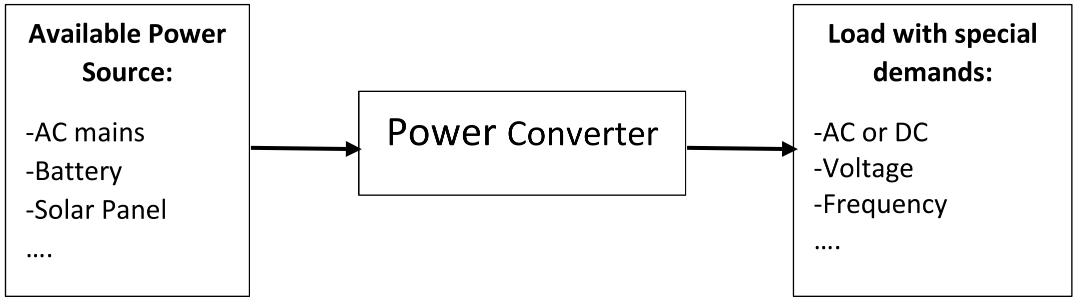


Fig. 1: Power converter to use available supply and meet load demands

Types of power converters

The general name “converter” is usually specified according to the type of operation as follows:

Input type	Output type	Converter name	Remarks
DC	DC	Chopper	To change voltage level (Chapter 5, 7)
DC	AC	Inverter	To obtain variable voltage and frequency AC (Chapter 6)
AC	DC	Rectifier	To obtain fixed or variable DC voltage (Chapter 3)
AC	AC	AC Controller	To change the voltage (fo=fin) (Chapter 4)
		Cyclovonverter	To change voltage and frequency (fo<<fin)
		Matrix Converter	To change both voltage and frequency

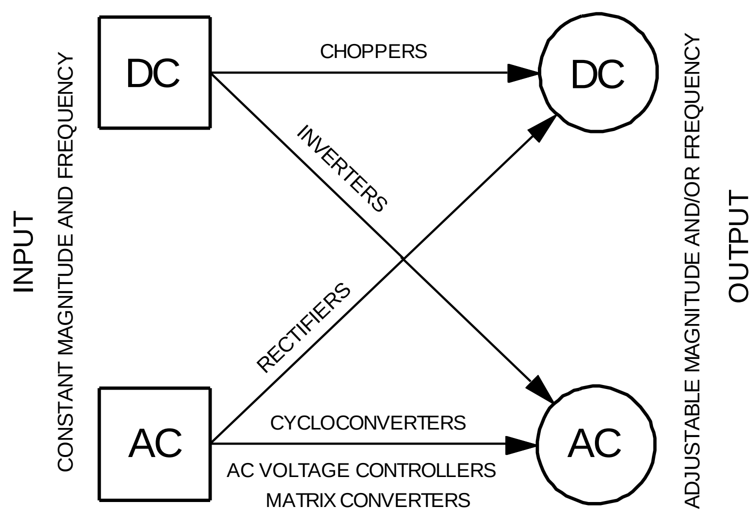


Fig. 2 Types of power conversion

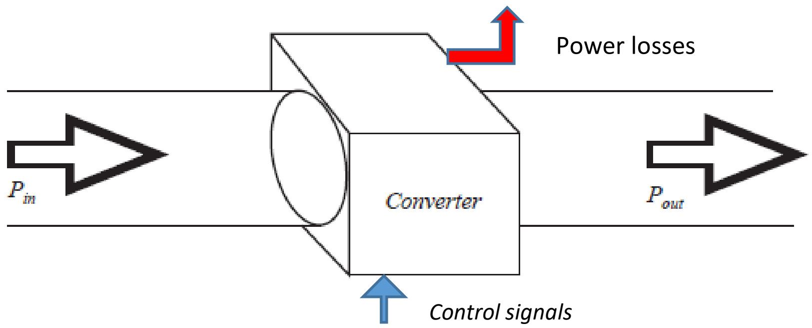


Fig.3 Power flow through power converter

3- <u>- Main characteristics of power converter.</u>

-As it transforms load power (Fig. 3) it is necessary for the converter to operate with: **High Efficiency**

-Low efficiency leads to losses of energy and the power associated losses are dissipated in the converter circuit causing temperature increase.

-To ensure high efficiency, power consuming elements, such as fixed or variable resistors, transistors in linear mode ($i_C = \beta i_B$),…etc; are **NOT** allowed in the converter circuits.

- Changing the output values as desired is done by "**control**" signals, therefore the converters usually have control ports to receive the signals that determine its output.

-Converters have been constructed using switching devises as the switch is (i) lossless and (ii) controllable, as shown in Fig. 4

Switch closed: $v(t) = 0$

Switch open: $i(t) = 0$

In either event: $p(t) = v(t)\, i(t) = 0$

Ideal switch consumes zero power

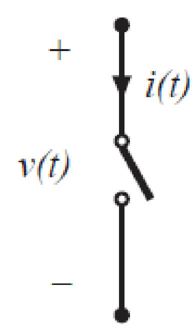


Fig. 4 Ideal switch is a lossless Element

4- Generic Power Converter

To explain how the converter can be operated using switches, a generic hypothetical 5-switch power converter is constructed as shown in Fig. 5. The converter is designed to operate in one of the three following states:

State	S1,S2	S3,S4	S5	v_o	i_i	Description
State 1	ON	OFF	OFF	v_i	i_o	Load is directly connected to source
State2	OFF	ON	OFF	$-v_i$	$-i_o$	Load is cross connected to source
State 0	OFF	OFF	ON	0	0	Source is opened and Load is Shorted (assuming inductive load)

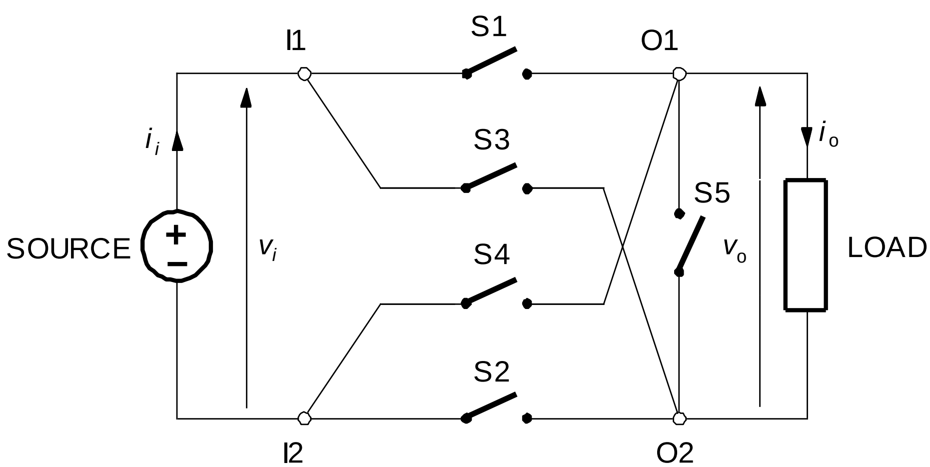


Fig. 5 A Generic 5-switch converter

Note that State 0 has been design to suit the common case of having (1) a voltage source which should never be short circuited and isolated by opening it; and (2) highly inductive load with a current that cannot be interrupted and always required to have a closed path for the current.

5- **Performing some conversion actions with generic converter**

To explain the tactics used in converters, this section shows two basic converter operations: rectification and inversion.

a. Rectifier operation:

The rectifier input is an AC voltage of the following form (assuming zero phase angle):

$$v_i = V_{i,p} \sin \omega t$$

Where $V_{i,p}$ is the amplitude (peak) of the sinusoidal input voltage and ω is its radian frequency. This voltage is positive for $0 < \omega t < \pi$ and negative for $\pi \le \omega t < 2\pi$. The generic converter is operated to connect the positive voltage to the output side and inversely connect the negative voltage. Selecting the suitable states as follows:

$$converter\ state = \begin{cases} state\ 1\ for\ 0 < \omega t < \pi \\ state\ 2\ for\ \pi < \omega t < 2\pi \end{cases}$$

This leads to an output voltage given by:

$$v_o = V_{i,p} |\sin \omega t|$$

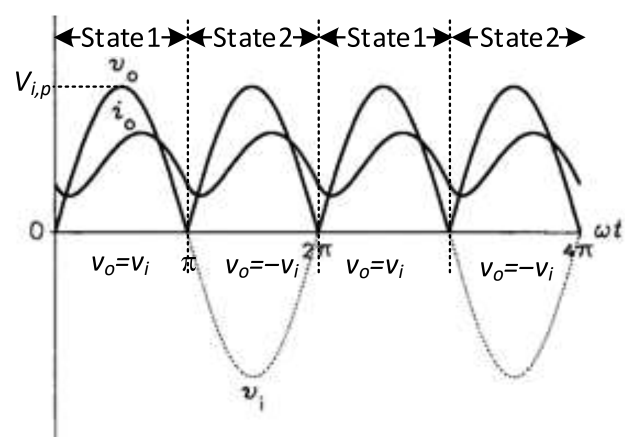


Fig 6

Both the output and input voltages are shown in Fig. 6. It is clear that the output voltage is not the ideal DC voltage! However this voltage has a positive average that can be taken as the DC voltage.

Fig. 6 also shows the output current waveform for an R-L load drawn considering the following data:

$v_i = 100 \sin 120\pi t$, R=1.3Ω, L=2.4mH

The analysis for obtaiing the current waveforms will be discussed later in this chapter. Now, it can be noted that the current waveform is closer to the ideal DC compared to the voltage wave as the variation around the "average" is smaller compared to the voltage.

b. Inverter Operation

This section shows how to obtain an AC voltage given a DC supply by operating the generic converter as inverter.

Assuming that the desired load frequency is fo; this implies that the load voltage period is T=1/fo.

The converter is operated periodically as follows:

$$State = \{ \begin{matrix} state\ 1\ \ for\ 0 < t < \frac{T}{2} \\ state\ 2\ for \frac{T}{2} < t < T \end{matrix}$$

The generic inverter operation is shown in Fig. 7.

Similar to the rectifier case, it can be seen that the resultant output voltage is square and not pure sinusoidal as for the ideal AC. However, according to Fourier series analysis, the fundamental component is of this wave is a pure sine. Therefore, the fundamental component of v_o is considered as required output voltage.

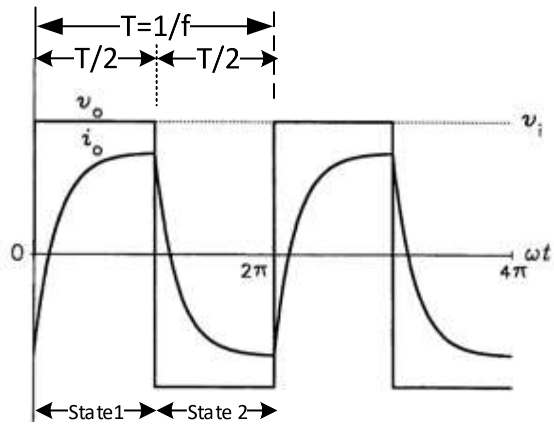


Fig 7

The output current has been shown is Fig. 7 for RL load. It can be seen that the output current is closer to the pure sine wave compared to the voltage.

Chapter 1

Performance Indicators

Lecture aims:

1- To determine the: average, rms and ripple components associated with a periodic quantity.
2- To define the ripple factor.
3- To review Fourier series equations and draw the spectrum.
4- To Define the Total Harmonic Distortion (THD).
5- To Define the Power Factor considering non-sinusoidal AC current.
6- To present the concept and calculation method of conversion efficiency.

1- Performance of DC supplying circuit:

The Average Component

Recall the output voltage waveform of the generic rectifier (lecture#1) and redrawn in Fig. 1. This waveform is NOT a pure DC. However, v_o has been accepted as a DC output because it has a non-zero average (DC) component.

The DC component of a periodic waveform can be determined by taking the average of the area under its curve over one complete cycle. As follows:

$$F_{dc} = \frac{1}{T}\int_T f(t)dt \qquad ...(1)$$

Where *f(t)* is the periodic function being voltage or current. *T* is the period and F_{dc} is the average value (the DC component) of *f(t)*.

Figure 1 marks $V_{o,dc}$ calculated using Eq. (1) as follows:

$$V_{o,dc} = \frac{1}{\pi}\int_0^{\pi} V_{i,p}\sin\omega t\, d\omega t = \frac{V_{i,p}}{\pi}(\cos 0 - \cos\pi) = \frac{2V_{i,p}}{\pi}$$

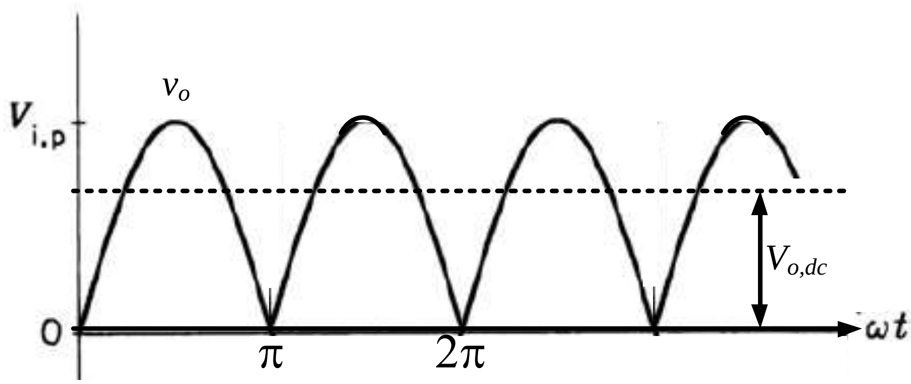


Fig. 1 The output voltage of the generic converter and its average

Notice the followings:

- The period (**π**) is the output voltage wave, this period may or may not equal to the input voltage period.
- For convenience, we have multiplied the time integration by **ω** (by using d**ω**t instead of dt) and divide it by **ω** (using 1/**π** instead of 1/T). The integration limits have been changed accordingly (**π** instead of T).

The Total RMS

The RMS has been used to describe periodic quantities as the effective value since it determines the power associated to this value. The RMS value of a periodic function is defined as the square root of the mean value of the squared function.

$$F_{rms}\ (or\ F) = \sqrt{\frac{1}{T}\int_T\ (f(t))^2 dt} \qquad \text{...(2)}$$

Where F_{rms} *or F* denotes the RMS value.

The RMS of the output voltage of Fig. 1, is derived as follows:

$$V_o = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(V_{i,p}\sin\omega t\right)^2 d\omega t} = \sqrt{\frac{{V_{i,p}}^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\,\omega t}{2}d\omega t}$$

$$V_o = V_{i,p}\sqrt{\frac{1}{2\pi}[\pi - \cos 2\pi - 0 - \cos 0]} = \frac{V_{i,p}}{\sqrt{2}}$$

RMS of functions with orthogonal components:

Assume that

$$g(t) = g_1(t) + g_2(t) + g_3(t) + \cdots \qquad \text{...(3)}$$

The components g_i , g_j are orthogonal if $\int_T g_i . g_j dt = 0$.

If all the components g indicated in Eq. (3) are orthogonal, the total RMS (G) can be determined given the RMS of the components (G_1, G_2, …) as follows:

$$G = \sqrt{G_1^2 + G_2^2 + G_3^2 + \cdots} \qquad \text{...(4)}$$

For periodic waveforms with multiple orthogonal components the total RMS can be determined using the RMS of the components.

<u>The Ripple (AC component)</u>

The AC component of a period wave is the total function minus the average value:

$$f_{ac} = f - F_{dc} \qquad \text{...(5)}$$

The AC component of the generic rectifier output is shown in Fig. (2). The RMS of the AC component can be determined using Eq. (2) or Eq. (4). However, it is much easier to use Eq. (4).

Assume that the function f has two components F_{dc} and f_{ac}; the RMS of the latter is F_{ac}, as the DC and AC components are orthogonal we can apply Eq. (4)

$$F = \sqrt{F_{dc}^2 + F_{ac}^2} \rightarrow F_{ac} = \sqrt{F^2 - F_{dc}^2} \qquad ...(6)$$

Substituting for the total RMS and $V_{o,dc}$

$$V_{o,ac} = \sqrt{\frac{V_{i,p}^2}{2} - \frac{2V_{i,p}^2}{\pi^2}} = V_{i,p}\sqrt{\frac{1}{2} - \frac{4}{\pi^2}} \approx 0.308V_{i,p}$$

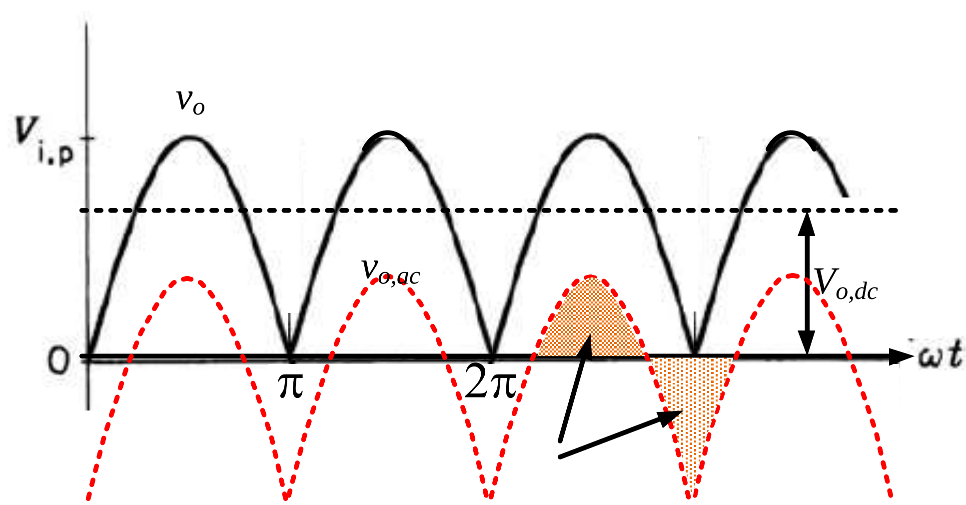


Fig. 2 The DC and AC components of the generic rectifier output voltage.

<u>The Ripple Factor, *RF*</u>

The ripple factor is the basic parameter used to determine the effect of distortion on DC quantities. The ripple factor is defined as:

$$RF_f = \frac{F_{ac}}{F_{dc}} \qquad ....(7)$$

The ripple factor is the ratio of the RMS of the AC to the DC components of a quantity. The higher ripple factor indicates more distorted DC quantity. A pure DC quantity has RF=0.

Example 1: Determine the ripple factor of the periodic DC voltage shown in fig. 3

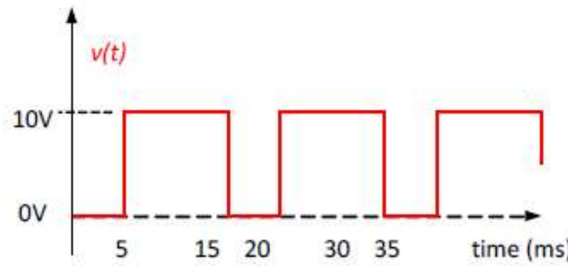


Fig. 3

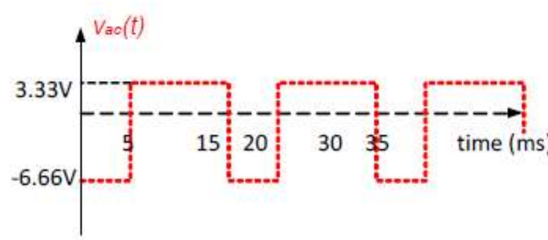
Ans.

$$The\ period, T = 15msec$$

$$V_{o,dc} = \frac{1}{T}\int_{0}^{15\times10^{-3}} v(t)dt = \frac{1}{15\times10^{-3}}\int_{5\times10^{-3}}^{15\times10^{-3}} 10dt$$

$$V_{dc} = \frac{1}{15\times10^{-3}}10[15\times10^{-3} - 5\times10^{-3}] = 6.67V$$

The AC component of v, is shown



The rms of the AC component:

$$V_{ac} = \sqrt{\frac{1}{0.015}\left[\int_{0}^{0.005}(-6.67)^2dt + \int_{0.005}^{0.015}(3.33)^2dt\right]}$$

$$V_{ac} = \sqrt{\frac{1}{0.015}[0.222 + 0.11]} \cong 4.7V$$

The ripple factor:

$$RF_V = \frac{V_{ac}}{V_{dc}} = \frac{4.7}{6.67} \cong 0.704$$

Drill: Find the ripple factor using the method described in Eq. (4)

2- Performance of AC supplying circuit:

Fourier series Analysis

You have learned from mathematics that a periodic function *f(t)* can be represented by an infinite Fourier series as:

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(\mathrm{n}\ \omega \mathrm{t}) + b_n \sin(\mathrm{n}\ \omega \mathrm{t})) \qquad \ldots(8)$$

Where $a_o = \frac{1}{T}\int_T f(t)dt$ which (= F_{dc})

T= period of f

$\omega = \frac{2\pi}{T}$ radian fundamental frequency=2πf_1

f_1 is the fundamental frequency (=1/T)

$$a_n = \frac{2}{T}\int_T f(t)\cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_T f(t)\sin(n\omega t)\,dt$$

Calculations of Fourier series can be simplifield if the function has one or two symmetries as given in the following table:

Symmetry	Condition	Simplified equations
Even	$f(t) = f(-t)$	$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega t)\,dt\ , \quad b_n = 0\ for\ all\ n$
Odd	$f(t) = -f(-t)$	$a_n = 0\ for\ all\ n,\ b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega t)\,dt$
Halfwave	$f(t) = -f(t \pm \frac{T}{2})$	$a_o = 0,\ a_n = b_n = 0\ for\ n = 2,4,6\ldots$
Even +Halfwave		$a_n = \frac{8}{T}\int_0^{\frac{T}{4}} f(t)\cos(n\omega t)\,dt\ \ n = 1,3,5\ldots$ $a_n\ for\ n = 0, 2,4,6\ldots b_n = 0\ for\ all\ n$
ODD +Halfwave		$a_n = 0\ for\ all\ n$ $b_n = \frac{8}{T}\int_0^{\frac{T}{4}} f(t)\sin(n\omega t)\,dt$ $for\ n = 1,3,5,\ldots$ $b_n = 0\ for\ n = 2,4,6,\ldots.$

Considering the output voltage of the generic inverter obtained in last lecture, and redrawn in Fig. 4. We can see that this waveform is ODD and HALFWAVE symmetrical, therefore:

$$V_{o,n,p} = b_n = \frac{8}{T}\int_0^{\frac{T}{4}} V_i \sin(n\omega t)\,dt = \frac{8V_i}{Tn\frac{2\pi}{T}}\left[\cos 0 - \cos n\frac{2\pi T}{4T}\right]$$

$$V_{o,n,p} = \frac{4V_i}{n\pi} \qquad\qquad n=1,3,5,\ldots$$

The fundamental component $V_{o,1}$

$$V_{o,1} = \frac{V_{o,1,p}}{\sqrt{2}} = \frac{2\sqrt{2}V_i}{\pi}$$

Fig. 4 also shows the difference between v_o and $v_{o,1}$ denoted $v_{o,h}$. This component represents the summation of all (unwanted) higher order harmonics $v_{o,h} = \sum_{n=2}^{\infty} v_{o,n}$.

//Remark: for sinusoidal wave: the RMS is the peak/√2…well known and no need to prove//

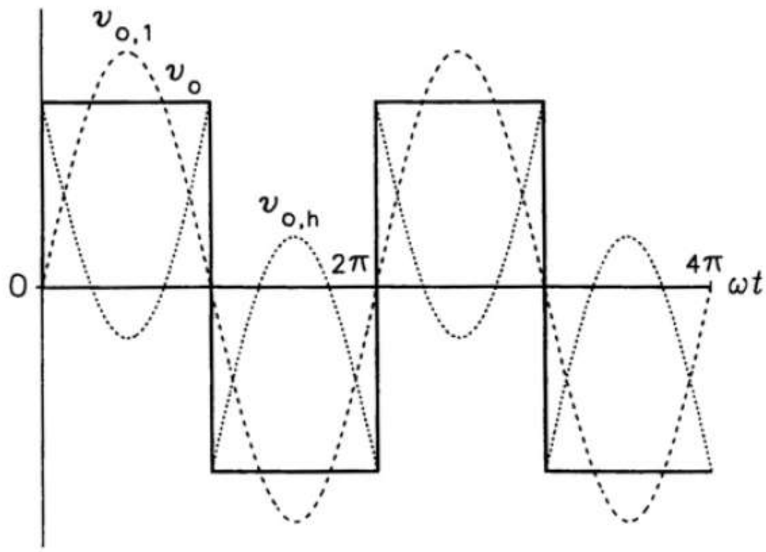


Fig. 4 the output voltage of the generic inverter and its fundamental and harmonic components

The Total Harmonic Distortion, THD

The total harmonics distortion is defined as

$$THD = \frac{F_h}{F_1} \qquad \text{…(9)}$$

Where F_h is the RMS of the harmonic of *f* except the fundamental harmonic and F_1 is the RMS of the fundamental harmonic.

The determination of the THD_{vo} of the generic inverter output voltage is as follows:

As $V_o = \sqrt{V_{o,1}^2 + V_{o,h}^2} \Rightarrow V_{o,h} = \sqrt{V_o^2 - V_{o,1}^2}$

$$V_{o,h} = \sqrt{V_i^2 - \left(\frac{2\sqrt{2}V_i}{\pi}\right)^2} = V_i\sqrt{1 - \frac{8}{\pi^2}} = 0.435V_i$$

$$THD_{vo} = \frac{V_{o,h}}{V_{o,1}} = \frac{0.435V_i}{\frac{2\sqrt{2}V_i}{\pi}} = 0.483$$

The spectrum is a very useful tool that is used to visualize the harmonic contents. It is a graph of the harmonic amplitude against the harmonic order (or frequency). Using the above result the spectrum of the generic inverter output voltage is drawn in Fig. 5. It can be seen that, for this waveform, as the harmonic order increases its amplitude decreases.

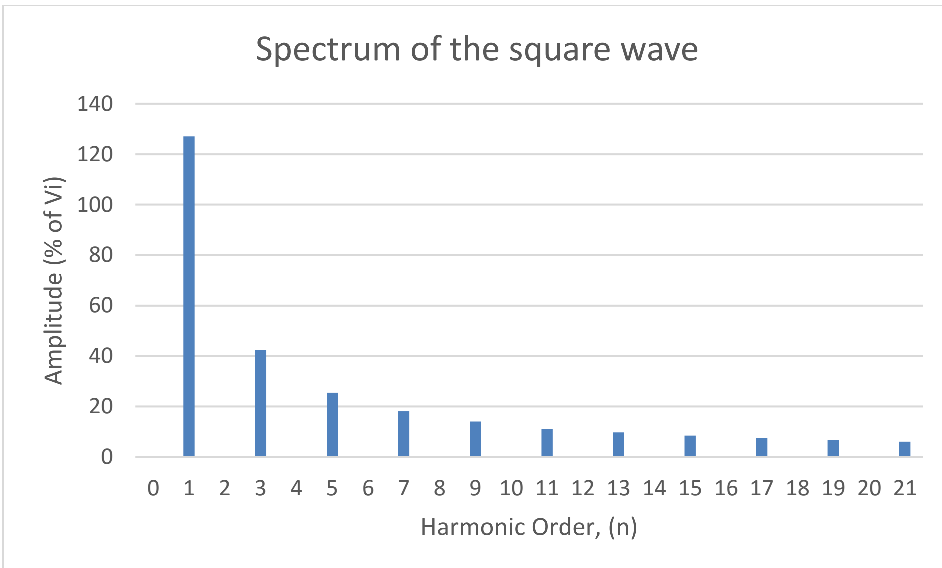
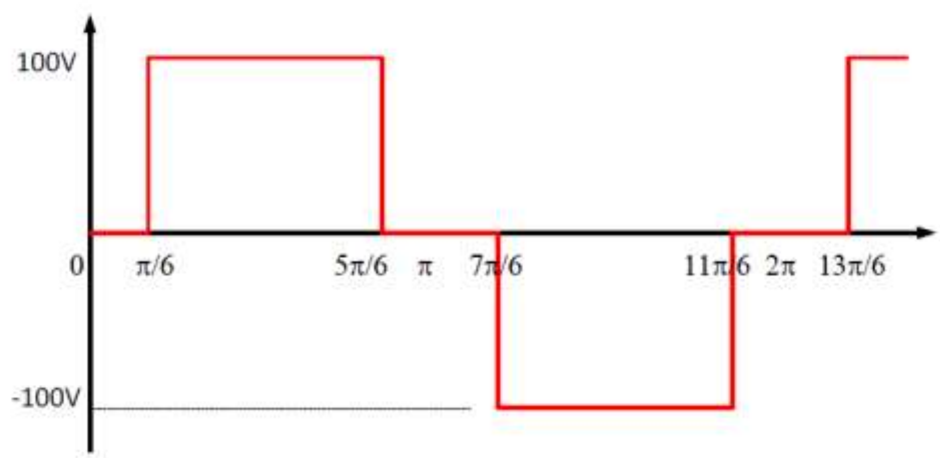


Fig. 5 Spectrum of the square wave voltage shown in Fig. 4

Drill Problem: Determine the THD of the voltage waveform shown in Fig. 6. Draw the spectrum corresponding to this voltage



ans:0.311

Fig. 6

3- **Power factor and efficiency:**

Input power factor, PF:

Power factor is a performance indicator used with converters supplied from (single-phase and three-phase) AC lines. The general definition of PF is:

$$PF \equiv \frac{P_i}{S_i} \qquad \text{...(10)}$$

where P_i and S_i are the real and apparent input powers respectively. Converter usually draws non-sinusoidal periodic power. In this case the famous relationship (pf=cosϕ) used in AC circuit is inapplicable. Instead, the power factor can be determined looking at the input voltage and current waveforms only using the following expression:

$$PF = K_d K_\emptyset \qquad (11)$$

Here, K_d denotes the so-called distortion factor, defined as the ratio of the RMS fundamental input current:

$$K_d = \frac{I_{i,1}}{I_i}$$

$K_\emptyset$ is the displacement factor:, $K_\emptyset = \cos \emptyset_1$the angle between the fundamentals of input voltage and current.

In coming chapters we are going to show that the two expressions of eq.(10) and eq.(11) are equivalent.

Example 2: Determine the power factor for the input current and voltage waveforms shown in Fig. 7

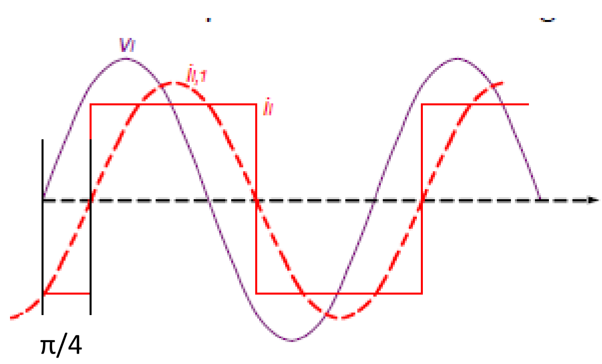


Fig.7

Sol. $K_{\emptyset} = \cos \emptyset_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$K_d = \frac{I_{i,1}}{I_i} = \frac{\frac{2\sqrt{2}I_i}{\pi}}{I_i} = \frac{2\sqrt{2}}{\pi}$$

$$PF = K_{\emptyset}K_d = \frac{1}{\sqrt{2}}\frac{2\sqrt{2}}{\pi} = \frac{2}{\pi}$$

Conversion efficiency, η_c

The conversion efficiency, η_c of the converter is defined as the ratio of the desired output power to the converter input voltage. The desired output power is the power associated to the DC quantity in DC producing converter and the fundamental component for the AC producing converter).

For DC-output converter:

$$\eta_c = \frac{P_{o,dc}}{P_{in}} \qquad \text{...(12)}$$

For AC-output converter:

$$\eta_c = \frac{P_{o,1}}{P_{in}} \qquad \text{...(13)}$$

Symbol $P_{o,dc}$ denotes the dc output power, that is, the product of the dc components of the output voltage and current, while $P_{o,1}$ is the ac output power carried by the fundamental components of the output voltage and current. In the cases discussed in this chapter using generic power converter; the converter does not incur any power losses and therefore the input power to the converter (P_{in} in Eq. (12) and (13)) is equals to the output power.

Chapter 1

Phase Control and Current Calculation Using Analytical Methods

Lecture aims:

1- To introduce the phase control concept using the hypothetical generic converter.
2- To present the analytical method of load current calculation based on natural and forced response.
3- To present the analytical solution method based on Fourier series and superposition.
4- To compare the RF and THD of the load current to those of the load voltage.

1- Phase control:

The aim of the control methods is to vary the output voltage as desired. The phase control is a systematic method characterized by low switching frequency which is usually equal to the output or input frequency. The term phase came from the fact that the output is varied by changing a control angle.

Applying the phase control on generic rectifier:

The average output voltage of the generic rectifier (studied in Lecture 1) can be varied by including zero level to produce an output voltage similar to that shown in Fig. 1. Where the converter is operated in state 0 to produce zero output voltage for the interval ($0 \le \omega t < \alpha_f$) before moving to state 1 (if v_i is positive) or state 2 (if v_i is negative).

α_f can be set to any value within the range $0 \le \alpha_f \le \pi$. Setting $\alpha_f = 0$ results an output voltage of full wave rectifier which is the maximum output voltage of this converter. On the other hand, $\alpha_f = \pi$ results an output voltage=0. Therefore the output voltage can be controlled through this angle, where:

$$V_{o,dc} = \frac{1}{\pi}\int_{\alpha_f}^{\pi} V_{i,p} \sin \omega t \, d\omega t = \frac{V_{i,p}}{\pi}\left(1 + \cos \alpha_f\right) \quad ....(1)$$

The variation of the output voltage with the control parameter $(\alpha_f)$is described in Fig. 2. The magnitude control ratio, M is defined as:

$$M \equiv \frac{V_{o,adj}}{V_{o,adj,max}} \qquad ...(2)$$

M is generally used to describe the effect of control parameter.

Fig. 1 also shows the output current for a series connected RL load, the following section shows how to determine the current.

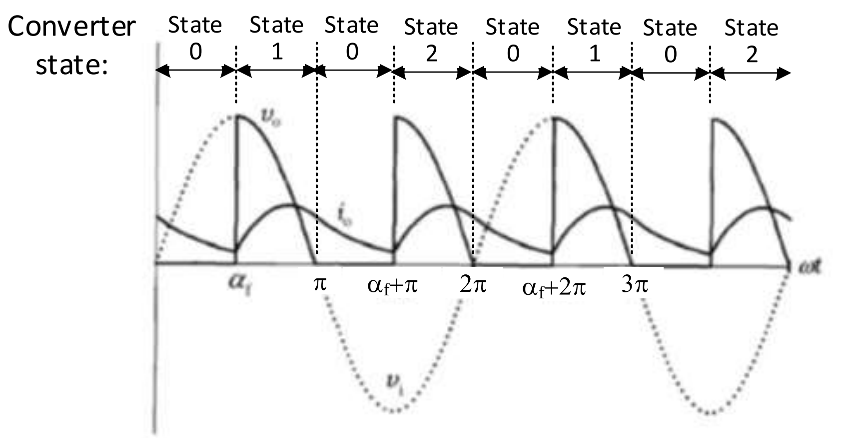


Fig. 1 The output voltage wave of a phase-controlled generic rectifier.

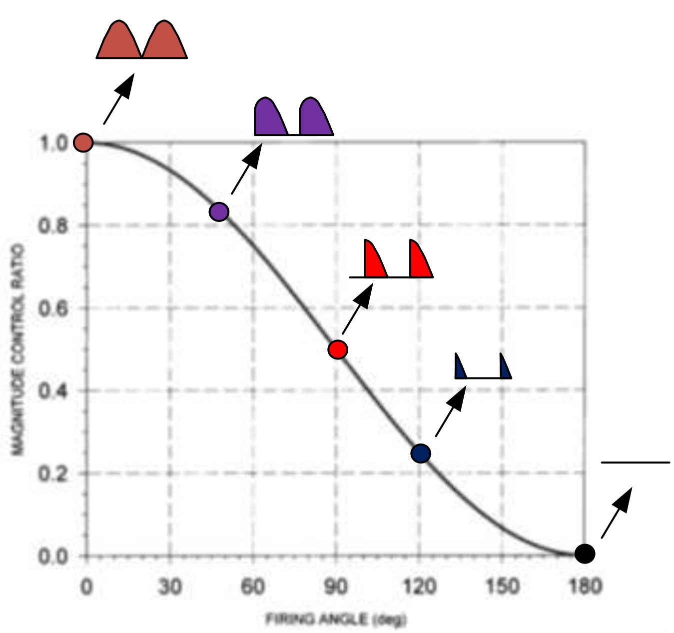


Fig. 2 The control characteristics of phase-controlled generic rectifier.

2-Current calculation using the analytical method

One cycle of the output current for the interval (0,π) is determined by dividing this interval according to the output voltage. The output voltage is given by:

$$v_o = \begin{matrix} 0 & for\ 0 < \omega t < \alpha_f \\ V_i \sin\omega t & for\ \alpha_f < \omega t < \pi \end{matrix} \quad ...(3)$$

For $0 < \omega t < \alpha_f$, the load RL circuit (Fig. 3) has zero voltage (source-free) and the current is known by the natural response only.

$$i_{o,1} = A_1 e^{-\frac{t}{\tau}} = A_1 e^{-\frac{\omega t}{\omega L/R}} = A_1 e^{-\frac{\omega t}{\tan\emptyset}} \quad ...(4)$$

$$0 < \omega t < \alpha_f$$

Where $A_1 = i_o(\omega t=0)$, R and L are the load resistance and inductance of the load as shown in Fig. 3. Subscript (1) denotes the first segment of current equation

For $\alpha_f < \omega t < \pi$, the circuit has a sinusoidal supply and the circuit current has two components, forced $i_{o,F}$and natural$i_{o,N}$:

$$i_{o,2} = i_{o2,F} + i_{o2,N} \quad ...(5)$$

Where

$$i_{o2,F} = \frac{V_{i,p}\sin(\omega t - \emptyset)}{|Z|} \quad ...(6)$$

$$i_{o2,N} = A_2 e^{-\frac{\omega t - \alpha_f}{\tan\emptyset}} \quad ...(7)$$

Where $|Z| = \sqrt{R^2 + (\omega L)^2}$ and $\emptyset = \tan^{-1}\left(\frac{\omega L}{R}\right)$

To specify the current expressions, we need to determine the integration constants A_1 and A_2, to be calculated using two (boundary) conditions:

$$i_{o,1}(\omega t = \alpha_f) = i_{o,2}(\omega t = \alpha_f) \quad ...(8)$$

$$i_{o,1}(\omega t = 0) = i_{o,1}(\omega t = \pi) \quad ...(9)$$

Gives:

$$A_1 e^{-\frac{\alpha_f}{\tan\emptyset}} = \frac{V_{i,p}\sin(\alpha_f - \emptyset)}{|Z|} + A_2 \quad ...(8)$$

$$A_2 = A_1 e^{-\frac{\alpha_f}{\tan\emptyset}} - \frac{V_{i,p}\sin(\alpha_f - \emptyset)}{|Z|} \quad ...(9)$$

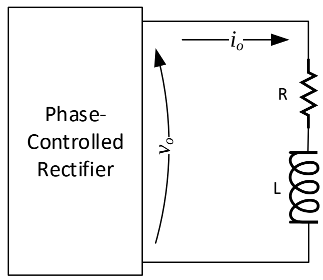


Fig. 3 RL load connected to the rectifier output side.

Example 1

A phase control generic converter is supplied by a 220V AC supply and the firing angle is adjusted to produce an average output voltage, $V_{o,dc} = 169V$. The load has series RL model with R=1.4Ω and L=4.5mH.

a. Calculate the firing angle, α_f
b. Calculate the magnitude control ratio
c. Calculate RF_{vo}
d. Derive the numerical expression of i_o
e. On the same time scale, draw the waveforms of:
   i. the supply voltage
   ii. the switching states
   iii. the output voltage
   iv. the output current
   v. the input current
f. Calculate RF_{io}
g. Calculate the input power factor

Solution:

a. $V_{o,dc} = \frac{V_{i,p}}{\pi}\left(1+\cos\alpha_f\right)$

$$169 = \frac{220\sqrt{2}}{\pi}\left(1+\cos\alpha_f\right)$$

$$\alpha_f = \cos^{-1}\left[\frac{169\pi}{220\sqrt{2}} - 1\right] \approx 45°$$

b. $M = \frac{V_{o,dc}}{V_{o,dc,max}} = \frac{169}{\frac{220\sqrt{2}*2}{\pi}} = 0.8532$

c. $V_o = \sqrt{\frac{1}{\pi}\int_{\alpha_f}^{\pi}\left(V_{i,p}\sin\omega t\right)^2 d\omega t} = V_{i,p}\sqrt{\frac{1}{2\pi}\int_{\alpha_f}^{\pi}(1-\cos 2\omega t)d\omega t}$

$$V_o = V_{i,p}\sqrt{\frac{1}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha_f}^{\pi}} = V_{i,p}\sqrt{\frac{1}{2\pi}\left[\left(\pi-\alpha_f\right) - \frac{\sin 2\pi}{2} + \frac{\sin 2\alpha}{2}\right]}$$

$$V_o = 220\sqrt{2}\sqrt{\frac{1}{2\pi}\left[\left(\pi-\frac{\pi}{4}\right) - 0 + \frac{1}{2}\right]} = 209.8V$$

$$V_{o,ac} = \sqrt{V_o^2 - V_{o,dc}^2} = 124.3V$$

$$RF_{vo} = \frac{V_{o,ac}}{V_{o,dc}} = \frac{124.3}{169} = 0.736$$

d.

$$\vec{Z} = R + j\omega L = 1.4 + j0.45\pi \approx 2\angle 45°$$

d. $i_{o,1} = A_1 e^{-\frac{\omega t}{\tan\emptyset}} = A_1 e^{-\omega t}$ for $0 \le \omega t < \frac{\pi}{4}$

$$i_{o,2} = \frac{220\sqrt{2}}{2}\sin(\omega t - \frac{\pi}{4}) + A_2 e^{-\left(\omega t - \frac{\pi}{4}\right)} = 155.6\sin(\omega t - \frac{\pi}{4}) + A_2 e^{-\left(\omega t-\frac{\pi}{4}\right)}$$

$$for\frac{\pi}{4} \le \omega t < \pi$$

To determine A_1 and A_2,use Eqs. (8) and (9)

$$i_{o,1}\left(\omega t = \alpha_f\right) = i_{o,2}\left(\omega t = \alpha_f\right)$$

$$A_1 e^{-\frac{\pi}{4}} = A_2$$

$$A_2 = 0.456A_1$$

$$i_{o,1}(\omega t = 0) = i_{o,1}(\omega t = \pi)$$

$$A_1 = \frac{155.6}{\sqrt{2}} + A_2 e^{-0.75\pi}$$

Gives $A_1 = 115, A_2 = 52.42$

$i_{o,1} = 115e^{-\omega t}$ f for $0 \leq \omega t < \frac{\pi}{4}$

$i_{o,2} = 155.6\sin(\omega t - \frac{\pi}{4}) + 52.42e^{-\left(\omega t - \frac{\pi}{4}\right)}$ for $\frac{\pi}{4} \leq \omega t < \pi$

ωt	i_o
0	115
15	88.5
30	68.1
45	52.42
60	80.6
75	108.9
90	133.9
105	153.1
120	164.5
130	166.9(max)
135	166.5
150	158.7
165	141.2
180	115

e.

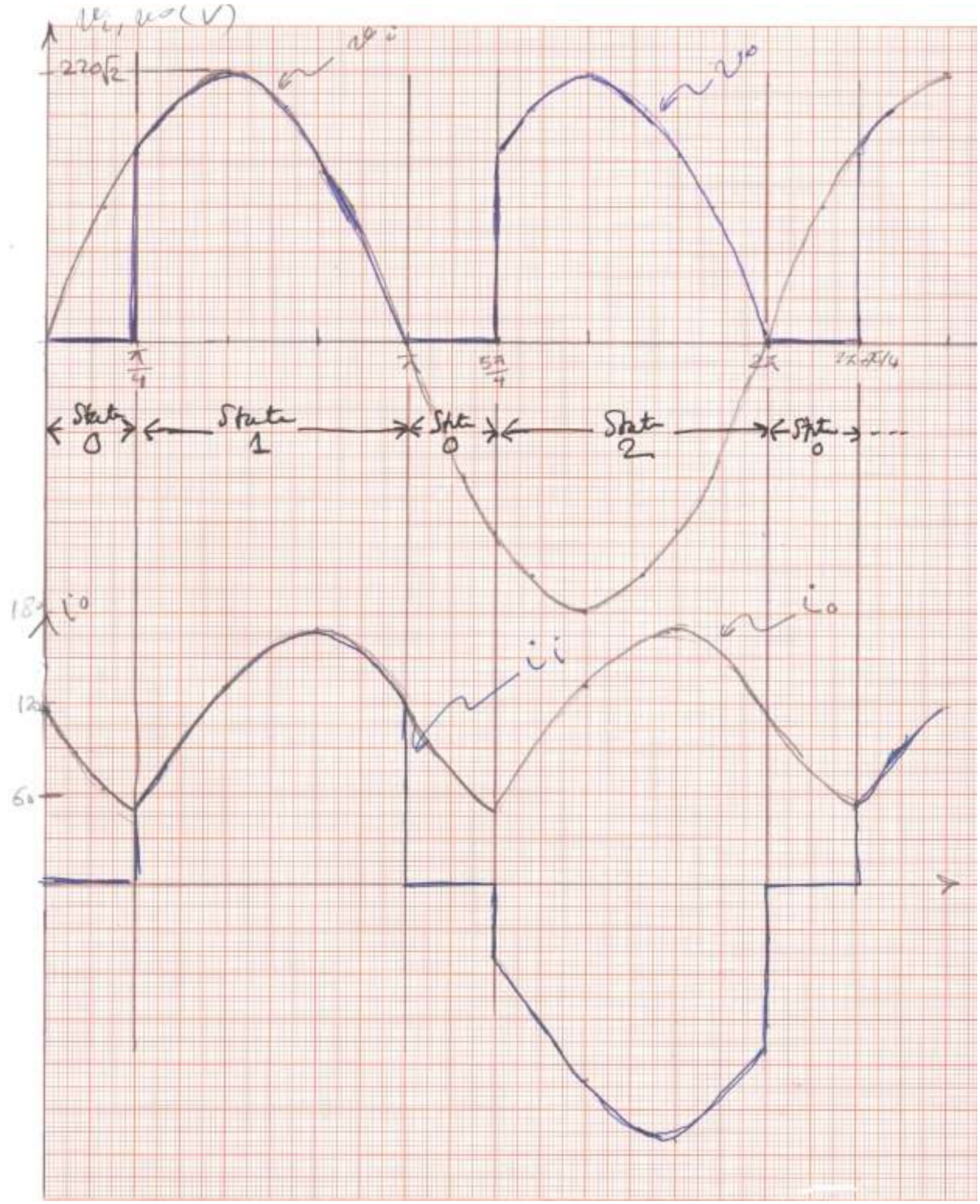


Fig. 4 Solution of Example 1 e

f.

To determine the current ripple factor,

$$I_{o,dc} = \frac{V_{o,dc}}{R} = \frac{169}{1.4} = 120.7A$$

we will approximate the current ripple to sinusoidal,

$$V_{o,ac,pp} = 166.9 - 52.4 = 114.5$$

Gives

$$V_{o,ac} = \frac{114.5}{2\sqrt{2}} = 40.5A$$

$$RF_{io} = \frac{I_{o,ac}}{I_{o,dc}} = \frac{40.5}{120.7} = 0.335$$

-To determine the apparent power, find I_o

$$I_o = \sqrt{I_{o,dc}^2 + I_{o,ac}^2} = 127.3A$$

We will use the approximation:

$$I_i \approx I_o\sqrt{\frac{3}{4}} = 110.3A$$

$$S_i = 220 * 110.3 = 24.3KVA$$

$$P_i = P_o = I_o^2 R = 22.7kW$$

$$PF = \frac{P_i}{S_i} = \frac{24.3}{26.1} = 0.93$$

$$\eta_c = \frac{I_{o,dc}^2}{I_o^2} \approx 0.9$$

3- Phase-Controlled Inverter

The generic inverter presented in Lecture 1 provides an output voltage with fixed amplitude. It has been shown in Lecture 2 that the output has a fundamental of ($V_{o,2} = \frac{2\sqrt{2}V_i}{\pi}$), which means that the output is constant. In order to control the amplitude of the inverter output voltage state 0 is inserted in the way shown in Fig. 5 to zero the voltage between positive and negative pulses. The operation cycle of this phase-controlled inverter is given by:

interval	Operation state	v_o
$0 \sim \alpha_d$	State 0	0
$\alpha_d \sim (\pi - \alpha_d)$	State 1	V_i
$(\pi - \alpha_d) \sim (\pi + \alpha_d)$	State 0	0
$(\pi + \alpha_d) \sim (2\pi - \alpha_d)$	State 2	$-V_i$
$(2\pi - \alpha_d) \sim 2\pi$	State 0	0

Where α_d denotes the “delay angle”. The resultant output voltage waveform is odd and half-wave symmetrical. And it is left to the student to show for this waveform that:

$$V_{o,n,p} = b_n = \frac{4}{n\pi} V_i \cos n\alpha_d \qquad ...(10)$$

By examining Fig. 5, we can realize that the delay angle (α_d) is adjustable in the range $0 \sim \frac{\pi}{2}$. The output voltage fundamental is maximum when $\alpha_d = 0 \left(V_{o,1,max} = \frac{2\sqrt{2}}{\pi} V_i \right)$ and minimum (=0) when $\alpha_d = \frac{\pi}{2}$. The control characteristics of the phase controlled generic inverter is shown in Fig. 6.

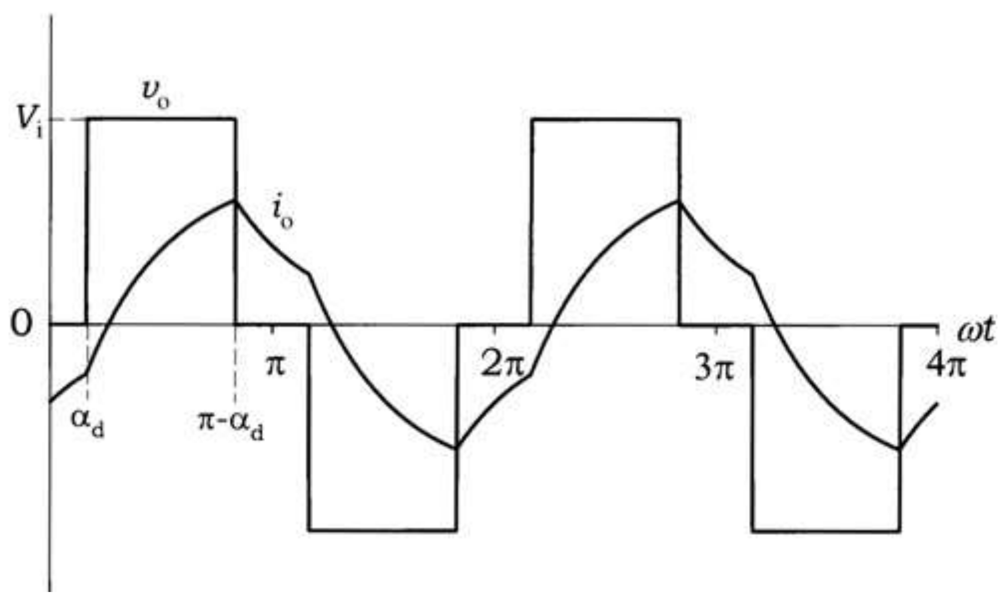


Fig. 5 output voltage of phase-controlled inverter

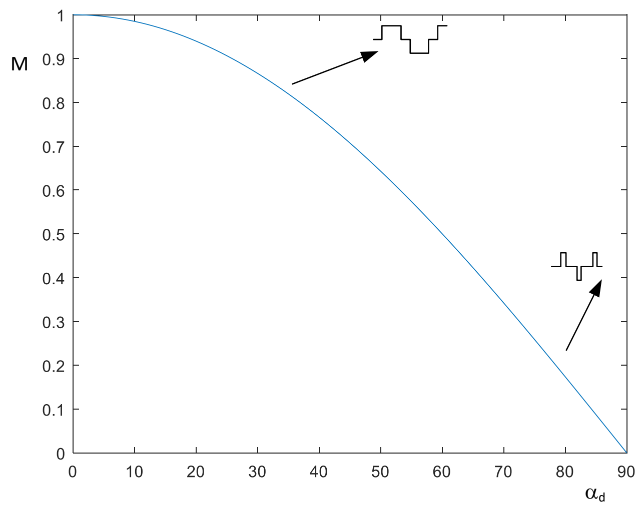


Fig. 6 Control characteristics of phase controlled inverter

4- Fourier series analysis:

Fourier series analysis use the superposition principle to carry out the analysis with any periodic non-sinusoidal supply to basic DC and AC analysis.

The idea is to replace the non-sinusoidal supply (the converter output voltage in our case) by its Fourier components which are DC and AC sinusoidal as shown in Fig. 7. The response of the circuit (i_o) is calculated by summing up the responses of the corresponding individual Fourier components.

$$i = \sum_{n=0}^{\infty} i_n \qquad \text{...(11)}$$

Where

$$i_n = \frac{V_{n,p}\ \sin(n\omega t + \emptyset_n)}{R + jn\omega_o L} \qquad \text{...(12)}$$

The following example demonstrates how to perform the analysis to determine the output current by applying superposition on Fourier components. Before presenting the example, it must be stressed that the analytical method based on DE solution similar to the one followed in Example 1 is similarly applicable here.

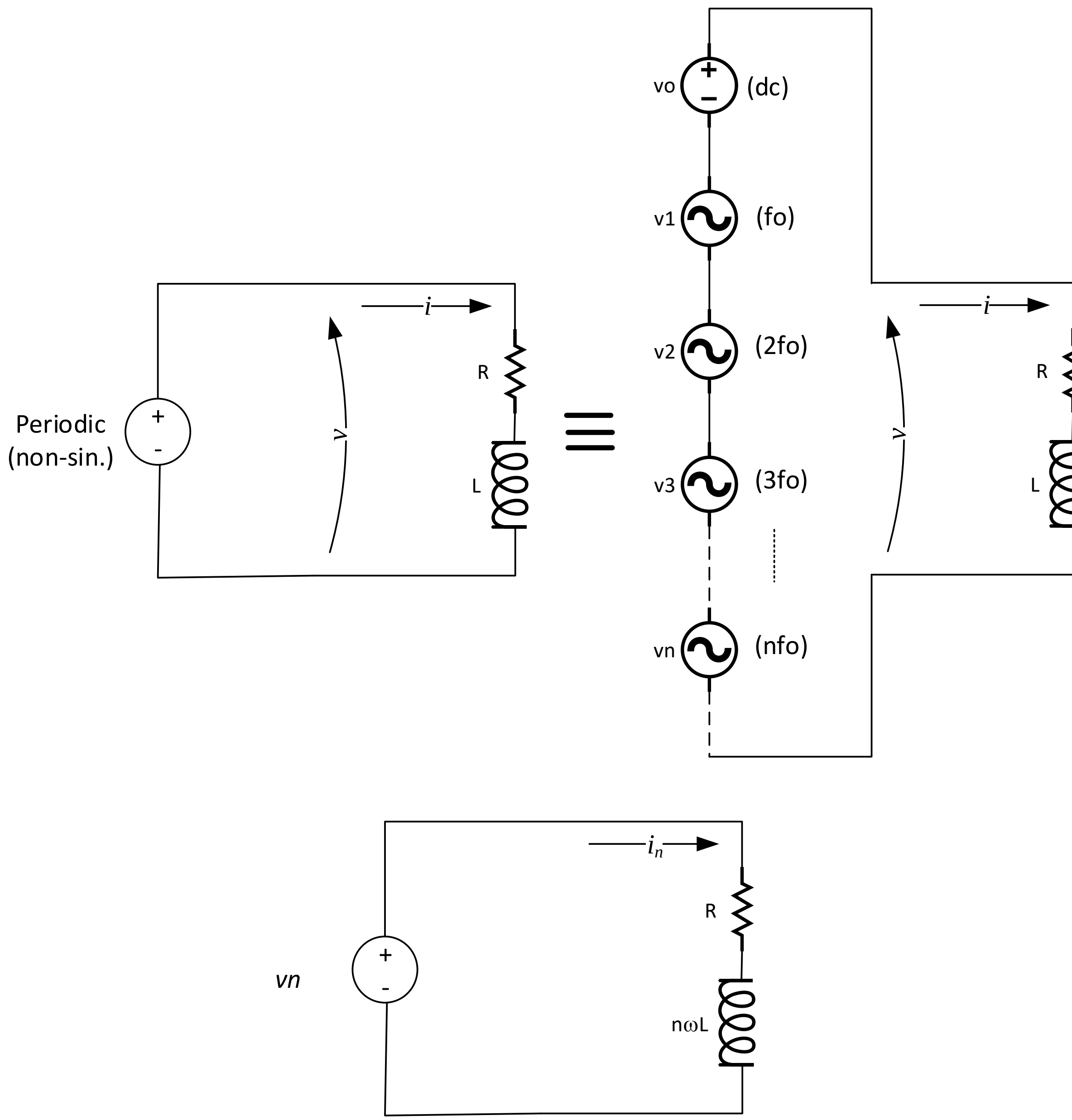


Fig. 7 Analysis of circuit supplied by periodic non-sinusoidal supply.

Example 2:

A generic converter is operated as a phase controlled generic inverter. The input DC voltage is 300V and the output fundamental voltage is 200V at 41.67Hz. The RL load has R=1.3Ω and 4.4mH.

a. Calculate the delay angle.

b. Determine the expression of the nth harmonic of the output voltage. Draw the spectrum of v_o.
c. Determine THD of the output voltage.
d. Use Fourier analysis to calculate the output current and its THD.

Sol.

a. To determine $\alpha_d(V_{o,1} = 200V, V_i = 300V)$, from Eq. (10)

$$V_{o,1,p} = \frac{4}{\pi} V_i \cos \alpha_d$$

$$200\sqrt{2} = \frac{4}{\pi}(300) \cos \alpha_d \rightarrow \alpha_d = 42.2°$$

b. From Eq. (10):

$$V_{o,n,p} = \frac{4}{n\pi} V_i \cos n\alpha_d$$

To draw the spectrum, we calculate $V_{o,n}$for different values of *n*

n	$V_{o,n,p}$	$V_{o,n}$	Accumulated Harmonics RMS
1	282.9	200	0
3	-75.9	53.66	53.6694
5	-65.48	46.3	70.88173
7	23.4	16.6	72.78736
9	39.9	28.2	78.06411
11	-8.5	6	78.29515
13	-29	20.5	80.93596
15	1.33	0.94	80.94143
17	22.4	15.8	82.47663
19	2.87	2.03	82.50159
21	-17.66	12.5	83.4413
23	-5.5	3.89	83.53189
25	13.84	9.79	84.1032
27	7.2	5.09	84.25716
29	-10.6	7.5	84.58989

Fig. 8 shows v_o spectrum, the green bars indicate negative amplitudes as calculated using Eq. (10), which means that the corresponding component is 180° out of phase.

For explanation only (and not a part of the solution), Fig. 9 shows the result of adding the harmonics calculated in the table above and shown in spectrum of Fig. 8. By comparing the three parts of Fig. 9 we can see that as more harmonics added the summation converges to the quasi square wave similar to the one shown in Fig. 5.

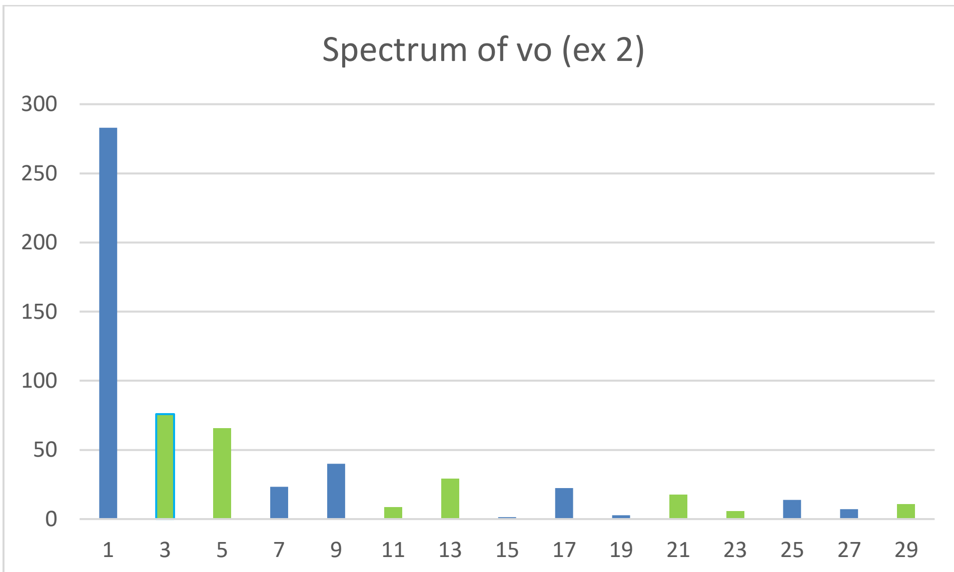


Fig. 8

THD_{vo} calculation:

$$V_o = 300\sqrt{\frac{(180 - 2 * 42.2)}{180}} = 218.63V$$

$$V_{o,h} = \sqrt{V_o^2 - V_{o,1}^2} = 88.3V$$

$$THD_{vo} = \frac{88.3}{200} = 44.1\%$$

We can notice that the ripple accumulation calculated in the Table above and drawn in Fig.10 is converging to $V_{o,h}$ determined above.

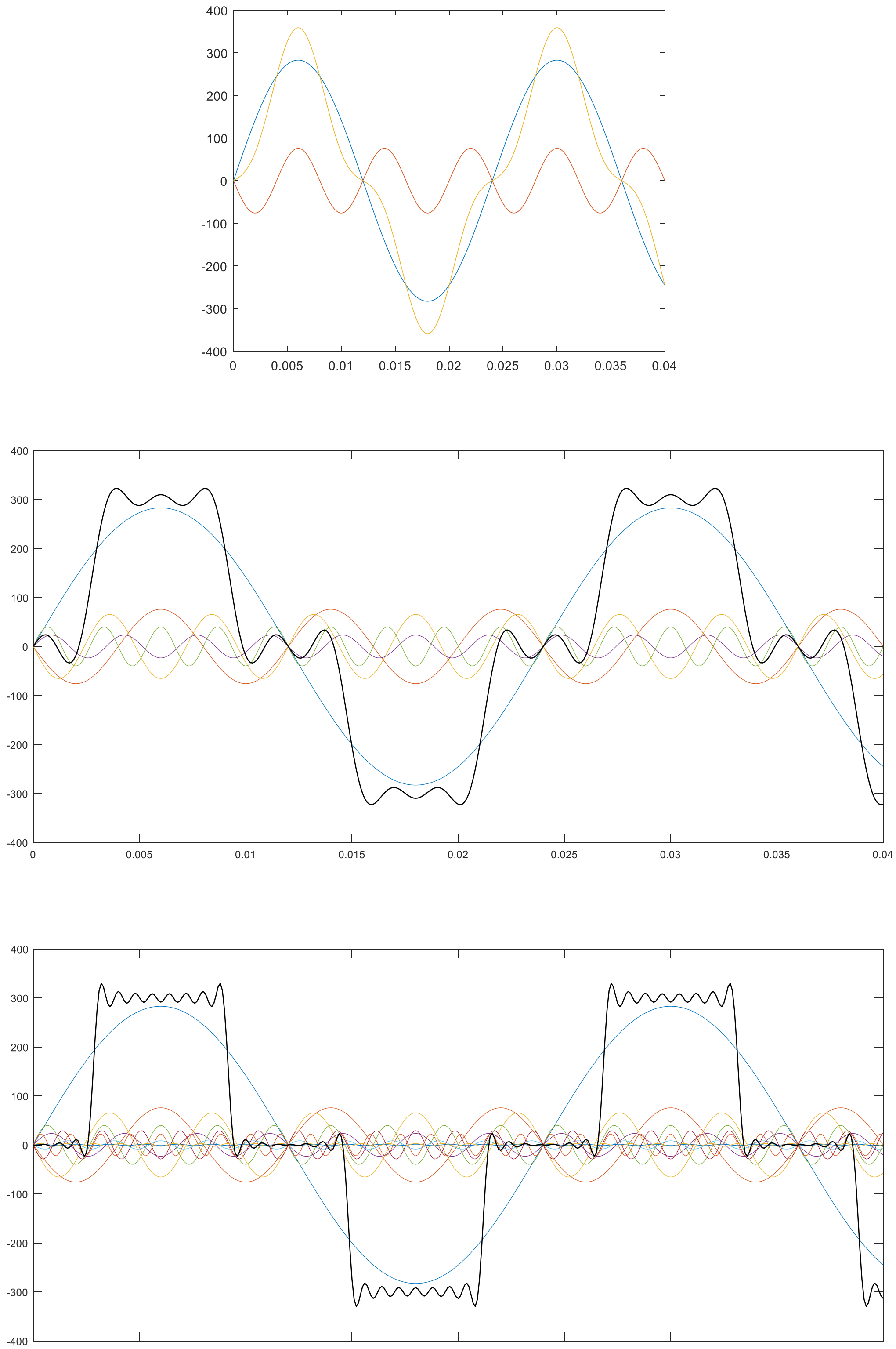


Fig. 9 the summation of obtained harmonics top: 1st and 3rd middle: up to 9th Bottom: up to 29th

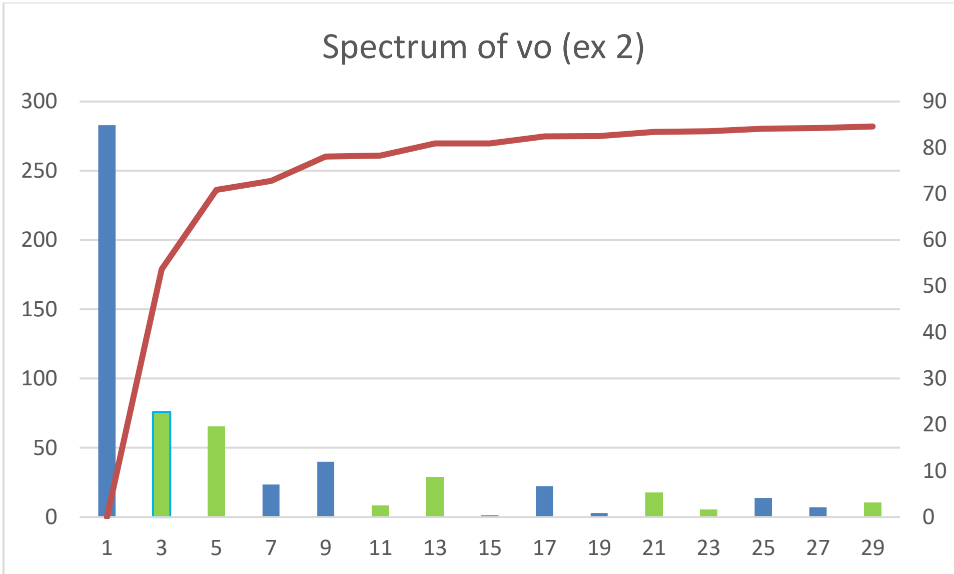


Fig. 10 the accumulation of Vh with the spectrum

d. Current Calculation

We are going to view deal the supply as an infinite series connected sinusoidal supplies as indicated in Fig.

The current in calculated by applying superposition on these supplies. Recall that while the load resistance does not affected by the supply frequency, the inductive reactance is proportional to the frequency. Therefore the impedance seen by the nth harmonic is:

$$Z_n = R + j\omega_n L = R + jn\omega L = 1.3 + j\frac{1.1n\pi}{3}$$

$Z_n = \sqrt{R^2 + (n\omega L)^2}\angle \tan^{-1}\frac{n\omega L}{R}$

The current Fourier components are given in the following table , and the current spectrum is shown in Fig. 10.

We can notice that the current harmonics amplitude reduces faster than the voltage as the harmonic order increase. This is basically due to the increase of the inductive reactance which is proportional to the frequency.

n	Vo,n,p	Zn	$\lvert I_{o,n} \rvert$	$\sum_{i}^{n} Io, h$
1	*282.9*	1.74∟41.54	115.1	0
3	*-75.9*	3.692177∟69.4	14.53598	14.53598
5	*-65.48*	5.904465∟77.3	7.841753	16.51629
7	*23.4*	8.167527∟80.8	2.025864	16.64007
9	*39.9*	10.44842∟82.9	2.70027	16.85774
11	*-8.5*	12.73758∟84.1	0.471864	16.86434
13	*-29*	15.03122∟85	1.364234	16.91943
15	*1.33*	17.32756∟85.7	0.054275	16.91952
17	*22.4*	19.62566∟86.2	0.807066	16.93875
19	*2.87*	21.92496∟86.6	0.092561	16.93901
21	*-17.66*	24.22512∟86.9	0.515478	16.94685
23	*-5.5*	26.52592∟87.2	0.146615	16.94748
25	*13.84*	28.8272∟87.36	0.339483	16.95088
27	*7.2*	31.12886∟87.6	0.163551	16.95167
29	*-10.6*	33.43082∟87.77	0.224204	16.95315

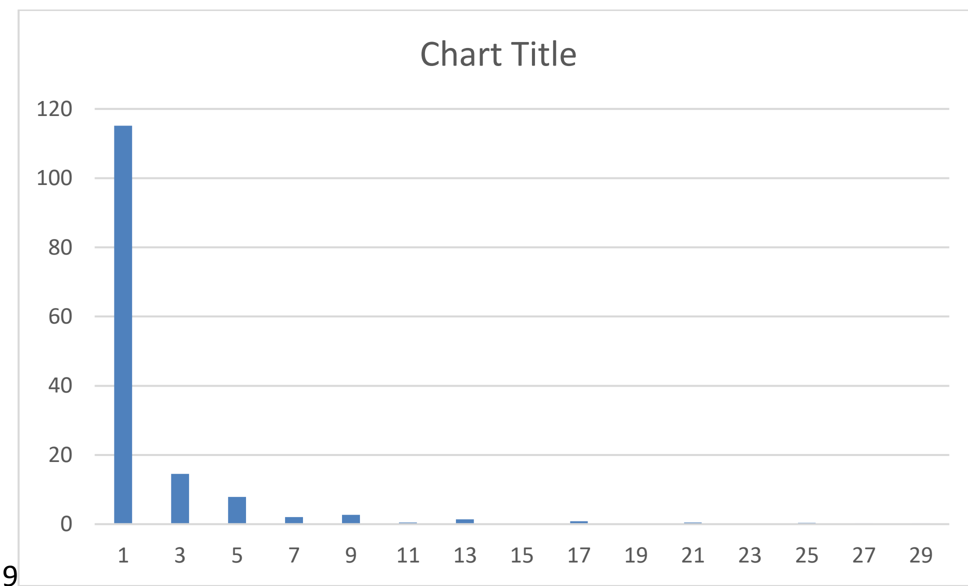


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Fig. 10 the spectrum of Io

The current ripple factor, $RF_{io} = \frac{I_{o,h}}{I_{o,1}} = \frac{16.95}{115.1} = 0.147$

Chapter 1

PWM Control and Practical Example

Lecture Aims

1- To introduce the concept of PWM control
2- To define the duty ratio as a control parameter
3- To relate the magnitude control ratio to the switching signal duty ratio
4- To highlight the effect of the switching frequency.
5- To present a practical circuit and compare it to the generic power converter.

1- PWM control of DC Chopper

The principle of PWM can be explained considering DC-to-DC power conversion performed by generic converter supplied by a DC voltage. The converter controls the dc component of the output voltage as shown in Fig. 1 by producing an output voltage consists of a train of pulses. The similar practical power electronic converters are called *choppers*.

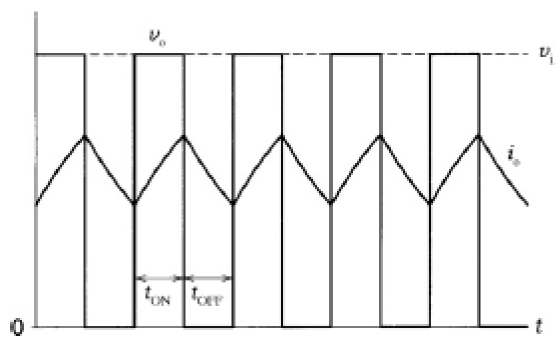


Fig. 1 PWM control applied to DC chopper.

In the case shown in Fig. 1, the pulses and notches are of equal duration, that is, switches S1 and S2 operate with the *duty ratio* of 0.5. A duty ratio, *d*, of a switch is defined as

$$d \equiv \frac{t_{on}}{t_{on}+t_{off}} \qquad \text{...(1)}$$

where t_{ON} is the on-time, and t_{OFF} is the off-time. Switch S5 in this case also operates with the duty ratio of 0.5. However, if the duty ratio of switches S1 and S2 were, for example,($d_{1,2}$=) 0.6, the duty ratio of switch S5 would have to be ($d_5 = 1 - d_{1,2}$=) 0.4.

The average value, *V*o,dc, of the output voltage is

$$V_{o,dc} = dV_i \qquad \ldots(2)$$

As $V_{o,dc(MAX)} = V_i$; then M=d (remember is M defined in Lecture 3 as $V_{o,adj}/V_{o,adj(MAX)}$)

The possible range of a duty ratio is from zero (switch open all the time) to 1 (switch closed all the time), changing the duty ratio, allows the adjustment of *V*o,dc to any level between zero and *V*i.

The effect of switching frequency

The switching frequency, f_{sw}, defined as:

$$f_{sw} = \frac{1}{T_{sw}} = \frac{1}{t_{on} + t_{off}}$$

The switching frequency does not affect $V_{o,dc}$. However, the quality of output current depends on f_{sw}. As illustrated in Fig. 2, where f_{sw} is doubled compared to that in Fig. 1, results in the reduction of the current ripple by about 50%. This can be reasoned in two ways:

First: If the time between state changes is short enough, it will not allow significant current changes between consecutive output voltage levels is.

Second: The fundamental output frequency equals the switching frequency. As a result, harmonics of the ac component of the voltage appear at frequencies that are integer multiples of *f*sw. The corresponding inductive reactances of the load are proportional to these frequencies.

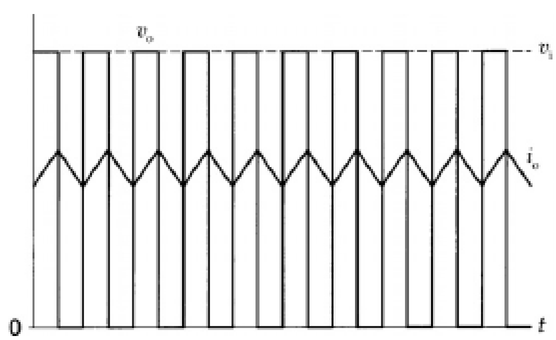


Fig. 2 The current ripple reduces to half as the frequency doubled compared to Fig. 1

If the switching frequency is sufficiently high, the ac component of the output current may be negligible that the current is practically of ideal dc quality.

PWM Application in Other types of Converters

PWM can be applied in all the other types of electric power conversion described earlier. The application of PWM control on rectifier and AC controller is shown in Fig. 3. Here, the ratio, *N*, of the switching to input frequency is 12. This is also the number of pulses of output voltage per cycle. The duration of one pulse cycle is called the switching interval (T_{sw}), the operating switching complete an operation cycle (ON-OFF-ON) with each interval. The switching intervals may or may not be synchronized to the input supply, this will be discusses in details in coming chapters.
As for the DC chopper case, Fig. 3 shown that the output current waveforms have higher quality compared to those of phase controlled converters.
Practically the switching frequency of a PWM converter is in the order of kHz, this is way higher than that shown in Fig. 3. Were the switching frequency is chosen to be low for clarity. Generally, a switching frequency should be several times higher than the reciprocal of the time constant of the load.

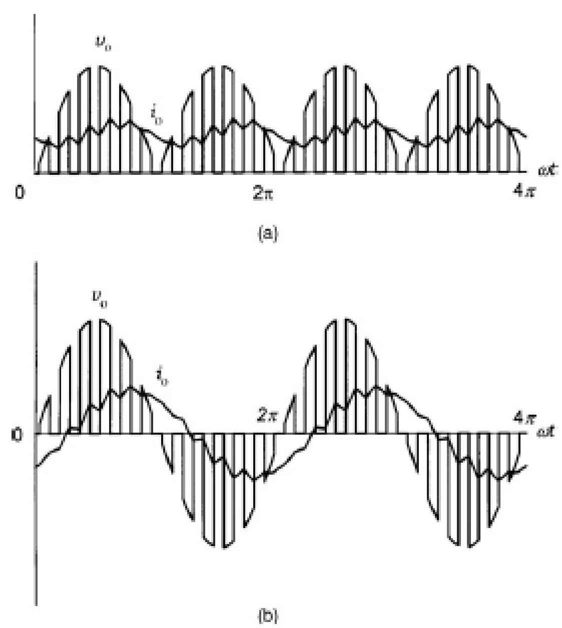


Fig.3 Voltage wave forms and RL load resultant current of PWM converters (a) Rectifier (b) AC controller

Duty ratio and Magnitude control ratio

It can be shown that the linear relation of to the chopper (M=d) can be extended on other types of PWM converters, such as rectifiers and ac voltage controllers. In all these converters:

$$V_{o,adj} = dV_{o,adj(MAX)} \qquad \ldots(3)$$

Depending on the type of converter, symbol *V*o,adj, represents the adjustable DC component or fundamental AC component of the output voltage, while *V*o,adj(max) is the maximum available value of this component. However, concerning the *rms value*, *V*o, of output voltage, the dependence on the duty ratio is radical, that is,

$$V_o = \sqrt{d}V_{o,(MAX)} \qquad \ldots(4)$$

where *V*o(MAX) is the maximum available rms value of output voltage.

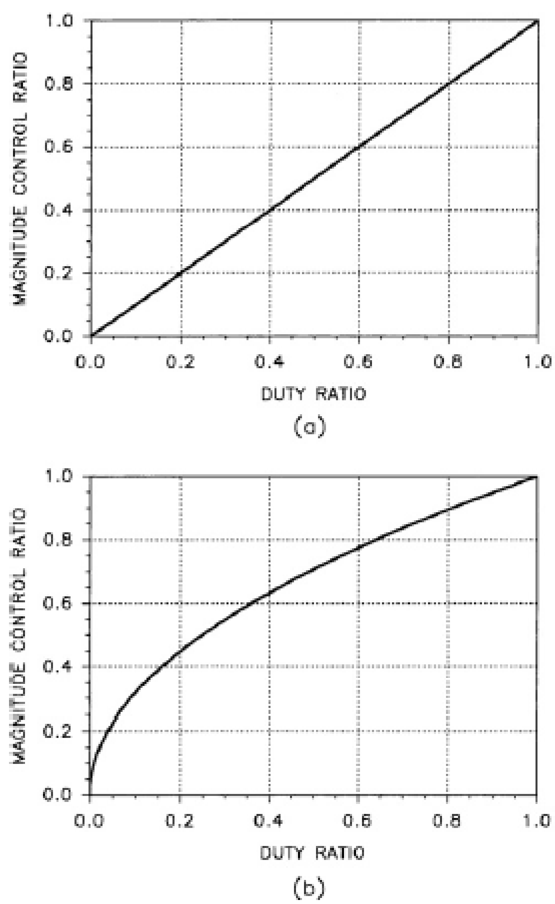


Fig. 4 Control characteristics of PWM converter (a) Rectifier (Vo=Vo,dc) (b) AC controller (Vo=Vo(rms))

Control characteristics of the generic PWM rectifier and AC voltage controller are shown in Fig. 4.

Harmonic spectra of the output voltage of the generic PWM rectifier and AC voltage controller are shown in Fig. 5. In AC controller, the switching frequency is 12 times higher than the input frequency, that is, the output voltage has 12 pulses per cycle. The duty ratio of converter switches is 0.5. In Fig. 5(b), we can notice that:

- Low order harmonics are almost eliminated.
- Higher harmonics appear in clusters centered about integer multiples of 24 (2N) for the ac voltage controller. The relationship between the number of pulses per cycle (N) and dominant harmonics order will be explained for each specific type of PWM converter in coming chapters.

The PWM rectifier spectra shows that the output voltage has all harmonics nonzero (including evens). This is due to the absence of any form of symmetry.
The reduced amplitudes of low-order harmonics (2nd to 10th) of the output voltage results in reduced ripple in the output current waveforms. The low order harmonics amplitudes is small compared to the harmonics of order around 12 and its multiples, and therefore current distortion is reduced.

2- Practical Example: Single-Phase Diode Rectifiers

Single-phase diode rectifiers are the simplest static converters of electrical power. The power diodes employed in the rectifiers can be thought of as uncontrolled semiconductor power switches. A diode in a closed circuit turns on when the diode is forward biased. The diode turns off when its current changes polarity.
The *single-pulse* (single-phase half-wave) diode rectifier is shown in Fig.6. It is equivalent to a reduced generic power converter having only switches S1 and S2, which stay closed as long as they conduct the load current. If the load is purely resistive (R-load), and the input voltage is sinusoidal, the output current waveform is similar that of the output voltage, as shown in Fig. 7(a). It can easily be seen that the average output voltage, $V_{o,dc}$, in the single-pulse rectifier is half as large as that in the generic rectifier,

that is, $V_{o,dc} = V_{i,p}/\pi \approx 0.32\ V_{i,p}$. The single "pulse" of output current per cycle of the input voltage explains the name of the rectifier.

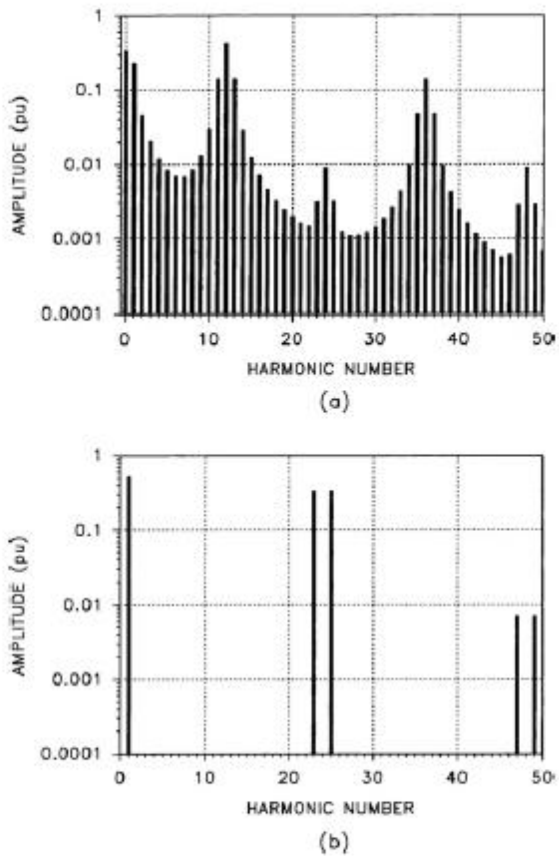


Fig. 5 Harmonic spectra of output voltage in a PWM controlled (a) rectifier (b) AC controller

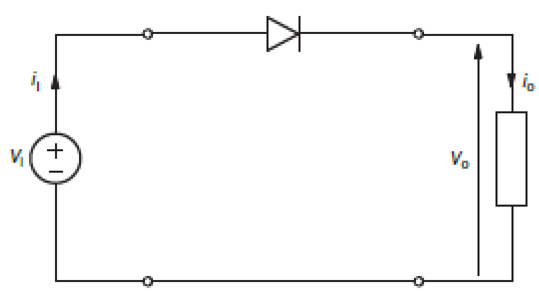
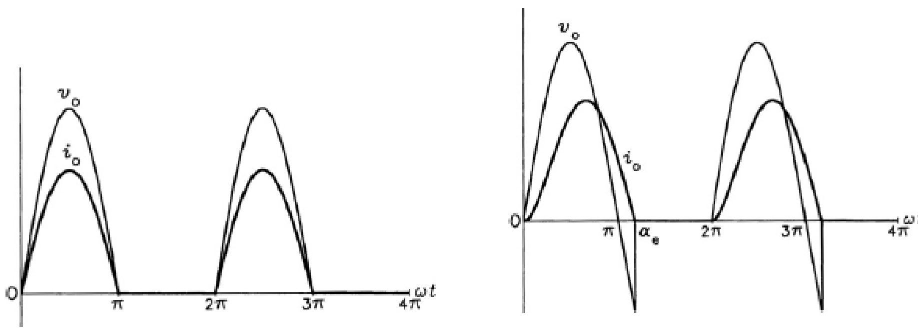


Fig. 6 Single pulse diode rectifier

The output voltage gets even lower when the load contains an inductance (RL-load). The output current, flows until it reaches zero at $\omega t = \alpha_e$, where α_e is called an *extinction* *angle*. Thus, when $i_o > 0$, that is, when $0 < \omega t \leq \alpha_e$. The output voltage and current waveforms of a single-pulse rectifier with an RL load are shown in Fig. 7(b). As the extinction angle is greater than 180°, the output voltage is negative in the $\omega t = \pi$ to $\omega t = \alpha_e$ interval, and its overall average value is lower than that with the resistive load.



(a) (b)

Fig. 7 The output voltage and current of a single pulse rectifier with (a) R load , (b) RL load

To calculate the extinction angle, we first obtain the output current expression as follows:

$$i_o(\omega t) = \frac{V_{i,p}}{Z}\sin(\omega t - \varphi) + Ae^{-\frac{\omega t}{tan\varphi}} \qquad \dots(5)$$

To find A, consider the initial condition: $i_o(0) = 0$, gives:

$$A = \frac{V_{i,p}}{Z}\sin(\varphi) \qquad \dots(6)$$

And therefore

$$i_o(\omega t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) + \sin(\varphi)\, e^{-\frac{\omega t}{tan\varphi}}\right] \qquad \dots(7)$$

To find the extinction angle we have to solve Eq. 7 for $i_o(\alpha_e) = 0 \; and \; \alpha_e > \pi$. This can be done only numerically (by iterations).

The negative part of the output voltage can be eliminated by connecting the *freewheeling diode*, DF, across the load, as shown in Fig. 8.

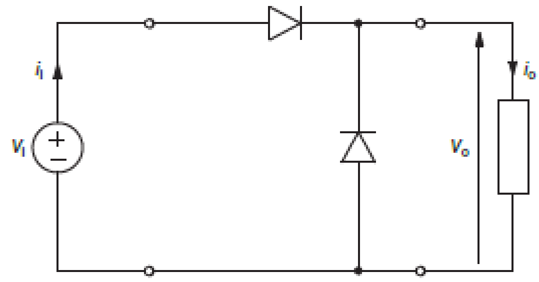


Fig. 8 Single pulse diode rectifier with freewheeling diode

The freewheeling diode, which corresponds to switch S5 in the generic power converter, shorts the output terminals when the output voltage reaches zero and provides a path for the output current in the π to α_e interval. Until the voltage reaches zero, the waveform of output voltage is the same as that in Fig. 7(b) and that of output current is described by Eq.(7) . Later, the current dies out while freewheeled by diode DF. As now $v_o = 0$, the equation of the current is simply

$$i_o(\omega t) = Ae^{-\frac{\omega t-\pi}{tan\varphi}} \quad for\ \pi < \omega t < \alpha_e \qquad \ldots(8)$$

where constant A can be found by equaling currents given by Eqs. (8) and (7)

at $\omega t = \pi$:

$$A = \frac{V_{i,p}}{Z}\left[\sin(\pi - \varphi) + \sin(\varphi)\, e^{-\frac{\pi}{tan\varphi}}\right] = \frac{V_{i,p}}{Z}\sin(\varphi)\left[1 + e^{-\frac{\pi}{tan\varphi}}\right]$$

Gives :

$$i_o(\omega t) = \frac{V_{i,p}}{Z}\sin(\varphi)\left[e^{-\frac{\omega t}{\tan\varphi}} + e^{-\frac{\omega t-\pi}{\tan\varphi}}\right] \quad for\ \pi < \omega t < \alpha_e$$

Waveforms of the output voltage and current of a single-pulse rectifier with the freewheeling diode are shown in Fig. 9.

Practical single-pulse rectifiers do not belong in the area of true power electronics, one reason is the need for large filtering capacitors.

The average output voltage, $V_{o,dc}$, can be increased to $2V_{i,p}/\pi \approx 0.64V_{i,p}$ in a two-pulse (single-phase full-wave) diode rectifier in Fig. 10. Diodes D1 through D4 correspond to the respective switches in the generic converter, directly connecting and cross-connecting the input terminals with the output terminals in dependence on polarity of the input.

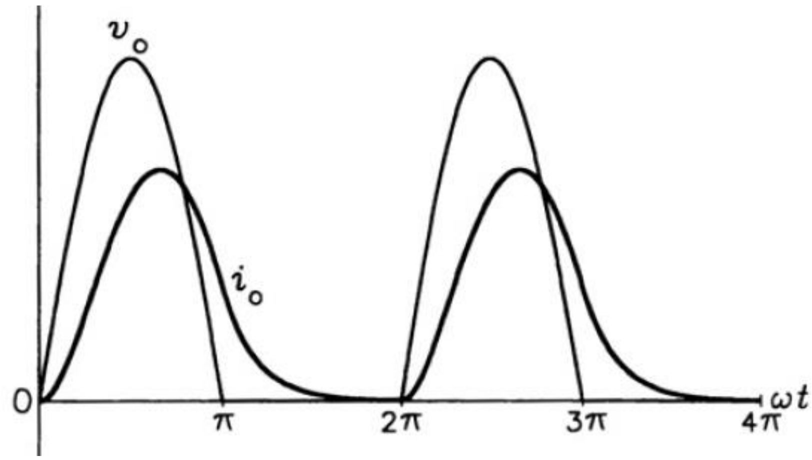


Fig. 9 The output voltage and current of a single pulse rectifier with freewheeling diode

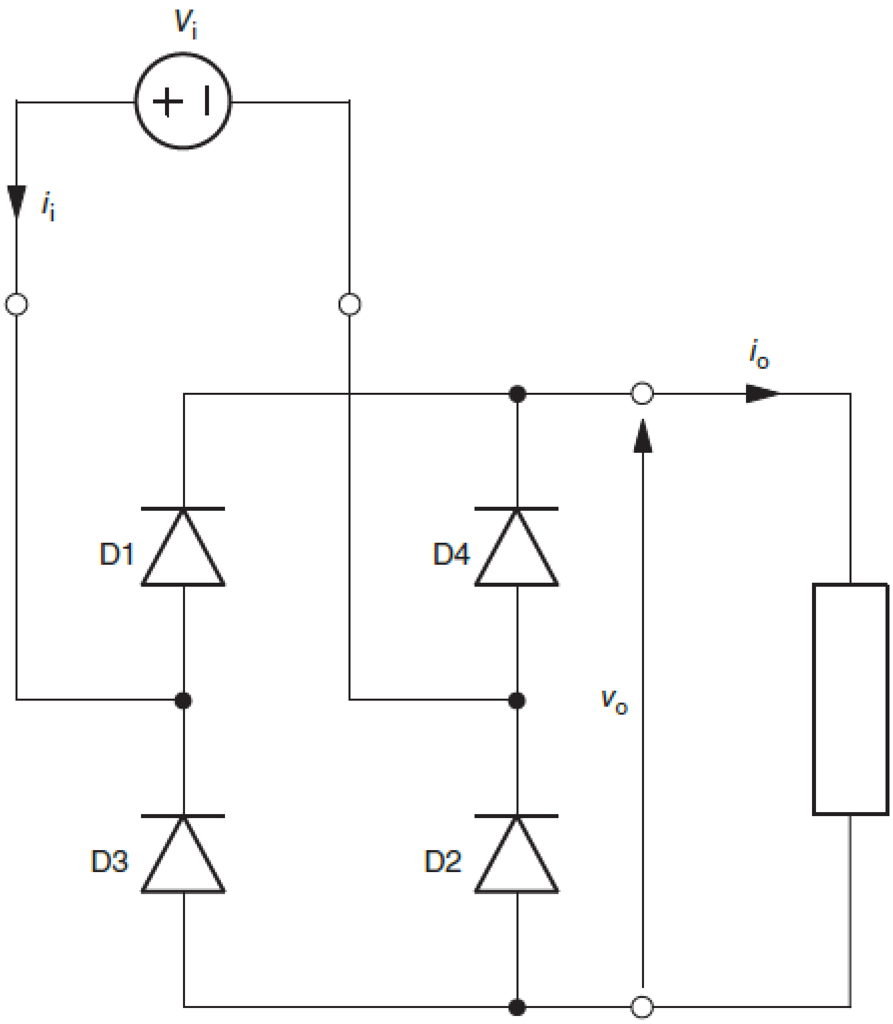


Fig. 10 2-pulse diode rectifier

Chapter 2

Semiconductor Power Switching Devices

Chapter Outcomes

- **Comparison between ideal and semiconductor switching devices.**
- **To study the static and dynamic characteristics of power diodes and define the diode ratings, and features of the special power diodes.**
- **To study the static and dynamic characteristics of SCR.**
- **To describe the operation, characteristics advantages and disadvantages of the most common fully controlled switching devices: GTO, BJT, MOSFET, IGBT and MCT.**

Semiconductor verses ideal switching devices

Ideal Switch	Semiconductor Switch
Conduction (ON-state) resistance = 0 Ω Or zero on-state voltage drop	At Conduction (ON)-state there is a voltage drop across the switch. (Typically 1~3Volts)
Blocking (OFF-state) resistance = ∞Ω Or zero Off-state leakage current.	At Blocking (OFF)-state there is a small leakage current, dependent on temperature.
When ON conducts current in both directions	usually conducts the current in one direction only
Instantaneous switching, or zero switching time.	Switching time is not zero and varies largely between different types/ratings of devices.
Fully Controlled	There are 3 types: Uncontrolled, Semi-controlled and Fully-Controlled
Zero control power	Needs a driving power for control signal

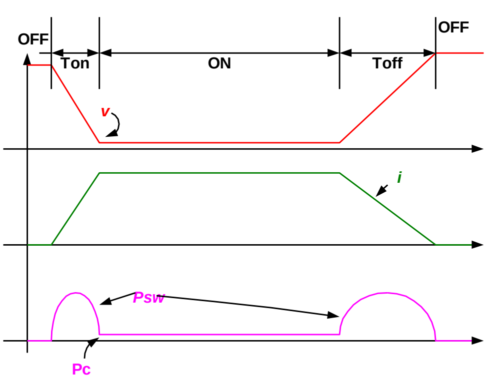
Power losses in semiconductor switches

1-Conduction Loss, P_c
Losses due to voltage drop during on state, the average of the conduction power losses is given by:

$$Pc = \frac{1}{T_{sw}} \int_0^{DT} (v_{ON} i) dt$$

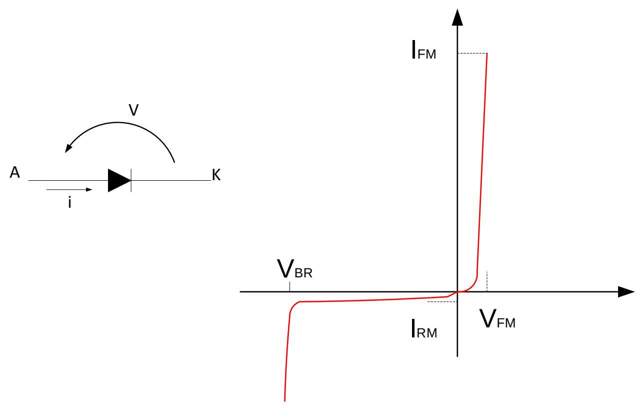
Where D is the duty ratio
2-Switching Loss, P_{sw}
Losses due to presence of significant voltage and current values during the ON-OFF transient. The average of switching losses is given by

$$P_{sw} = f_{sw} \left(\int_0^{t_{on}} pdt + \int_0^{toff} pdt \right)$$



Power Diodes

A 2-layer, 2-terminal semiconductor device, compared to the (basic) rectifying diode there are some differences, like higher current and voltage capabilities and higher ON-state voltage drop.



Schematic diagram | Static characteristics

Diode Ratings

Breakdown voltage, V_{BR}:

- The maximum allowable reverse voltage
- Applying a voltage > V_{BR} sets the diode in the avalanche breakdown region and damage the diode.
- This figure represents the device voltage rating.

Maximum Ratings and Electrical Characteristics (@T_A = +25°C unless otherwise specified.)

Single phase, half wave, 60Hz, resistive or inductive load.
[illegible]

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
Peak Repetitive Reverse Voltage Working Peak Reverse Voltage DC Blocking Voltage	V_{RRM} V_{RWM} V_R	50	100	200	400	600	800	1000	V
[illegible]	$V_{R(RMS)}$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O				1.0				A
Non-Repetitive Peak Forward Surge Current 8.3ms Single Half Sine-Wave Superimposed on Rated Load	I_{FSM}				30				A
Forward Voltage @ I_F = 1.0A	V_{FM}				1.0				V
Peak Reverse Current @T_A = +25°C at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}				5.0 50				µA

Diode Ratings

Full cycle average forward current, I_{FM} :

- Maximum allowable average current for one cycle of the periodic diode current.
- This Figure represents the diode current rating.
- There are other current ratings such as : $I_{F(rms)}$, I_{FSM} (Non-repetitive

Maximum Ratings and Electrical Characteristics (@T_A = +25°C unless otherwise specified.)

Single phase, half wave, 60Hz, resistive or inductive load.
For capacitive load, derate current by 20%.

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
Peak Repetitive Reverse Voltage Working Peak Reverse Voltage DC Blocking Voltage	V_{RRM} V_{RWM} V_R	50	100	200	400	600	800	1000	V
[illegible]	$V_{R(RMS)}$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O	1.0							A
[illegible] Single Half Sine-Wave Superimposed on Rated Load	I_{FSM}	[illegible]							[illegible]
Forward Voltage @ I_F = 1.0A	V_{FM}	1.0							V
Peak Reverse Current @T_A = +25°C at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}	5.0 50							μA

Diode Ratings

Maximum Forward Voltage Drop, V_{FM}:

- The On-state voltage drop when the device current $= I_{FM}$

Maximum Ratings and Electrical Characteristics (@T_A = +25°C unless otherwise specified.)

Single phase, half wave, 60Hz, resistive or inductive load.
For capacitive load, derate current by 20%.

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
Peak Repetitive Reverse Voltage Working Peak Reverse Voltage DC Blocking Voltage	V_{RRM} V_{RWM} V_R	50	100	200	400	600	800	1000	V
RMS Reverse Voltage	$V_{R(RMS)}$	35	70	140	280	420	560	700	V
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O	1.0							A
Non-Repetitive Peak Forward Surge Current 8.3ms [illegible]	I_{FSM}	30							A
Forward Voltage @ I_F = 1.0A	V_{FM}	1.0							V
[illegible] at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}	5.0 50							μA

Diode Ratings

<u>Junction and case temperature, θ_{JM} and θ_{CM}</u>

- Maximum operating temperature and maximum and minimum storage temperatures are specified.
- The maximum temperature is around 150-200°C. The minimum storage temperature around –75 to –50°C.

Maximum Ratings and Electrical Characteristics (@T_A = +25°C unless otherwise specified.)

Single phase, half wave, 60Hz, resistive or inductive load.
For capacitive load, derate current by 20%.

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
Peak Repetitive Reverse Voltage Working Peak Reverse Voltage DC Blocking Voltage	V_{RRM} V_{RWM} V_R	50	100	200	400	600	800	1000	V
RMS Reverse Voltage	$V_{R(RMS)}$	35	70	140	280	420	560	700	V
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O	1.0							A
Non-Repetitive Peak Forward Surge Current 8.3ms Single Half Sine-Wave Superimposed on Rated Load	I_{FSM}	30							A
Forward Voltage @ I_F = 1.0A	V_{FM}	1.0							V
Peak Reverse Current @T_A = +25°C at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}	5.0 50							µA
Typical Junction Capacitance (Note 2)	C_j	15				8			pF
Typical Thermal Resistance Junction to Ambient	$R_{\theta JA}$	100							K/W
Maximum DC Blocking Voltage Temperature	T_A	+150							°C
Operating and Storage Temperature Range	T_J, T_{STG}	-65 to +150							°C

Notes: 1. Leads maintained at ambient temperature at a distance of 9.5mm from the case.

Diode Ratings

<u>Thermal resistance, $R_{\theta JC}$ and $R_{\theta CS}$</u>

- Given in (°C/Watt)
- Specifies how much the junction temperature increases when the loss power is dissipated to the ambient.
- Used in the design stage to determine the specifications of the heatsink (or the need for other means for cooling).

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
RMS Reverse Voltage	$V_{R(RMS)}$	35	70	140	280	420	560	700	V
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O	1.0							A
Non-Repetitive Peak Forward Surge Current 8.3ms Single Half Sine-Wave Superimposed on Rated Load	I_{FSM}	30							A
Forward Voltage @ I_F = 1.0A	V_{FM}	1.0							V
Peak Reverse Current @T_A = +25°C at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}	5.0 50							µA
Typical Junction Capacitance (Note 2)	C_j	15				8			pF
Typical Thermal Resistance Junction to Ambient	$R_{\theta JA}$	100							K/W
Maximum DC Blocking Voltage Temperature	T_A	+150							°C
Operating and Storage Temperature Range	T_J, T_{STG}	-65 to +150							°C

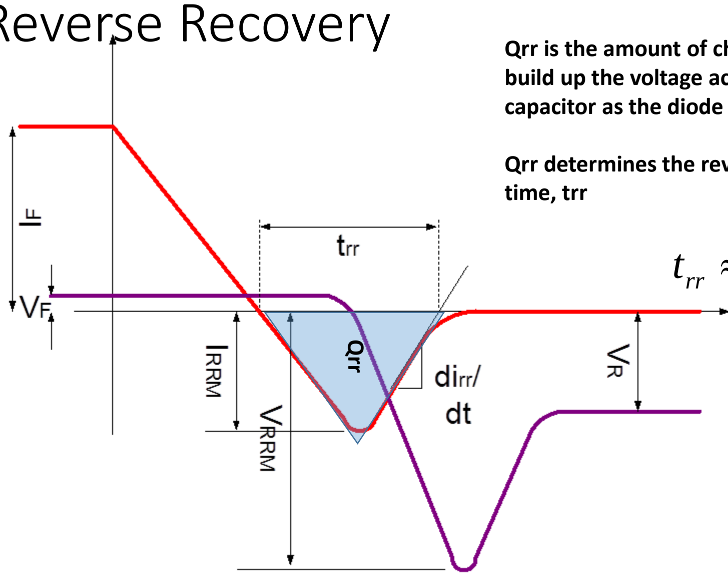
Notes:
1. Leads maintained at ambient temperature at a distance of 9.5mm from the case.
2. Measured at 1.0 MHz and applied reverse voltage of 4.0V DC.
3. EU Directive 2002/95/EC (RoHS). All applicable RoHS exemptions applied, see EU Directive 2002/95/EC Annex Notes.

Diode Ratings

Thermal Capacity, I^2t

- This parameter is needed for fuse selection.
- I^2t of the fuse must be less than that of the device.
- The fuse is important to protect the device against faults.

Diode Reverse Recovery



Qrr is the amount of charge required to build up the voltage across the junction capacitor as the diode turns OFF.

Qrr determines the reverse recovery time, trr

$$t_{rr} \approx \frac{2Q_{rr}}{I_{RRM}}$$

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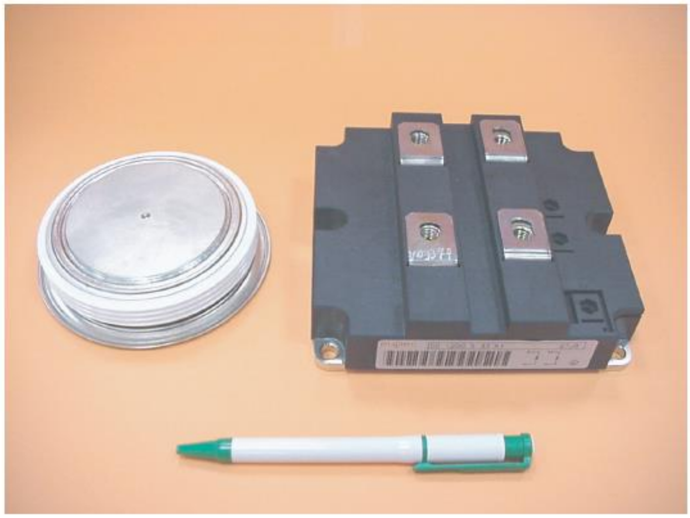


4.5-kV/0.8-kA press-pack and 1.7-kV/1.2-kA module diodes.

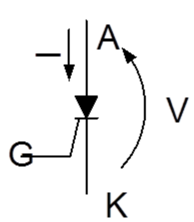
Special Diodes: 1 Fast Reverse Recovery

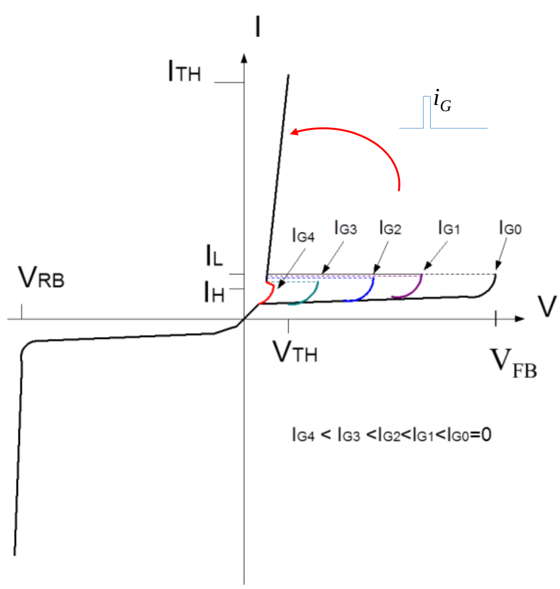
- Required for power electronics circuits that operates at frequencies higher than line frequency.
- For ultra Fast recovery t_{rr} in the order of 100ns, which is about 1000 times shorter than that for line frequency diodes.
- Disadvantage: Higher on-state voltage drop

Special Diodes: 2 Shottky diodes

- Based on metal-semiconductor junction.
- Required for power electronics circuits that operates at low voltage level
- The ON-state forward voltage drop is 0.2~0.4V
- Disadvantage: The maximum breakdown voltage is about 200V

SCR





Normal operating regions:
-Forward and reverse blocking regions
-Conduction region

How the SCR can be turned ON?

1. Through the Gate: When the SCR voltage is positive, A positive gate current pulse with sufficient duration and amplitude turns ON the SCR, If the Anode current (I_T) increased to higher than the device latching current *(I_L)*, while the gate pulse is being applied.
2. If the forward voltage (V_{AK}) acceded the forward blocking voltage of the device (V_{FB})
3. When the OFF-state voltage changes (suddenly) at a rate higher than the device's rated *(dv/dt).*
4. By injecting energy to the gate circuit using other means such as light, the typical way to trigger the LASCR (light activated SCR)
5. High temperature might cause undesirable triggering

How the SCR can be turned OFF?

- For SCR Turn-OFF (***Commutation***), the forward conduction current must be reduced to a value below the holding current (I_h) of the devices for sufficient time.
- This reduction in the current can be done only through the external circuit.
- In AC circuits SCR turns OFF as the currents tends to change its polarity (***natural commutation***)
- In DC circuits, to turn OFF the SCR an additional auxiliary circuit is applied to bring the SCR to zero (***forced commutation***)

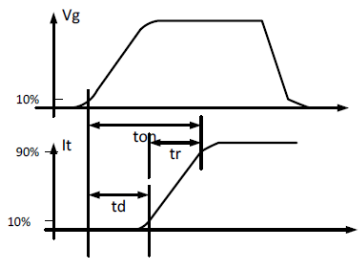
Additional Thyristor Ratings

- *All diode ratings (discussed previously) are applicable for SCR, However SCR has additional rations, the common additional ratings are*
- **Latching current, I_L:** The minimum anode current that will cause the device to remain in forward conduction as the thyristor moves from forward-blocking to forward conduction. The external circuit must allow sufficient anode current (>I_L) to flow to keep the device in conduction region.
- **Holding current, I_H:** If the thyristor is already in forward conduction and the anode current is reduced, the device can move its operating mode from forward-conduction back to forward-blocking. The minimum value of anode current necessary to keep the device in forward-conduction after it has been operating at a high anode current value is called the holding current I_H.

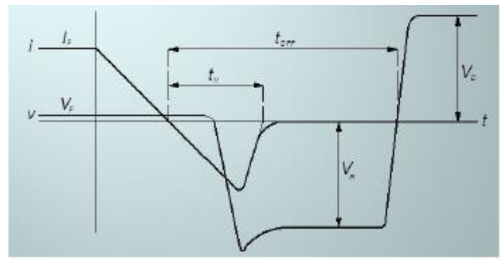
- **Critical *dv/dt* rating:** Critical rate of rise of off-state voltage *dv/dt* : this specifies the maximum rate of rise of off-state voltage that will not drive the device from an off-state to an on-state when an exponential off-state voltage of specified amplitude is applied to the device.
- **Maximum Allowable *di/dt*** *(repetitive, non-repetitive)* Critical rate of Rise of on-state current. At specified case temperature, specified off-state voltage, and specified gate conditions, indicates the maximum rate of rise of on-state current which the thyristor will withstand when switching from an off-state to an on-state, when using recommended gate drive.
- **Gate ratings**
  - I_{GT}**,** Gate current to trigger: At a junction temperature of 25°C, and with a specified off-voltage, and a specified load resistance, indicates the minimum gate dc current required to switch the thyristor from an off-state to an on-state.
  - V_{GD}**,** At a junction temperature of 25°C, and with a specified off-state voltage, indicates the minimum dc gate voltage required to switch the thyristor from an off-state to an on-state.

SCR Dynamics

- Turning On



- Turning OFF

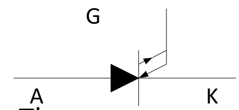


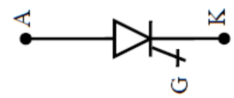
Fully Controlled SCSDs

- GTO
- BJT
- MOSFET
- IGBT
- GCT
- Power Modules

GTO

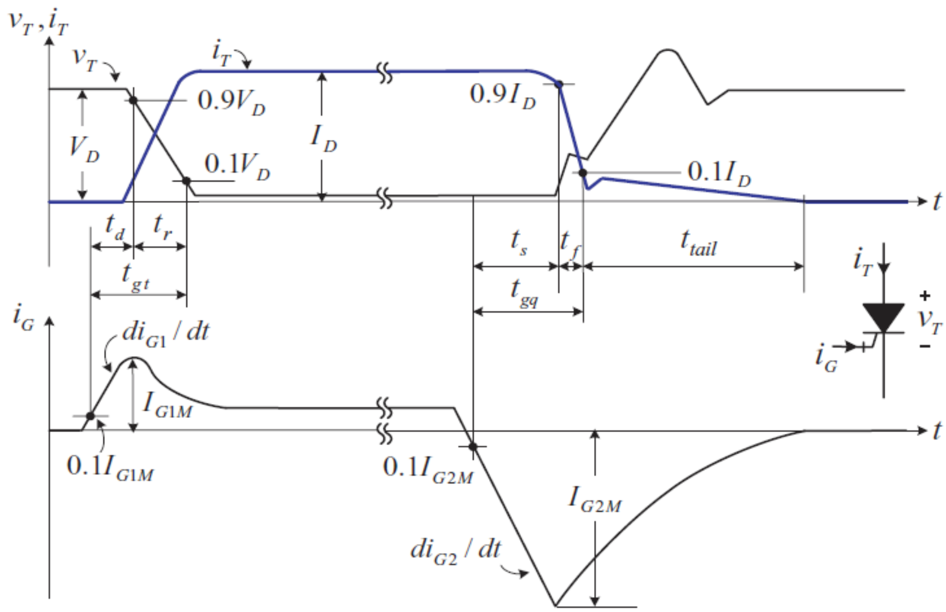
- The Forward (Turn-ON) characteristics are similar to those of the SCR.
- Negative gate is able to turn off the device.
- The gate cathode junction is divided many segments. The most popular design features multiple segments arranged in concentric rings around the device center.
- There are two types symmetrical (V_{RRM}=V_{DRM}) and asymmetrical (V_{RRM}<<V_{DRM}).
- The positive gate pulse required to turn on the device has an amplitude of few hundreds of milliamperes.
- To ensure reliable turn off, the rate of change of negative gate current (d_{iG2}/dt) must meet the device specifications.





GTO Characteristics

- The latching current of a GTO is much higher than a SCR's.
- The forward leakage current is also considerably higher.
- GTO can block rated forward voltage only when the gate is negatively biased with respect to the cathode in forward blocking state.
- Asymmetric GTOs have small (20-30 V) reverse break down voltage.
- Fast turning ON (high dia/dt), we need high (diG/dt) which requires high V_{GK}.
- Fast turn OFF (storage time reduction) requires high negative diG/dt.



V_{DRM}	=	4500	V
I_{TGQM}	=	4000	A
I_{TSM}	=	25×10^3	A
V_{T0}	=	1.2	V
r_T	=	0.65	mΩ
V_{Dclink}	=	2800	V

Asymmetric Gate turn-off Thyristor

5SGF 40L4502

Doc. No. 5SYA1209-04 Feb. 05

- **Patented free-floating silicon technology**
- **Low on-state and switching losses**
- **Annular gate electrode**
- **Industry standard housing**
- **Cosmic radiation withstand rating**

Blocking

Maximum rated values [1)]

Parameter	Symbol	Conditions	min	typ	max	Unit
Repetitive peak off-state voltage	V_{DRM}	$V_{GR} \geq 2$ V			4500	V
Repetitive peak reverse voltage	V_{RRM}				17	V

5SGF 40L4502

GTO Data

On-state

Maximum rated values [1)]

Parameter	Symbol	Conditions	min	typ	max	Unit
Max. average on-state current	$I_{T(AV)M}$	Half sine wave, T_C = 85 °C			1180	A
Max. RMS on-state current	$I_{T(RMS)}$				1850	A
Max. peak non-repetitive surge current	I_{TSM}	t_p = 10 ms, T_{vj} = 125°C, sine wave After Surge: $V_D = V_R$ = 0 V			25×10^3	A
Limiting load integral	I^2t				3.1×10^6	A^2s
Max. peak non-repetitive surge current	I_{TSM}	t_p = 1 ms, T_{vj} = 125°C, sine wave After Surge: $V_D = V_R$ = 0 V			40×10^3	A
Limiting load integral	I^2t				800×10^3	A^2s

Characteristic values

Parameter	Symbol	Conditions	min	typ	max	Unit
On-state voltage	V_T	I_T = 4000 A, T_{vj} = 125°C			3.8	V
Threshold voltage	$V_{(T0)}$	T_{vj} = 125°C I_T = 400...5000 A			1.2	V
Slope resistance	r_T				0.65	mΩ
Holding current	I_H	T_{vj} = 25°C			100	A

Turn-on switching

Maximum rated values [1)]

Parameter	Symbol	Conditions		min	typ	max	Unit
Critical rate of rise of on-state current	di_T/dt_{cr}	T_{vj} = 125°C, I_T = 4000 A, I_{GM} = 50 A, di_G/dt = 40 A/μs	f = 200 Hz			500	A/μs
Critical rate of rise of on-state current	di_T/dt_{cr}		f = 1 Hz			1000	A/μs

Power Bipolar Junction Transistor (BJT)

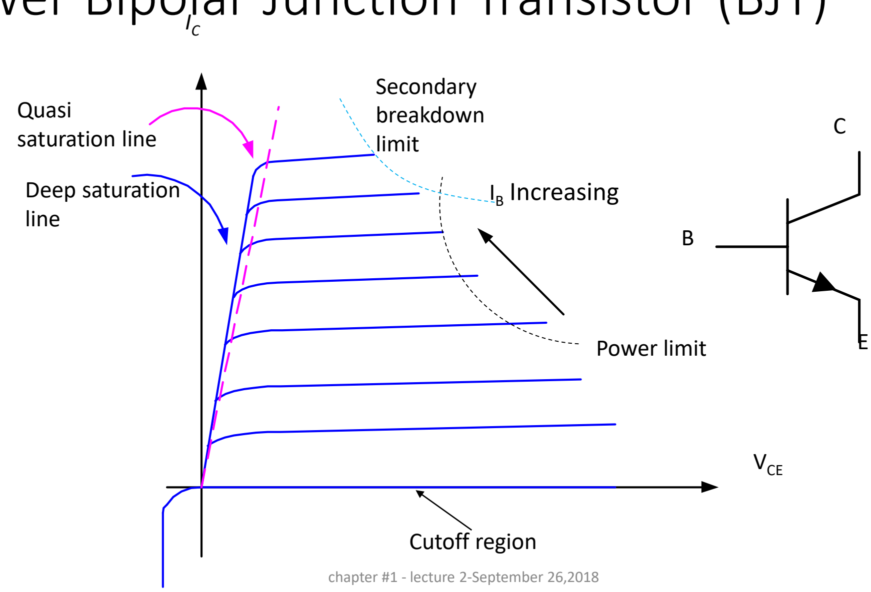
BJT is the first SCSD to allow full control. Subsequently, many other devices (that over perform BJT) classified as "Transistors" have been developed. They have almost completely replaced BJTs.

The construction and operating characteristics of a Power BJT differs significantly from the signal BJT due to the (V and I) ratings requirements.

In the cut off region ($i_B \leq 0$) the collector current is almost zero. The maximum voltage between collector and emitter under this condition is denoted by V_{CEO} (the rated voltage) .

Power transistors have very small reverse voltage withstanding capability.

Power Bipolar Junction Transistor (BJT)



BJT Characteristics

•Current-Controlled Device

•Low current gain compared to signal amplifiers 10<β<20

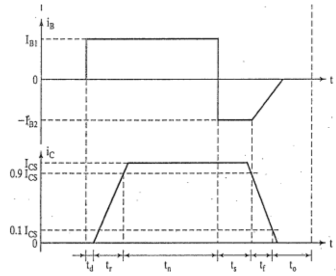
•Moderate switching frequency (up to 10kHz), voltage (up to 1.5kV) and current (up to 1kA) ratings.

•Subjected to secondary breakdown

•The saturation region of BJT can be subdivided into a quasi saturation and a hard saturation regions.

•In the quasi saturation region, the resistivity depends to some extent on the base current. The total voltage drop across the device in this mode of operation is higher for a given collector current compared to hard saturation region.

BJT switching characteristics (turn ON)



- The delay time (t_d) is determined by the collector current and collector voltage during switching, both determined by external circuits.
- The rise time (t_r) is determined by the base current waveform.
- The switching characteristics of BJT is specified in relation to the external load circuit and the base current waveform.

BJT Turn OFF

- By applying negative V_{BE} , negative base current starts removing carriers during the storage time (ts).
- After a further time –the fall time- “t_f” the BJT completes traversing through the quasi saturation region and enters the active region towards cutoff. Turn OFF process of the transistor ends at this point.
- Turn OFF time intervals of BJT are strongly influenced by the operating conditions and the base drive design. Manufacturers usually specify these values as functions of collector current for given positive and negative base current and case temperatures.

BJT requirements

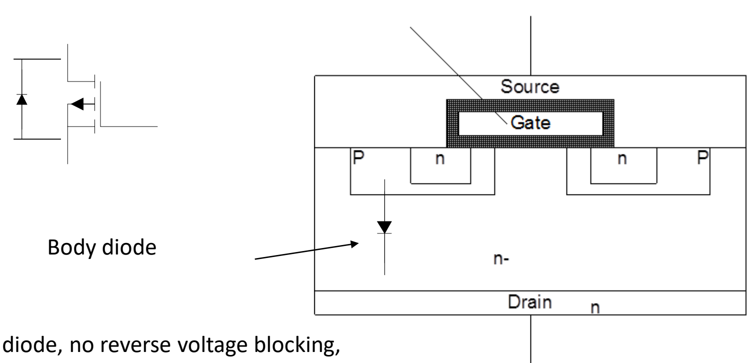
For fast switching:
- Avoid deep saturation
- Apply negative base current pulse to switch-off

To minimize leakage current:
Apply small negative V_{BE} during Off interval

Due to small β:
BJT always used as a Darligton pair or triple.

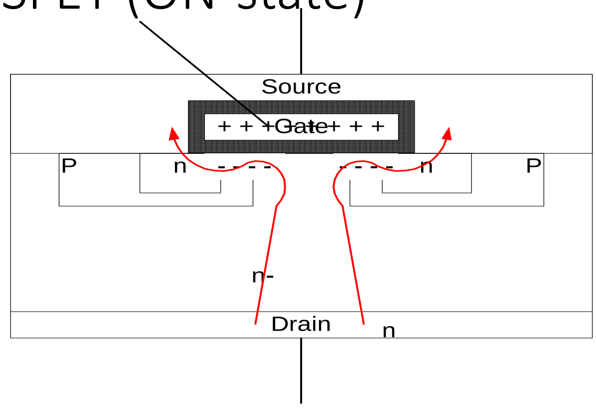
Due to its limitations:
BJT is rarely used nowadays and almost completely replaced by more advanced "Transistors"

Power MOSFET



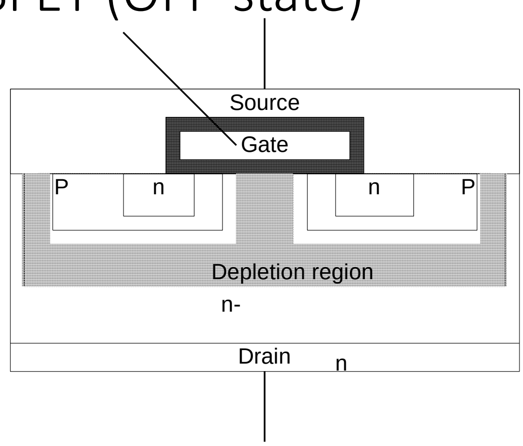
Due to body diode, no reverse voltage blocking, therefore the device is suitable for DC-supplied converters

Power MOSFET (ON-state)



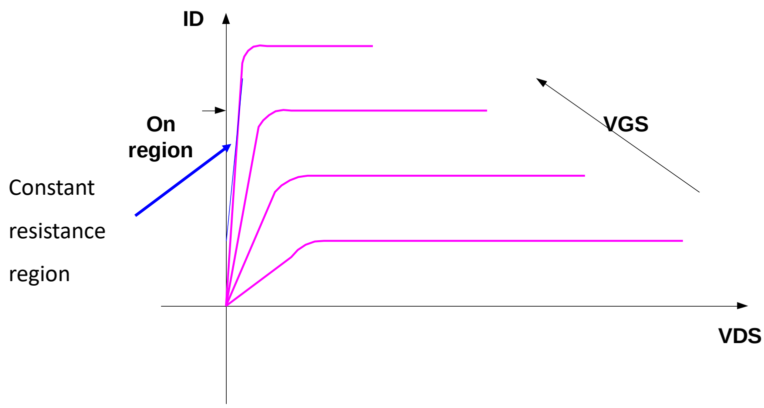
- Positive gate voltage induces conducting channel
- Drain Current Flows through n^- region and conducting channel.

Power MOSFET (OFF-state)



- Pn junction is reverse-biased
- off-state voltage appears across n^- region

Power MOSFET Characteristics



Power MOSFET

Compared to BJT,

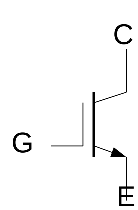
- power MOSFET does not undergo second break down.
- The primary break down voltage of a MOSFET remains same in the cut off and in the active modes.
- The ON state resistance of a MOSFET in the Ohmic region has positive temperature coefficient which allows paralleling of MOSFET without any special arrangement for current sharing. On the other hand, BJT has negative temperature coefficient making parallel connection of BJTs very complicated.

MOSFET Characteristics

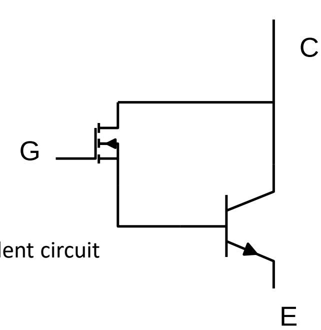
- High switching speed; the highest among all other devices.
- Voltage controlled; low control power.
- Simple control circuit requirements; no deep saturation and zero voltage is required for off-state.
- Negative thermal coefficient leads to equal current division among parallel-connected devices.
- Higher Conduction Losses than the BJT.
- Available with small-to-small medium power ratings (100s of Amps and <1000V)

IGBT

Hybrid Device that combines the gate advantages of the MOSFET and the conduction and high ratings advantages of the BJT



Schematic symbol

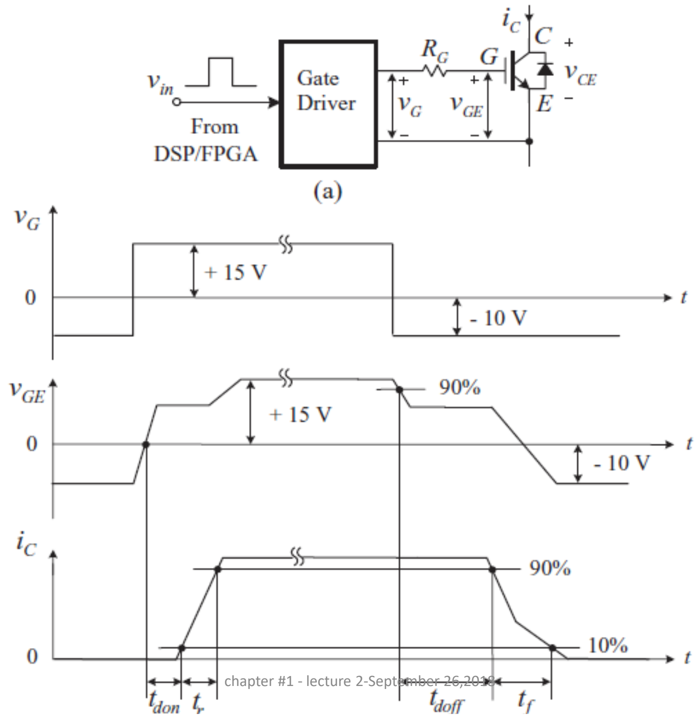


Equivalent circuit

IGBT characteristics

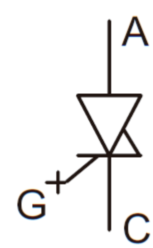
- The available ratings are up to 6.5kV, and 3.6kA (different devices)
- Typical switching frequencies 3-30kHz
- Slower than MOSFET but faster than BJT, GTO, SCR.
- Easy to drive; like MOSFET.
- Conduction losses less than MOSFET and higher than BJT and CGT.

1.7-kV/1.2-kA and 3.3-kV/1.2-kA IGBT modules.



(GCT/IGCT) Gate-Commutated Thyristor

- A Hybrid-Driven GTO
- Gate driver is build-in to the Module
- Voltage blocking is symmetric, asymmetric or reverse conducting
- Conduction voltage drop < IGBT
- Switching frequency in kHz
- No snubber needed



6.5-kV/1.5-kA symmetrical GCT.

Type	Antiparallel Diode	Blocking Voltage	Example (6000V GCT)	Applications
Asymmetrical GCT	Excluded	$V_{RRM} \ll V_{DRM}$	$V_{DRM} = 6000$ V $V_{RRM} = 22$ V	For use in voltage source converters with antiparallel diodes.
Reverse-conducting GCT	Included	$V_{RRM} \approx 0$	$V_{DRM} = 6000$ V	For use in voltage source converters.
Symmetrical GCT (reverse blocking)	Not required	$V_{RRM} \approx V_{DRM}$	$V_{DRM} = 6000$ V $V_{RRM} = 6500$ V	For use in current source converters.

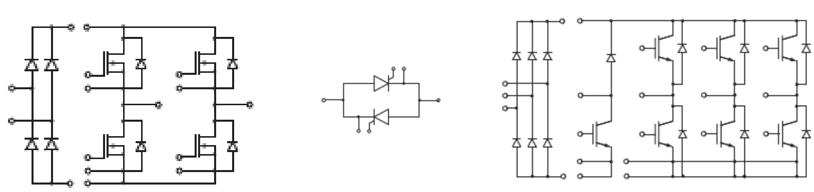
Maximum Rating	V_{DRM}	V_{RRM}	I_{TQRM}	I_{TAVM}	I_{TRMS}	—
	6000 V	22 V	6000 A	2000 A	3100 A	—
Switching Characteristics	**Turn-on Switching**	**Turn-off Switching**	di_T/dt	dv_T/dt	di_{G1}/dt	di_{G2}/dt
	$t_d < 1.0\ \mu s$ $t_r < 2.0\ \mu s$	$t_s < 3.0\ \mu s$ t_f – N/A	1000 A/μs	3000 V/μs	200 A/μs	10,000 A/μs
On-state Voltage	$V_{T(on\text{-}state)} < 4$ V at $I_T = 6000$ A					
Part number: FGC6000AX120DS (Mitsubishi)						

Parameters considered in switching device selection

- Voltage/current rating.
- Required switching frequency.
- Conduction and OFF-state losses.
- Switching losses.
- Gate requirements.
- Negative temperature coefficient: when parallel operation is needed.
- dv/dt and di/dt capabilities
- low price

Power Modules

- A power module is a set of semiconductor power switches interconnected into a specific topology and enclosed in a single case.
- Most popular topologies are the single- and three-phase bridges, or their sub-circuits and several switches of the same type connected in series, parallel, or series-parallel, to increase the overall voltage and/or current ratings.
- In intelligent power modules (IPMs), power components are accompanied by protection circuits and gate drives.



Chapter 2

Power Semiconductor Switches

Lecture Aims:

1- **To furnish a comparison between ideal and semiconductor switching devices.**
2- **To formulate the equations of power losses in switching devices**
3- **To study the static and dynamic characteristics of power diodes and define the diode ratings.**
4- **To highlight salient features of the most common special power diodes.**
5- **To introduce the SCR construction and principle of operation using two transistor model.**
6- **To study the static and dynamic characteristics of SCR and present the SCR ratings.**
7- **To describe the turn ON and Turn OFF process of SCR.**
8- **To present the LASCR and TRAIC compared to SCR.**

1-Comparison between ideal and semiconductor switching devices

Ideal Switch	Semiconductor Switch
Conduction (ON-state) resistance = 0 Ω Or zero on-state voltage drop	At Conduction (ON)-state there is a voltage drop across the switch. (Typically 1~3Volts)
Blocking (OFF-state) resistance = $\infty\Omega$ Or zero Off-state leakage current.	At Blocking (OFF)-state there is leakage current, normally very small but highly dependent on temperature.
When conducting (ON) conducts current in both directions	Basically it conducts the current in one direction only
When OFF blocks positive and negative voltage	Some types block bidirectional voltage, others block the voltage in one direction only.
Instantaneous switching, or zero switching time.	Switching time is not zero and varies largely between different types/ratings of devices.
Fully Controlled	There are 3 types: Uncontrolled, Semi-controlled and Fully-Controlled
Zero control power	Needs a driving power for control signal

2- Power losses in semiconductor switches

The nonzero on-state voltage drop and leakage current, and the non-zero turn ON and turn OFF time result in power losses in semiconductor switching device as shown in Fig. 1

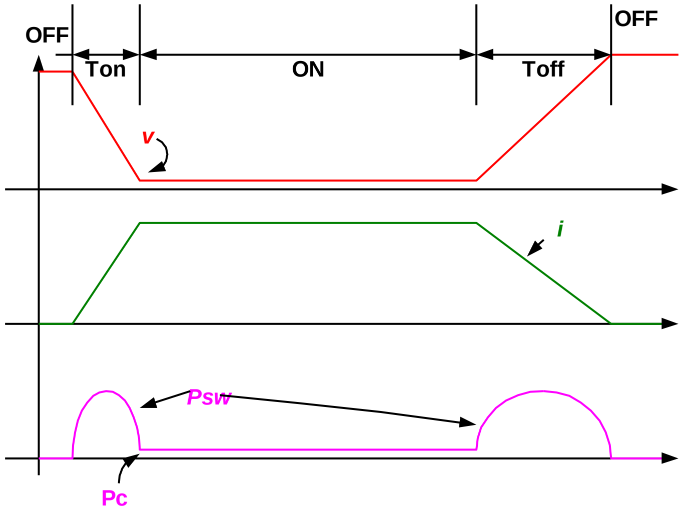


Fig. 1 Voltage, current and dissipated power in semiconductor switch

There are two types of losses in the semiconductor switch:

1-Conduction Loss, P_c

Losses due to voltage drop during on state, the average of the conduction power losses is given by:

$$Pc = \frac{1}{T_{sw}} \int_{0}^{DT} (v_{ON} i) dt$$

Where D is the duty ratio

2-Switching Loss, P_{sw}

Losses due to presence of significant voltage and current values during the ON-OFF transient. The average of switching losses is given by

$$P_{sw} = f_{sw} \left(\int_{0}^{t_{on}} p dt + \int_{0}^{toff} p dt \right)$$

The OFF-state leakage current, and therefore, power losses are usually negligible.

The total switch losses are the SUM of the P_C+P_{sw}

$$P_s = f_{sw}\left(\int_0^{DT} v_{ON} i dt + \int_0^{ton} pdt + \int_0^{toff} pdt\right)$$

Example 1: Calculate the switching losses and conduction losses in a semiconductor switching device considering the following data:

Switching frequency: 500 kHz, duty ratio: 0.4, On-state voltage drop: 2V , On-state current : 100A , off-state voltage: 300V, turn –on time: 1 nsec, turn-off time: 3nsec

Answer:

Conduction losses, $P_{con} = \frac{1}{T_{sw}}\int_{Ton} v_{on} i\, dt = f_{sw} d T_{sw} v_{on} I = 0.4 * 2 * 100 = 80W$

Switching losses,$P_{sw} = \int_0^{ton} ivdt + \int_0^{toff} ivdt =$

$$= 50 * 10^3\left[\int_0^{10^{-9}} (10^{11}t)(300 - 3 * 10^{11}t)dt + \int_0^{3*10^{-9}} (100 - 3.3 * 10^{10}t)(10^{11}t)dt\right]$$

$$= 500 * 10^3[0.5 * 10^{-5} + 4.2 * 10^{-5}] = 23.5W$$

Total switch losses=80+23.5=103.5W

3-Power Diodes

A 2-layer, 2-terminal semiconductor device, compared to the (basic) rectifying diode there are some differences, like higher current and voltage capabilities and higher ON-state voltage drop. The schematic diagram and static characteristics of the diode are given in Fig. 2.

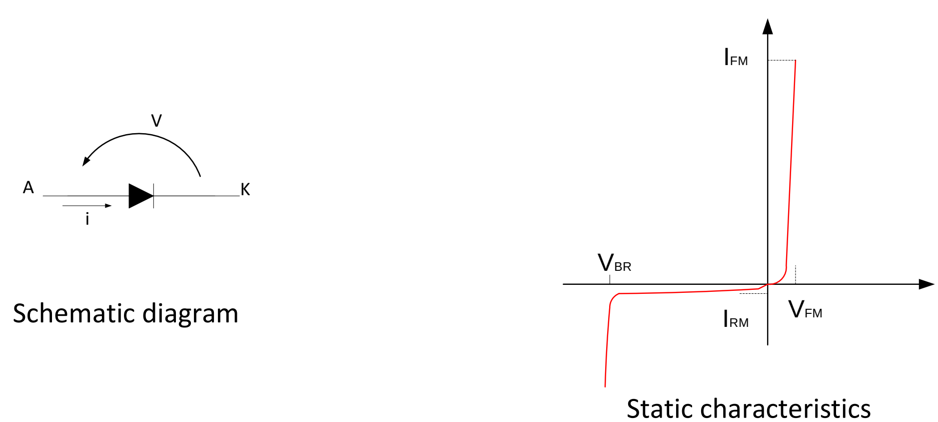


Fig. 2

Diode Static Ratings:

Ratings of the switching devices describe the numerical limits of the device withstand.

Breakdown voltage, V_{BR}:

- The maximum allowable reverse voltage
- Applying a voltage > V_{BR} sets the diode in the avalanche breakdown region and damage the diode.
- This figure represents the device voltage rating.
- The manufacturers specify other voltage ratings in the data sheet as shown in Fig. 3.

Full cycle average forward current, I_{FM} :

- Maximum allowable average current for one cycle of the periodic diode current.
- This Figure represents the diode current rating.
- There are other current ratings such as : $I_{F(rms)}$, I_{FSM} (Non-repetitive peak surge current).

Maximim Forward Voltage Drop, V_{FM}:

- The On-state voltage drop when the device current $= I_{FM}$

Junction and case temperature, θ_{JM} and θ_{CM}

- Maximum operating temperature and maximum and minimum storage temperatures are specified.
- The maximum temperature is around 150-200°C. The minimum storage temperature around –75 to –50°C.

Maximum Ratings and Electrical Characteristics (@T_A = +25°C unless otherwise specified.)

Single phase, half wave, 60Hz, resistive or inductive load.
For capacitive load, derate current by 20%.

Characteristic	Symbol	1N4001	1N4002	1N4003	1N4004	1N4005	1N4006	1N4007	Unit
Peak Repetitive Reverse Voltage Working Peak Reverse Voltage DC Blocking Voltage	V_{RRM} V_{RWM} V_R	50	100	200	400	600	800	1000	V
RMS Reverse Voltage	$V_{R(RMS)}$	35	70	140	280	420	560	700	V
Average Rectified Output Current (Note 1) @ T_A =+75°C	I_O	1.0							A
Non-Repetitive Peak Forward Surge Current 8.3ms Single Half Sine-Wave Superimposed on Rated Load	I_{FSM}	30							A
Forward Voltage @ I_F = 1.0A	V_{FM}	1.0							V
Peak Reverse Current @T_A = +25°C at Rated DC Blocking Voltage @ T_A = +100°C	I_{RM}	5.0 50							μA
Typical Junction Capacitance (Note 2)	C_j	15				8			pF
Typical Thermal Resistance Junction to Ambient	$R_{\theta JA}$	100							K/W
Maximum DC Blocking Voltage Temperature	T_A	+150							°C
Operating and Storage Temperature Range	T_J, T_{STG}	-65 to +150							°C

Notes:
1. Leads maintained at ambient temperature at a distance of 9.5mm from the case.
2. Measured at 1.0 MHz and applied reverse voltage of 4.0V DC.
3. EU Directive 2002/95/EC (RoHS). All applicable RoHS exemptions applied, see EU Directive 2002/95/EC Annex Notes.

Fig. 3 Description of static characteristics in Data sheet

Thermal resistance, $R_{\theta JC}$ and $R_{\theta CS}$

- Given in (°C/Watt)
- Specifies how much the junction temperature increases when the loss power is dissipated to the ambient.
- Used in the design stage to determine the specifications of the heatsink (or the need for other means for cooling).

Thermal Capacity, I^2t

- This parameter is needed for fuse selection.
- I^2t of the fuse must be less than that of the device.
- The fuse is important to protect the device against faults.

Diode Dynamic Characteristics:

Reverse recovery: When a conducting diode becomes suddenly reverse biased, the device does not regain its reverse-blocking capability until the reverse recovery charge, Qrr, is removed from the junction capacitance. To discharge that capacitance, for a short time the diode passes a high current in the reverse direction. This phenomenon is illustrated in Fig.4 which shows the voltage and current waveforms when a reverse voltage, VR, is applied to a conducting diode at t = 0. The reverse recovery time, trr, is also shown.

Within this period, a negative current overshoot, I_{rrM}, occurs, closely followed by a voltage overshoot, V_{RM}. The latter overshoot is proportional to the slope, dirr/dt, of the current tail, that is, the decaying portion of the current waveform.

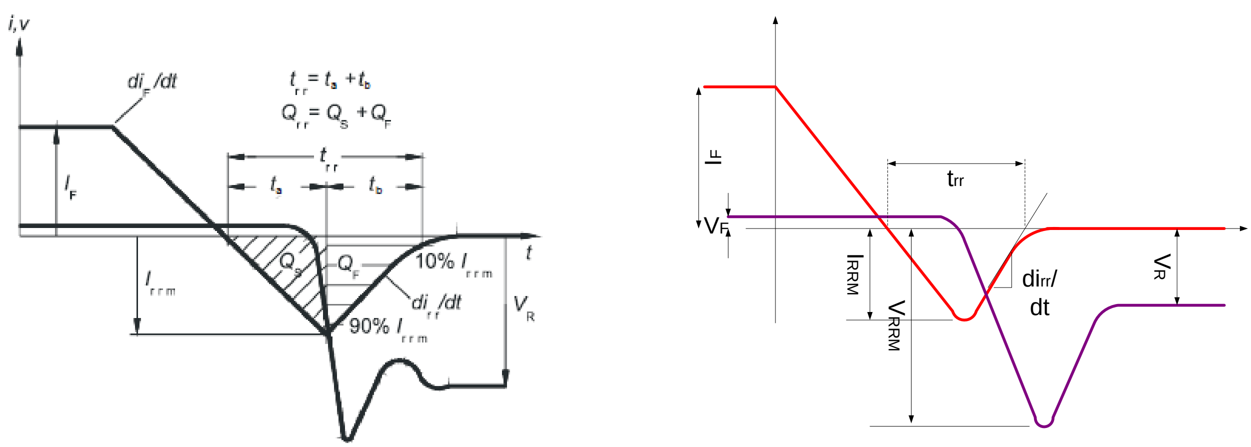


Fig. 4 diode reverse recovery

- When conducting diode goes to blocking region, it passes considerable amount of current in the reverse region for short time before it becomes fully blocking.
- The reverse recovery time, t_{rr} is the time taken by the diode to recover its blocking capability. Or the time interval between the instant the declining current passes through zero and the moment the reverse current has decayed to 25% of its peak reverse value.
- t_{rr} depends on di_F/dt and junction temperature.
- Reverse recovery charge, Q_{rr}: the amount of charge carries that flows across the diode in the reverse direction.
- $Q_{rr} \cong \frac{1}{2} I_{rr} t_{22}$

A datasheet shows dynamic parameters is given in Fig.5

Switching Characteristic, Inductive Load

Parameter	Symbol	Conditions	Value			Unit
			min.	typ.	max.	
Diode Characteristic, at T_{vj} = 25°C						
Diode reverse recovery time	t_{rr}	T_{vj} = 25°C, V_R = 400V, I_F = 15.0A, di_F/dt = 1000A/µs, $L\sigma$ = 35nH, $C\sigma$ = 32pF, switch IPW60R045CP	-	30	-	ns
Diode reverse recovery charge	Q_{rr}		-	0.20	-	µC
Diode peak reverse recovery current	I_{rrm}		-	12.8	-	A
Diode peak rate of fall of reverse recovery current during t_b	di_{rr}/dt		-	-6500	-	A/µs
Diode reverse recovery time	t_{rr}	T_{vj} = 25°C, V_R = 400V, I_F = 15.0A, di_F/dt = 200A/µs, $L\sigma$ = 35nH, $C\sigma$ = 32pF, switch IPW60R045CP	-	47	-	ns
Diode reverse recovery charge	Q_{rr}		-	0.12	-	µC
Diode peak reverse recovery current	I_{rrm}		-	3.3	-	A
Diode peak rate of fall of reverse recovery current during t_b	di_{rr}/dt		-	-1500	-	A/µs

Fig. 5

4- Special Types of Power Diode

<u>Fast Recovery diodes :</u>

- **Required for power electronics circuits that operates at frequencies higher than line frequency.**
- **For ultra Fast recovery t_{rr} in the order of 10s of nano seconds, which is about 1000 times shorter than that for line frequency diodes.**
- **Disadvantage: Higher on-state voltage drop.**

Shottky diode:

- **Based on metal-semiconductor junction.**
- **Required for power electronics circuits that operates at low voltage level**
- **The ON-state forward voltage drop is 0.2~0.4V.**
- **Available with current ratings up to several hundreds of Amperes**
- **Disadvantage: The maximum breakdown voltage is about 200V.**

6-The Silicon Controlled Rectifier, SCR

The SCR is a member of the thyristor family. Where the thyristor family includes devices composed of 4-layers p-n-p-n, as shown in Fig. 6

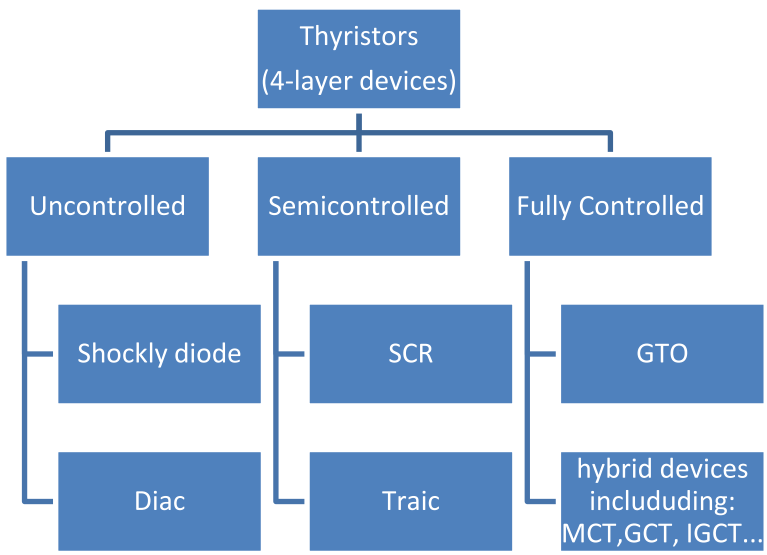


Fig. 6

- The anode side is the p-layer side of the device and the cathode side is the n-layer of the device (Fig. 7). The gate is connected to the p-layer next to the cathode.
- The main device current passes from anode-to-cathode, while the gate terminal is intended to be a control terminal. The gate current circulates through the cathode.

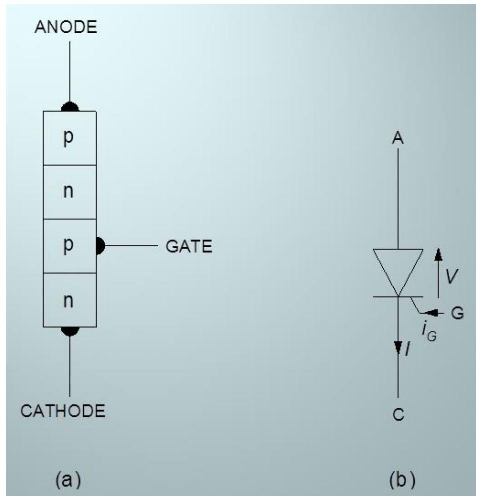


Fig. 7

SCR static characteristics

The SCR has three operation regions: forward conduction, forward blocking reverse blocking and breakdown. (Fig. 8)

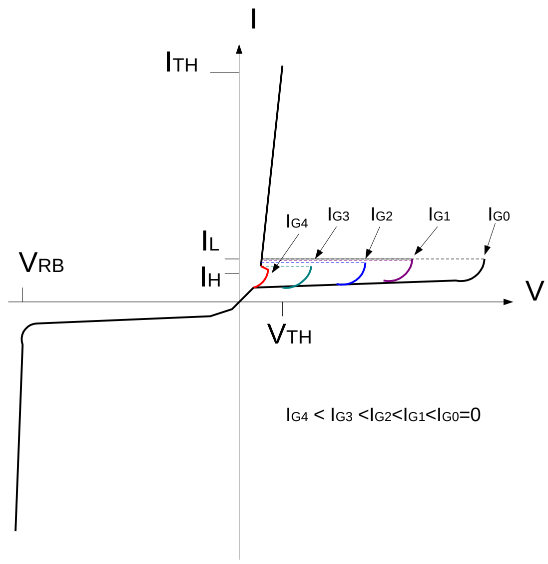


Fig. 8

What makes the SCR turns ON with a gate pulse? 2-transistor model explanation is given in Fig. 9

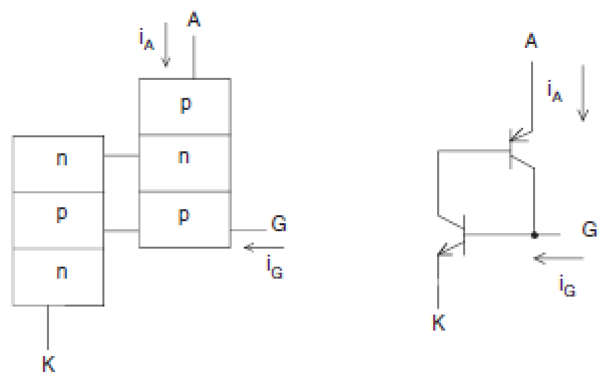


Fig. 9

How the SCR can be turned ON?

1. Through the Gate: When the SCR voltage is positive, A positive gate current pulse with sufficient duration and amplitude turns ON the SCR, If the Anode current (I_T) increased to higher than the device latching current *(I_L),* while the gate pulse is being applied.
2. If the forward voltage (V_{AK}) acceded the forward blocking voltage of the device (V_{FB})
3. When the OFF-state voltage changes (suddenly) at a rate higher than the device's rated *(dv/dt).*
4. By injecting energy to the gate circuit using other means such as light, the typical way to trigger the LASCR (light activated SCR)
5. High temperature might cause undesirable triggering

How the SCR can be turned OFF?

- For SCR Turn-OFF (***Commutation***), the forward conduction current must be reduced to a value below the holding current (I_h) of the devices for sufficient time.
- This reduction in the current can be done only through the external circuit.
- In AC circuits SCR turns OFF as the currents tends to change its polarity (***natural commutation***)
- In DC circuits, to turn OFF the SCR an additional auxiliary circuit is applied to bring the SCR to zero (***forced commutation***)

Additional Thyristor Ratings

All diode ratings (discussed previously) are applicable for SCR, However SCR has additional rations, the common additional ratings are

- **Latching current, I_L:** The minimum anode current that will cause the device to remain in forward conduction as the thyristor moves from forward-blocking to forward conduction. The external circuit must allow sufficient anode current (>I_L) to flow to keep the device in conduction region.
- **Holding current, I_H:** If the thyristor is already in forward conduction and the anode current is reduced, the device can move its operating mode from forward-conduction back to forward-blocking. The minimum value of anode current necessary to keep the device in forward-conduction after it has been operating at a high anode current value is called the holding current *IH*.
- **Critical *dv/dt* rating:** Critical rate of rise of off-state voltage *dv/dt* : this specifies the maximum rate of rise of off-state voltage that will not drive the device from an off-state to an on-state when an exponential off-state voltage of specified amplitude is applied to the device.
- **Maximum Allowable *di/dt*** *(repetitive, non-repetitive)* Critical rate of Rise of on-state current. At specified case temperature, specified off-state voltage, and specified gate conditions, indicates the maximum rate of rise of on-state current which the thyristor will withstand when switching from an off-state to an on-state, when using recommended gate drive.
- **Gate ratings**
    - I_{GT}**,** Gate current to trigger: At a junction temperature of 25°C, and with a specified off-voltage, and a specified load resistance, indicates the minimum gate dc current required to switch the thyristor from an off-state to an on-state.
    - V_{GD}**,** At a junction temperature of 25°C, and with a specified off-state voltage, indicates the minimum dc gate voltage required to switch the thyristor from an off-state to an on-state.

SCR Dynamics

Turning On and turning off are shown in Fig 10 and 11.

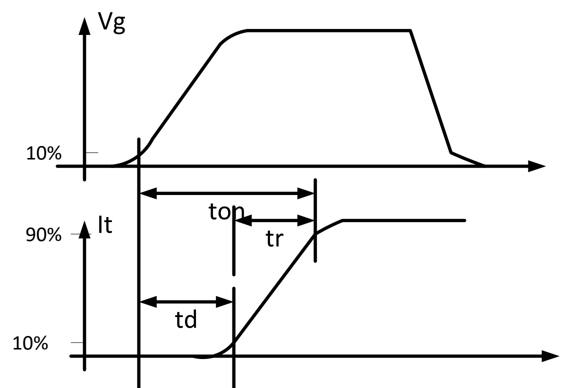


Fig. 10 SCR Turn ON

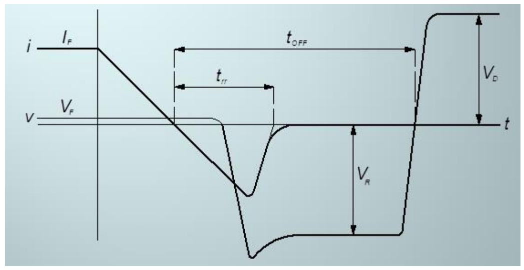
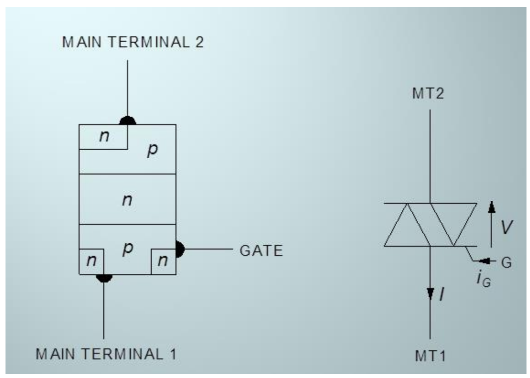


Fig. 11 SCR Turn OFF

<u>The Triac</u>



Chapter 3

AC-to-DC Converters

1- Single-Phase Full-Bridge Controlled Rectifier

Lecture aims:

1- To present the circuit diagram and operation of full-bridge rectifier.
2- To analyze the full bridge rectifier circuit considering a highly inductive load.
3- To derive the performance indicators of the converter circuit with highly inductive load.
4- To compare the operation of the converter supplying R-load and highly inductive load.

1- Circuit Diagram, Control Signals and States

The single-phase full converter has four SCRs connected in the form of bridge as shown in Fig. (1). This circuit is also known as:

- Single-phase full converter,
- 2-pulse converter,
- Single-phase fully-controlled rectifier, or
- Phase-controlled single-phase rectifier.

Assuming the supply voltage has the form:

$$v_s = V_p \sin(\omega t) \tag{1}$$

Where V_p and ω are the supply peak voltage and radian frequency respectively.

Thyristors T1 and T2 (Fig.1) are triggered at the same instant at ($\omega t = \alpha_f$). SCRs T3 and T4 triggered at $\omega t=\alpha_f+\pi$. The load voltage and current are denoted by v_o *and* i_o respectively. Other assumptions made in the following analysis include:

- The SCRs on state voltage and leakage current are zeros.
- The switching times: the turn-on time and the turn-off time are both zeros.

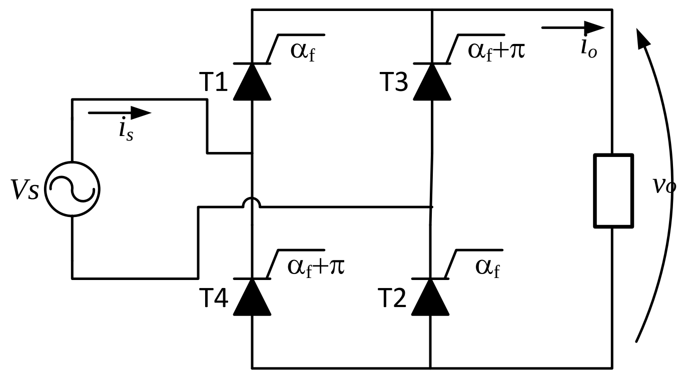
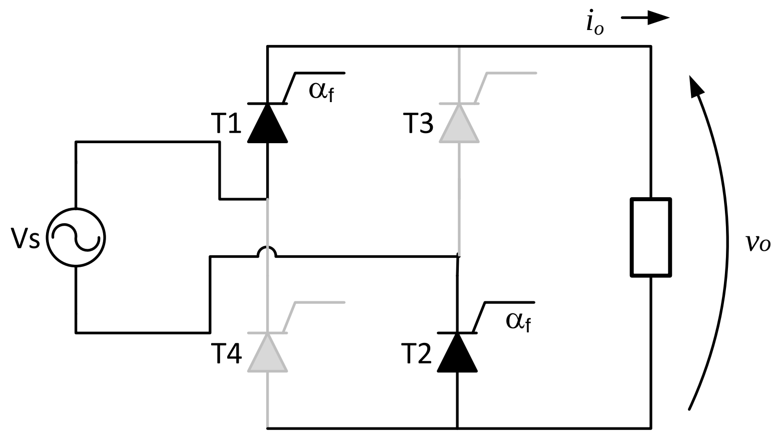


Fig. 1 The fully controlled single phase rectifier.

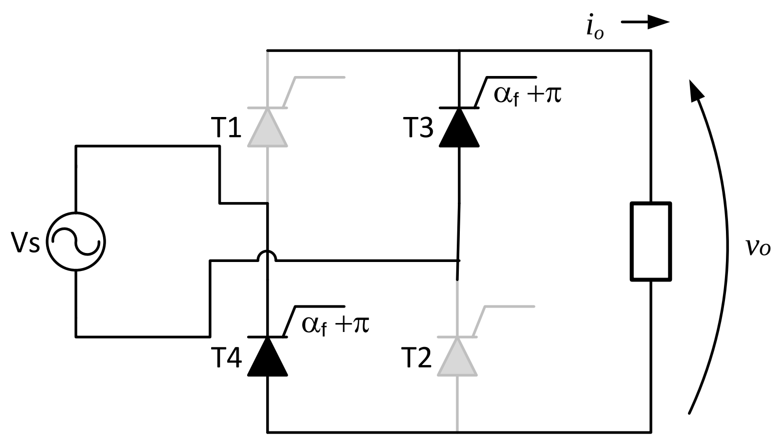
The Firing angle, α_f:

- The angle at which thyristors T1 and T2 are triggered by applying the gate pulse is known as the *triggering* or *firing* angle and denoted by α_f
- This angle is synchronized with the input AC voltage and measured on ωt axis.
- Thyristors T3 and T4 are triggered at angle ($\alpha_f + \pi$), that is 180° after triggering T1 and T2. , the difference of π between the two firing angles is important to ensure symmetry at DC side and AC side as follows:
    - At DC side: To make sure that the output voltage for the interval $\pi < \omega t < 2\pi$ is identical to that of interval $0 < \omega t < \pi$.
    - At AC side: To make sure that AC supply current i_s is half wave symmetrical to eliminate the DC component in AC supply current.
- When α_f=0 the SCR act as a diode.
- The SCR can be triggered as long as v_s positive and therefore ($0 < \alpha_f < \pi$)

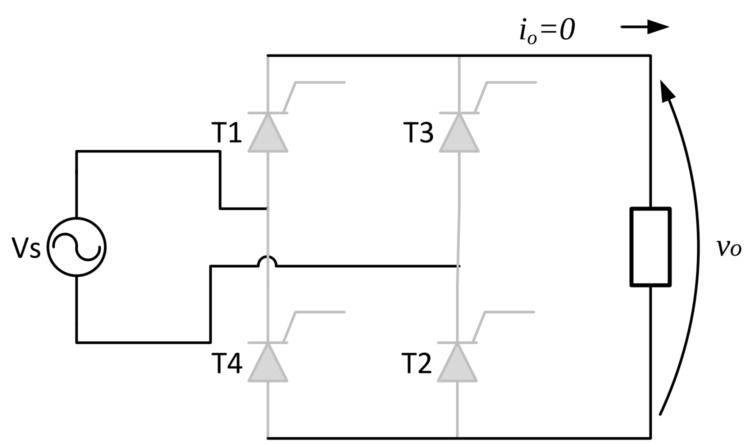
When T1 and T2 are ON; the equivalent circuit is shown in Fig. 2-a. When T3 and T4 are ON, the equivalent circuit is shown in Fig. 2-b. If the load current reduces to zero, the SCRs will turn-OFF and the equivalent circuit in this case is shown in Fig. 2-c. The three states are summarized in Table-1.



(a) State 1: T1 and T2 Conducting



(b) State 2: T3 and T4 Conducting



(c) State 0 the four thyristors are OFF

Fig. 2 The three states of the full converter

Table the states of the full converter circuit as shown in Fig. 2				
Fig	T1, T2	T3, T4	v_o	i_i
2-a	ON	OFF	v_i	i_o
2 b	OFF	ON	$-v_i$	$-i_o$
2-c	OFF	OFF	-	0

2- Analysis of Full Converter with Constant Load Current

In this section we will consider a highly inductive load. High inductance reduces the current ripple. We will assume the idealized case of pure DC load current; therefore the load is drawn as a DC current source as shown in Fig. 3. The analysis of this circuit includes:

- determination of $v_o(\alpha_f)$ and hence ($V_{o,dc}$, V_o, RF_{Vo})
- determination of $i_s(\alpha_f)$ and hence the supply power factor.

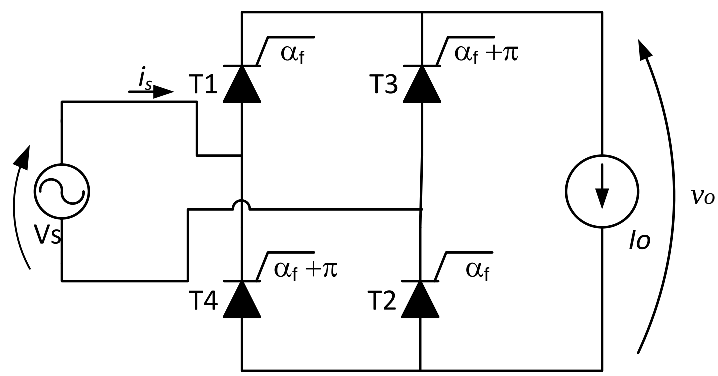


Fig. 3 single phase full converter with constant current load

The continuous load current assures that at any time there are two SCRs ON to carry load current. The conducting SCR are as follows:

T1 and T2 conduct for $\alpha_f < \omega t < \alpha_f + \pi$: the supply is directly connected to the load makes $v_o = v_s$ and $i_s = I_o$ (equivalent to State 1 of the generic converter)

T3 and T4 conduct for $\alpha_f + \pi < \omega t < \alpha_f + 2\pi$: the supply is cross connected to the load makes $v_o = -v_s$ and $i_s = -I_o$. (equivalent to State 2 of the generic converter)

From that the waveforms of the load voltage and input current can be drawn as show in Fig. 4.

DC Side Voltage Calculations:

$$V_{o,dc} = \frac{1}{\pi} \int_{\alpha_f}^{\alpha_f+\pi} V_p sim(\omega t) d\omega t$$

$$\boldsymbol{V_{o,dc} = \frac{2}{\pi} V_p cos\alpha_f} \qquad ...(2)$$

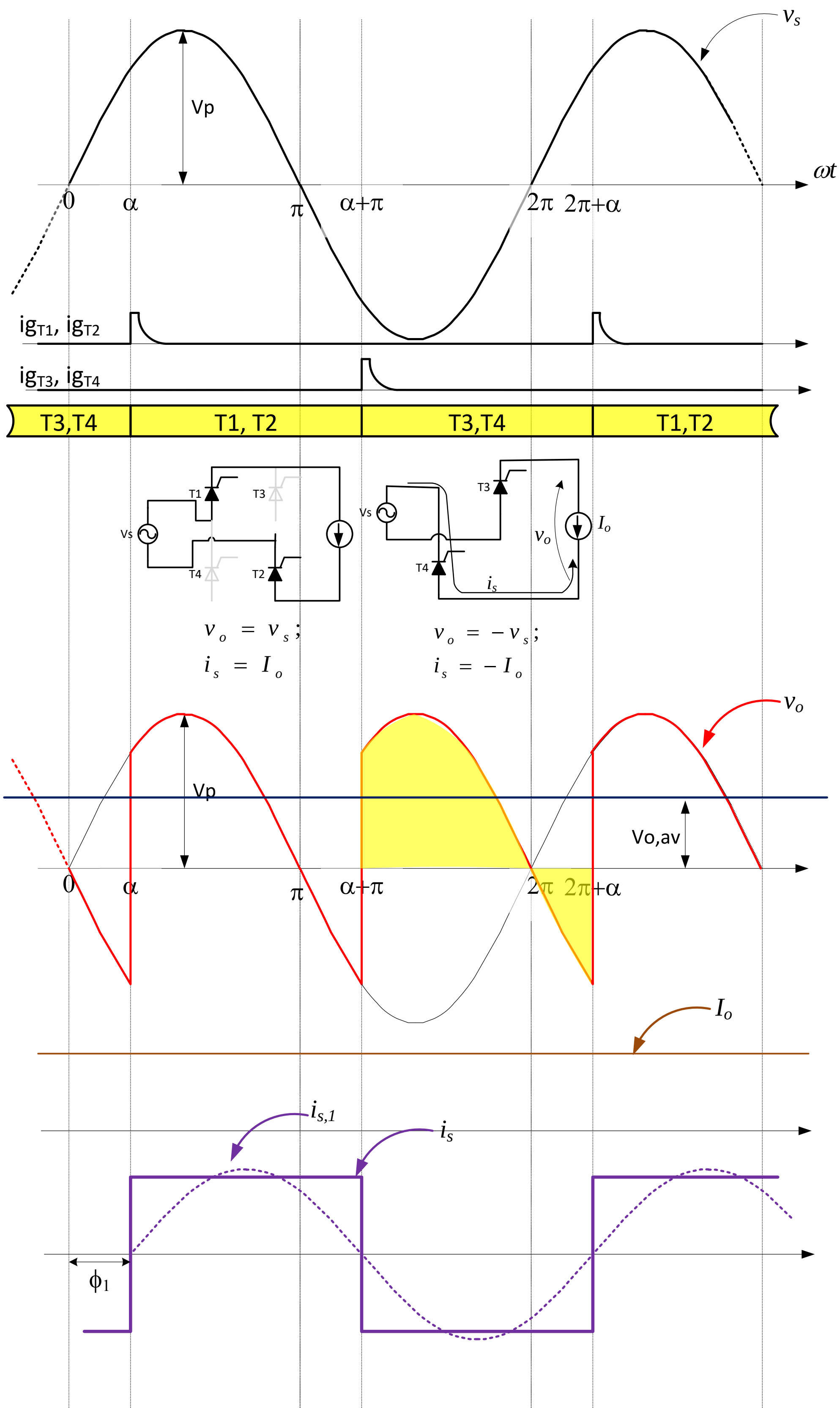


Fig. 4 waveforms of voltages and currents of converter

The variation of the DC output voltage against α_f is shown in Fig. 5.

Notice that the average voltage becomes negative for $\alpha_f > \pi/2$ (Fig. 5). The reason is that the negative region is greater than the positive region under the voltage curve. In this case the average load power becomes negative and this means that the power is being transferred from the load (the DC side) to the supply (the AC side). Therefore, converter is said to be operating in **inverting** mode in contrast to the **rectification** mode for positive $V_{o,dc}$ as indicated in Fig. 5.

The inversion mode is possible only if the load has the capacity to produce power (Example: DC machine in generation mode). For passive load (R / RL) the converter can only operate in rectification region.

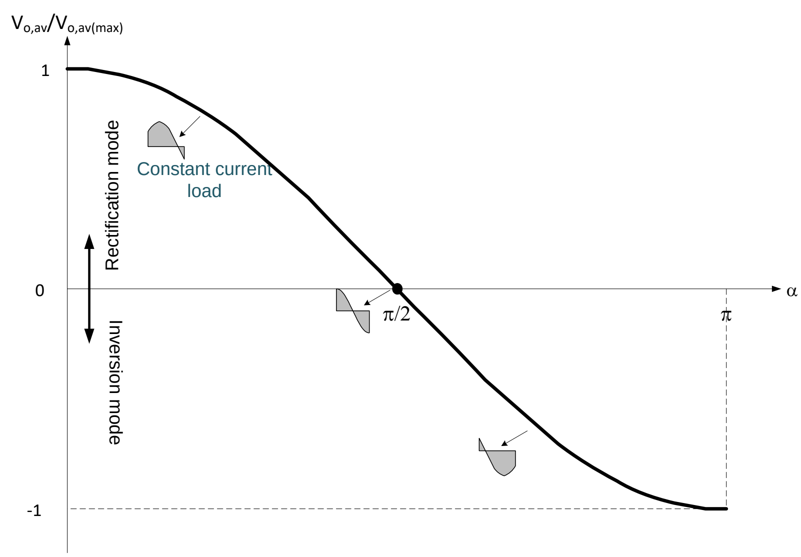


Fig. 5

The output voltage RMS expression is similar to that of the sine wave as the squares of the two voltages are identical:

$\boldsymbol{V_o = \frac{1}{\sqrt{2}} V_p}$ (independent of α_f)

The ripple factor is given by:

$$RF_{vo} = \frac{V_{o,ac}}{V_{o,dc}}$$

$$V_{o,ac} = \sqrt{V_{o,rms}^2 - V_{o,dc}^2}$$

$$V_{o,ac} = V_p \sqrt{\frac{1}{2} - \frac{4}{\pi^2} cos^2 \alpha}$$

$$\boldsymbol{RF = \sqrt{\frac{\pi^2}{8\, cos^2 \alpha} - 1}} \qquad ...(3)$$

The variation of the ripple factor with α_f is shown in Fig. 6. The ripple factor increases dramatically for α around $\pi/2$. And the minimum RF is for α_f=0°, ($RF(\alpha_f = 0) = 0.483$).

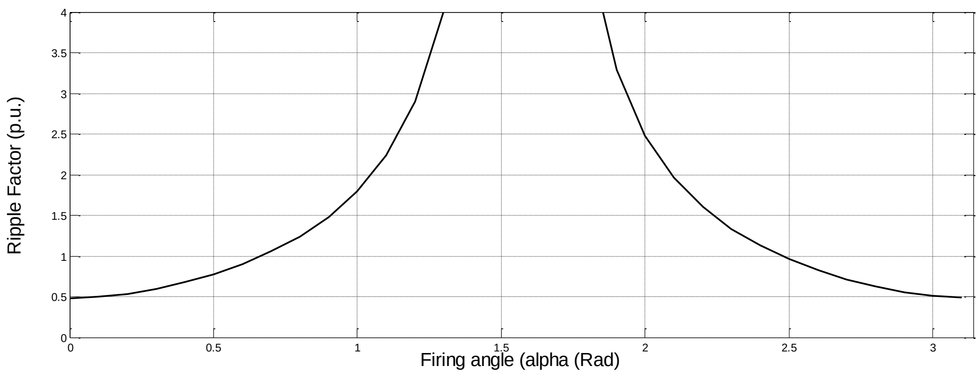


Fig. 6

<u>Power factor Calculation:</u>

Without much mathematical analysis, the supply power factor is determined using i_s waveform drawn Fig. 4 and the definition of the power factor for sinusoidal supply having non-sinusoidal current given earlier as:

$$PF = cos\emptyset_1 \frac{I_{s,1}}{I_s}.$$

From Fig, 4

$$\emptyset_1 = \alpha_f$$

And we can show that for the square-wave i_s, $\frac{I_{s,1}}{I_s} = \frac{4}{\sqrt{2}\pi} \approx 0.9$ (left for student to verify as a practice)

Therefore:

$$\boldsymbol{PF = 0.9\, cos\alpha_f}$$

3- Analysis of Full Converter with R Load

The converter circuit with R load is shown in Fig. 7. The following discussion focuses on the differences between the R load and the constant load current cases.

As the instantaneous voltage reaches a negative the range at $\omega t > \pi$, the resistive load current also tends to follow the voltage to negative. But as the current reaches zero[1], SCRs (T1 and T2) turn-off. This leaves the converter with all four switches at off state until the next triggering action at $\omega t = \alpha_f + \pi$. In other words, for ($\pi < \omega t < \alpha_f + \pi$), converter will be with four blocking switches or in state 0 shown in Fig. 2 c.

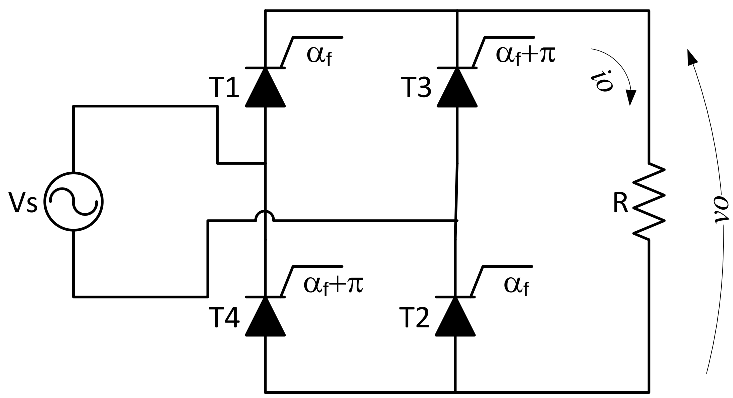


Fig. 7 Single-phase Fully-controlled rectifier with R-Load

Fig. 8 shows the waveforms of the voltages and currents for R load. Obviously the analysis and performance indicators calculations determined earlier for constant current load are not applicable for R load. Therefore we need to re-calculate it:

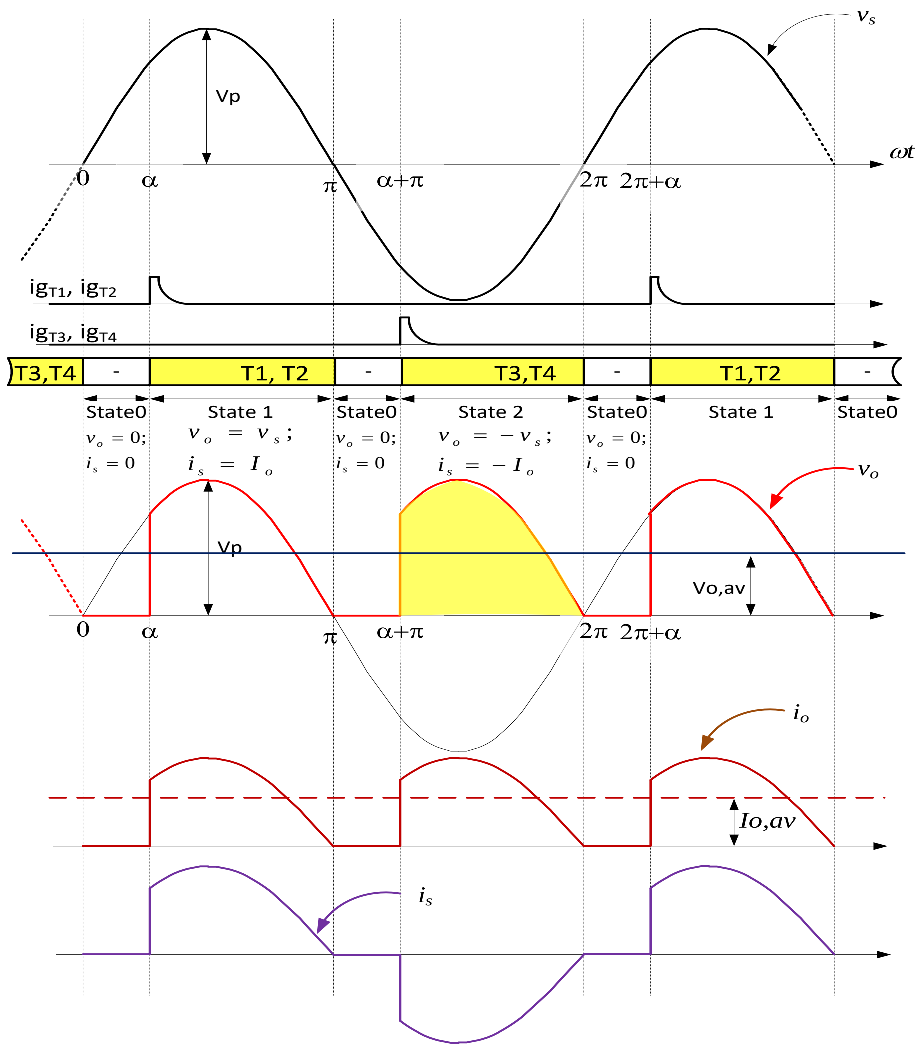
$$V_{o,dc} = \frac{1}{\pi} \int_{\alpha_f}^{\pi} V_p sin\omega t d\omega t = \frac{V_p}{\pi}\left(1 + cos\alpha_f\right)$$

The variation of $V_{o,dc}$ with α_f is shown in Fig. 9. This fig. shows that the converter with R load does not go operate in inversion mode.

It is left for the student as practice to determine the following parameters for R-load case:

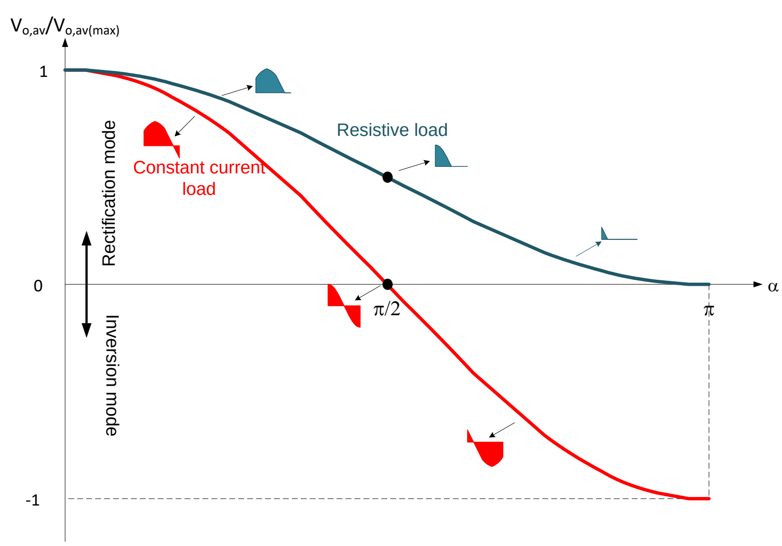
- $V_{o,rms}$, $RF\ (v_o)$, I_s, $I_{s,1}$, $\emptyset_1$, The AC supply power factor.

[1] Recall that the SCR turns off as its forward current goes bellow the holding current (I_H). The holding current is very small compared to the load current; therefore in this discussion and almost all future discussions we will approximate I_H to zero.



v_s
Vp
ωt
0
α
π
α+π
2π
2π+α
ig_{T1}, ig_{T2}
ig_{T3}, ig_{T4}
T3,T4
-
T1, T2
-
T3,T4
-
T1,T2
-
State0
$v_o = 0;$
$i_s = 0$
State 1
$v_o = v_s;$
$i_s = I_o$
State0
$v_o = 0;$
$i_s = 0$
State 2
$v_o = -v_s;$
$i_s = -I_o$
State0
$v_o = 0;$
$i_s = 0$
State 1
State0
v_o
Vp
Vo,av
i_o
Io,av
i_s

Fig. 8



$V_{o,av}/V_{o,av(max)}$
1
0
-1
Rectification mode
Inversion mode
Resistive load
Constant current load
π/2
π
α

Fig. 9

Chapter 3

AC-to-DC Converters

2- Analysis of Single-Phase Full-Bridge Controlled Rectifier with RL Load

Lecture aims:

1- To distinguish between continuous and discontinuous load current modes.
2- To perform analysis of the converter in continuous mode.
3- To perform analysis of the converter in discontinuous mode.

1- Operation in continuous and discontinuous modes

The single-phase full converter with a series connected RL load is shown in Fig. (1). As indicated earlier, T1 and T2 are triggered at $(\omega t = \alpha)$

The AC supply voltage has the following form:

$$v_s = V_p \sin(\omega t) \tag{1}$$

The load voltage will take the value of v_s as long as T1 and T2 are ON. The conduction of T1 and T2 starts at $(\omega t = \alpha_f)$ and continues as long as the current is positive[1] $i_o > 0$. The current is determined by the output voltage and load. RL load satisfies the equation:

$$v_o = R i_o + L \frac{di_o}{dt} \tag{2}$$

When T1 and T2 are ON, the output voltage equals to the supply voltage given in Eq. (1). In this case, the solution of Eq. (2) has the following form

$$i_o = i_{o,F} + i_{o,N} \tag{3}$$

[1] More precisely: "as long as their current is greater than I_H"

Where:

$$i_{o,F} = \frac{V_p}{|Z|}\sin(\omega t - \emptyset) \text{ (T1 and T2 are ON)} \quad (4)$$

And

$$i_{o,N} = Ae^{-\frac{t-\alpha f/\omega}{L/R}} = Ae^{-\frac{\omega t-\alpha f}{\tan\emptyset}} \quad (5)$$

In order to calculate A, we need to impose initial, or boundary, conditions. And in order to specify the condition we need to know if the rectifier is operating in continuous or discontinuous mode. Fig. 2 explains the boundary and initial conditions for the two cases.

The subsequent derivation initially assumes continuous current and then examines the condition: ($i_{o,\min} > 0$). If true, then $i_o$is indeed continuous; otherwise it is discontinuous and A has to be determined using the appropriate initial condition.

Also in subsequent discussion, one cycle of the periodic current for the interval $\left(\alpha_f < \omega t < \alpha_f + \pi\right)$

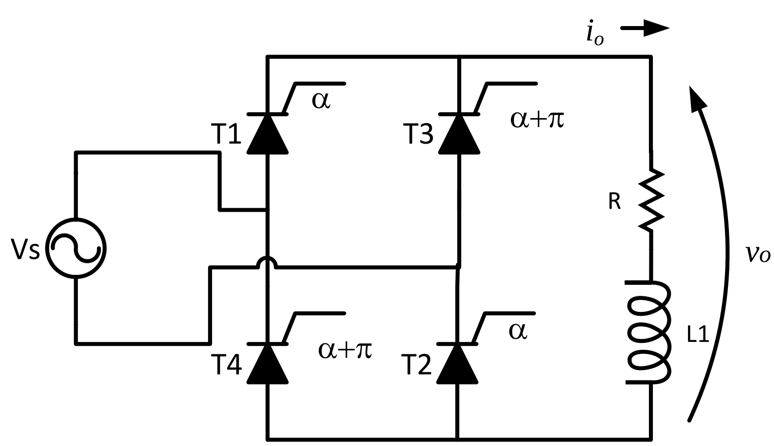


Fig. 1 The fully controlled single phase rectifier with RL load

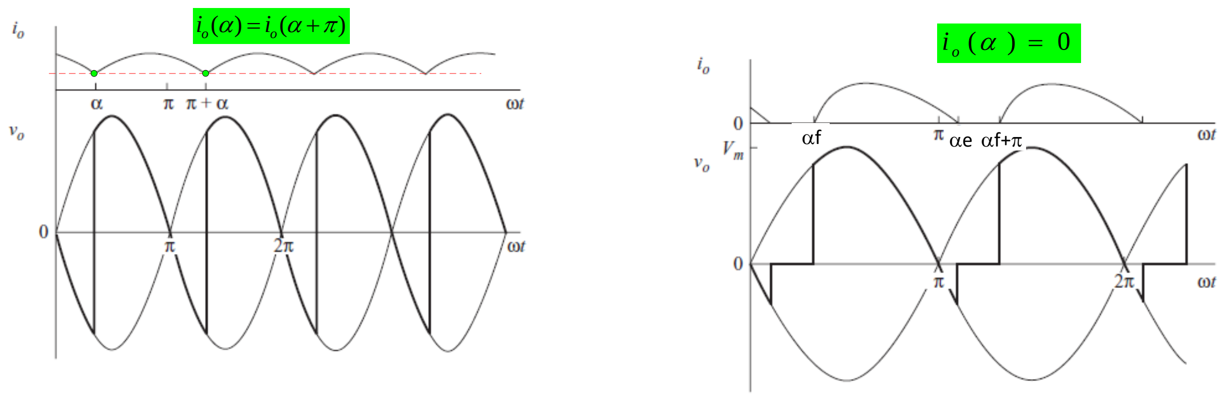


Fig. 2 Boundary and initial conditions for continuous and discontinuous modes

2- Continuous conduction mode

To determine the current expression assuming continuous current mode:

Substitute (4) and (5) into (3):

$$i(\omega t) = \frac{V_p}{|Z|}\sin(\omega t - \emptyset) + Ae^{-\frac{\omega t - \alpha f}{\tan \emptyset}} \qquad ...(6)$$

The boundary conditions at steady state (see Fig. 2a):

$$i(\omega t = \alpha f) = i(\omega t = \alpha f + \pi) \qquad ...(7)$$

Results by substituting into (6):

$$\frac{V_p}{|Z|}\sin(\alpha_f - \emptyset) + A = \frac{V_p}{|Z|}\sin(\alpha_f + \pi - \emptyset) + Ae^{-\frac{\pi}{\tan \emptyset}}$$

Gives:

$$A = \frac{2V_p \sin(\alpha_f - \emptyset)}{|Z|\left(e^{-\frac{\pi}{\tan \emptyset}} - 1\right)} \qquad ...(8)$$

Substitute (8) into (6)

$$i(\omega t) = \frac{V_p}{|Z|}\left[\sin(\omega t - \emptyset) + \frac{2\sin(\alpha_f - \emptyset)}{\left(e^{-\frac{\pi}{\tan\emptyset}} - 1\right)} e^{-\frac{\omega t - \alpha f}{\tan\emptyset}}\right] \quad ...(9)$$

For $\alpha_f < \omega t < (\alpha_f + \pi)$

Eq. (9) is applicable providing that $i_{min} > 0$.

3- Determination of conduction mode

Considering that $i_{min} = i(\omega t = \alpha_f)$, for continuous conduction mode

$$i(\omega t = \alpha_f) > 0$$

$$\sin(\alpha_f - \emptyset) + \frac{2\sin(\alpha_f - \emptyset)}{\left(e^{-\frac{\pi}{\tan\emptyset}} - 1\right)} > 0$$

Since $e^{-\frac{\pi}{\tan\emptyset}} < 1$; the condition reduces to

$$\boldsymbol{\alpha_f} < \emptyset \quad (10)$$

Example 1: A single-phase full bridge rectifier is supplied from a 220V, 50Hz AC source and its RL load has a 50mH inductor and 10Ω resistor. The firing angle is set to 45°

(a) Show that the rectifier is operating in continuous conduction mode.

(b) Calculate the following:

i. $i_o(\omega t)$
ii. $V_{o,dc}$
iii. $I_{o,dc}$
iv. I_o
v. RF_{vo}
vi. RF_{io}
vii. PF

Sol

From given data $Z = 10 + j100\pi * 0.05 = 18.6\angle 57.5°$

$$V_p = 220\sqrt{2} = 311V$$

(a) To show that the current is continuous check the condition given in (10)

Since $\emptyset > \alpha_f$, therefore the rectifier is operating in continuous mode

(b) The solution…

i. $i(\omega t) = \frac{311}{18.6}\left[\sin(\omega t - 57.5) + \frac{2\sin(45-57.5)}{\left(e^{-\frac{\pi}{0.5\pi}}-1\right)} e^{-\frac{\omega t - \frac{\pi}{4}}{0.5\pi}}\right]$

$$i(\omega t) = 16.72\left[\sin(\omega t - 57.5) + 0.5e^{-\frac{4\omega t-\pi}{2\pi}}\right]$$

ii. $V_{o,dc} = \frac{2}{\pi}V_p\cos\alpha_f = \frac{2}{\pi} * 220\sqrt{2} * \frac{1}{\sqrt{2}} = 140V$

iii. $I_{o,dc} = \frac{V_{o,dc}}{R} = 14A$

iv. $I_o = \sqrt{\frac{16.72^2}{\pi}\int_{\alpha}^{\alpha+\pi}\left[\sin(\omega t - 57.5) + 0.5e^{-\frac{4\omega t-\pi}{2\pi}}\right]^2 d\omega t}$=14.7A

v. RF_{vo}

$$V_{o,ac} = \sqrt{V_o^2 - V_{o,dc}^2} = \sqrt{220^2 - 140^2} = 169.7V$$

$$RF_{vo} = \frac{V_{o,ac}}{V_{o,dc}} = 1.212$$

vi. RF_{io}

$$I_{o,ac} = \sqrt{I_o^2 - I_{o,dc}^2} = \sqrt{14.7^2 - 14^2} = 4.63A$$

$$RF_{io} = \frac{4.63}{14} = 0.33$$

vii. $PF = \frac{P}{S} = \frac{I_o^2 R}{I_o V_i} = \frac{14.7*10}{220} = 0.668$

4- Discontinuous of conduction mode

If the condition given in (10) is not satisfied, the current falls to zero before (α_f+π). And the four SCRs become OFF as shown in Fig. 2.

In this case the initial condition $i_o\left(\omega t=\alpha_f\right)=0$ is used to determine A in Eq. (6)

$$0=\frac{V_p}{|Z|}\sin\left(\alpha_f-\emptyset\right)+A$$

Gives:

$$A=\frac{V_p}{|Z|}\sin\left(\emptyset-\alpha_f\right) \qquad \text{...(11)}$$

Note: as $\emptyset<\alpha_f$, the value of A determined in (11) is always negative.

Example 2 Repeat Example 1 with $\alpha_f\ =90°$.

Sol

(a) Since $\emptyset<\alpha_f$, therefore the rectifier is operating in continuous mode

(b) The solution…

i. $A=\frac{V_p}{|Z|}\sin\left(\emptyset-\alpha_f\right)=16.72*-\frac{1}{\sqrt{2}}=-11.82$

$$i(\omega t)=16.72\sin(\omega t-57.5)-11.82e^{-\frac{2\omega t-\pi}{\pi}}$$

For $\alpha_f<\omega t<\alpha_e$

Where $\alpha_e$is the angle at which the current becomes 0 and the SCRs turn OFF

To determine α_e we solve the equation

$$i(\alpha_e)=16.72\sin(\alpha_e-57.5)-11.82e^{-\frac{2\alpha_e-\pi}{\pi}}=0$$

Gives $\alpha_e = 228.57$

$$i(\omega t) = 16.72\sin(\omega t - 57.5) - 11.82e^{-\frac{2\omega t-\pi}{\pi}}$$
$$90° < \omega t < 228.75°$$

ii. $V_{o,dc} = \frac{1}{\pi}\int_{90}^{228.75} V_p \sin\omega t\, d\omega t = \frac{220\sqrt{2}}{\pi}[\cos 90 - cos228.75] = 65.3V$

iii. $I_{o,dc} = \frac{V_{o,dc}}{R} = 6.53A$

iv. $I_o = \sqrt{\frac{1}{\pi}\int_{\alpha f}^{\alpha e}\left[16.72\sin(\omega t - 57.5) - 11.82e^{-\frac{2\omega t-\pi}{\pi}}\right]^2 d\omega t}$=6.69A

v. RF_{vo}

$$V_o = 220\sqrt{2}\sqrt{\frac{1}{\pi}\int_{90}^{228.75}[\sin\omega t\, d\omega t]^2} = 172.25V$$

$$V_{o,ac} = \sqrt{V_o^2 - V_{o,dc}^2} = 159.4V$$

$$RF_{vo} = \frac{V_{o,ac}}{V_{o,dc}} = 2.44$$

vi. RF_{io}

$$I_{o,ac} = \sqrt{I_o^2 - I_{o,dc}^2} = 1.44A$$

$$RF_{io} = \frac{1.44}{6.53} = 0.22$$

vii. $PF = \frac{P}{S} = \frac{I_o^2 R}{I_o V_i} = \frac{6.69*10}{220} = 0.3$

Chapter 3

AC-to-DC Converters

3- Three-Phase Diode Rectifiers

Lecture aims:

1- To present the circuit of diode three-pulse rectifier.
2- To describe the operation of the diode six-pulse rectifier.
3- To analyze a diode six-pulse rectifier supplying a highly inductive load.

1- The three-pulse diode rectifier

The circuit diagram of a three-pulse diode rectifier is shown in Fig. 1. Supplied from a three-phase, four-wire AC power line, the rectifier consists of three power diodes, DA through DC. The load is connected between the common-cathode node of the diode and the neutral, N, of the supply line.

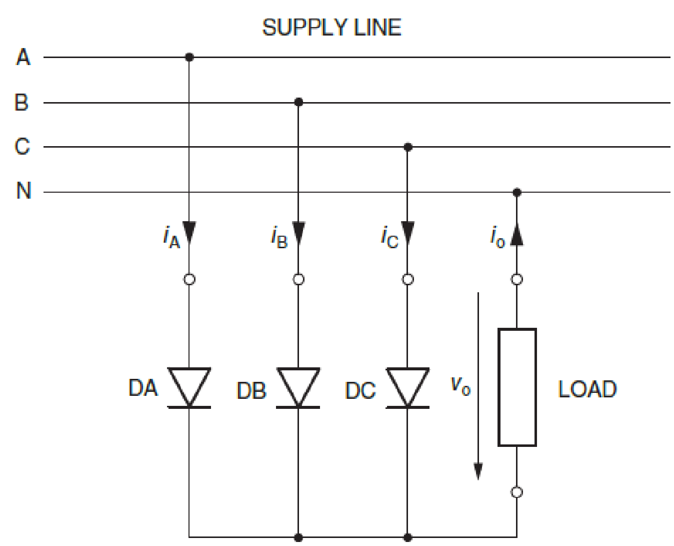


Fig. 1 three pulse diode rectifier

At any time only the diode that is supplied with the highest line-to-neutral voltage is conducting the output current, io. For Example when the highest voltage is that of phase B ($v_{BN} > v_{AN}$ and $v_{BN} > v_{CN}$)

then it is diode DB that is conducting, while diodes DA and DC are reverse biased by line-to-line voltages v_{BA} and v_{BC}, respectively.

Each of the three line-to-neutral voltages is higher than the other two for 1/3 of the cycle of input voltage. Consequently, each diode conducts the current within a 120◦ interval. Voltage and current waveforms of the three-pulse rectifier with a resistive load (R-load) are shown in Fig. 2. The waveform pattern of the output voltage is repeated every 120◦ ($2\pi/3$ rad).

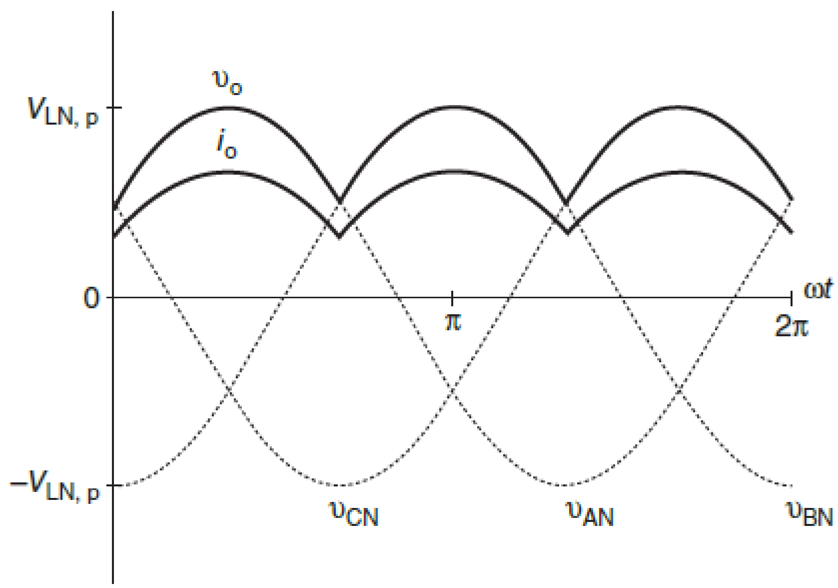


Fig. 2 three-pulse rectifier output voltage and output current (R-load)

The average of the output voltage is determined as follows:

$$V_{o,dc} = \frac{1}{2\pi/3}\int_0^{2\pi/3} v_{AN}d\omega t \qquad ...(1)$$

Notice that if we consider the start of the conduction of DA at ($\omega t = 0$) it means that the positive zero crossing of v_{AN} is $\frac{\pi}{6}$ before zero. Therefore

$$v_{AN} = V_{LN,p}\sin(\omega t + \frac{\pi}{6}) \qquad ...(2)$$

Substitute (2) into (1)

$$V_{o,dc} = \frac{3V_{LN,p}}{2\pi}\int_{0}^{2\pi/3} \sin(\omega t + \pi/6)\, d\omega t = \frac{3V_{LN,p}}{2\pi}\left[\cos\frac{\pi}{6} - cos\frac{5\pi}{6}\right]$$

$$V_{o,dc} = \frac{3\sqrt{3}V_{LN,p}}{2\pi} \approx 0.827V_{LN,p} \qquad ...(3)$$

Drill derive the expression of the output voltage ripple factor of the three-pulse rectifier

Example

1- A three-pulse diode rectifier, fed from a 460-V ac line, supplies a 5-Ωresistive load. Calculate the average output voltage and current of the rectifier.
2- The rectifier charges a 360-V battery pack through a 5Ω. Is the charging current continuous or discontinuous? Sketch the output voltage and current.

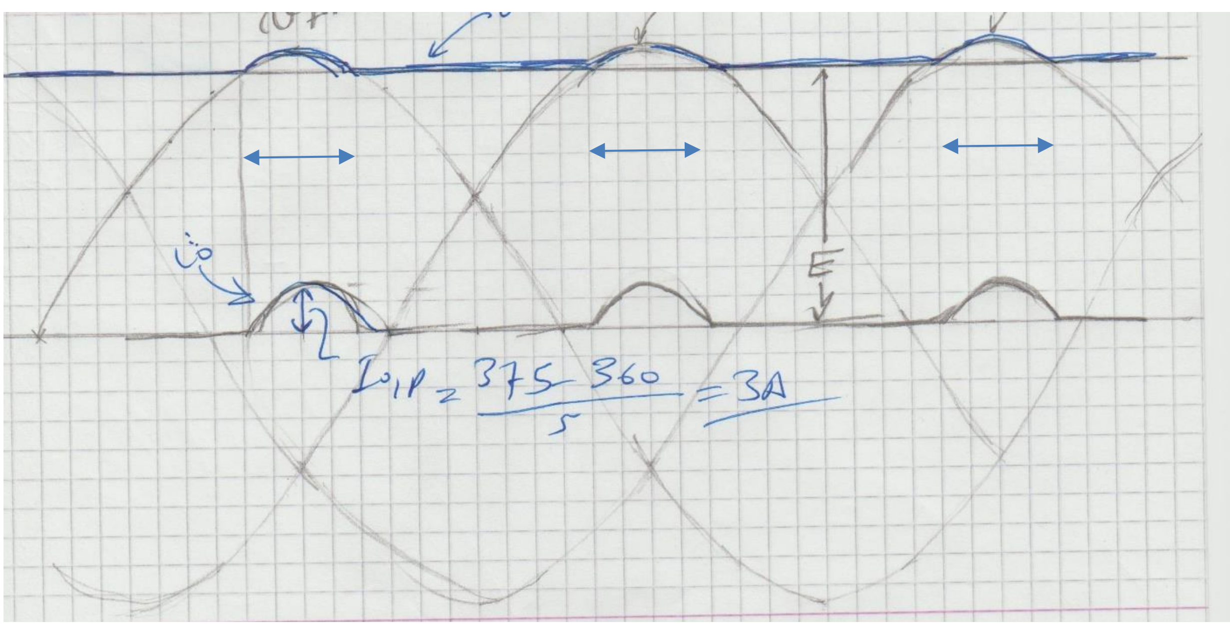
Sol.

1-) $V_{LL,p} = 460\sqrt{2} = 650V$

$$V_{LN,p} = \frac{650}{\sqrt{3}} = 375V$$

$$V_{o,dc} = 0.827V_{LN,p} = 310V$$

2-) Discontinuous



The waveform of the phase-A line current, i_A, is shown in Fig. 3. Clearly, it is completely different from a sinewave. Such currents drawn by the rectifier would disturb the operation of protection systems.

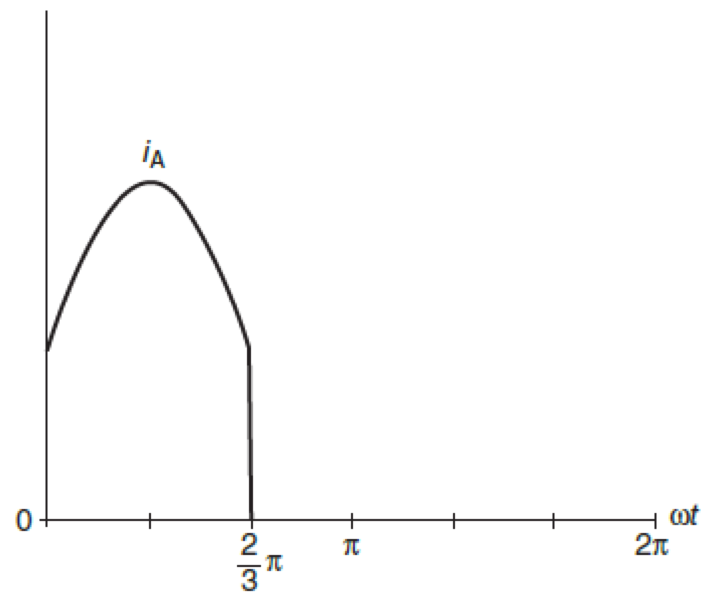


Fig. 3 current of phase A

3- Six-pulse Diode Rectifier

The six-pulse (three-phase, full-wave) diode rectifier, shown in Fig. 4 is the most commonly used ac-to-dc power converter producing a fixed dc voltage. The circuit of the rectifier consists of six power diodes in a three-phase bridge configuration. Diodes DA, DB, and DC form *the common-cathode* group, and diodes DA', DB', and DC' constitute the *common-anode* group. At any instant, only one diode in the common-cathode group and one in the common-anode group conduct the current.

In Fig. 4 the load is considered highly inductive that draws a ripple-free current and therefore represented by a dc current supply.

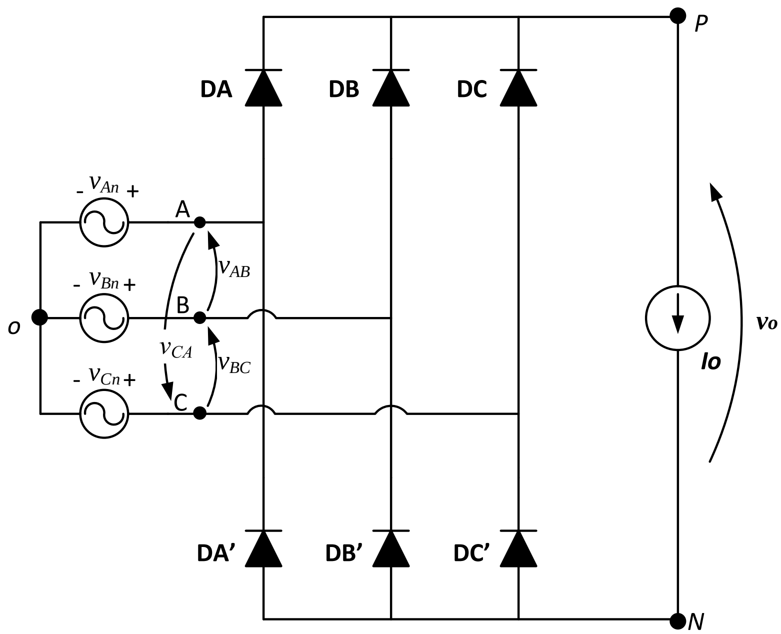


Fig. 4 six pulse diode rectifier

Referring to Fig. 5, by doubling the function of the three pulse rectifier we can deduce that:

- at any time, only one diode from the common cathode group: is forward biased which is the diode with the highest voltage at its anode.
- Also one diode from the common-anode diodes is forward biased: the diode which has the lowest voltage at its cathode.

Or we can say that the conducting diode pair is that supplied with the highest line-to-line voltage, as is illustrated in Fig.5. Note that the output voltage takes six line-to-line voltages.

For the first interval when diodes DA and DB’ form a path for the output current. The other four diodes are subjected to voltages v_{AC} (diode DC), v_{CB} (diode DC’), and v_{AB} (diodes DA’ and DB).

It can be seen that each diode conducts for 120°, and then another diodes in the same group becomes forward-biased forcing the former one to turn OFF. The process of a diode taking over conduction of current from another diode is called natural commutation.

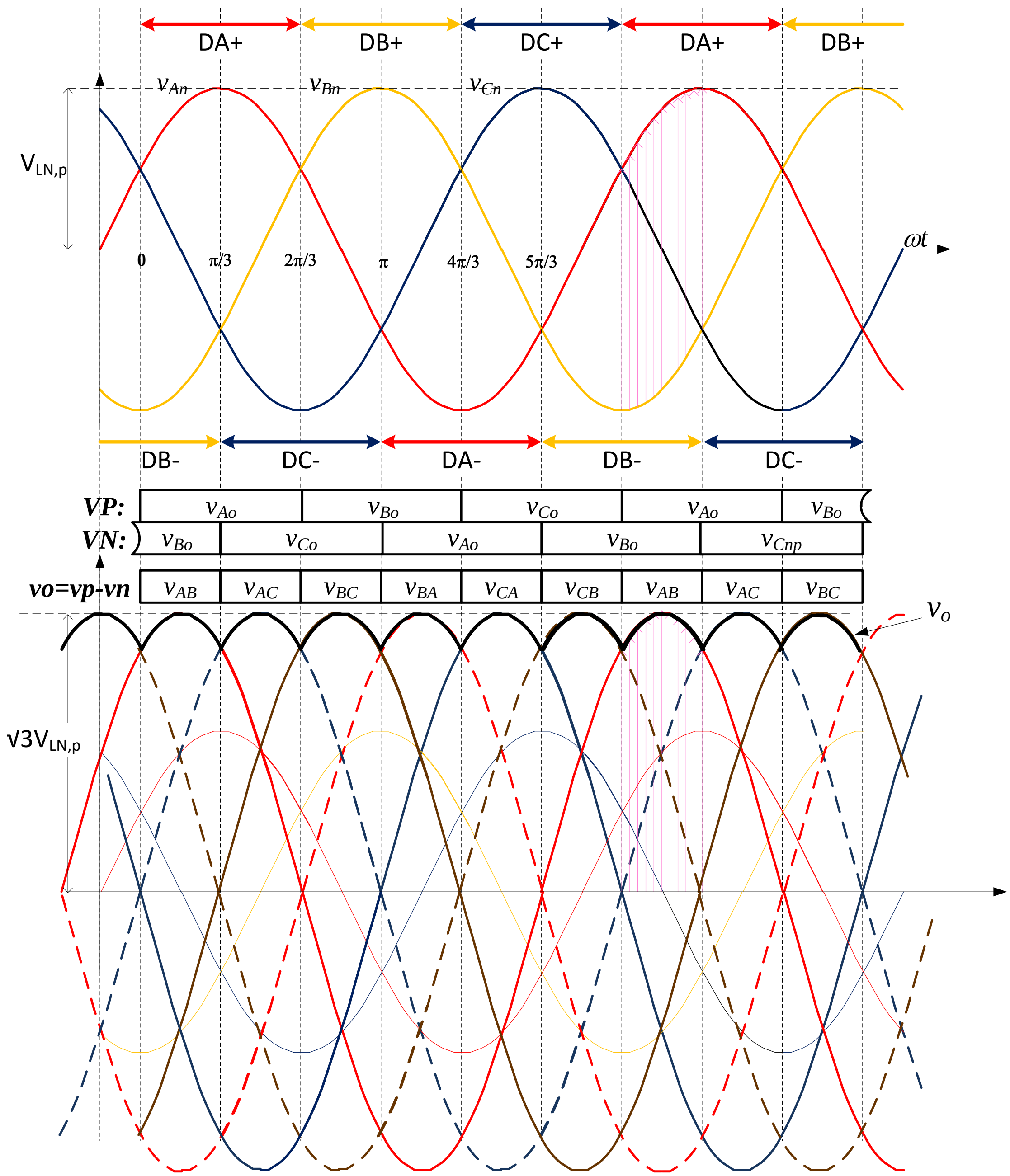
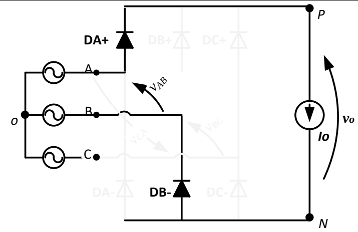
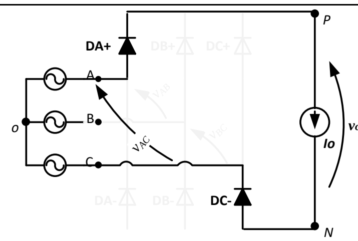
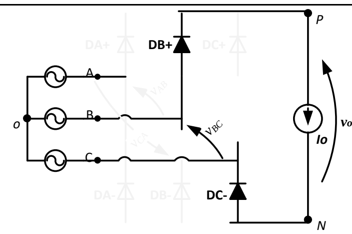
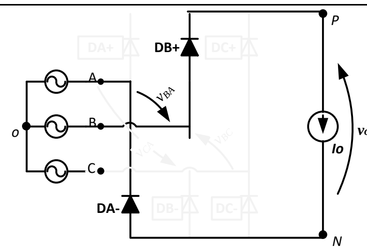
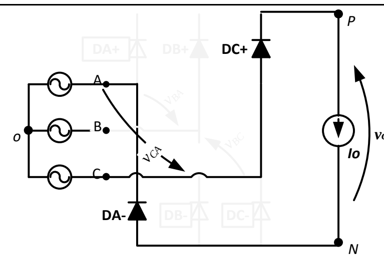
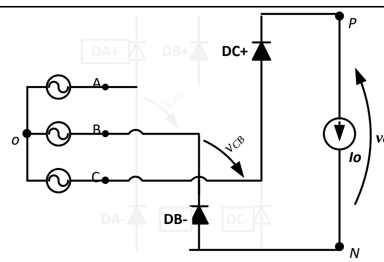


Fig. 5 The operation cycle of 6-pulse rectifier

A detailed description of the circuit states is given in table 1. This list of all possible states is referred to as the circuit topologies.

Table 1					
Interval	Diodes ON upper	Diodes ON lower	Equivalent Circuit	v_o	Input currents
1 $0 < \omega t < \frac{\pi}{3}$	**DA,**	**DB'**		v_{AB}	$i_A = i_o$ $i_B = -i_o$ $i_C = 0$
2 $\frac{\pi}{3} < \omega t < \frac{2\pi}{3}$	**DA**	**DC'**		v_{AC}	$i_A = i_o$ $i_B = 0$ $i_C = -i_o$
3 $\frac{2\pi}{3} < \omega t < \pi$	**DB**	**DC'**		v_{BC}	$i_A = 0$ $i_B = i_o$ $i_C = -i_o$
4 $\pi < \omega t < \frac{4\pi}{3}$	**DB**	**DA'**		v_{BA}	$i_A = -i_o$ $i_B = i_o$ $i_C = 0$
5 $\frac{4\pi}{3} < \omega t < \frac{5\pi}{3}$	**DC**	**DA'**		v_{CA}	$i_A = -i_o$ $i_B = 0$ $i_C = i_o$
6 $\frac{5\pi}{3} < \omega t < 2\pi$	**DC**	**DB'**		v_{CB}	$i_A = 0$ $i_B = -i_o$ $i_C = i_o$

3- Determination of the output voltage average and ripple factor

The output voltage period is $\pi/3$; from Fig.4 the average of the output voltage:

$$V_{o,av} = \frac{1}{\pi/3}\int_{\omega t=0}^{\omega t=\frac{\pi}{3}} V_{LL,p}\sin\left(\omega t+\frac{\pi}{3}\right)d\omega t$$

$$V_{o,av} = \frac{3}{\pi}V_{LL,p}\left[-\cos\left(2\frac{\pi}{3}\right)+-\cos\left(\frac{\pi}{3}\right)\right]$$

$$\boldsymbol{V_{o,av} = \frac{3}{\pi}V_{LL,p}}$$

Or

$$\boldsymbol{V_{o,av} \cong 0.955V_{LL,p}}$$

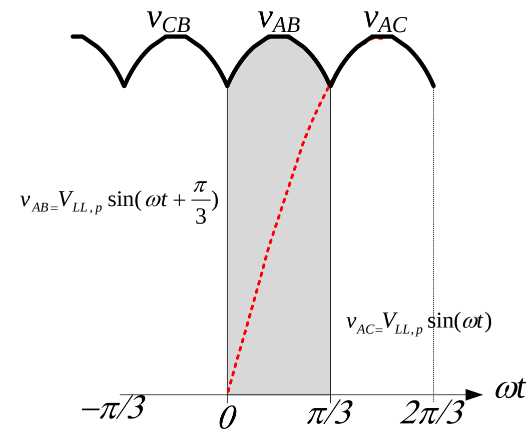


Fig. 6

The RMS of the output voltage:

$$V_{o,rms} = \sqrt{\frac{3}{\pi}\int_{\omega t=\frac{\pi}{3}}^{\omega t=2\frac{\pi}{3}}\left(V_{LL,p}\sin(\omega t)\right)^2 d\omega t}$$

(The reference has been changed OR we have used the second output pulse (v_{AC}) to obtain simpler sine expression!)

$$V_{o,rms} = V_{LL,p}\sqrt{\frac{3}{2\pi}\int_{\omega t=\frac{\pi}{3}}^{\omega t=2\frac{\pi}{3}}(1-\sin(2\omega t))\quad d\omega t}$$

$$V_{o,rms} = V_{LL,p}\sqrt{\frac{3}{2\pi}\left[\left(\frac{2\pi}{3}-\frac{\pi}{3}\right)-\frac{1}{2}\left(cos\frac{4\pi}{3}-cos\frac{2\pi}{3}\right)\right]}$$

$$\boldsymbol{V_{o,rms} = V_{LL,p}\sqrt{\frac{3}{2\pi}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)} \cong 0.956V_{LL,p}}$$

The RMS and the average values are very close, which implies a small ripple factor.

Practice: show that the ripple factor =4.2%

4- Input power factor of a highly inductive load:

The highly inductive load current is considered to be ripple-free or pure DC for simplification.

Fig. 7 is drawn to identify the phase relationship between the AC supply voltage and current. Fig. 7 is continuation of Fig. 5 and follows the same style. To draw an AC line current (say i_A) we need to look at (i) the load current (*Io*) and the state of the diodes connected to the corresponding line (DA and DA' for line A). The current of line A is determined as follows:

- $i_A = I_o$ if DA+ is conducting.
- $i_A = -I_o$ if DA- is conducting. Otherwise;
- $i_A = 0$ if DA+ and DA- are both blocking.

The other 2 lines currents can be determined by extending the same rule.

Fig. 7 shows that the line current has a quasi-square waveform which has a $2\pi/3$ pulse every half cycle of the corresponding phase voltage. The fundamental component of the as current is in-phase with the phase voltage, or ϕ_1= 0.

The resultant power factor: $PF = \cos \emptyset_1 \frac{I_{A1}}{I_A}$

$\cos \emptyset_1 = 1$ (since $I_{A,1}$ is in=phase with v_{Ao})

$$\frac{I_{A1}}{I_A} = \frac{\sqrt{6}/\pi}{\sqrt{2/3}} = \frac{3}{\pi} = 0.955$$

PF=0.955

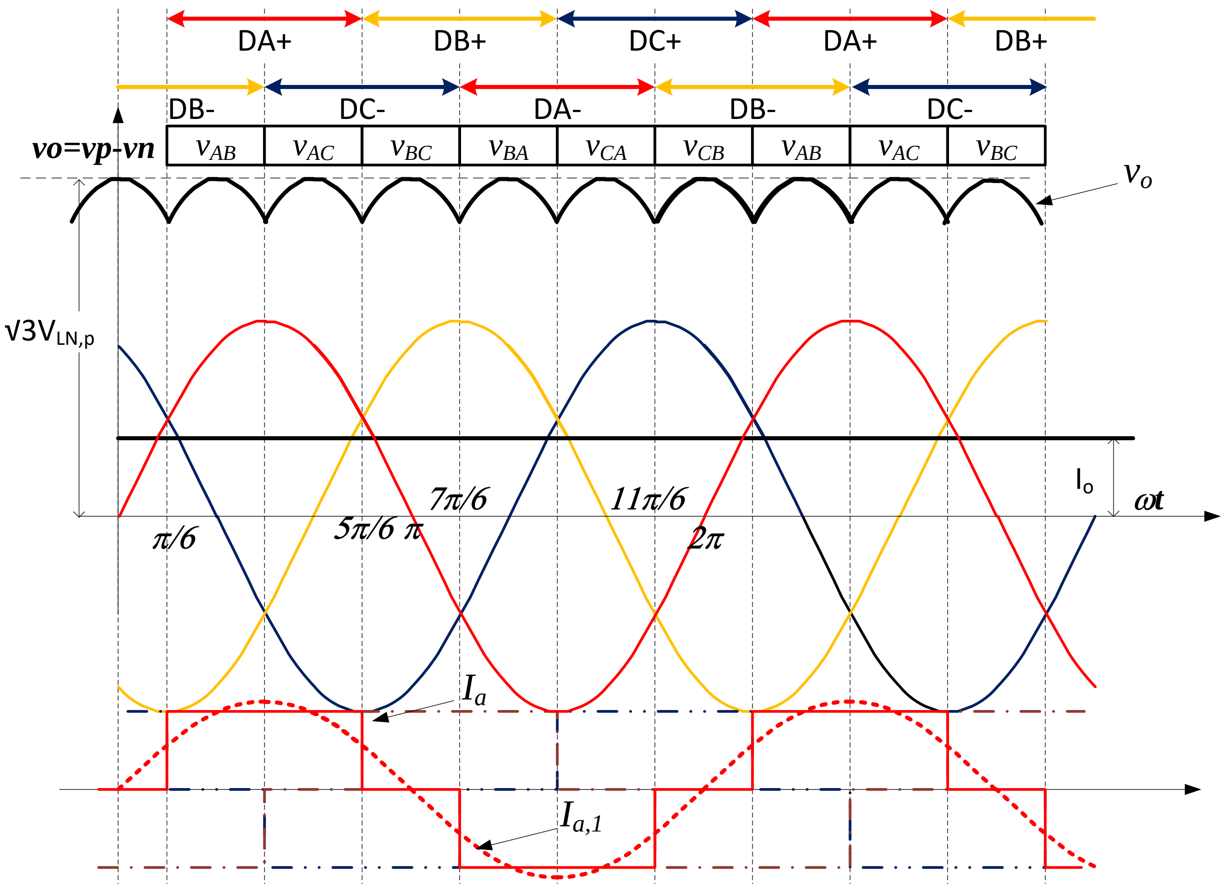

DA+
DB+
DC+
DA+
DB+
DB-
DC-
DA-
DB-
DC-
vo=vp-vn
vAB
vAC
vBC
vBA
vCA
vCB
vAB
vAC
vBC
vo
√3VLN,p
Io
ωt
π/6
5π/6 π
7π/6
11π/6
2π
Ia
Ia,1

Fig. 7

Chapter 3

AC-DC Converters

3-Three-phase full bridge controller rectifier

(constant current load)

Lecture aims:

1- To present the circuit diagram and control pulses of the three-phase fully controlled rectifier.
2- To conduct analysis leading to output voltage and input currents waveform considering a constant, ripple-free load current.
3- To determine the output voltage average and the performance indicators.

1. The Circuit Diagram

The three-phase controlled rectifier (also known as six-pulse controlled converter) is the basic AC-to-variable DC supply. The circuit, as shown in Fig. 1, is similar to the three-phase diode rectifier except it replaces the diodes with SCRs.

The SCRs provides adjustable output voltage that can be controlled by changing the firing angle (turn-on instant of the SCR). The turn-on point of each SCR is delayed by an angle (α_f), compared to that of the corresponding diode. The operation sequence of the SCRs is similar to that of the diodes in the three-phase rectifier (Chapter 3 -2). ALL the six SCRs are delayed by the same angle; this keeps:

1- The conduction interval of 120° for each SCR, and
2- The output voltage 60° interval.

This constant delay of all switches conduction is important to maintain the output voltage symmetry and minimum ripple.

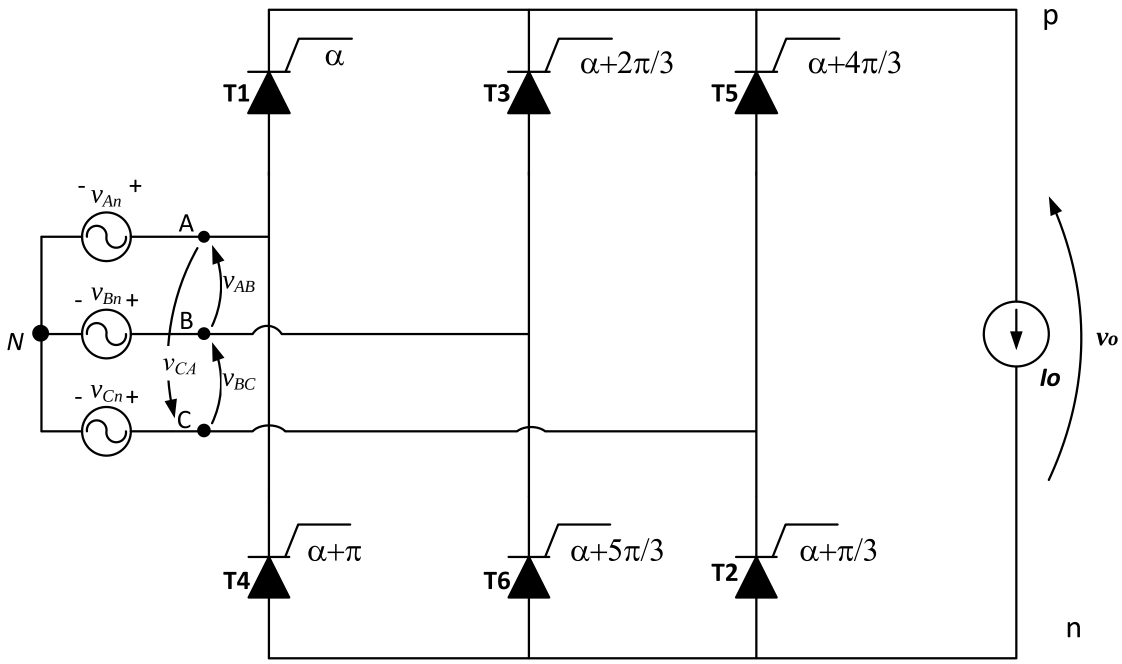


Fig. 1 Three phase controlled rectifier

2. Control pulses and circuit topologies

<u>The range of α_f</u>

For successful SCR triggering, the firing signal must be applied while the SCR is in the **forward blocking region**. This means that **the anode voltage is higher than cathode voltage**.

Since the load current is continuous, the SCRs in common cathode group operates in the sequence (TA, TB, TC, TA, …),-each conducts for 120°-. When triggering TA, TC is ON this means that line C is connected to point p (the cathode of TA) as the gate pulse is applied to TA. Therefore, TA can be triggered only when $v_A > v_C$. The range that satisfiers this condition is explained in Fig. 2. Notice that the earliest instant for triggering, denoted by (α_f=0) makes the SCR operates as a diode.

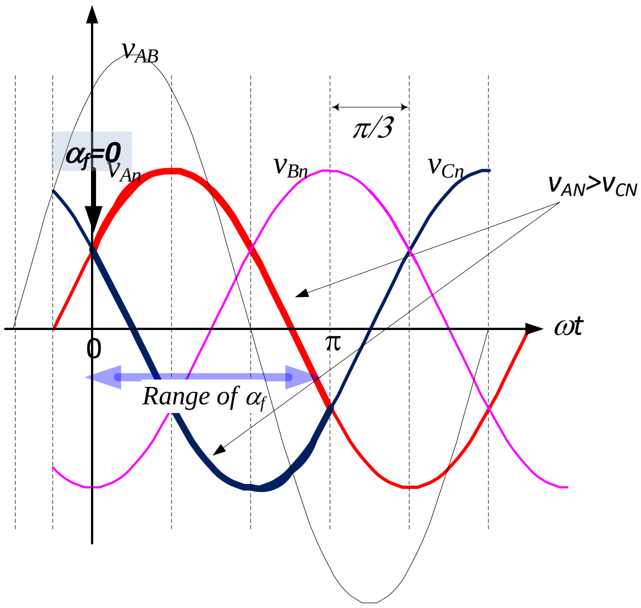


Fig. 2

Remember that the reference point (ωt=0), has been defined in the same way used for the 6-pulse diode rectifier. This means at (ωt=0°) $v_{AN}(+ve) = v_{CN}(+ve)$. This implies that $v_{AN} = V_{LN,p} \sin\left(\omega t + \frac{\pi}{6}\right)$.

<u>The effect of changing α_f.</u>

We can notice that the output voltage takes a π/3 portion of v_{AB} starting form ωt=α_f; where:

$$v_o = v_{AB} = V_{LL,p} \sin(\omega t + \tfrac{\pi}{3}) \qquad \alpha_f < \omega t < \alpha_f + \pi/3 \qquad \text{...(1)}$$

Equation (1) describes one period of v_o, which repeats by taking similar $\frac{\pi}{3}$ portions from the subsequent line-to-line voltages: $v_{AC}; v_{BC}; v_{BA}; v_{CA}; v_{CB}|v_{AB};$ ...as explained in Fig. 3. Figure 4 shows the output voltage waveforms for selected values of α_f.

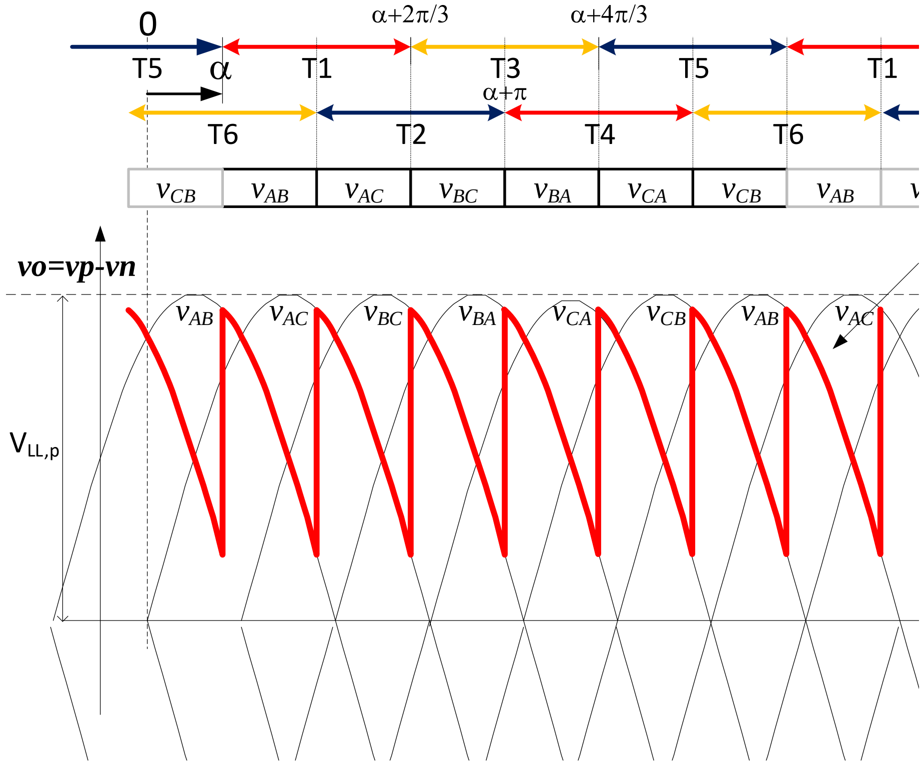


Fig. 3 the output voltage waveform

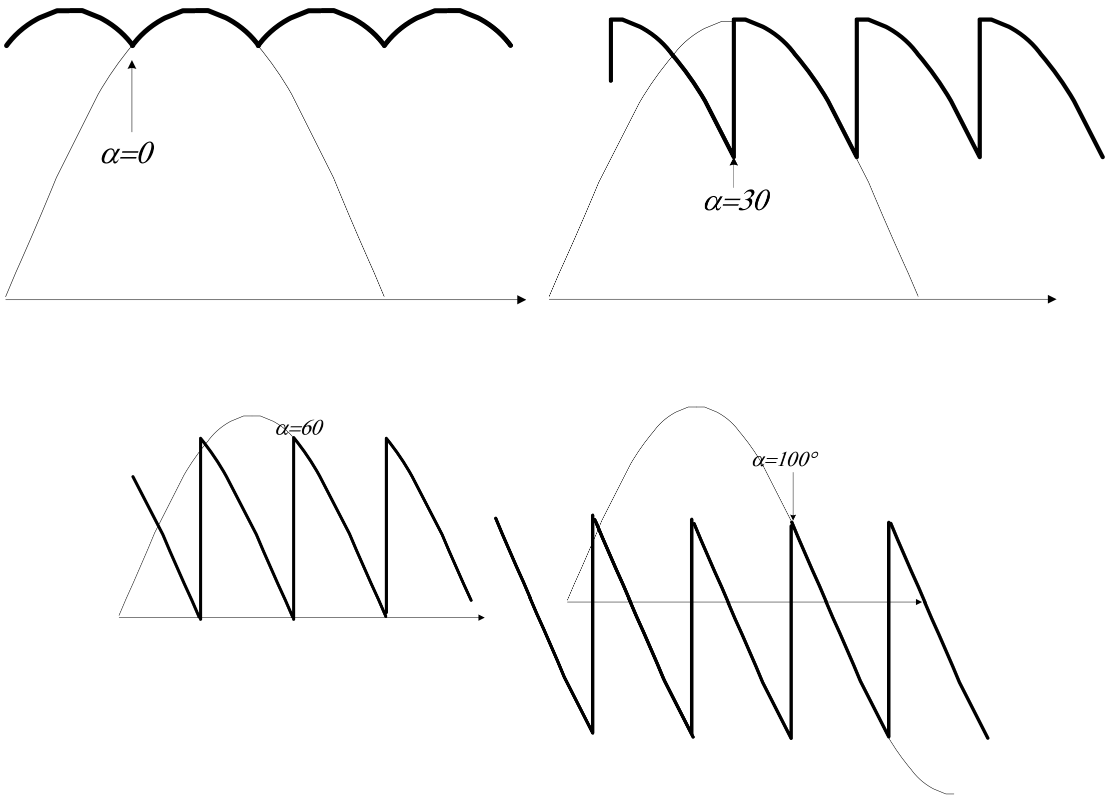


Fig. 4 the output voltage with four values of α

3- Determination of the output voltage average:

Refer to Fig. 5.

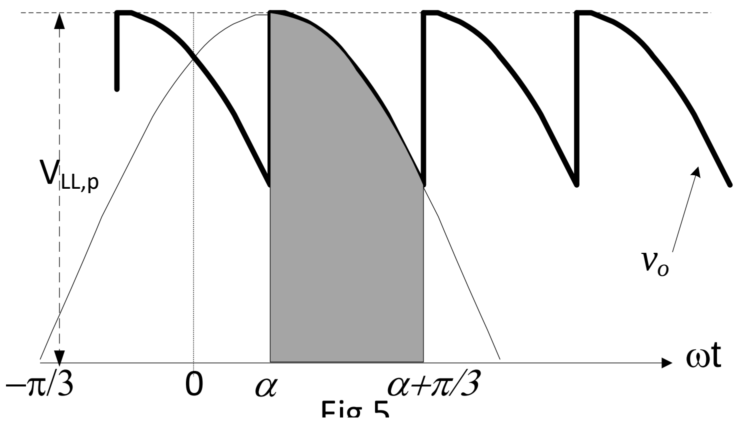


Fig. 5

$$V_{o,dc} = \frac{1}{\frac{\pi}{3}} \int_{\alpha_f}^{\alpha_f + \pi} V_{LL,p} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t$$

$$V_{o,dc} = \frac{3}{\pi} V_{LL,p} \cos \alpha_f \ldots \qquad (2)$$

Equation (2) is applicable for continuous conduction mode (i.e. $i_o(\omega t) > 0$,for all ωt)

Fig, 6 shows the variation of $V_{o,av}$ against α_f as given in Eq. 2.

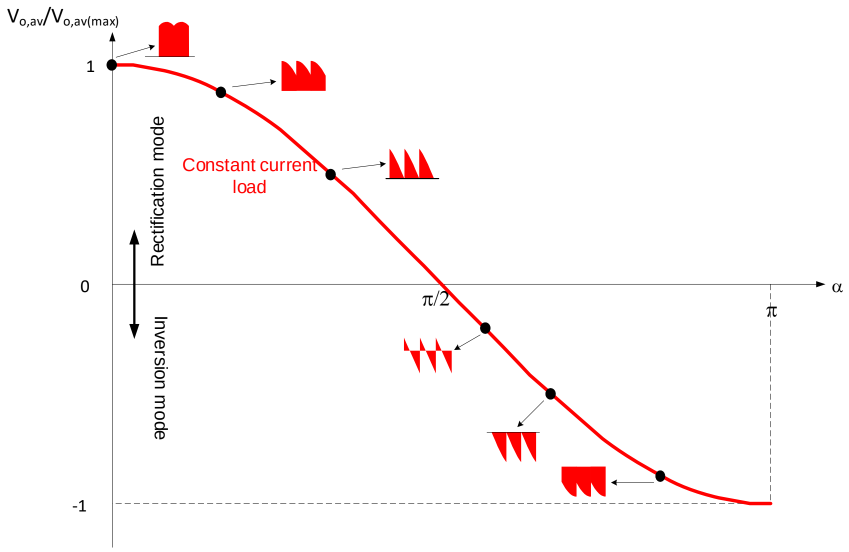


Fig. 6

Fig.7 shows the phase and line-to-line voltages waveforms together with the output voltage (α=45°). On the same time scale the input currents are also shown. By comparing the current waveform to that of the corresponding phase voltage; it can be noted that the current fundamental component lags the phase voltage by α_f. Therefore:

The displacement factor: $DF = \cos\emptyset_1 = \cos\alpha_f$

And by comparing to the diode case, the power factor (for this ripple-free load current):

$$PF = \frac{3}{\pi}\cos\alpha_f \qquad \text{...(3)}$$

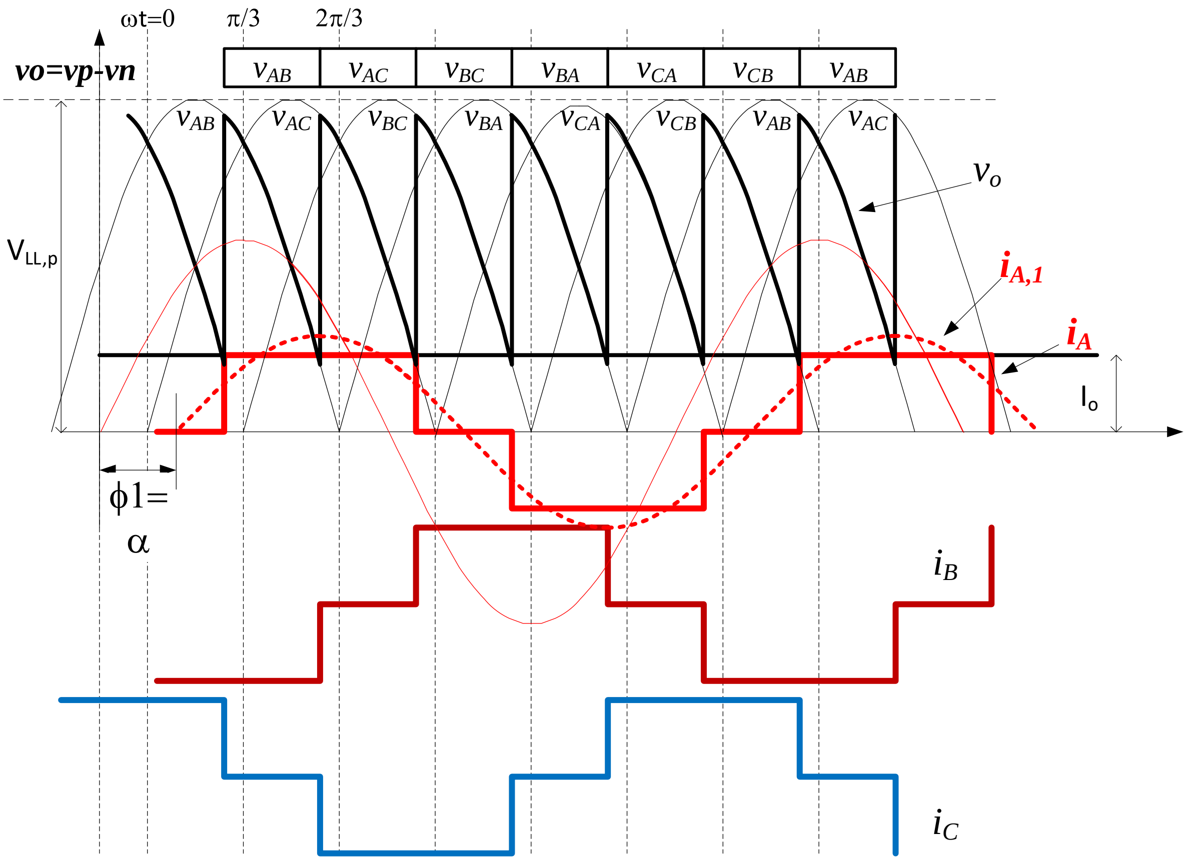


Fig. 7

Example 1: A three-phase converter is operated from a three-phase, 208-V, 60Hz supply a highly inductive load modeled by 10A current source. It is required to obtain an average output voltage of 50% of the maximum possible output voltage. Calculate:

a. The delay angle, α_f.
b. The output voltage ripple factor.
c. The average and RMS thyristor currents.
d. And the input PF.

Ans:

a. The maximum output voltage is corresponding to $\alpha_f=0$.

$$V_{o,av,max} = \frac{3}{\pi} V_{LL,p}$$

$$50\% \left(V_{o,av,max}\right) = \frac{3}{\pi} V_{LL,p} \cos \alpha_f$$

$$\alpha_f = \cos^{-1} 0.5 = 60^{\circ}$$

b. For any α_f:

$$V_{o,rms} = \sqrt{\frac{3}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} \left(V_{LL,p} \sin(\omega t + \frac{\pi}{3})\right)^2 d\omega t}$$

$$V_{o,rms} = V_{LL,p} \sqrt{\left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} (\sin(2\alpha))\right]}$$

At $\alpha_f=60^{\circ}$

$$V_{o,rms} = 208\sqrt{2}\sqrt{\left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}(\sin 120)\right]}$$=272.48V;

$$I_o = 10A$$

C. $I_{T(RMS)} = \frac{I_{o,rms}}{\sqrt{3}} \approx 5.77$A

. $I_{T,av} = \frac{I_{o,av}}{3} \approx 3.33$A

d. Rectification efficiency=$\frac{V_{o,av}I_{o,av}}{V_{o,rms}I_{o,rms}} = 51.4\%$

f. $PF = \frac{I_{s,1}}{I_s} cos\emptyset_1$

$$PF = \frac{3}{\pi}\cos\alpha = 0.477$$

Chapter 3

AC-DC Converters

3-Three-phase full bridge controller rectifier

(RLE –Load-2)

Lecture aims:

1- To derive the continuous conduction condition of the rectifier with RLE load.
2- To obtain the current and voltage expression in discontinuous mode.
3- To layout full analysis flow diagram and apply it in numerical example.
=

1. Continuous conduction condition

It can be seen from the example of the last lecture that:

$$i_{o,MIN} \approx i_o(\alpha_f) \qquad \text{...(1)}$$

We will use equation 1 as a basis to check if the output current continuous or discontinuous. Accordingly, the continuous conduction condition is given by:

$$i_o(\alpha_f) > 0$$

From Eq.(8) last lecture …

$$i_o(\omega t) = \frac{V_{LL,p}}{|Z|}\left[\sin\left(\omega t + \frac{\pi}{3} - \emptyset\right) + \frac{\sin(\emptyset - \alpha_f)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}} e^{-\frac{(\omega t - \alpha_f)}{\tan\emptyset}}\right] - \frac{E}{R}$$

$$i_o(\alpha_f) = \frac{V_{LL,p}}{|Z|}\left[\sin\left(\alpha_f + \frac{\pi}{3} - \emptyset\right) + \frac{\sin(\emptyset - \alpha_f)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}}\right] - \frac{E}{R} > 0$$

Gives:

$$\sin\left(\alpha_f + \frac{\pi}{3} - \emptyset\right) + \frac{\sin(\emptyset - \alpha_f)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}} > \frac{E}{\cos\emptyset V_{LL,p}} \qquad ...(2)$$

Rearrange Eq. (2)

$$\frac{E}{V_{LL,p}} < \left[\sin\left(\alpha_f + \frac{\pi}{3} - \emptyset\right) + \frac{\sin(\emptyset - \alpha_f)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}}\right]\cos\emptyset \qquad ...(3)$$

Fig. 1 shows the condition for the entire range of ($E/V_{LL,P}$) and ϕ.This Fig. 1

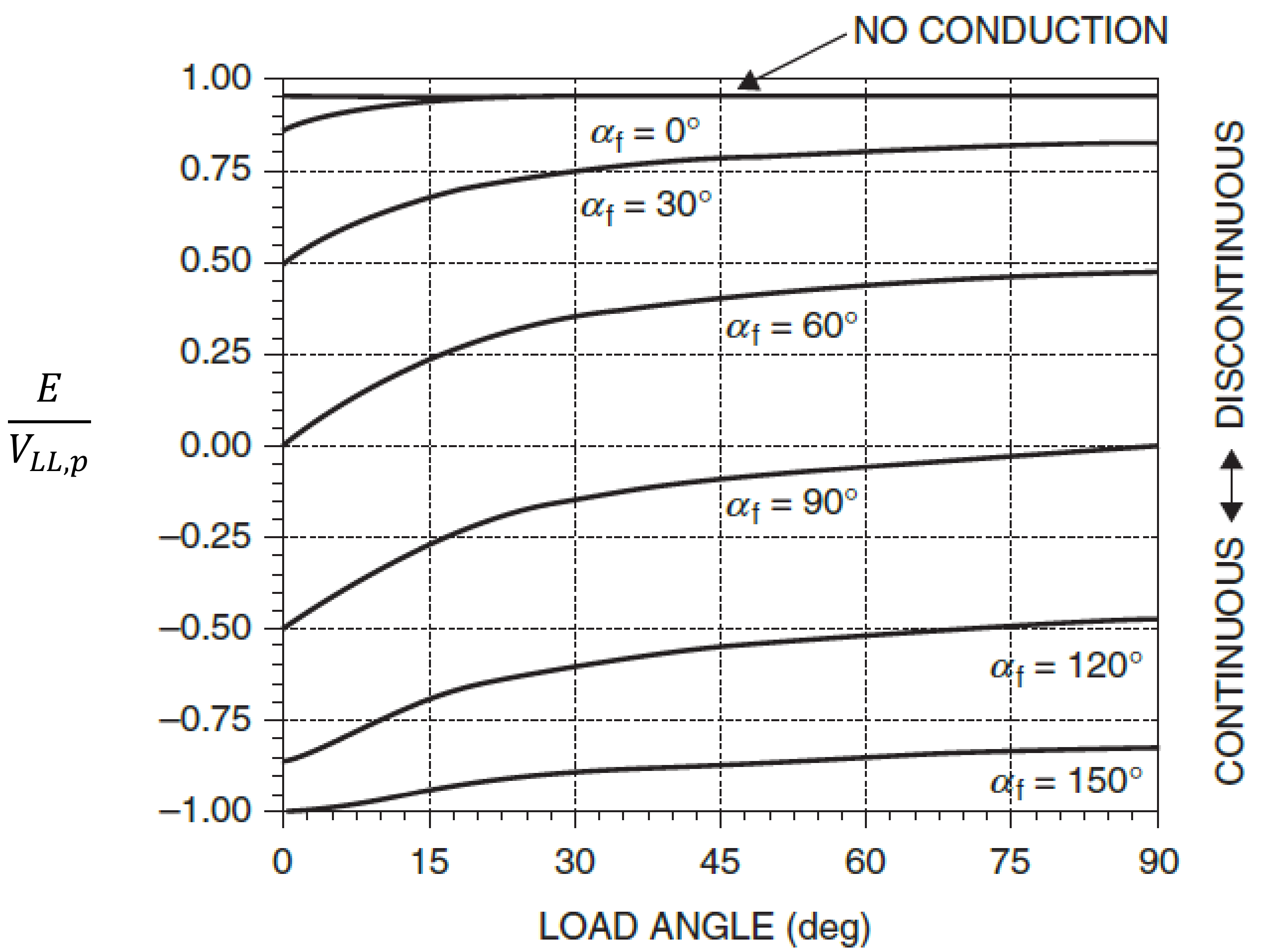


Fig.1 Conduction mode areas of three-phase controlled rectifier

Example 1: A 240V battery bank has an internal resistance of 2Ω, is required to be charged using a three-phase controlled rectifier. The rectifier is connected to a 380V, 50Hz ac line.

A “smoothing “inductor is connected in series with the battery to ensure that the charging current is always continuous.

By adjusting the firing angle the charging current average is changed from 8-to-100A.

a) Determine the range of α_f
b) Determine the minimum inductance required to ensure continuous charging current.

Sol:

a. $V_{o,dc(MIN)} = E + I_{o,dc(MIN)}R = 240 + 8 \times 2 = 256V$

$$V_{o,dc(MAX)} = E + I_{o,dc(MAX)}R = 240 + 100 \times 2 = 340V$$

As the rectifier in continuous mode:

$$V_{o,dc} = \frac{3}{\pi} V_{LL,p} \cos \alpha_f$$

Gives:

$$\alpha_{f(MIN)} = \cos^{-1}\left(\frac{\pi}{3}\frac{340}{380\sqrt{2}}\right) = 48.5°$$

$$\alpha_{f(MAX)} = \cos^{-1}\left(\frac{\pi}{3}\frac{256}{380\sqrt{2}}\right) = 60°$$

For the converter to operate always in continuous mode; the operation mode must be continuous when α_f is maximum (60°). The ratio $E/V_{LL,p}$ =0.4466.

Using Fig. 1 we can find the minimum $\phi_{(MIN)}$ to operate in continuous mode is 58.5°.

(we can obtain a more accurate result by solving for ϕ of Eq. 3)

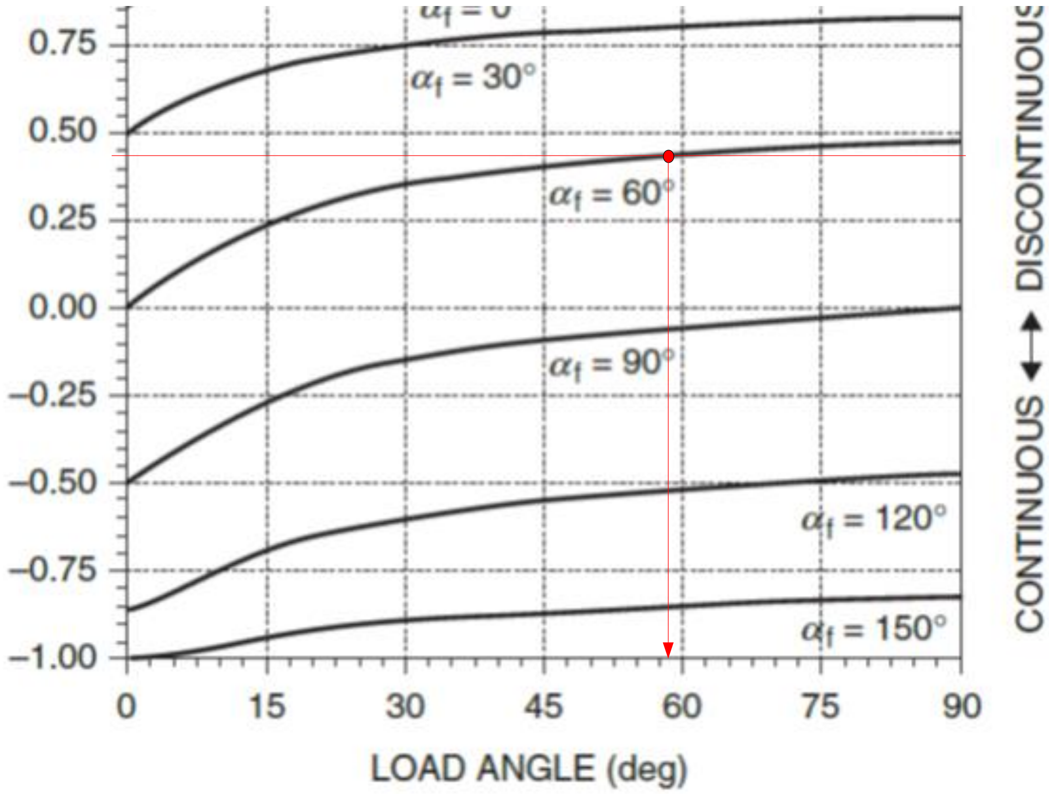


Fig. 2

$$\emptyset = \tan^{-1}\frac{\omega L}{R}$$

$$L_{min} = \frac{R\tan\emptyset_{min}}{\omega} = \frac{2\tan(58.5)}{100\pi} = 10.4mH$$

2. Discontinuous mode :

The rectifier current expression given in Eq. (8), last lecture, is applicable if i_o >0 for the entire range (α_f<ωt< α_f+60°). If not, the condition described in Eq. (3) will not be satisfied. The discontinuous current equation is derived by solving the general equation:

$$i_o = \frac{V_{LL,p}}{|Z|}\sin\left(\omega t + \frac{\pi}{3} - \emptyset\right) - \frac{E}{R} + Ae^{-\frac{(\omega t-\alpha_f)}{\tan\emptyset}} \qquad ...(4)$$

With initial condition $i_o(\omega t = \alpha_f) = 0$. Solving (4) for A gives:

$$A = \frac{E}{R} - \frac{V_{LL,p}}{|Z|}\sin\left(\alpha_f + \frac{\pi}{3} - \emptyset\right) \qquad ..(5)$$

Substitute (5) into (4) gives

$$i_{o,D} = \frac{V_{LL,p}}{|Z|}\left[\sin\left(\omega t + \frac{\pi}{3} - \emptyset\right) - \sin(\alpha_f + \frac{\pi}{3} - \emptyset)\ e^{-\frac{(\omega t - \alpha_f)}{\tan\emptyset}}\right] - \frac{E}{R}\left[1 - e^{-\frac{(\omega t - \alpha_f)}{\tan\emptyset}}\right]; for\ (\alpha_f < \omega t < \alpha_e)$$

...(6)

Where the subscript ($_D$), indicates that the current is discontinuous, and α_e is the extinction angle, the angle it which the current return to zero. An example of the output voltage and current is shown in Fig. 3. It can be seen that the voltage average is different in this case.

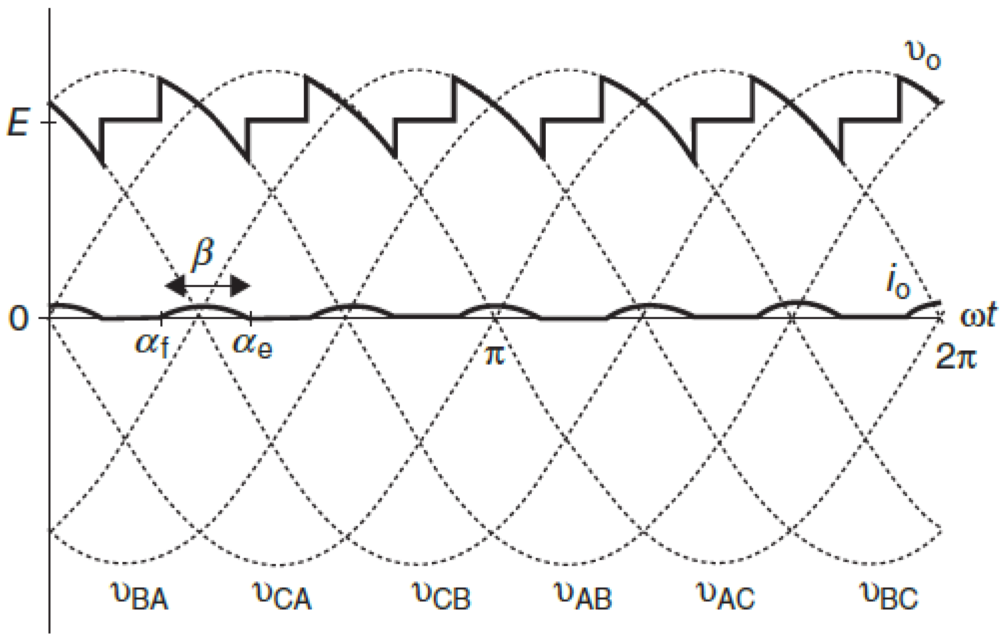


Fig. 3

Derivation of $V_{o,dc}$

$$V_{o,dc} = \frac{1}{\frac{\pi}{3}}\left[\int_{\alpha_f}^{\alpha_e} V_{LL,p}\sin\left(\omega t + \frac{\pi}{3}\right) + E\left(\frac{\pi}{3} - \beta\right)\right] \qquad ...(7)$$

Where $\beta = \alpha_e - \alpha_f$ the conduction angle

$$V_{o,dc} = \frac{3}{\pi}\left[2V_{LL,p}\sin(\alpha_f + \frac{\beta}{2} + \frac{\pi}{3})\sin\left(\frac{\beta}{2}\right) + E\left(\frac{\pi}{3} - \beta\right)\right] \qquad ...(8)$$

3. Analysis Procedure

The analysis procedure presented so far has been assembled shorty in the flow diagram shown in Fig. 4. Without further explanation we will follow this procedure in the following example.

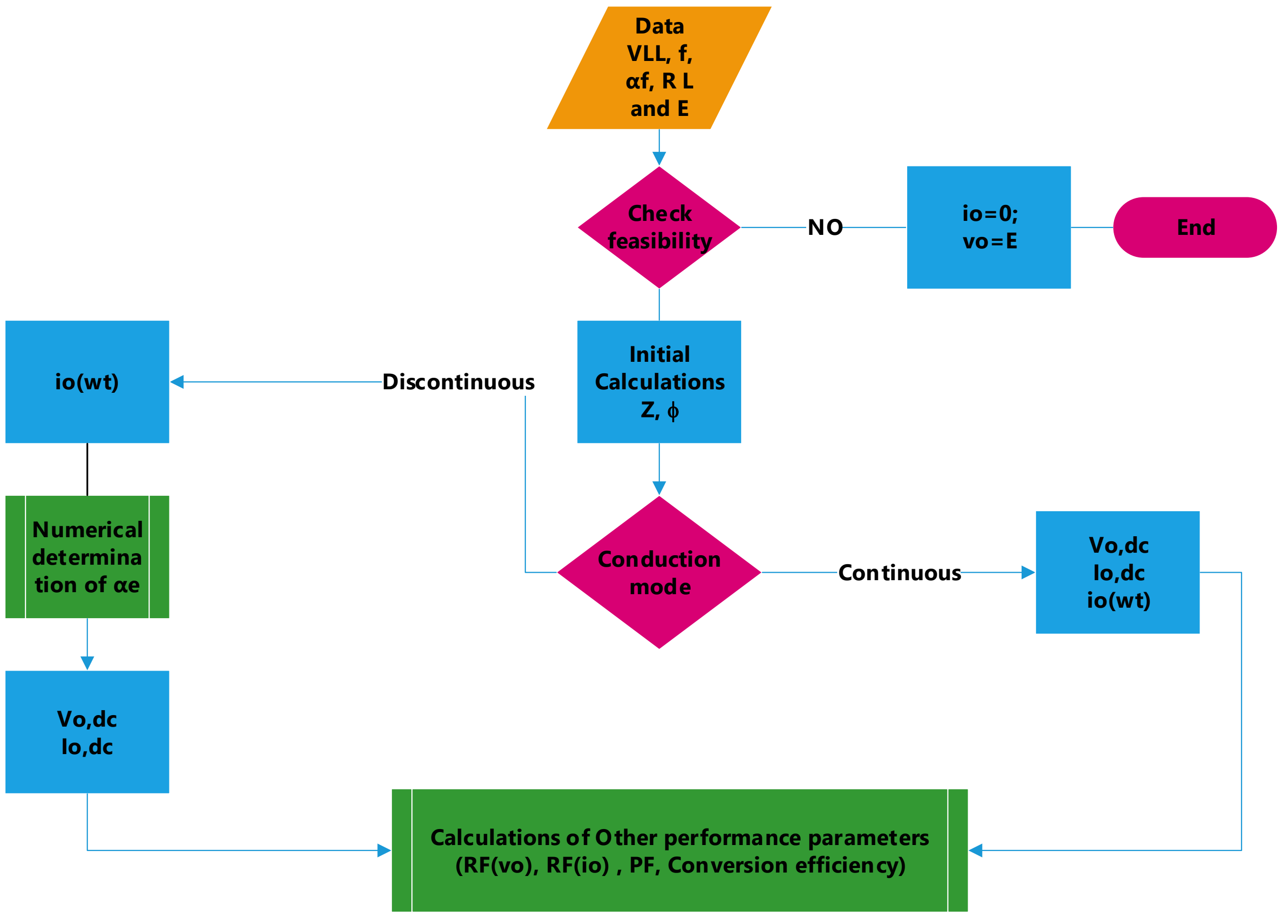


Fig. 4

Example2: A three-phase converter is supplied from a 600V, 50Hz line. The load model has E=650, R=12Ω, L=22mH. With $\alpha_f = 30$, answer the following

a. Check the triggering feasibility.
b. Determine the conduction mode
c. Determine the average output voltage and current
d. Determine the power factor
e. Determine the conversion efficiency

Ans.

a. The range of feasible firing angle,

$$\sin^{-1}\left(\frac{E}{V_{LL,p}}\right) - 60 < \alpha_f < 120 - \sin^{-1}\left(\frac{E}{V_{LL,p}}\right)$$

$$-10 < \alpha_f < 70$$

As the firing angle within the above range, the triggering is ***feasible***.

Before proceeding to the following, determine |Z| and ϕ

$Z=R+j\omega L=12+j100\pi*0.022=13.85\angle 29.94^\circ$

b. The continuous conduction condition

We can check the mode using the condition given in Eq. 3

Approximate ϕ to 30 …

$$\frac{E}{V_{LL,p}} < \left[\sin\left(\alpha_f + \frac{\pi}{3} - \emptyset\right) + \frac{\sin\left(\emptyset - \alpha_f\right)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}}\right]\cos\emptyset$$

$$\frac{650}{600\sqrt{2}} < \left[\sin(60) + \frac{\sin(0)}{1 - e^{-\frac{\pi}{3\tan\emptyset}}}\right]0.866$$

0.766<(3/4)?………not satisfied--→ discontinuous mode

c. We have to start by evaluating α_e

The discontinuous current:

$$i_{o,D} = \frac{V_{LL,p}}{|Z|}\left[\sin\left(\omega t + \frac{\pi}{3} - \emptyset\right) - \sin(\alpha_f + \frac{\pi}{3} - \emptyset)\ e^{-\frac{(\omega t-\alpha_f)}{\tan\emptyset}}\right] - \frac{E}{R}\left[1 - e^{-\frac{(\omega t-\alpha_f)}{\tan\emptyset}}\right]$$

$$i_{o,D} = 61.27\left[\sin(\omega t + 30) - 0.866\ e^{-\frac{(\omega t-30)}{33.1}}\right] - 54.17\left[1 - e^{-\frac{(\omega t-30)}{33.1}}\right]$$

Evaluate $i_{o,D}(\alpha_e) =$ 0 gives α_e=88.2° //using calculator//

$$\beta = 88.2 - 30 = 58.2°$$

$$V_{o,dc} = \frac{3}{\pi}\left[2 \times 600\sqrt{2}\sin\left(30 + \frac{58.2}{2} + 60\right)\sin\left(\frac{58.2}{2}\right) + 650\left(1.8 \times \frac{\pi}{180}\right)\right]$$

$$V_{o,dc} = 708.2V$$

$$I_{o,dc} = \frac{V_{o,dc} - E}{R} = 4.85A$$

d. Power Factor, $PF = \frac{P}{S} = \frac{Po}{\sqrt{3}V_{LL}I_L} = \frac{{I_o}^2R + I_{o,dc}E}{\sqrt{3}V_{LL}(\frac{\sqrt{2}}{\sqrt{3}}I_o)}$

To determine I_o,

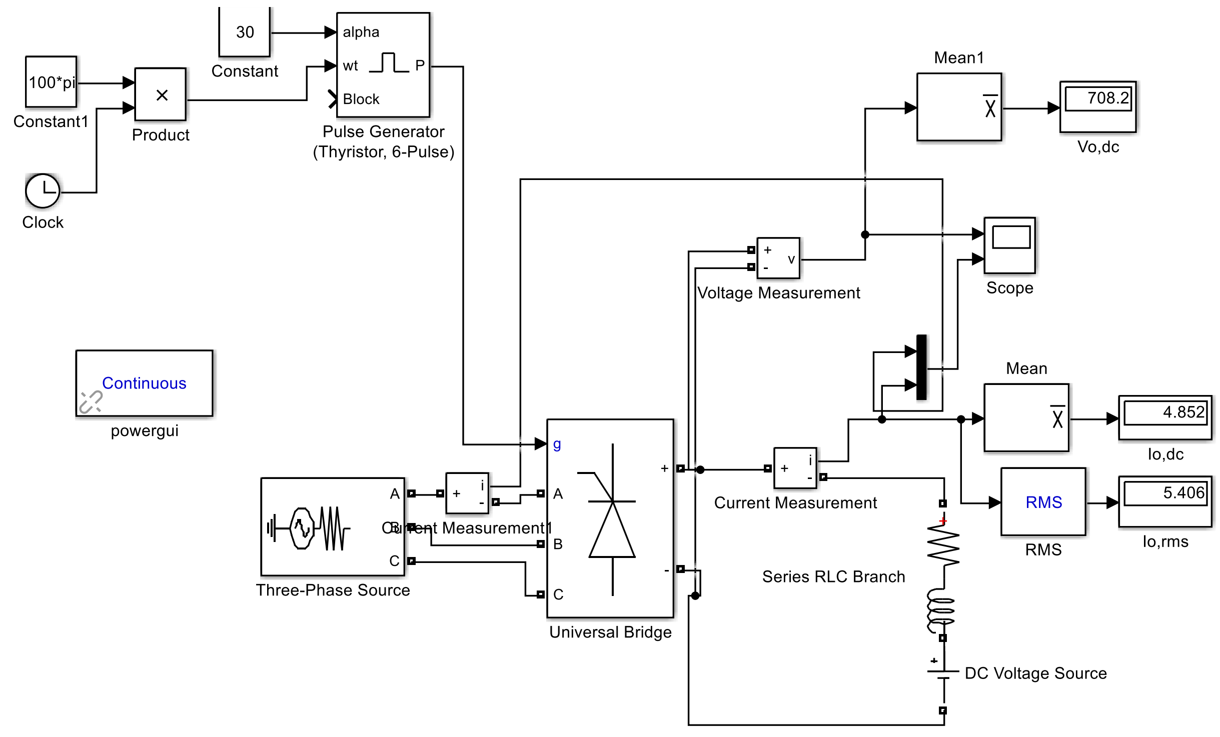
$$i_{o,D} = 61.27\left[\sin(\omega t + 30) - 0.866\ e^{-\frac{(\omega t-30)}{33.1}}\right] - 54.17\left[1 - e^{-\frac{(\omega t-30)}{33.1}}\right]$$

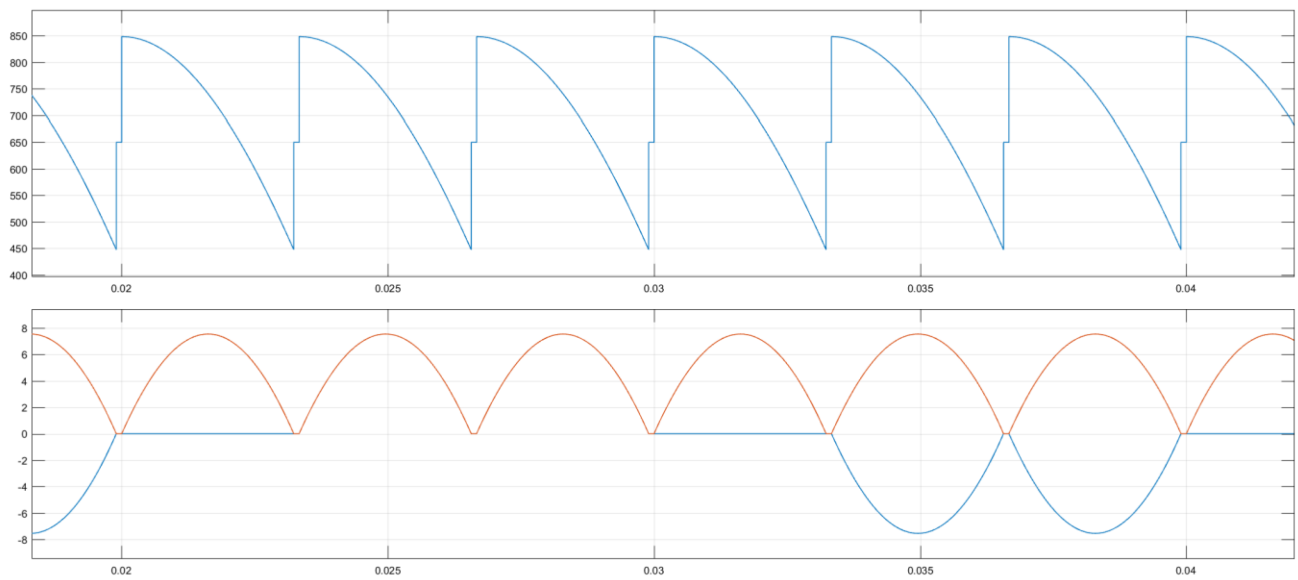
$$I_o = \sqrt{\frac{3}{\pi}\int_{\pi/6}^{1.54}\left[61.27\left[\sin(\omega t + \pi/6) - 0.866\ e^{-\frac{(\omega t-\pi/6)}{0.577}}\right] - 54.17\left[1 - e^{-\frac{(\omega t-\pi/6)}{0.577}}\right]\right]^2 d\omega t}$$

$$= 5.42A$$

$$PF = \frac{5.42^2 \times 12 + 4.85 \times 650}{\sqrt{2} \times 600 \times 5.42} = 0.762$$

The case described in this has been modeled using Simulink, the following figure shows that results and the output voltage, current and input current wavefroms


30
Constant
alpha
wt
P
Block
Pulse Generator
(Thyristor, 6-Pulse)
100*pi
Constant1
Product
Clock
Mean1
708.2
Vo,dc
Scope
Voltage Measurement
Continuous
powergui
Mean
4.852
Io,dc
RMS
5.406
Io,rms
Current Measurement
Current Measurement1
Three-Phase Source
Universal Bridge
Series RLC Branch
DC Voltage Source



Chapter 4

AC Controllers

4.1 **Single-phase AC Controllers**

Lecture aims:

1- To identify the circuit and switching signals of single-phase controller.
2- To analyze of the AC controller with RL load.

1. Single-phase AC Controllers

If the amplitude of an ac voltage needs to be controlled, an ac voltage controller is used. The fundamental output frequency equals the input frequency.

The circuit diagram of an ac voltage controller is shown in Fig.1. A Triac may be used instead of the two antiparallel connected SCRs. When one of the SCRs is conducting, the other SCR is reverse biased. Always the input current, i_i, equals the output current, i_o. With either SCR conducting, the output voltage, v_o, equals the input voltage, v_i.

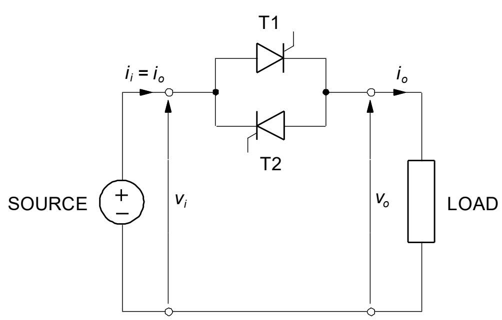


Fig. 1 Single-phase ac voltage controller

الفصل الرابع المسيطرات

4-1 المسيطرات أحادية الطور

أهداف المحاضرة

1- التعرف على دائرة المسيطر أحادي الطور و إشارات القدح
2- تحليل عمل المسيطر أحادي الطور مع حمل حثيRL.

1- المسيطر أحادي الطور.

يستخدم المسيطر المتناوب عندما يكون المطلوب هو التحكم بفولتية الإخراج المتناوبة لدائرة مجهزة بفولتية متناوبة ثابتة المقدار و يكون تردد الإخراج (الأساسي) مساو لتردد الإدخال. يبين الشكل (1) دائرة المسيطر أحادي الطور, حيث يمثل الثايرستوران المربوطان بطريقة التوازي المتعاكس دائرة المسيطر - و يمكن استخدام عنصر ترايك واحد بدلا من الثايرستورين-. في هذه الدائرة لو فرضنا أن فولتية المصدر جيبية بالشكل التالي:

$$v_i = V_{i,p} \sin \omega t$$

يتم قدح الثايرستور T1 عند زاوية قدح α_f و الثايرستور T2 عند $\alpha_f+\pi$. عندما يكون أحد الثايرستورين في حالة توصيل يكون الثايرستور الأخر في حالة انحياز عكسي. و في حالة توصيل أحد الثايرستورين تكون فولتية الإخراج مساوية لفولتية الإدخال و تيار الإدخال مساو لتيار الإخراج.

Output Voltage and Current.

Voltage and current waveforms for a controller with an RL load are shown in Fig.2. When T1 is forward biased and fired at α_f, the conducted current initially increases then drops to zero, as tending to follow the voltage by the phase angle (φ). When the input voltage is negative, T2 is forward biased and ready for firing. The firing occurs at $\omega t = \alpha_f + \pi$. The waveform of current through T2 is a mirror image of that through T1, so both the output current and the voltage have the half-wave symmetry and no dc component.

In Fig. 2a, the firing angle is 45° and in Fig. 2b 135°. An RL load with the load angle, φ, of 30° is assumed. The *minimum feasible firing angle equals the load angle*. It can be seen that when the firing angle increases, the conduction angle, β, decreases and the current pulses become smaller and shorter. Consequently, the rms value of the current and output voltage decreases, as does the rms value of output voltage. The rms value of the output voltage can be found as

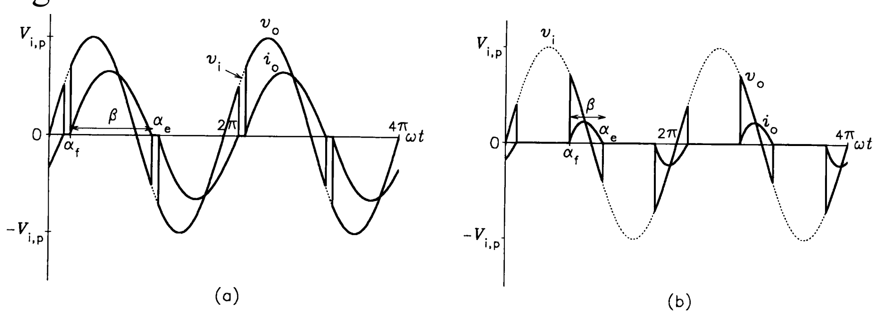


Fig. 2 Output voltage and current of a single-phase ac controller (φ=30°) (a) α_f=45°, (b) α_f=135°

فولتية وتيار الإخراج:

يبين الشكل (2) فولتية و تيار مسيطر يجهز حمل حثي. عند قدح T1 عند زاوية α_f, تظهر فولتية الإدخال الوجبة على الجمل و يبدأ التيار بالزيادة التدريجية ليأخذ تدريجيا الشكل الجيبي متخلفا عن الفولتية بزاوية الطور ϕ. و عند وصول التيار الى الصفر في طريقه للنصف السالب من الموجة الجيبية يتحول الثايرستور T1 الى حالة القطع عند α_e. و عندها تكون فولتية المصدر سالبة و الثايرستور T2 في حالة إنحياز أمامي. يتم قدح T2 عند زاوية $\alpha_f+\pi$. , لينتج عن ذلك القدح نبضة تيار سالبة مطابقة للنبضة الموجبة التي بدأت بعد α_e أي :

$$i_o(\omega t + \pi) = -i_o(\omega t)$$

$$\text{For } \alpha_f < \omega t < \alpha_e$$

في الشكل (2) تم رسم التيار و الفولتية لحمل ذو زاوية طور تساوي 30° و زاوية قدح تساوي 45° (الشكل a) و زاوية قدح 135° (الشكل b). و الملاحظ أنه لزاوية قدح أقل تكون زاوية التوصيل ($\beta = \alpha_e - \alpha_f$) أكبر. و عليه تزداد القيمة الفعالة RMS لكل من فولتية و تيار الإخراج. و فيما يلي حساب القيمة الفعالة لفولتية الإخراج (معادلة (1)).

$$V_o = \sqrt{\frac{1}{\pi}\int_{\alpha_f}^{\alpha_e}\left(V_{i,p}\sin\omega t\right)^2 d\omega t}$$

$$V_o = V_{i,p}\sqrt{\frac{1}{\pi}\left(\alpha_e - \alpha_f - \frac{1}{2}[\sin 2\alpha_e - \sin 2\alpha_f]\right)} \quad ..(1)$$

where $V_{i,p}$ denotes peak of the input voltage, and α_e is the extinction angle. The extinction angle depends on the firing angle, α_f, and the load angle, φ. The load current, $i_o(\omega t)$, for ($\alpha_f < \omega t < \alpha_e$) is given by:

$$i_o(\omega t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) - e^{-\frac{\omega t - \alpha_f}{tan\varphi}}\sin(\alpha_f - \varphi)\right] \quad ..(2)$$

where Z is the load impedance. As for the rectifier case, α_e is the angle at which the current reaches zero.

If $\alpha_f = \varphi$, the current becomes purely sinusoidal, as if the load was directly connected to the supply source.

Voltage Control.

The magnitude control ratio, M, for ac voltage controllers is defined as:

$$M \equiv \frac{V_o(\alpha_f)}{V_{o,max}} \quad \ldots(3)$$

Control characteristics for load angles in the 0–$\pi/2$ range can be determined. If $\varphi = 0$ (purely resistive load), the extinction angle is π. If $\varphi = \pi/2$ (purely inductive load), then and the extinction angle is $2\pi - \alpha_f$. Substituting these values of α_e in Eq. (1), the envelope of control characteristics can be expressed as:

$$V_{o,(\varphi=0)}(\alpha_f) = V_{i,p}\sqrt{\frac{1}{\pi}\left(\pi - \alpha_f + \frac{1}{2}\sin(2\alpha_f)\right)} \quad ..(4)$$

$$V_{o,\left(\varphi=\frac{\pi}{2}\right)}(\alpha_f) = \sqrt{2}V_{o,(\varphi=0)} \quad \ldots(5)$$

تبين المعادلة (1) أن فولتية الأخراج تعتمد - إضافة الى إتساع فولتية الإدخال $V_{i,p}$ و زاوية القدح- على زاوية الإخماد α_e. و زاوية الإخماد بدورها تعتمد على زاوية القدح و زاوية الحمل و تحددها معادلة التيار , المعادلة (2):

تبين معادلة التيار (2), ان الحد الثاني داخل الاقواس [ ] سيساوي صفر إذا كانت $\alpha_f = \varphi$, مما يجعل التيار جيبي خالص. أما إذا كانت $\alpha_f < \varphi$ فإن هذه المعادلة لا تنطبق و ستناقش هذه الحالة في الجزء القادم.

التحكم بالفولتية:

تقدم المعادلة رقم (3) نسبة التحكم M و التي تعرف على أنها نسبة فولتية الإخراج عند قيمة معينه من زاوية القدح الى فولتية الإخراج العظمى. لاحظ أن زاوية الإخماد إذا كان الحمل مقاومي خالص هي π وعليه تكون فولتية الإخراج كما في المعادلة (4) . و الحالة المتطرفة اللأخرى هي حالة الحمل الحثي الخالص و التي تكون زاوية إخماده $2\pi - \alpha_f$

The relationships of Eq. (4) and (5) are shown in Fig.3.

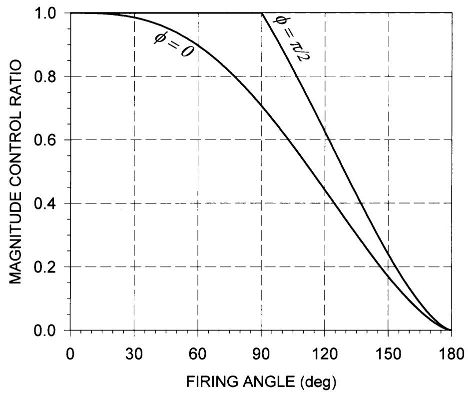


Fig. 3 Control characteristics of a single-phase ac voltage controller

When the firing angle is less than the load angle SCRs and triacs should be fired using a long multi-pulses, otherwise the controlled will (undesirably) operate as a rectifier, since one of the SCRs will be reversed biased when receiving the firing signal, this phenomenon is explained in Fig. 4a. The solution of this problem using multiple pulses is shown in Fig. 4b. The multiple pulses extend over 90° angle.

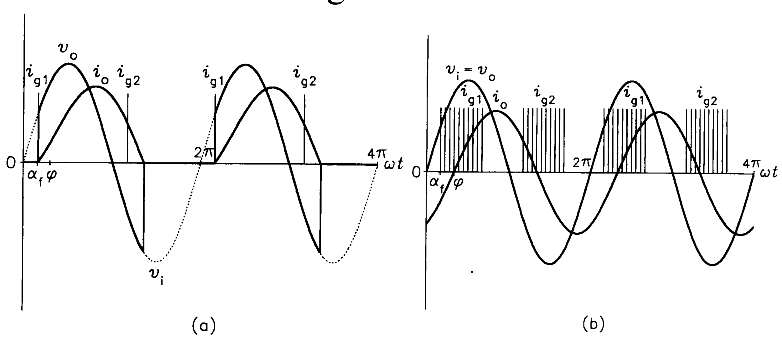


Fig. 4 Operation of the single-phase ac voltage controller with: (a) single-pulse gate signal, (b) multipulse gate signal

تمثل المعادلة (5) تغير فولتية الإخراج للحمل الحثي معطاة مع زاوية القدح.

رسمت المعادلات (4) و (5) في الشكل 3 . لتبين علاقة فولتية الإخراج بزاوية القدح. و نلاحظ أن أصغر زاوية قدح للحمل الحثي الخالص هي 90° (أي زاوية الحمل ϕ). و هنا لابد من الإشارة الى الحالة التي تطبق فيها زاوية قدح أقل من ϕ. و هذه الحالة موضحة في الشكل 4a . فإذا قدح T1 عند زاوية أقل من زاوية الحمل فإن قدح T2 سيكون قبل أن يخمد T1 مما يعني أن T2 سوف لن يستجيب لإشارة القدح.

من أساليب القدح هو تسليط أشارات قدح متتالية كما مبين في الشكل 4 b تبدأ عند α_f و تستمر لفترة 90° . عند القدح بهذه الطريقة فإن القدح بزاوية أقل من زاوية الحمل تجعل فولتية الإخراج مساوية لفولتية الإدخال أو نسبة التحكم M=1.

With the multipulse firing, AC controlled operates correctly never mind if α_f is greater or smaller than φ.

Example 1: A single-phase ac voltage controller is used in a movie theater to control the incandescent lighting. Neglecting the temperature-related changes of resistance of the lamps, find the reduction in power consumed by the lighting when the firing angle is 60°. What firing angle would result in a 50% reduction of that power?

Solution: The incandescent lighting constitutes a resistive load. Consequently, the ratio of rms values of the output and input voltages, that is, the magnitude control ratio, can be found from Eq. (4) as

$$V_{o,(\varphi=0)}\left(\frac{\pi}{3}\right) = V_{i,p}\sqrt{\frac{1}{\pi}\left(\pi - \frac{\pi}{3} + \frac{1}{2}\sin\left(2\frac{\pi}{3}\right)\right)} = 0.897V_{i,p}$$

With a constant resistance, the power is proportional to the squared rms value of voltage, so the resultant power constitutes $0.897^2 = 0.805$ of the full power. Thus, the consumed power is reduced by 19.5%.

The magnitude control ratio for the 50% power reduction is:

$$M = \sqrt{0.5} = 1/\sqrt{2}$$

The control characteristic in Fig. 3 for $\varphi = 0$, the corresponding value of the firing angle is determined as 90°. Notice that at this firing angle, exactly half of the sinusoidal waveform of the input voltage is passed to the load.

مثال 1:

مسيطر فولتية متناوب أحادي الطور أستخدم للسيطرة على الإنارة في قاعة مسرح (من التطبيقات الشائعة لهذه الدوائر). على فرض أن مقاومة المصابيح ثابتة (مع تغير درجة الحرارة)

ما نسبة التقليل في قدرة الحمل عند زاوية قدح 60°.

عند أي زاوية قدح تكون نسبة تقليل القدرة 50%.

Chapter 4

AC Controllers

4.2 Three-phase AC Controllers

Lecture aims:

1- To identify the circuit and switching signals of three-phase Y-connected AC controller.
2- To analyze of the AC controller with R-load.
3- Describe the circuit operation with L load.

1. Three-phase AC Controller

Among several possible circuits, only the most common one: the fully controlled topology with a Y-connected load will be described in this course. Operation of the fully controlled three-phase ac voltage controller, shown in Fig. 5, is more complicated than that of the single-phase controller. To get the controller started, two triacs must be fired simultaneously to provide the path for the current necessary to maintain their on-state.

Operation with a single triac conducting a current is impossible. Therefore, the controller can operate with two or three triacs conducting or none at all. If no triac is conducting, the load is cut off from the supply and all currents and output voltages are zero. The two- and three-triac conduction cases are illustrated in Fig. 6 balanced load is assumed and, the triacs are represented by switches. It can be seen in Fig. 6a that with two triacs, TA and TB, conducting, the line-to-line voltage, v_{AB}, is equally divided between the load impedances in phases A and B. Consequently, $va = v_{AB}/2$, $vb = -v_{AB}/2 = v_{BA}/2$, and $vc = 0$.

4.2 المسيطر ثلاثي الطور

أهداف المحاضرة:

1- التعرف على دائرة المسيطر ثلاثي الطور المربوط بطريقة Y
2- تحليل دائرة المغير عند العمل بحمل مقاومي.
3- وصف عمل الدائرة مع حمل حثي

المسيطر ثلاثي الطور:

في هذا الموضوع سنتطرق الى مسيطر ثلاثي الطور مربوط على شاكلة " Y ". و هذا هو الربط الأكثر شيوعا رغم أن هناك عدة مسيطرات ثلاثية الطور تنتج بطرق ربط أخرى الا أن دراستنا في هذه المادة ستقتصر على المغير Y. يبين الشكل 5 دائرة المسيطر حيث أستخدمت مفاتيح الترایاك ليعوض كل مفتاح عن زوج الثايرسترات المربوطة بطريقة التوازي المتعاكس التي وصفت في المحاضرة السابقة.

إن عمل المسيطر ثلاثي الطور أكثر تعقيدا من عمل ألمسيطر أحادي الطور. فمثلا لكي يبدأ المسيطر بالعمل فعلا يجب قدح مفتاحين (على الأقل) في نفس الوقت و ذلك لتوفير المسار المغلق للتيار الضروري لغلق مفاتيح الترایاك.

لا يمكن للدائرة أن تعمل بمفتاح واحد فقط لعدم وجود مسار مغلق للتيار. لذا فالمسيطر يكون بإحدى ثلاث حالات و التي هي:

-ثلاث مفاتيح في حالة توصيل: الدائرة في هذه الحالة موضحة في الشكل 6b و تكون فولتيات الأطوار للحمل مساوية لفولتيات الأطوار للمصدر (على فرض أن الحمل و المصدر متزنان).

-مفتاحين في حالة توصيل: رسمت الدائرة المكافئة في الشكل 6a و التي تبين أن فولتية الطور للحمل تساوي نصف فولتية الخط للمصدر. في حالة غلق المفتاحين TA , TB مثلا تكون فولتيات الطور للحمل:

$$v_a = \frac{v_{AB}}{2};\ v_b = -\frac{v_{AB}}{2} = \frac{v_{BA}}{2};$$

$$v_c = 0$$

-كل المفاتيح في حالة قطع: في هذه يكون الحمل في حالة قطع من المصدر و تكون فولتيات الأحمال مساوية للصفر و كذلك (بطبيعة الحال) التيارات.
و هذه الحالات الثلاثة هي كل الحالات الممكنة لعمل المسيطر.

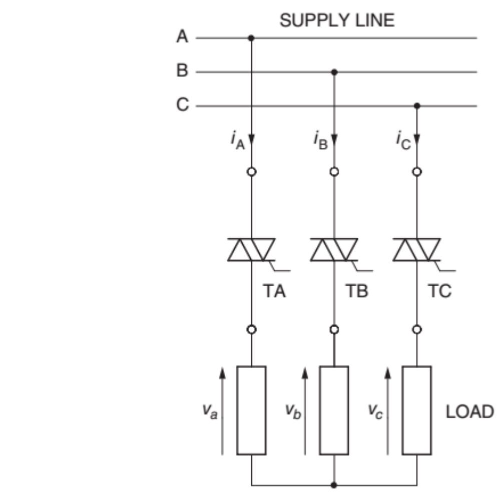


Fig. 5 Fully controlled three-phase ac voltage controller

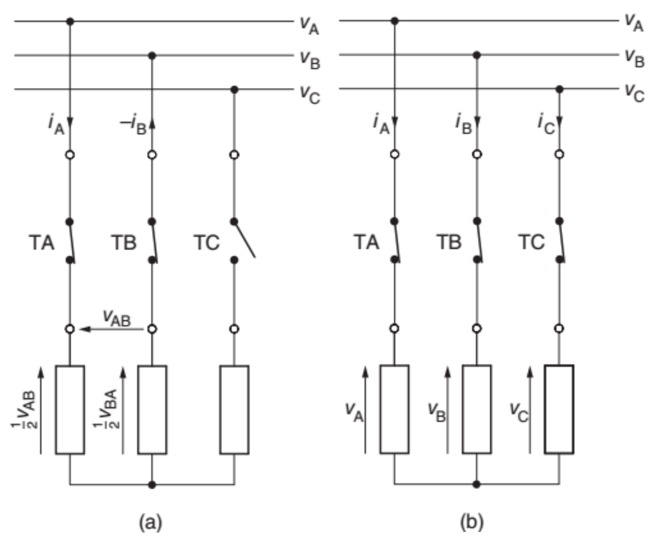


Fig. 6 Voltage and current distribution in a fully controlled three-phase ac voltage controller: (a) two triacs conducting, (b) three triacs conducting.

When all three triacs are conducting, as in Fig. 6b, the output voltages, v_a, v_b, and v_c, equal their input counterparts, v_A, v_B, and v_C.

Switching variables, a, b, and c, can be introduced for triacs TA, TB, and TC, respectively and are equal to 1 when a given triac is conducting and is equal 0 otherwise. It can be shown that the output voltages of the controller are given by:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} * \left[\frac{1}{2} \begin{bmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & c \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} \right]' \quad .(5)$$

Where the prime ($'$) denotes the transposition. If no triac is conducting, then $a = b = c = 0$ and $v_a = v_b = v_c = 0$.

When, for example, triacs TA and TB are conducting, then $a = b = 1$, $c = 0$ and $v_a = a(v_A - v_B)/2 = v_{AB}/2$, $v_b = b(-v_A + v_B)/2 = v_{BA}/2$, and $v_c = 0(-v_A - v_B)/2 = 0$. Finally, if all triacs are conducting, then $a = b = c = 1$ and $v_a = (v_A - v_B - v_C)/2 = [2v_A - (v_A + v_B + v_C)]/2 = (2v_A - 0)/2 = v_A$, and, similarly, $v_b = v_B$, and $v_c = v_C$.

For simplicity, a purely resistive load is assumed, and only the waveform of phase A output voltage, v_a, will be considered. It can be seen from Eq. (5) that, in general, the waveform in question is composed of segments of the v_A, $v_{AB}/2$, and $v_{AC}/2$ waveforms. For example, in Fig. 7, the firing angle, α_f, is zero and all three triacs conduct all the time connecting the source with the load. With the firing angle of 30°, in Fig. 8, the triacs cease to conduct when their currents reach zero. As a result, either two or three triacs are simultaneously conducting and the output voltage , v_a, sequentially equals v_A, $v_{AB}/2$, v_A, $v_{AC}/2$, and zero.

بالإمكان كتابة معادلة للتعبير عن فولتيات الإخراج بدلالة فولتيات المصدر و حالات المفاتيح كما في المعادلة رقم 5. حيث تمثل a, b, c متغيرات ثنائية تمثل حالات مفاتيح التراياك الثلاثة. يأخذ المتغير قيمة 0 إذا كان المفتاح في حالة قطع و القيمة 1 إذا كان المفتاح في حالة غلق.

(ملاحظة المعادلة 5 هي تعديل للمعادلة الموجودة في الكتاب المقرر (5.16))

تغير فولتية الإخراج مع تغير زاوية القدح:

قبل وصف تغير فولتية الإخراج مع زوايا القدح نشير الى أن زاوية القدح تقاس بالنسبة الى فولتية الطور. فمثلا لو كانت

$$v_A = V_{LN,p} \sin \omega t$$

فإن الترياك TA يقدح عند α و $\alpha+\pi$ و يكون القدح على شكل سلسلة من النبضات تبدأ عند α و تنتهي عندما تتغير قطبية الفولتية (أي عند π)

في الفقرات التالية سنعتبر أن الحمل مقاومي صرف و ترسم شكل الفولتية v_a. يبين الشكل 7 حالات مفاتيح التراياك مع فولتية الطور عند زاوية قدح تساوي 0 . و الشكل 8 يبين هذه الموجات عند زاوية قدح 30°.

إعتمادا على زاوية القدح, يمكن تحديد ثلاث صيغ لعمل المسيطر ثلاثي الطور:

-<u>الصيغة الأولى زاوية القدح 0~60°:</u>

في أي وقت هناك 2 أو 3 مفاتيح في حالة غلق.

-<u>الصيغة الثانية زاوية القدح 60° ~ 90°:</u>

في أي وقت هناك مفتاحين في حالة غلق.

-<u>الصيغة الثالثة زاوية القدح 90°~150°</u>

في أي وقت هناك مفتاحين أو 0 في حالة غلق.

تم بيان عمل المسيطر في الصيغة الأولى في الشكلين 7 و 8. أما الصيغة الثانية فمبينة في الشكل 9a (αf = 75°). و الشكل 9b يبين حالات المفاتيح في الصيغة الثالثة (αf = 120°).

تبين المعادلات (6, 7, 8) القيمة الفعالة لفولتيات الإخراج.

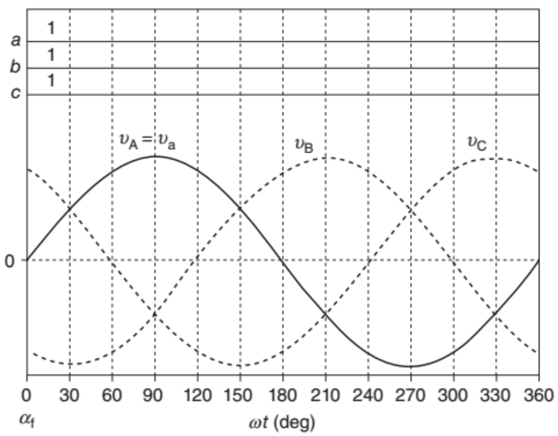


Fig. 7 Traic states and output voltage ($\alpha_f = 0$)

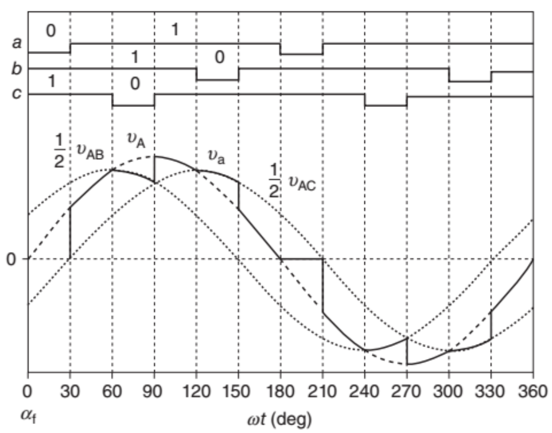


Fig. 8 Traic states and output voltage ($\alpha_f = 30°$)

Operation of the controller with large firing angles is illustrated in Fig. 9. An example waveform of the output voltage at αf = 75° is shown in Fig. 9a and that at αf = 120° in Fig. 9b.

For completeness, formulas for the rms output voltage, V_o, of the fully controlled ac voltage controller with purely resistive and purely inductive loads are provided below without derivation.

Resistive load:

$$V_o = V_i\sqrt{\frac{1}{\pi}\left[\pi - \frac{3}{2}\alpha_f + \frac{3}{4}\sin(2\alpha_f)\right]} \qquad (6)$$

for $0 \le \alpha_f < 60°$,

$$V_o = V_i\sqrt{\frac{1}{\pi}\left[\frac{\pi}{2} + \frac{3\sqrt{3}}{4}\sin\left(2\alpha_f + \frac{\pi}{6}\right)\right]} \qquad (7)$$

for $60° \le \alpha_f < 90°$, and

$$V_o = V_i\sqrt{\frac{1}{\pi}\left[\frac{5}{4}\pi - \frac{3}{2}\alpha_f + \frac{3}{4}\sin\left(2\alpha_f + \frac{\pi}{3}\right)\right]} \qquad (8)$$

for $90° \le \alpha_f < 150°$.

Inductive load:

$$V_o = V_i\sqrt{\frac{1}{\pi}\left[\frac{5}{2}\pi - 3\alpha_f + \frac{3}{2}\sin(2\alpha_f)\right]} \qquad (9)$$

for $90° \le \alpha_f < 120°$

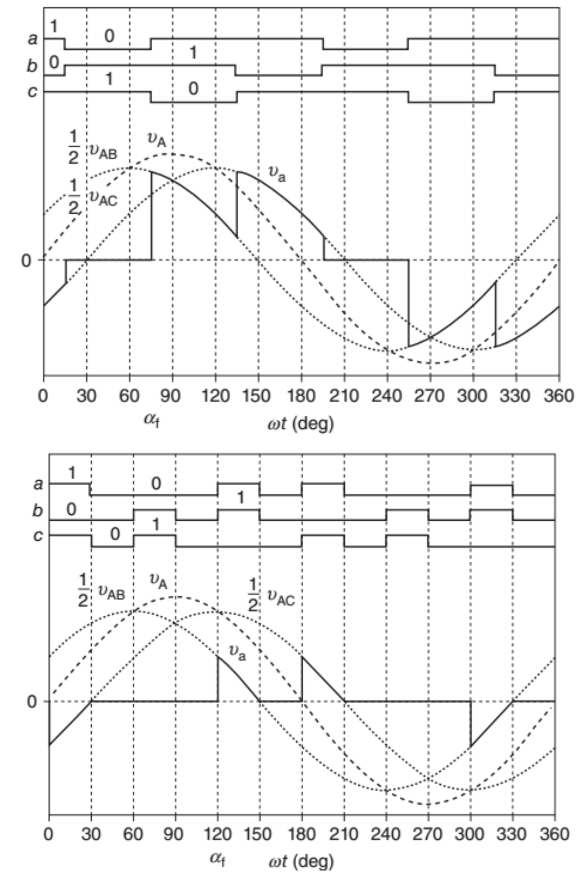


Fig. 9 phase voltage of a three-phase controller with triggering angle (a) 75° and (b) 120°.

$$V_o = V_i\sqrt{\frac{1}{\pi}\left[\frac{5}{2}\pi - 3\alpha_f + \frac{3}{2}\sin(2\alpha_f+)\right]} \qquad (10)$$

for $120° \le \alpha_f < 150°$.

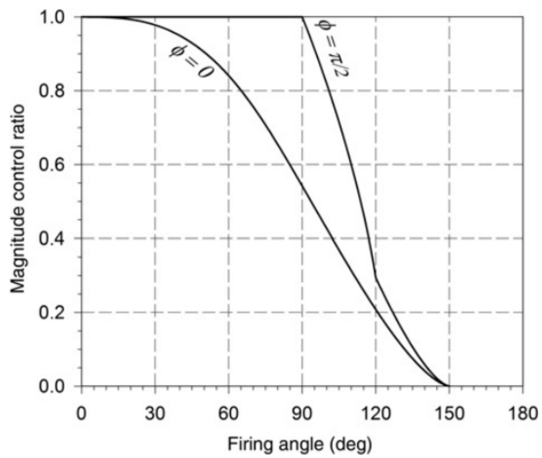


Figure 5.11 Envelope of control characteristics, $V_o = f($

Chapter 5

DC Choppers

First-Quadrant Choppers

Power electronic choppers are dc-to-dc converters with adjustable average output voltage. This lecture covers the circuit diagram, control and calculations of the first-quadrant chopper.

Operation Quadrant

The operation quadrant refers to the possible directions/polarities of the converter output voltage and current. As explained in Fig. 1, the four quadrants are defined as follows:

- First Quadrant: when $v_o$and $i_o$are positive.
- Second Quadrant: when $v_o$is positive and $i_o$is negative.
- First Quadrant: when $v_o$and $i_o$are negative.
- First Quadrant: when v_o is negative and $i_o$is positive.

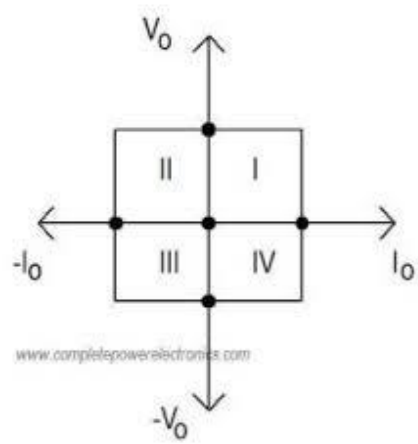


Fig. 1 Operation quadrant based on voltage and current polarity.

The converters can be classified according to the operation quadrants, for instance the six-pulse diode rectifier is a first quadrant; while fully controlled rectifier is a first and fourth quadrant converter. All converter have one, two or four operation quadrants.

The switching variable

The concepts of the switching variables and switching functions have been presented in Chapter 1. Recall that a switching variable (*x*) is a binary variable that determines the switch state being turns on or off, when *x* is 1 or zero respectively.

The switching function *x(t)* determines the value of the switching variable using some (mathematical or logical) basis. Referring to Fig.2, the periodic switching function has two basic attributes:

- the switching frequency, f_{sw}:

$$f_{sw} = \frac{1}{T_{sw}} = \frac{1}{t_{ON}+t_{OFF}} \quad ..(1)$$

And the duty ratio,

$$d \equiv \frac{t_{ON}}{T_{sw}} = \frac{t_{ON}}{t_{ON}+t_{OFF}} \quad ..(2)$$

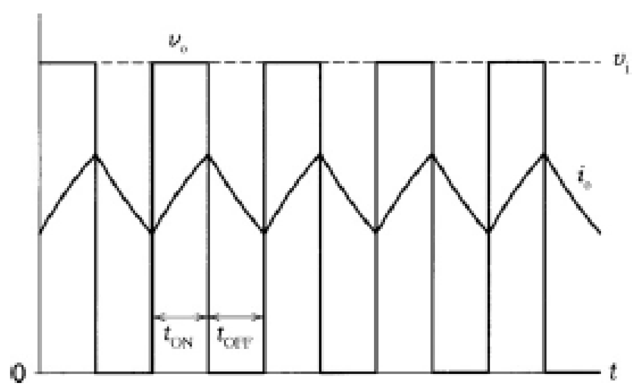


Fig. 2 A switching function

Step-down choppers

Choppers are dc-to-dc converters with adjustable $V_{o,dc}$. The dc source is periodically connected and disconnected results a pulsed output voltage waveform. Neglecting switch voltage drops, the magnitude of pulses of the output voltage equals the input voltage, Vi. The average output voltage, *Vo,dc*, depends linearly on the duty ratios of chopper switches and, generally, can be adjusted in the –Vi to +Vi range.

The magnitude control ratio, *M*, can be defined as

$$M = \frac{V_{o,dc}}{V_i} \qquad \text{...(3)}$$

Block diagram of a chopper and the equivalent circuit for dc components of the output voltage and current are shown in Fig, 3.

The differential equation of the load is

$$L\frac{di_o}{dt} + Ri_o + E = v_o \qquad \text{...(3)}$$

Which has a solution of the general form:

$$i_o(t) = \frac{v_o - E}{R} + \left[\frac{E - v_o}{R} + i_o(t_o)\right] e^{-\frac{R}{L}(t - t_o)} \qquad \text{...(4)}$$

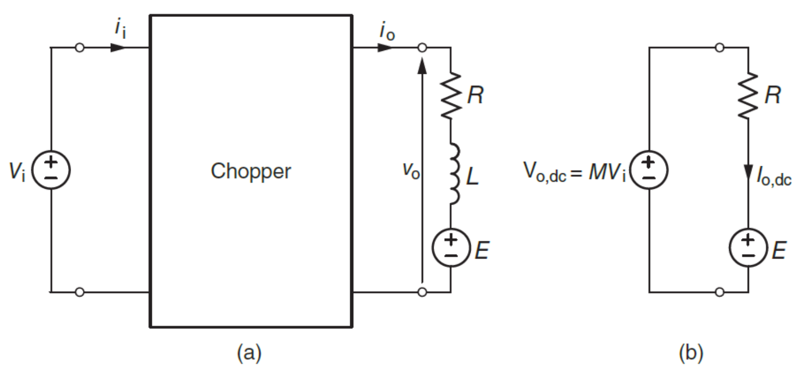


Fig. 3 Step-down chopper: (a) block diagram, (b) dc equivalent circuit for the output.

The load average current is found by considering the dc components at the output circuit as follows:

$$I_{o,dc} = \frac{V_{o,dc}\_E}{R} = \frac{MV_{i}\_E}{R} \qquad ..(5)$$

As high-frequency on–off switching is applied to limit the current ripple; this chapter, applies a linearized approximate method to determine load current (i_o) as explained in the next section.

First-quadrant chopper

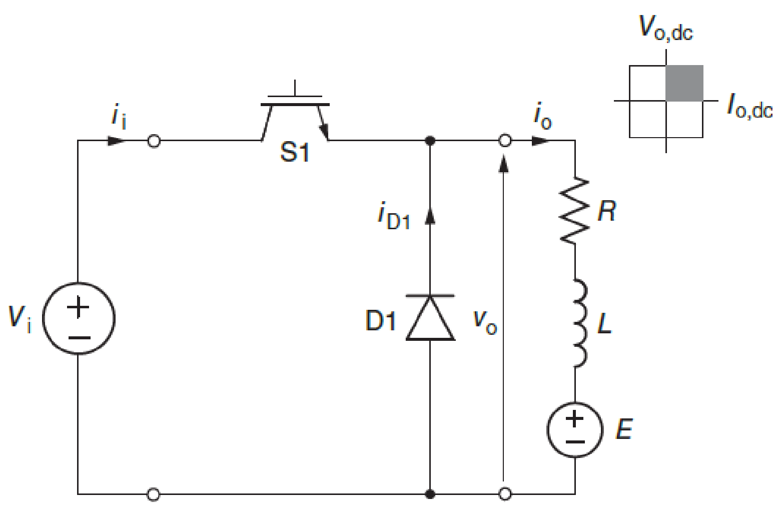


Fig. 4

The first-quadrant chopper can only produce positive average voltage and current, and the average power flows always from the source to the load. Fig. 4 shows the circuit diagram of the chopper. A periodic binary switching variable, x_1, can be assigned to the fully controlled switch S1, and the output voltage of the chopper can be expressed as:

$$v_o = x_1 V_i \qquad ..(6)$$

The switching variable determines the state of chopper. Equivalent circuits of the chopper in state 1 and state 0 are shown in Fig. 5a and 5b, respectively.

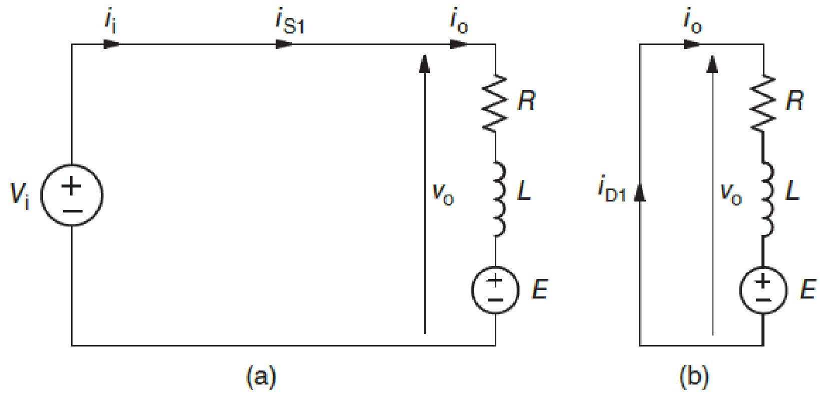


Fig. 5

In state 1, $v_o = V_i$, and the output current follows a growth function as in Eq. (4) with $v_o = V_i$. In state 0, and the output current decays according to Eq. (4) with $v_o = 0$.

Denoting by t_{ON} and t_{OFF} the times during which switch S1 is on and off, that is, the chopper is in state 1 and state 0, respectively, the dc-output voltage, Vo,dc, can be calculated as a time-weighted average of output voltages in the equivalent circuits in Fig. 5. Specifically,

$$V_{o,dc} = \frac{t_{on}V_i + t_{OFF}*0}{t_{ON}+t_{OFF}} = \frac{t_{on}V_i}{T_{sw}} = d_1 V_i \qquad ..(6)$$

where d_1 is the duty ratio of switch S1. Substitute, into (3) gives for first quadrant converter:

$$M = d_1 \qquad ...(7)$$

As the output current is always positive, this defines the range of the M as

$$\frac{E}{V_i} \leq M \leq 1 \qquad ..(8)$$

E/Vi represents the minimum duty ratio of the switch for continuous conduction. If operation with a lower value of d is attempted, the output current becomes discontinuous.

Output current in choppers can be assumed piecewise linear if high switching frequencies employed. This approximation simplifies the analysis of choppers, as shown below.

A single cycle of the output voltage and current in the steady state of a first quadrant chopper is shown in Fig. 6. The output current, i_o, increases by Δio during the t_{ON} and decreases by the same amount during the t_{OFF}.

The waveform of Δio is triangular, and its rms value, Io,ac = Δio/√3.

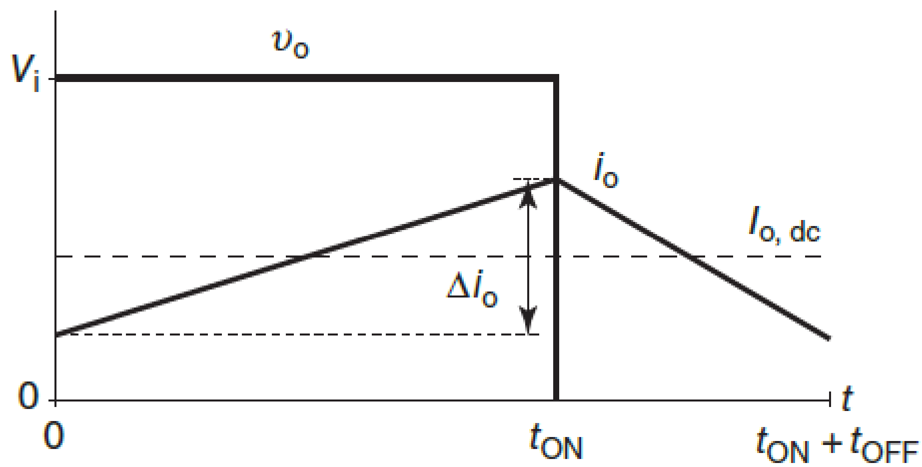


Fig. 6

From Eq. 3

$$di_o = \frac{1}{L}(v_o - E - i_o R)dt \qquad ..(9)$$

$$di_o \approx \frac{1}{L}\left(v_o - E - I_{o,dc} R\right)dt \qquad ..(10)$$

When the switch is on

$$\Delta i_o = \frac{1}{L}\left(V_i - E - I_{o,dc} R\right)t_{on} = \frac{1}{L}\left(V_i - V_{o,dc}\right)t_{on} \qquad ..(11)$$

When the switch is off

$$-\Delta i_o = \frac{1}{L}\left(0 - E - I_{o,dc}R\right)t_{OFF} = \frac{1}{L}\left(-V_{o,dc}\right)t_{OFF}$$

$$\Delta i_o = \frac{1}{L}\left(V_{o,dc}\right)t_{OFF} \qquad \text{...(12)}$$

We can show that (11) and (12) are equivalent as follows:

$$\Delta i_o = \frac{1}{L}\left(V_i - V_{o,dc}\right)t_{on} = \frac{1}{L}(V_i - MV_i)MT_{sw} = \frac{T_{sw}V_i}{L}M(1-M)$$

$$\Delta i_o = \frac{1}{L}\left(V_{o,dc}\right)t_{OFF} = \frac{1}{L}(MV_i)(1-M)T_{sw} = \frac{T_{sw}V_i}{L}M(1-M)$$

Example 1 A step-down chopper fed from a 200-V source operates a dc motor whose armature EMF is 70 V and armature resistance is 0.5 Ω. With the magnitude control ratio of 0.4, find the average output voltage and current of the chopper and determine the quadrant of operation.

The chopper operates with a switching frequency of 1 kHz and the armature inductance of the motor is 20 mH. Find the ac component of the armature current and ripple factor of the current.

Chapter 5

DC Choppers

Second, First & Second and First & Fourth Quadrant Choppers

This lecture presents three types of DC Choppers, for each type, the circuit diagram is presented; the control signals and the output voltage are defined and the magnitude control ratio limits are defined.

Second quadrant chopper

The second quadrant chopper produces a positive voltage and negative current. The circuit diagram is shown in Fig. 1 and the equivalent circuits when the switch is on and off are shown in Fig 2 (a) and (b) respectively. When the switch is on (for interval of $d_2 Tsw$) the load is short circuited and the output voltage is zero. When the switch is off the load current passes through the diode and connects the load to the supply which results an output voltage $=V_i$. Therefore, the average of the output voltage:

$$V_{o,dc} = \frac{(1-d_2)T_{sw}V_i}{T_{sw}} = (1-d_2)V_i \qquad ..(1)$$

The magnitude control ratio is given by:

$$M = (1-d_2) \qquad ..(2)$$

The output current must be maintain negative, otherwise the chopper will fall into the discontinuous region. This imposes the basic condition:

$$I_{o,dc} < 0 \rightarrow E > V_{o,dc} \rightarrow MV_i < E$$

Or more precisely

$$0 < M < \frac{E}{V_i} \qquad ..(3)$$

The corresponding range of d2

$$\left(1 - \frac{E}{V_i}\right) < d_2 < 1 \qquad ..(4)$$

Notice that the absolute minimum of d is 0. So if E>Vi the left term of Eq. (4) becomes zero.

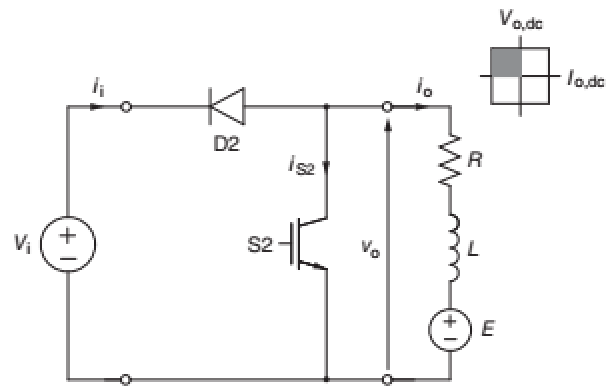


Fig. 1 Second Quadrant Choppers

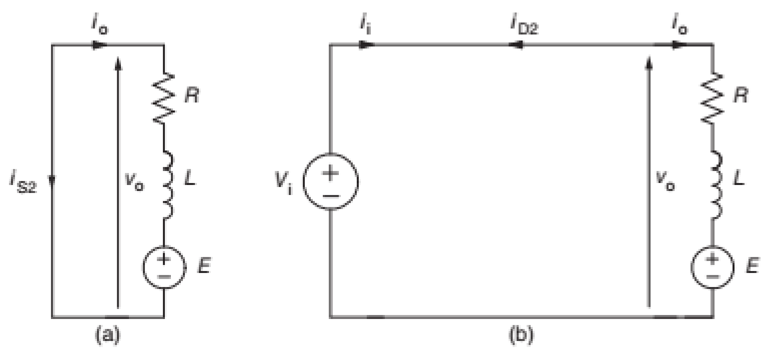


Fig. 2 The two states of second quadrant chopper

A second quadrant chopper has been simulated using Simulink with the following parameters: V_i=100V, E=80V, d2= (a) 0.25 and (b) 0.75 R=10Ω, L=20mH and f_{sw}=2kHz.

Simulink model is shown in Fig. 3; and the results for d2=0.25 and d2=0.75 are shown in Figs. 4(a) and 4(b) respectively. The callout boxes comments to compare the simulation results to the equation indicated earlier.

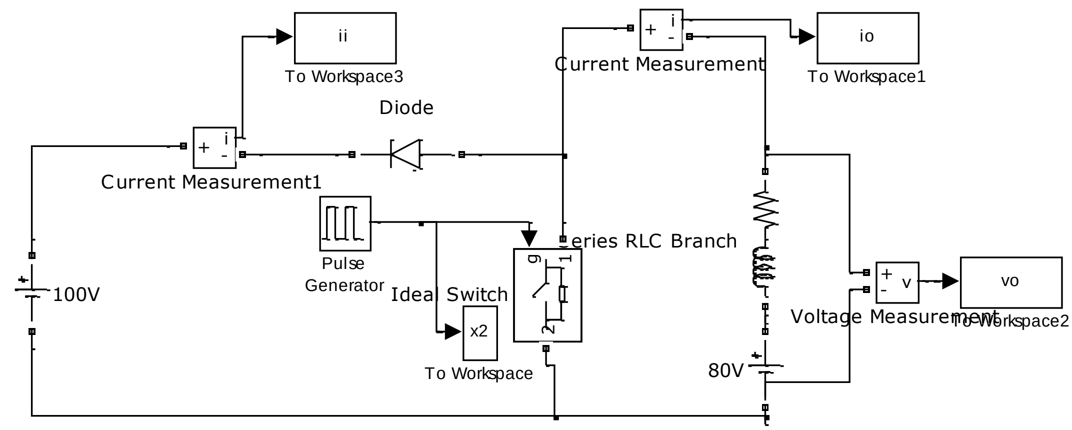
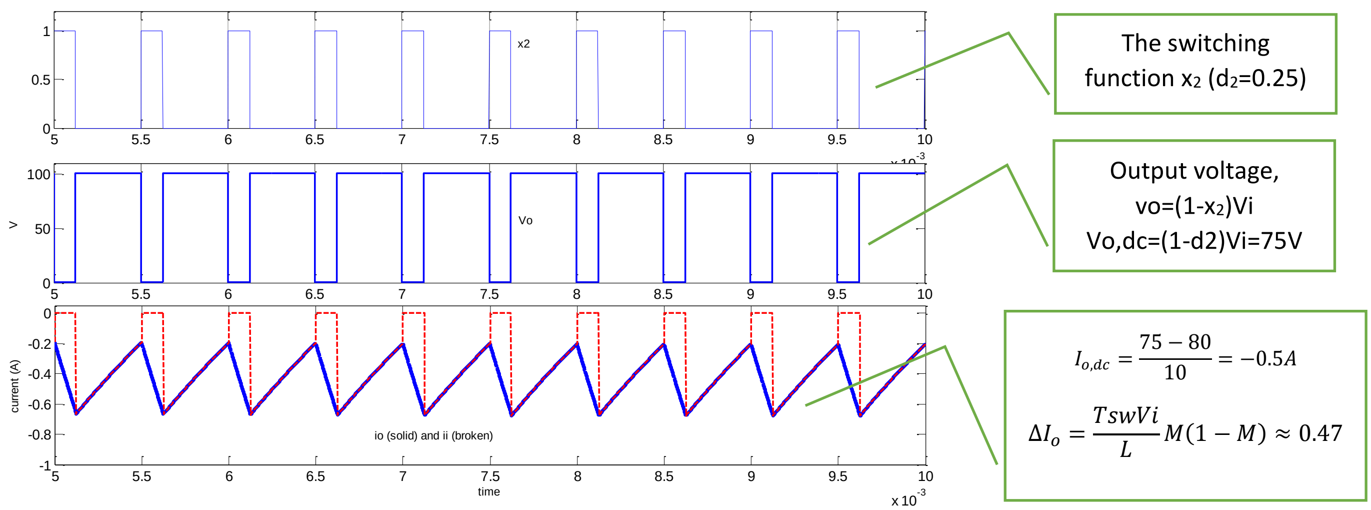
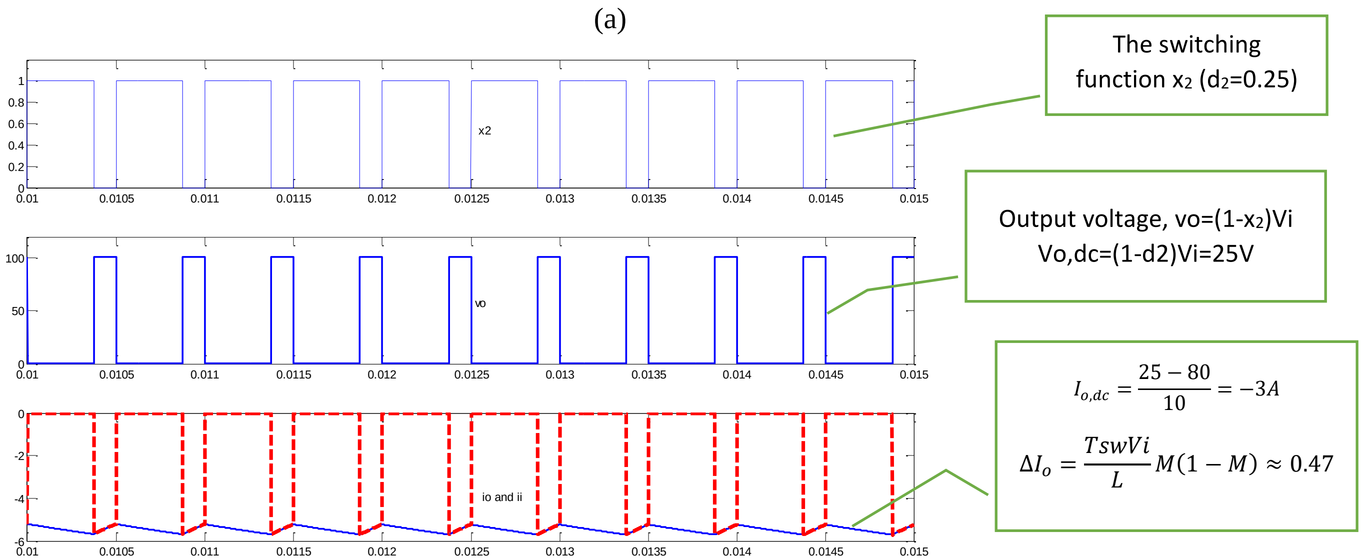


Fig. 3 Simulink model for second quadrant chopper



(a)



(b)

Fig. 4 Simulation of second quadrant chopper

First and second quadrant chop

The first and second quadrant chopper produces output current in two directions. Circuit diagram of a first-and-second quadrant chopper is shown in Fig. 5. The chopper is a combination of the first- and second quadrant choppers. Denoting by $x1$ and $x2$ switching variables of switches S1 and S2, respectively. The chopper can be operated as two independent choppers by keeping the inactive switch off. In this case the equations derived for the first- and second quadrant choppers are directly applied. To control the chopper in this way, we need to know the current direction. So this method of control is not used.

The usual method to control the first and second quadrant chopper is to apply a switching function to S2 that is a logical inverse of the switching function of S1. i.e:

$$x_2 = \overline{x_1} \quad ..(5)$$

Leads to the relationship $d_2 = 1 - d_1$

This does not affect the desired output voltage as

$$M = d_1 = 1 - d_2 \quad ..(6)$$

If the condition given in Eq. (5) is applied, the chopper can smoothly move between the first and second quadrants and the load current will be always continuous.

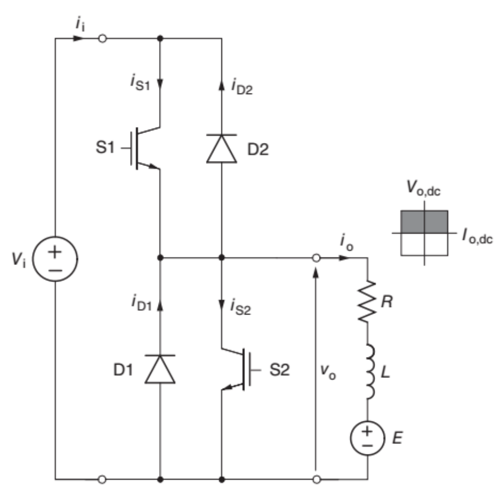


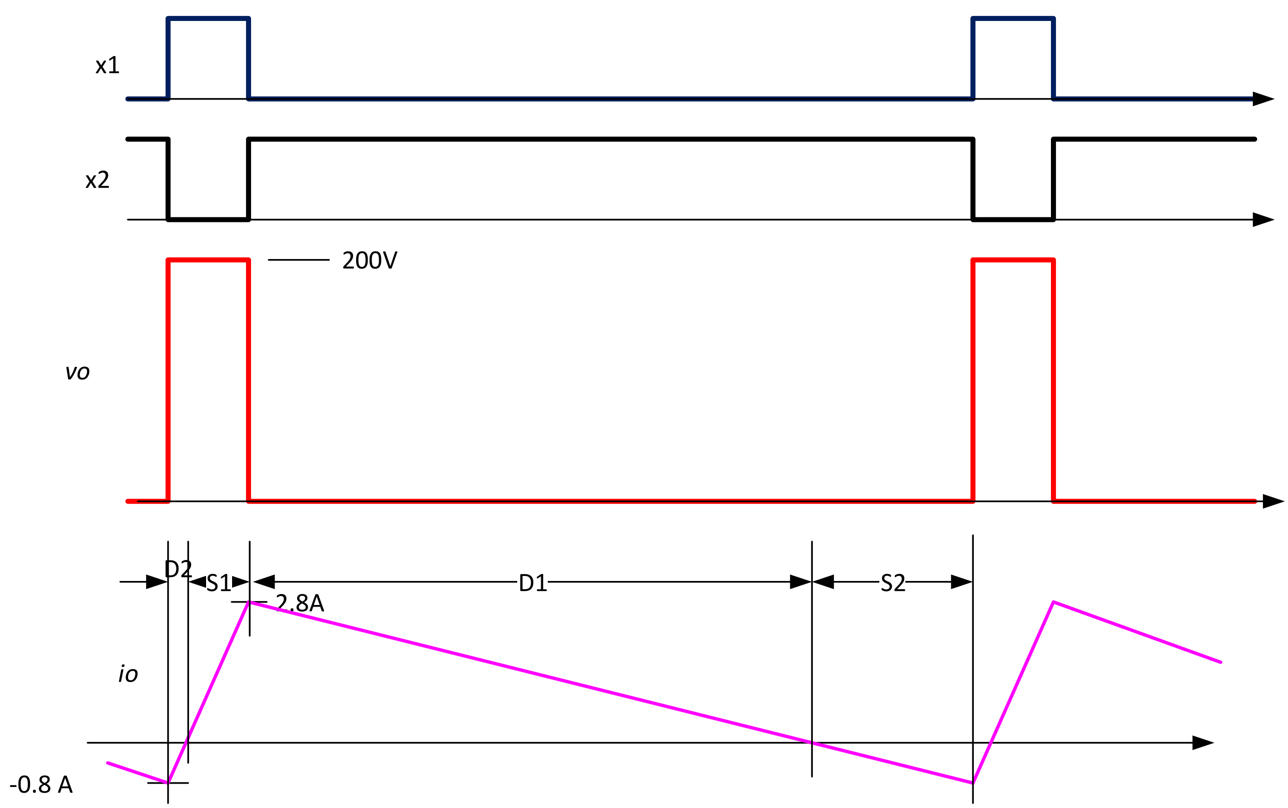
Fig. 5 First and second quadrant chopper

Example 1: A first and second quadrant chopper is supplied from a 200V dc source is operated with M=0.1 and supplying an RL load of 20Ω and 5mH and switching frequency of 1kHz. Calculate:

a. $V_{o,dc}$ and $I_{o,dc}$
b. Δi_o
c. Draw on the same time scale the waveforms of the following quantities:
   The two switching variables; the output voltage; the output current. Mark the conducting switch (or diode) over the switching period

Ans.

a. $V_{o,dc} = MV_i = 20V;\ I_{o,dc} = \frac{V_{o.dc}}{R} = 1A$
b. $\Delta i_o = \frac{V_i}{Lf_{sw}} M(1-M) = \frac{200}{0.005*1000} 0.1 * 0.9 = 3.6A$
c.



First and fourth quadrant chopper

The first-and-fourth-quadrant chopper (Fig. 6) can produce positive and negative output voltage with positive output current. The load emf (E) must be positive for the first-quadrant operation and negative when the chopper is to operate in the fourth quadrant.

The state of the chopper is designated as (x1x4)2, where x1 and x4 denote switching variables of switches S1 and S4, respectively.

Fig. 7 explains the control method. For the first-quadrant operation, switch S4 must remain on to provide a path for the output current. Switch S1 performs the chopping, with the duty ratio d1, so the chopper operates alternatively in states 1 (01) and 3 (11). In the fourth quadrant, switch S1 is off, S4 operates with the duty ratio d4, and the chopper alternates between states 0 (00) and 1 (01). This is explained in Fig. 7

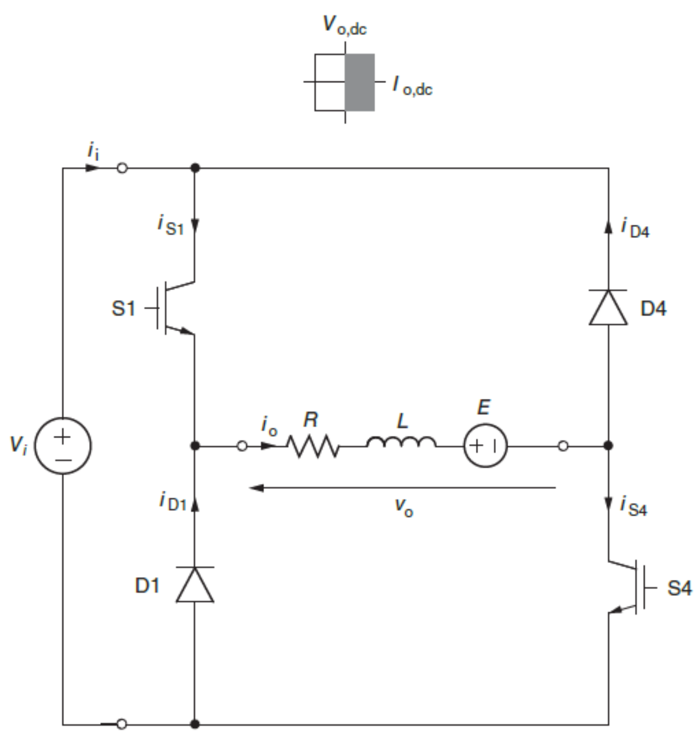


Fig. 6 First and fourth Quadrant Chopper

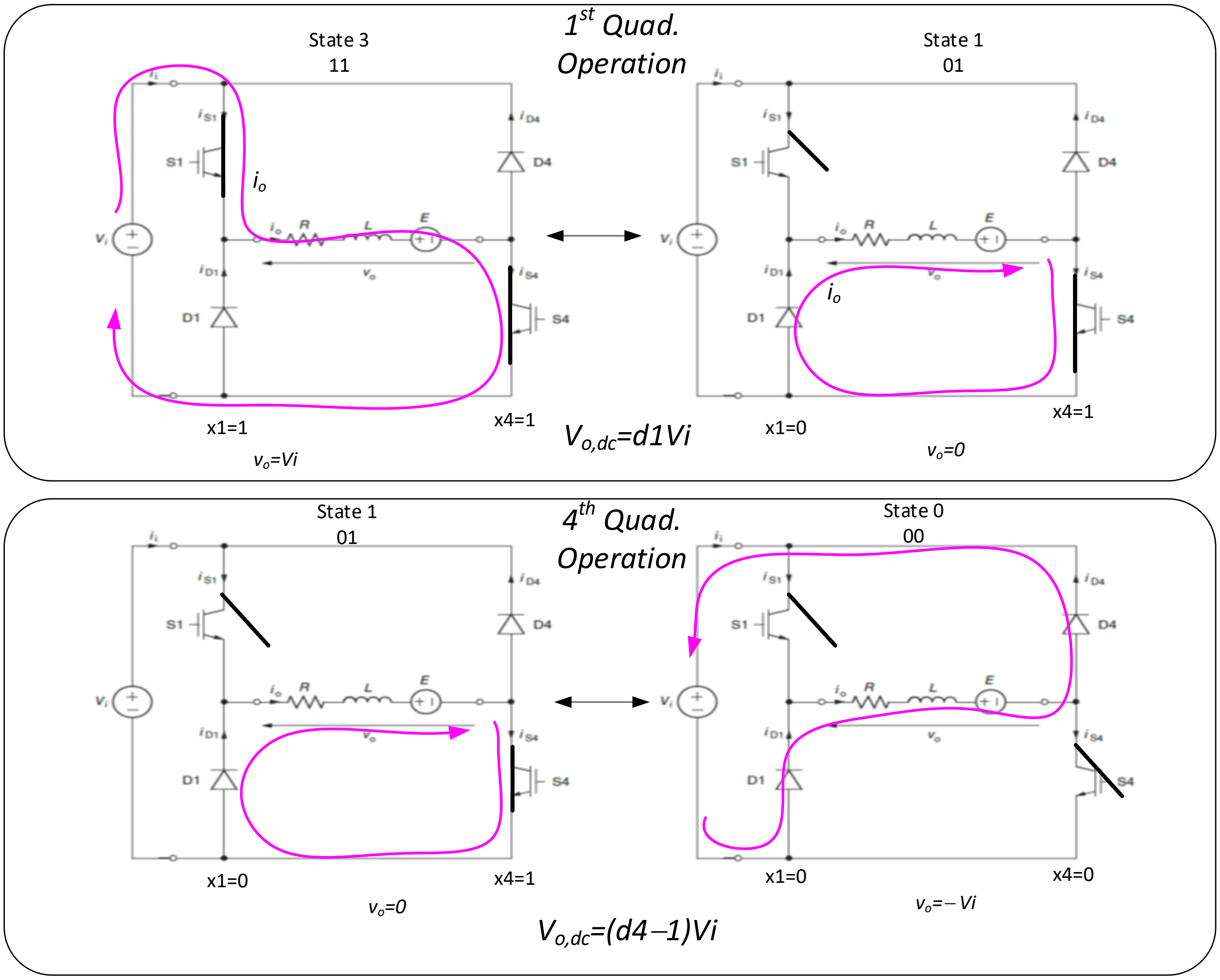


Fig. 7 Operation of 1st and 4th Quadrant Chopper

Output voltage averages expression for the two quadrants are indicated in Fig. 7, from which it can be seen that

For first quadrant: $M = d1$

And for 4th quadrant: $M = d4 - 1$

It has to be stressed that the output current is always positive, i.e. E<Vo,dc. This implies that E must be negative for fourth quadrant operation.

Chapter 5

DC Choppers

Four Quadrant Choppers

This lecture presents the four quadrant DC Choppers, the circuit diagram and its control for four quadrants operation are given. Analysis are explained by two examples.

The four quadrant chopper circuit

The four-quadrant chopper is the most flexible of all step-down choppers. Its power circuit, shown in Fig.1, is composed of four power switches (S1 through S4) and four freewheeling diodes (D1 through D4).

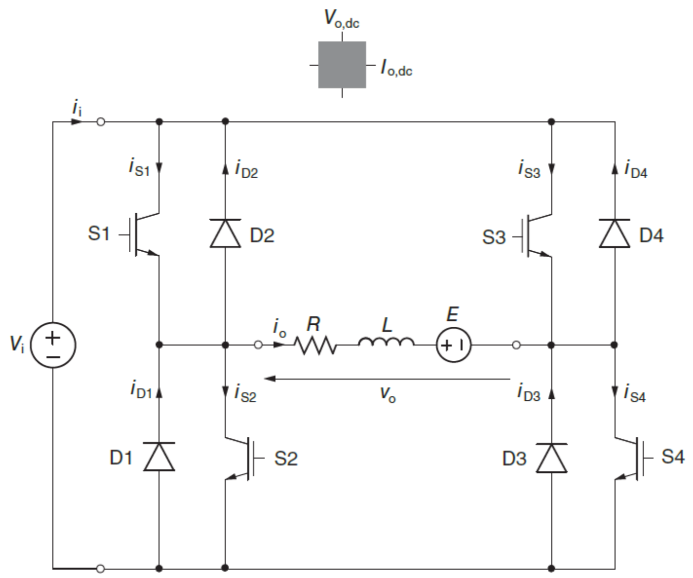


Fig. 1 Four-quadrant chopper

<u>Switching state</u>

Denoting a state of the chopper by the 4-digit binary number $(x_1x_2x_3x_4)_2$, where x_1 through x_4 are switching variables of the switches S1 to S4. A total of 16 states can be represented. However, only states 0, 1, 4, 6, and 9 are utilized. The remaining states would either short-circuit the supply source or spoil the symmetry of control.

Four Quadrant Operation

A summary of operating conditions for each quadrant is provided in Table 1. You can notice that there is always one chopping switch and the remaining three switches hold their (ON or OFF) state. For k^{th} quadrant operation, switch *Sk* is the chopping switch. The table also defines the ON and OFF state for each quadrant. The following explanation discusses in details the first column of Table 1. Other columns are not discussed as they can be realized in a similar way.

Table 1

Quadrant:	*I*	*II*	*III*	*IV*
E	≥ 0	>0	≤ 0	<0
x_1	0, 1, 0, 1, …	0	0	0
x_2	0	0, 1, 0, 1, …	1	0
x_3	0	0	0, 1, 0, 1, …	0
x_4	1	0	0	0, 1, 0, 1, …
ON state	9	4	6	1
OFF state	1	0	4	0
ON circuit	S1-D4	S2-D3	S2-S3	S4-D1
OFF circuit	S4-D1	D2-D3	S2-D3	D1-D4
v_o	$x_1 V_i$	$(1-x_1)V_i$	$-x_3 V_i$	$(x_4-1)V_i$
M	d_1	$1-d_2$	$-d_3$	d_4-1
M range	E/V_i to 1	0 to E/V_i	-1 to E/V_i	E/V_i to 0

First Quadrant Operation

When operating in 1st quadrant, the ON state is state 9 (or 1001) that is S1 and S4 are ON. The off state is state 1 (0001), this means that only S4 is ON.

This agrees with other values given in Table 1 in the column corresponding to 1st quadrant. In this column x_1 is given as (0,1, 0,1,...) which means that the switch S1 is turned (ON-OFF) every switching cycle. x_2 x_3 x_4 have constant values of 0, 0 and 1 respectively. This makes the state when S1 is ON $=(1001)_2$ i.e. state 9 and when S1 is OFF $=(0001)_2$ as stated in the same table.

The ON and OFF circuits for the first quadrant are shown in Fig. 2. This figure shows the ON circuit and OFF circuit as indicated in Table 1. Fig. 2 shows that the output voltage, in first quadrant:

$$v_o = V_i \; when \; x_1 = 1$$

$$v_o = 0 \; when \; x_1 = 0$$

Or, in general, ***for 1st quadrant operation***:

$$v_o = x_1 V_i$$

Which implies that

$$V_{o,dc} = d_1 V_i$$

Equivalently

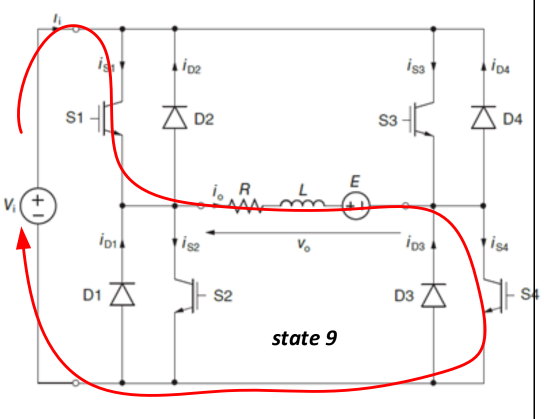
$$M = d_1$$

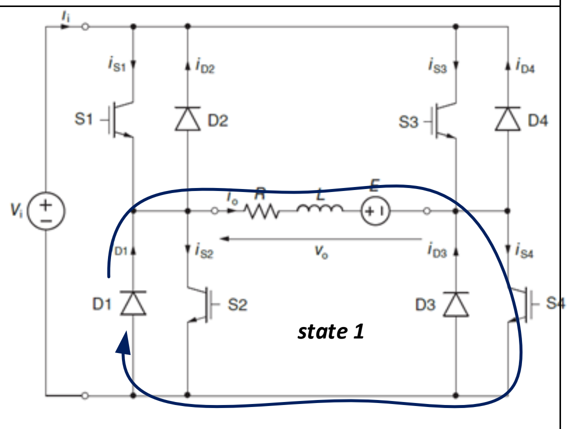
Where d_1 is the duty ratio of x_1

Finally to maintain the first quadrant operation, the current must be positive, or:

$$V_{o,dc} > E$$

$$M > \frac{E}{V_i}$$

<table>
<tr><th>Q</th><th>On State</th><th>Off-State</th></tr>
<tr><td>1</td><td>

$v_o = V_i$</td><td>

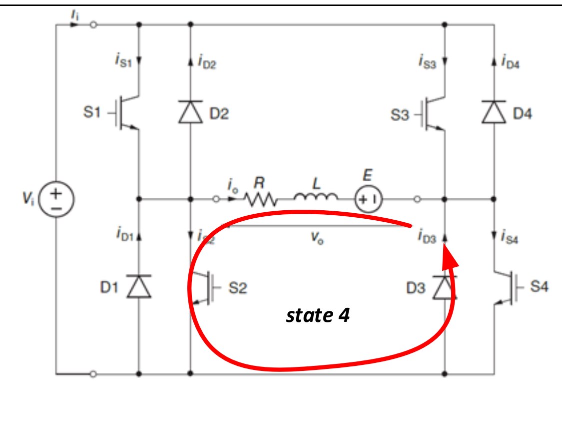
$v_o = 0$</td></tr>
</table>

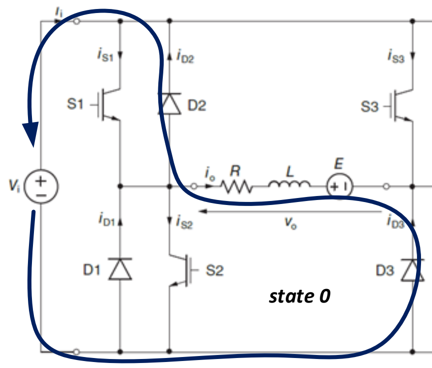
Fig. 2 First Quadrant Operation

Second, Third and Fourth Quadrants Operation

Figures 3, 4 and 5 show the four quadrant chopper operation in 2nd, 3rd and 4th quadrant respectively.

These Figures match Table -1 and each shows the circuit state in ON and OFF states.

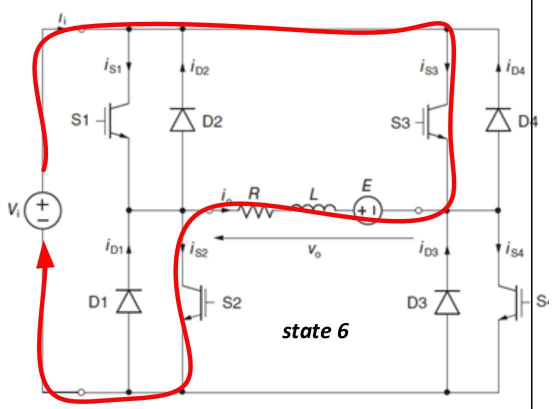
<table>
<tr><th>Q</th><th>On State</th><th>Off-State</th></tr>
<tr><td>2</td><td>

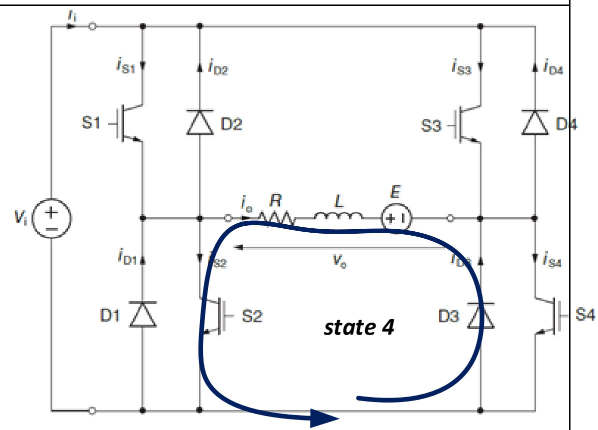
$v_o = 0$</td><td>

$v_o = V_i$</td></tr>
</table>

Fig. 3 Second Quadrant Operation

You are encouraged to conduct a detailed comparison between Table 1 and Figures 3-5 in a way similar to that conducted in the last section for first Quadrant operation.

<table>
<tr><th>Q</th><th>On State</th><th>Off-State</th></tr>
<tr><td>3</td><td>

$v_o = -V_i$</td><td>

$v_o = 0$</td></tr>
</table>

Fig. 4 Third Quadrant Operation

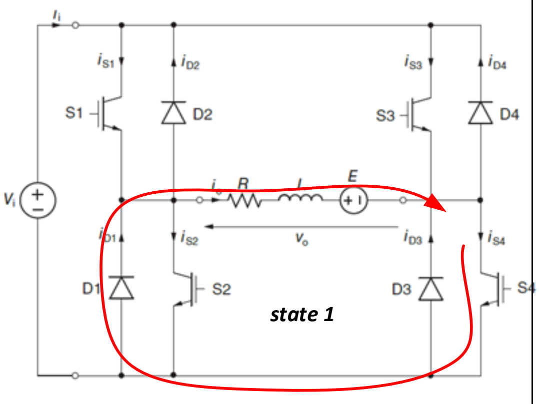
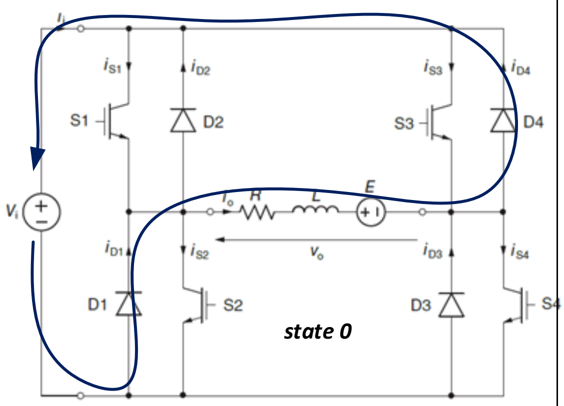
Q	On State	Off-State
4	 $v_o = 0$	 $v_o = -V_i$

Fig. 5 Fourth Quadrant Operation

Example 1: A four-quadrant chopper operates with a switching frequency of 1 kHz and supplies power to a dc motor whose armature EMF is –216 V. The input voltage of the chopper is 240 V. Find:

(a) Voltage control ranges for the third and fourth quadrants of operation of the chopper

(b) on-time and off-time of the modulating switch when the chopper operates in the third quadrant with the magnitude control ratio of –0.95

(c) on-time and off-time of the modulating switch when the chopper operates in the fourth quadrant with the magnitude control ratio of –0.75.

Solution:

(a)

Third quadrant operation,

$$I_{o,dc} = \frac{V_{o,dc} - E}{R} < 0$$

$V_{o,dc} < E$, which implies: $-240 < V_{o,dc} < -216$

As $M = \frac{V_{o,dc}}{V_i} = -d_3$

Giving: $-1 < M < -0.9$ and $1 > d_3 < 0.9$

Fourth quadrant operation,

$$I_{o,dc} = \frac{V_{o,dc} - E}{R} > 0$$

$V_{o,dc} > E$, which implies: $-216 < V_{o,dc} < 0$

As $M = \frac{V_{o,dc}}{V_i} = d_4 - 1$

Giving: $-0.9 < M < 0$ and $0.1 > d_4 < 1$

(b) The modulating switch in the third quadrant is S3. From Table 1, the duty ratio, d3, of this switch is d3 = −M = 0.95.

Consequently, the on-time, $t_{ON,3}$, of the switch is:

$t_{ON,3} = d_3T_{sw} = 0.95 \times 1$ ms = 0.95 ms,

and the off-time, $t_{OFF,3}$, is

$t_{OFF,3} = T_{sw} - t_{ON,3} = 1$ ms − 0.95 ms = 0.05 ms,

where $T_{sw} = 1/f_{sw} = 1$ ms

(c) In 4th quadrant, S4 performs the modulation. Since its duty ratio, d_4, is

$d_4 = (M + 1) = -0.75 + 1 = 0.25$

then $t_{ON,4} = 0.25 \times 1$ ms = 0.25 ms

and $t_{OFF,4} = 1$ ms − 0.25 ms = 0.75 ms

Example (2): A two-quadrant chopper, fed from a 400-V dc source, supplies power to a dc motor whose armature resistance is 0.2 Ω.

(a) The armature current is to be maintained constant at 250 A, independently of the motor speed that affects the armature EMF. Find the magnitude control ratio of the chopper and duty ratio of the modulating switch when the armature EMF is (i) 320 V and (ii) (b) –320 V

(b) Find the average output current when: (i) the armature EMF is 200 V and magnitude control ratio is 0.6 and (ii) the armature EMF is –200 V and magnitude control ratio is –0.6.

(c) The armature inductance of the motor is 0.75 mH. Find the switching frequency of the chopper such that the ripple current ($I_{o,ac}$) does not exceed 5A.

Sol.:

(a) i. $V_{o,dc} = E + I_{o,dc}R = 320 + 250 * 0.2 = 370V$

$$M = \frac{V_{o,dc}}{V_i} = \frac{370}{400} = 0.925$$

1st quadrant operation $d_1 = M = 0.925$, and $x_4 = 1$

ii. $V_{o,dc} = E + I_{o,dc}R = -320 + 250 * 0.2 = -270V$

$$M = \frac{V_{o,dc}}{V_i} = \frac{-270}{400} = -0.675$$

4th quadrant operation $d_4 = 1 - M = 0.325$, and $x_1 = 0$

(b) i. $I_{o,dc} = \frac{V_{o,dc}-E}{R} = \frac{MV_i-E}{R} = \frac{240-200}{0.2} = 200A$

ii. $I_{o,dc} = \frac{V_{o,dc}-E}{R} = \frac{-240-(-200)}{0.2} = -200A$

(c) $I_{o,ac,max}$=5A

$$I_{o,ac} = \frac{\Delta I_o}{2\sqrt{3}} \; gives \; \Delta I_{o,max} = 10\sqrt{3}A$$

$$\Delta I_o = \frac{M(1-M)V_i}{f_{sw}L}$$

$$\Delta I_{o,max} = \left[\frac{M(1-M)V_i}{f_{sw}L}\right]_{max} = \frac{0.5(1-0.5)V_i}{f_{sw}L}$$

$$f_{sw,min} = \frac{0.25 * 400}{10\sqrt{3} * 0.75 * 10^{-3}} = 7.7kHz$$

Chapter 6

Inverters

1-Single-phase inverter: square-wave mode

This lecture presents the full bridge single-phase inverter circuit and its operation in square wave mode.

Voltage Source Inverter (VSI)

The dc-to-ac power conversion is performed by inverters. An inverter is supplied from a dc source while the ac-output voltage has a fundamental component with adjustable frequency and amplitude.
There are two types of inverters inverter according to the dc supply circuit: The voltage source inverter (VSI) and current source inverter (CSI).
VSIs have a dc supply circuit with constant voltage and therefore in modeled by a pure DC voltage source. The constant dc voltage can be obtained by a large shunt capacitor as shown in Fig. 1.
CSIs have a dc supply circuit with constant, or slowly changing, current and therefore in modeled by a pure DC current source. The constant dc current is obtained by a large series inductor in the supply line as shown in Fig. 2.
Notes:

1- **Practically VSIs is way more common than CSIs. Therefore in this chapter only the VSIs are studied.**
2- **As the power line is the most accessible electric energy source; the inverters usually supplied by rectifying the power line source as indicated in Figs. 1 and 2. The six-pulse diode rectifier in the most**

common for industrial VSIs, while the two-pulse diode rectifier in used for smaller VSIs.

3- Due to the arrangement described in Note 2; the dc supply of the inverter is referred to by “dc link”!

4- Certainly if a DC source is available (e.g. solar cell or battery in electric vehicle), inverter will be simply connected to the dc supply and no rectifier stage (as indicated in note 2) is required.

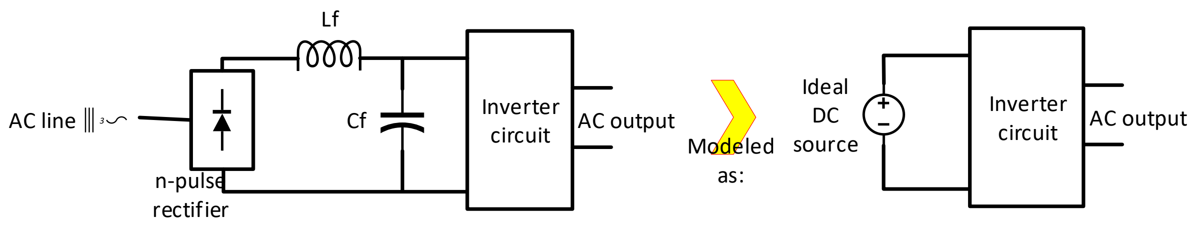


Fig. 1 voltage source inverter: a practical circuit and model

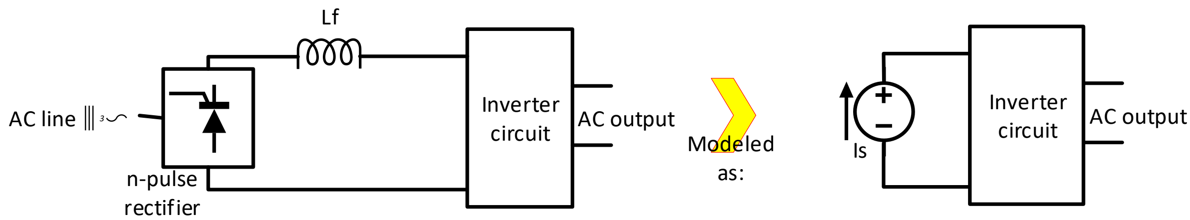


Fig. 2 Current source inverter: a practical circuit and model

Single-phase inverter circuit

The circuit diagram of the single-phase full bridge inverter is shown in Fig. 3. Although, the switches and diodes have been repositioned and renamed; the circuit is indeed identical to the 4-quadrant chopper. This is logical as the inverter is supplied by dc and produces AC i.e. positive or negative voltage and current. This is exactly the four quadrants operation repeating within each cycle of output voltage. However the inverter control is different than the chopper control as shown in this chapter.

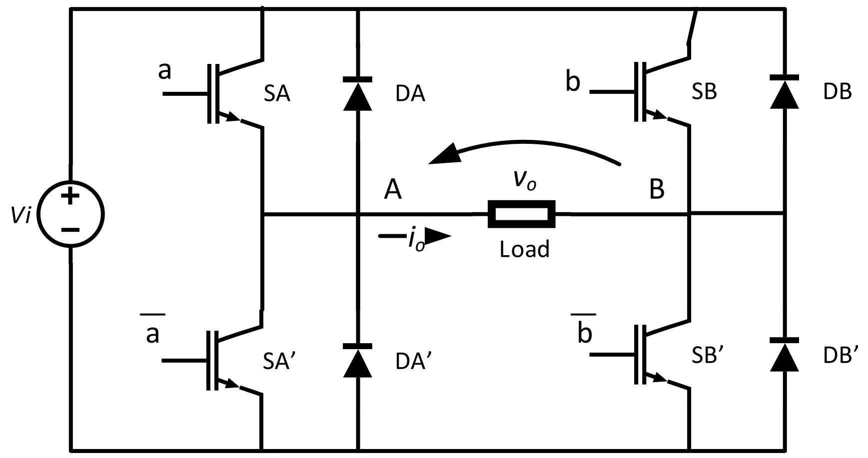


Fig. 3 single phase inverter circuit

Switches in each leg of the inverter cannot be turned on at the same time, as they would short circuit the supply. Semiconductor switches, have nonzero on-off and off-on transition times. Therefore, in practice, to avoid short circuit, a switch is turned off shortly **before** the turn-on of the other switch in the same leg. The interval between the turn-off and turn-on signals is called a dead time or blanking time. For simplicity, in the subsequent discussion, the dead time will be assumed zero.

The switching variables and the switching state

The two switches (IGBTs) in each leg is operated in complementary states that is one switch is on and the other is off. An inverter branch (or leg) can have one of two states:

- either the upper switch is on and the lower switch is off, or
- the other way around.

Thus, two switching variables *a* and *b* have been assigned as the control variables to the inverter legs and defined as follows:

$$a = \begin{cases} 0 & \text{if SA is OFF and SA}' \text{ is ON} \\ 1 & \text{if SA is ON and SA}' \text{ is OFF} \end{cases}$$

$$b = \begin{cases} 0 & \text{if SB is OFF and SB}' \text{ is ON} \\ 1 & \text{if SB is ON and SB}' \text{ is OFF} \end{cases}$$

…(6.1)

An inverter state is designated as ab_2. If, for instance, a = 1 and b = 1, the inverter is said to be in state 3 as $11_2 = 3$. Clearly, four states are possible, from state 0 to state 3.

The inverter output voltage

When a given switching variable is set to 1, the positive terminal of the supply source is connected to the corresponding output terminal of the inverter. Otherwise, a value of 0 implies connection of the negative terminal of the source to the output terminal of the inverter.

When the output current, i_o, is of such polarity that it cannot flow through a switch that is turned on, the freewheeling diode parallel to the switch provides a path for the current. If, for instance, the output current is negative, and switch SA is on, the current passes through diode DA because switch SA' must be off. Therefore the freewheeling diodes assure that the output voltage of a leg is determined by its switching variable and independent on the polarity of the current.

Consequently, the output voltage, v_o, of the inverter can be expressed as:

$$v_o = V_i(a - b) \qquad ..(6.2)$$

By substituting all possible combinations of (ab_2) we can see that the output voltage can take one of three values only: V_i, 0, and $-V_i$, corresponding to state 2, states 0 and (state 3, and state 1), respectively. By referring to the circuit topologies of Fig. 4 we can see the output voltage for the four possible states Figure 4 also relates the supply current to the switching variable and output current. The supply current can be represented in terms of the output current and the switching variables as follows:

$$i_i = i_o(a - b) \quad ..(6.3)$$

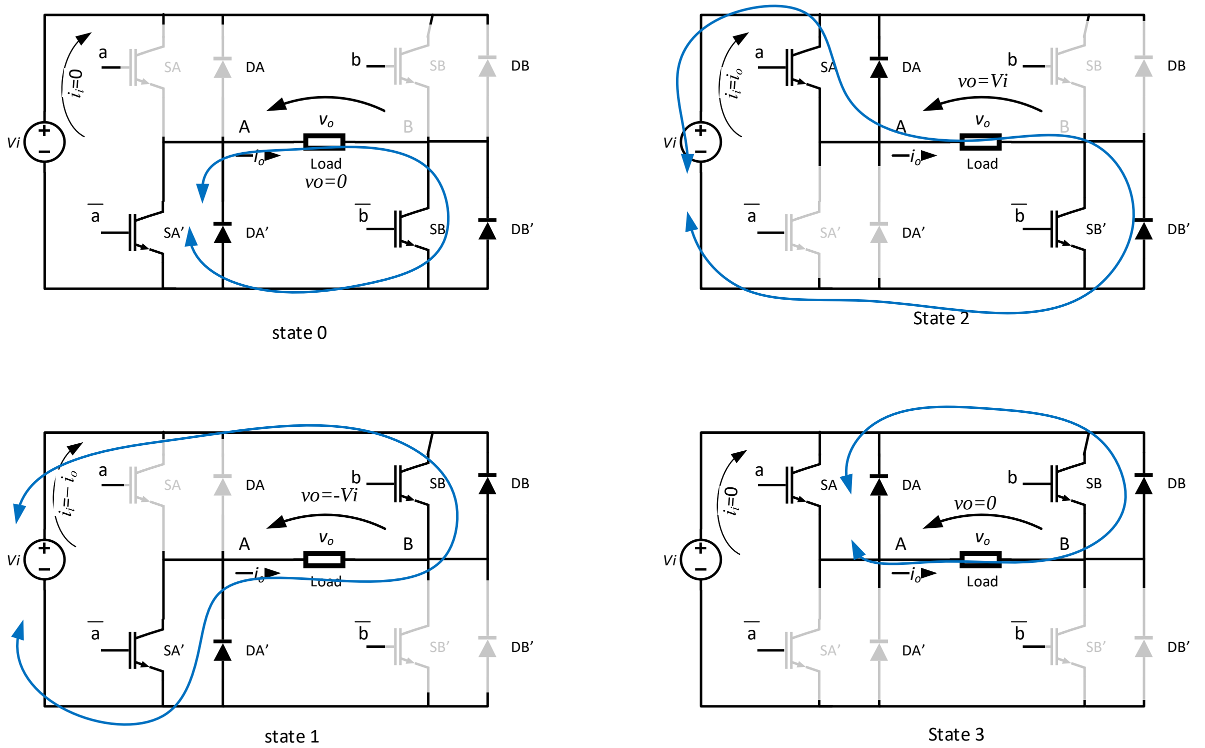


Fig. 4 Inverter States

Square-wave mode

In square-wave operation mode of the inverter, the control law is given by:

$$a = \begin{cases} 1 & for\ 0 < \omega t < \pi \\ 0 & for\ \pi < \omega t < 2\pi \end{cases} \quad and\ b = \begin{cases} 0 & for\ 0 < \omega t < \pi \\ 1 & for\ \pi < \omega t < 2\pi \end{cases}$$

. …(6.4)

where ω denotes the fundamental output radian frequency of the inverter ($2\pi f_1$). Only states 1 and 2 are used in this mode. Waveforms of the output voltage and current in an RL load of an inverter in the basic square-wave mode are shown in Fig. 5. As already derived in chapter 1; the rms fundamental output voltage, $V_{o,1}$, and harmonic content, $V_{o,h}$, of this voltage equal 0.9Vi and 0.435Vi, respectively. The total harmonic distortion, THD, is 0.483.

Another important result has been obtained in chapter 1 is the amplitude of the k[th] harmonics of v_o, given by $V_{o,k} = \frac{2\sqrt{2}V_i}{k\pi}$; where k=1,3,5,...

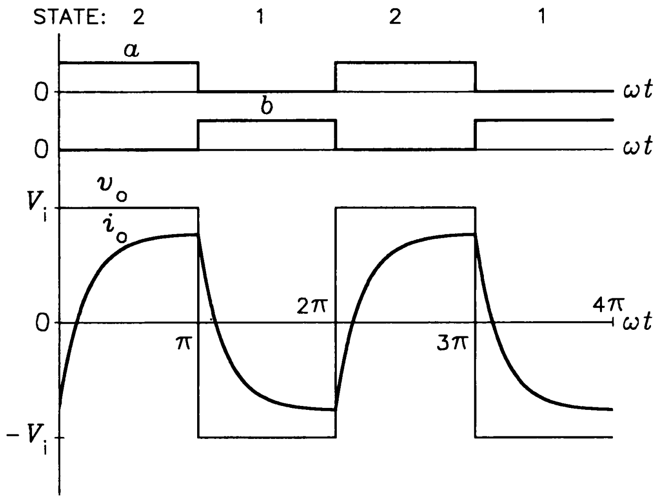


Fig. 5 switching variables and output voltage of single-phase inverter operating in square wave mode.

Square wave mode control of the inverter suffers from two main problems:

- 1- The amplitude of the fundamental component is constant, and second. (remember it is always required to control the amplitude of the fundamental component)
- 2- Highly distorted output voltage and current.

Example 1:

The single-phase inverter of has a switching sequence that produces a square wave voltage across a series RL load. The switching frequency is 60 Hz. The dc supply is (Vi=) 100 V, and the RL load has R=10 Ω and L = 25 mH. Determine (a) the expression for load current ($i_o(t)$) , (b) the power absorbed by the load, and (c) the average current in the dc source. (d) Over one operation cycle draw the waveforms of the load voltage and current and identify the conducting switch and the operating quadrant.

Solution

Refer to Fig. 6, the output current of the RL load is denoted by i_1 and i_2 when the output voltage = Vi and $-Vi$ respectively. Gives:

$$i_o = i_1 = \frac{V_i}{R} - \left(\frac{V_i}{R} + I_p\right) e^{-\frac{\omega t}{\tan \emptyset}} \quad for\ 0 < \omega t < \pi$$

$$i_o = i_2 = -\frac{V_i}{R} + \left(\frac{V_i}{R} + I_p\right) e^{-\frac{\omega t - \pi}{\tan \emptyset}} \quad for\ \pi < \omega t < 2\pi$$

Applying the boundary condition:

$$i_1(\omega t = \pi) = i_2(\omega t = \pi)$$

Gives:

$$I_p = \frac{V_i}{R}\left(\frac{1 - e^{-\frac{\pi}{tan\emptyset}}}{1 + e^{-\frac{\pi}{tan\emptyset}}}\right)$$

Accordingly:

$$\tan \emptyset = \frac{\omega L}{R} = \frac{120\pi * 0.025}{10} = 0.942$$

$$I_p = 9.31A$$

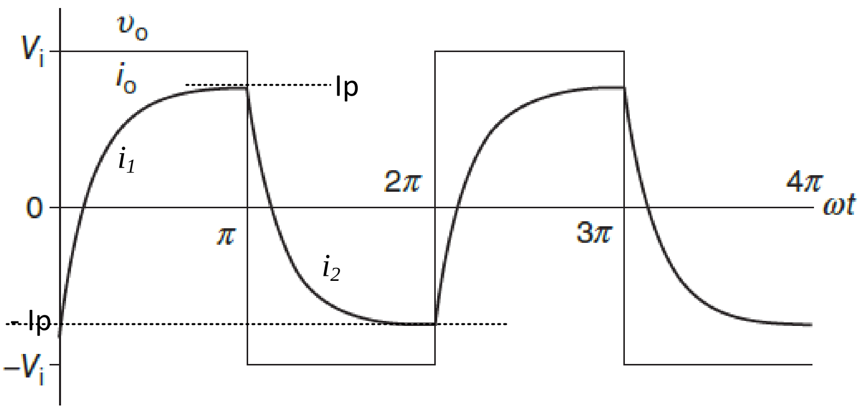


Fig. 6

(a) The load current expression

$$i_o = \begin{cases} 10 - 19.31e^{-\frac{\omega t}{0.942}}, & for\ 0 < \omega t < \pi \\ -10 + 19.31e^{-\frac{\omega t - \pi}{0.942}}, & for\ \pi < \omega t < 2\pi \end{cases}$$

(b) To determine the power absorbed by the load, find I_o

$$I_o = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(10 - 19.31e^{-\frac{\omega t}{0.942}}\right)^2 d\omega t}$$

Solved numerically using calculator, $I_o = 6.644A$

The load power is the power dissipated in the resistor, $P = I_o^2 R = 441.6W$

(c) The dc source current is equivalent to the load current for 0<π<ωt and repeats this period; therefore:

$$I_{i,dc} = \frac{1}{\pi}\int_0^{\pi}\left(10 - 19.31e^{-\frac{\omega t}{0.942}}\right)\ d\omega t$$

$$= \frac{1}{\pi}\left[10\pi + 18.19\left(e^{-\frac{\pi}{0.942}} - 1\right)\right] = 4.416A$$

(d) The drawing …

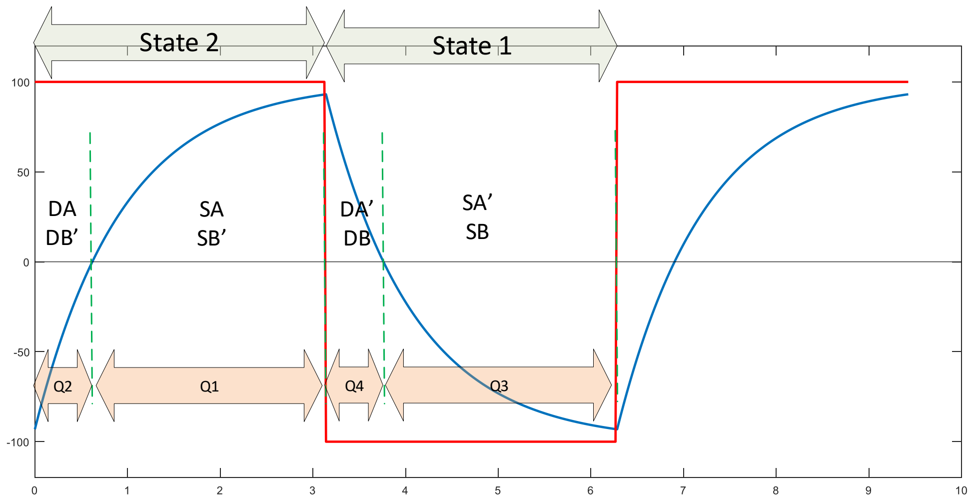


Fig. 7

Drill Problem:

Derive the expression of the k^{th} harmonic of the load current and determine the current harmonic distortion taking into consideration (i) the first three harmonics (3,5,7), (ii) the first 6 harmonics (3rd – 13th) and (iii) the first ten harmonics (3rd -21st)

Chapter 6

Inverters

3-Single-phase inverter: PWM Control

The quality of operation of the inverter can be improved by Pulse Width Modulation (PWM) control. This lecture introduces the basic concept of sinusoidal PWM. The basic terms of modulation index (m) and frequency modulation index, m_f have been defined. The relationship between the PWM waveform and its Fourier components has been described. The comparator-based implementation of PWM is shown and the traditional bipolar and unipolar methods are given.

Aim of PWM control

In Chapter 5 (dc-choppers), we have controlled the chopper to produce variable output voltage or magnitude control ratio (denoted then by M) and we have implemented this through the switching signals. M has been defined as ($V_{o,dc}/V_i$).

In this lecture we are going to extend this method to control inverters. For the inverter the desired output is sinusoidal, let's say it is given by:

$$v_o^* = V_p^* \sin \omega t \qquad ..(6.9)$$

Where the asterisk (*) superscript indicates a reference or desired value.

Define, the ***modulation index***, *m,* as:

$$m \equiv \frac{V_p^*}{V_i} \qquad ..(6.10)$$

m is closely related to magnitude control ratio (M), and typically *m(max)=1*.

Fig. 9 explains the approach of PWM control, where the desired sinusoidal output voltage is denoted by *v**. This voltage cannot be produced by the inverter as the inverter produces voltages of (+Vi, 0 and –Vi) only. The method followed to

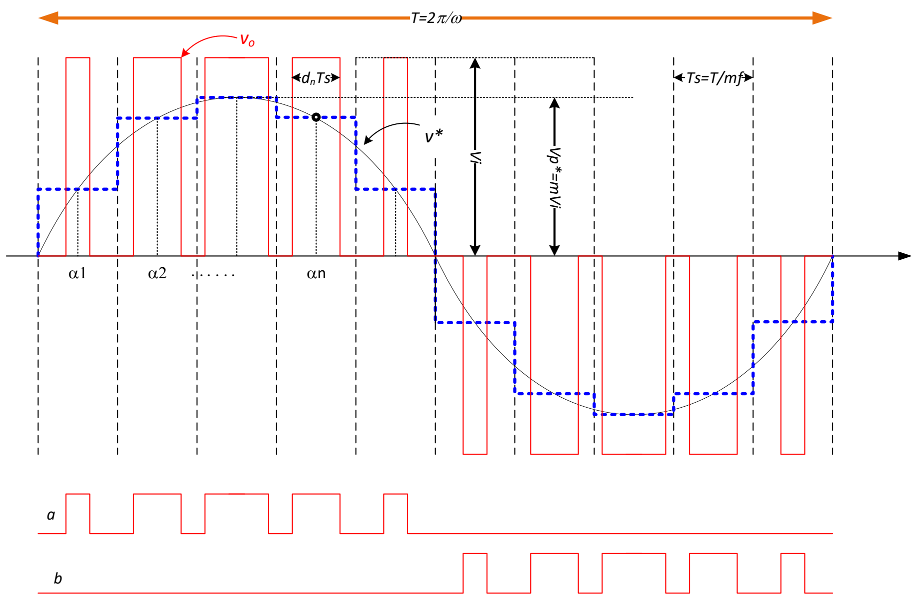


Fig. 9 Sinusoidal PWM

approximate the desired output voltage is to divide the period of the voltage v^* into m_f switching intervals. m_f is defined as the ***frequency modulation index***.

$$m_f = \frac{T}{T_{sw}} \qquad ..(6.11)$$

Within each switching interval the inverter approximates the desired output voltage by producing a voltage that has a "*local average*" equals to the desired output voltage or:

$$\overline{v_{o,n}} = v^*(\omega t = \alpha_n) = V_p^* \sin(\alpha_n)$$

The resultant voltage, v_o, has a sequence of pulses as shown in Fig. 9. The advantages of this voltage lies in its spectrum, specifically:

- $V_{o,1,p} = mV_i$: this implies that the output fundamental can be linearly controlled by changing m.
- The harmonics of v_o appear as a sidebands around orders: m_f , $2m_f$ … which suggests that this method allows the control of the dominant harmonics frequency.

Fig. 9 also shows the switching variables a, b to produce the corresponding v_o.

In practice, PWM has been implemented using a commonly known (analog or digital) electronic system by a method known as carrier comparison.

There are famous two carrier-comparison PWM schemes for single phase inverter as described in subsequent sections.

Bipolar PWM

This method is explained in Figures 10 and 11. The switching signals is produced by comparing a sinusoidal reference (v_{ref}) and a triangular carrier (v_{tri}). The sinusoidal reference has:

-frequency equals to the desired output frequency.

- an amplitude adjusted in such a way that $m = \frac{V_{ref,p}}{V_{tri,p}}$, which means that: *(0<m<1)*➔ *($0<V_{ref,p}<V_{tri,p}$)*

The triangular waveform has a constant amplitude and its frequency is m_f multiples of the sinusoidal reference frequency.

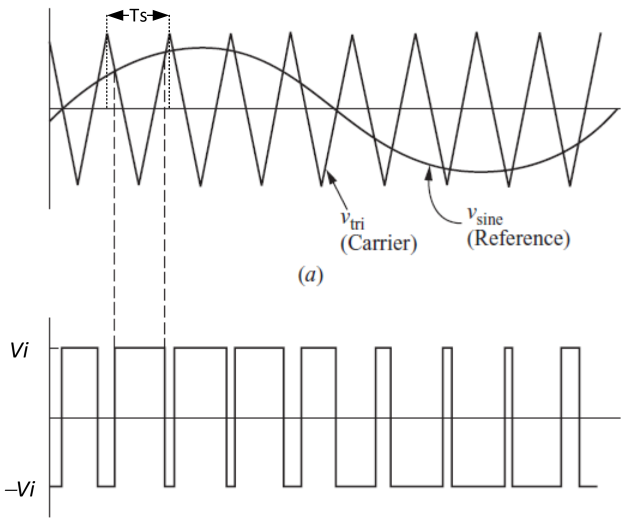


Fig. 10 Carrier comparison and bipolar output voltage of the inverter

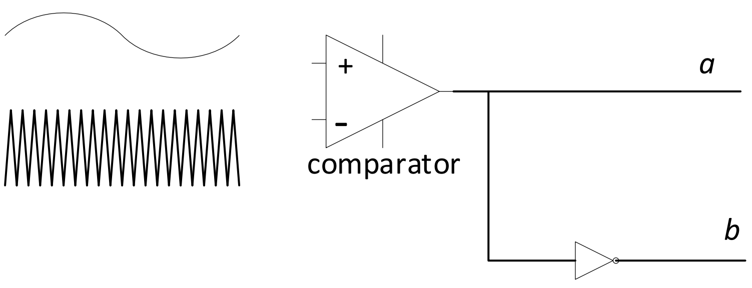


Fig. 11 Electronic system to control the inverter using bipolar PWM

The criteria for producing the switching signals is:

$$a = \begin{cases} 0 & if\ v_{ref} < v_{tri} \\ 1 & if\ v_{ref} > v_{tri} \end{cases} \qquad ..(6.12)$$

$$b = \bar{a} \qquad ..(6.13)$$

The switching signals generated as shown in Fig. 11 produce a bipolar output voltage, i.e. switches between positive and negative polarities every interval and no zero output.

To assure half wave symmetry, m_f must be odd, therefore the most dominant harmonic will be, as shown in Fig. 12 of order m_f , other harmonics appear at (m_f±2). The next bunch of harmonics appear as side bands around 2 m_f (but not at $2m_f$ as this is an even number).

Figure 12 also shows that fundamental output voltage amplitude peak is equals to mV_i.

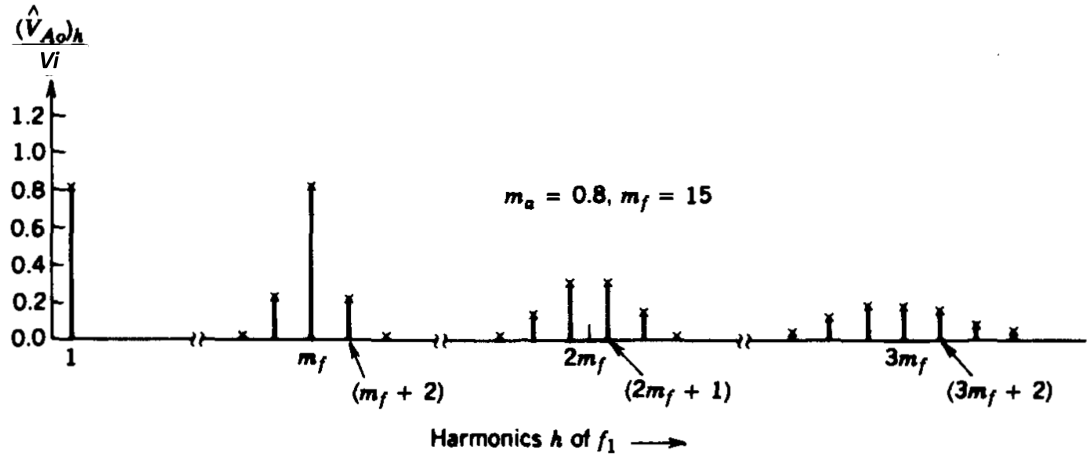


Fig. 12 Harmonics spectrum of the bipolar PWM voltage.

Table 8-1 Generalized Harmonics of v_{Ao} for a Large m_f.

h \ m_a	0.2	0.4	0.6	0.8	1.0
1 Fundamental	0.2	0.4	0.6	0.8	1.0
m_f	1.242	1.15	1.006	0.818	0.601
$m_f \pm 2$	0.016	0.061	0.131	0.220	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.190	0.326	0.370	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.113
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.157	0.008	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.050

Table 1 lists the amplitudes of various Fourier components for the entire range of *m* and $m_f = 39$. The table reflects the various PWM characteristics.

The bipolar PWM provides simple implementation method, however its output voltage has high harmonic distortion. The distortion can be reduced by following the unipolar PWM strategy.

Unipolar PWM

Unipolar method, is achieved by applying two reference voltages which are 180° out of phase. Where, *a* is produced in the way described in Eq. (6.12) while *b* is produced by comparing $-v_{ref}$ to the carrier as follows:

$$b = \begin{cases} 0 & if - v_{ref} < v_{tri} \\ 1 & if -v_{ref} > v_{tri} \end{cases} \qquad \ldots(6.14)$$

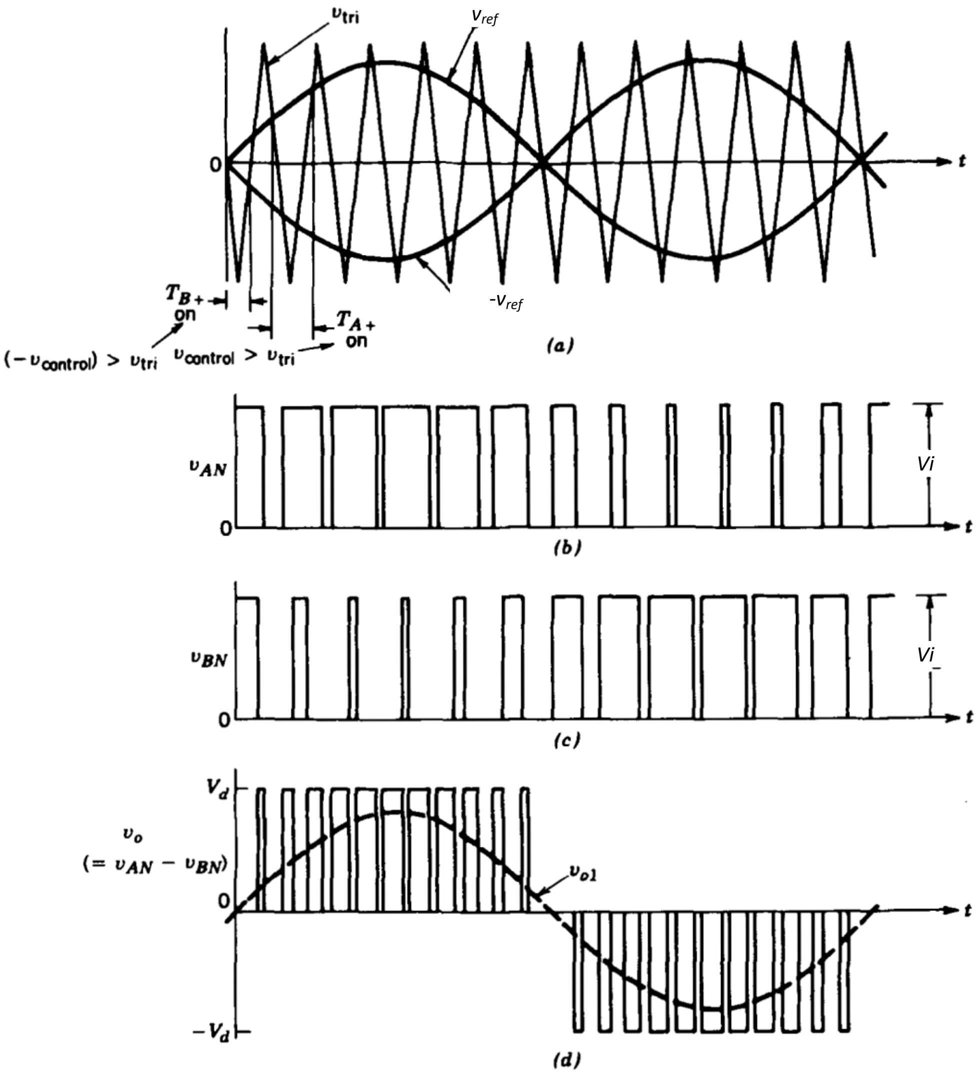


Fig 13 Unipolar PWM

In unipolar PWM, the duty ratio for the two switching functions during the n^{th} interval are given by:

$$d_{an} = \frac{1}{2}[1 + F(m, \alpha_n)] \qquad ..(6.15)$$

$$d_{bn} = \frac{1}{2}[1 + F(m, \alpha_n)] \qquad ..(6.16)$$

For sinusoidal modulating function (reference) the duty ratios are

$$d_{an} = \frac{1}{2}[1 + m \sin \alpha_n] \qquad ..(6.17)$$

$$d_{bn} = \frac{1}{2}[1 - m \sin \alpha_n] \qquad ..(6.18)$$

Figure 13 shows the unipolar PWM method operation. It can be seen that with each switching interval (triangular cycle) there is one pulse of *a* and one pulse *b* but two pulses of the output voltage v_o, with switching frequency of $f_s = m_f \times$(the reference frequency, *f*), the resultant output voltage has two pulses every T_s and therefore its effective switching frequency = $2f_s$ and the harmonics appear around $2m_f$, $4m_f$, …., as shown in Fig. 13. So for the same switching frequency we can roughly expect that the THD in the current will be reduced to around half.

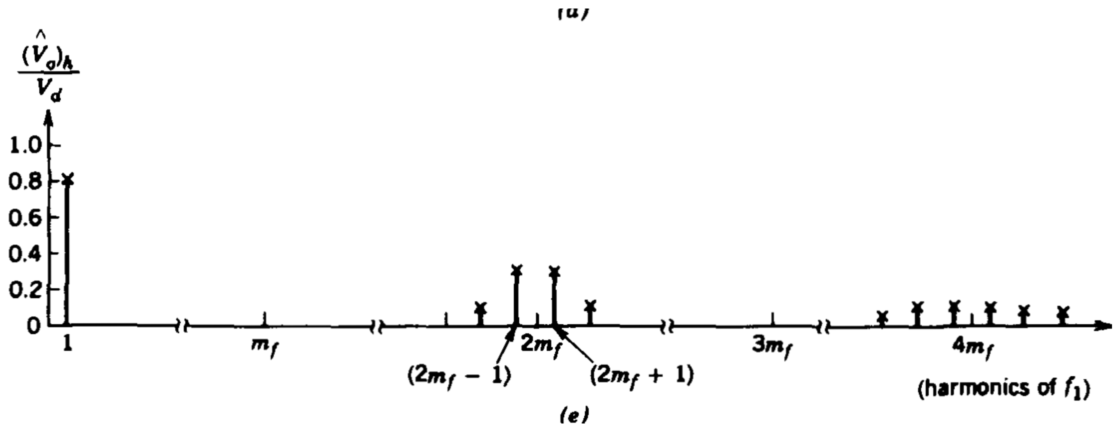


Fig. 14 Spectrum of the unipolar output voltage ($m=0.8$, $m_f=15$)

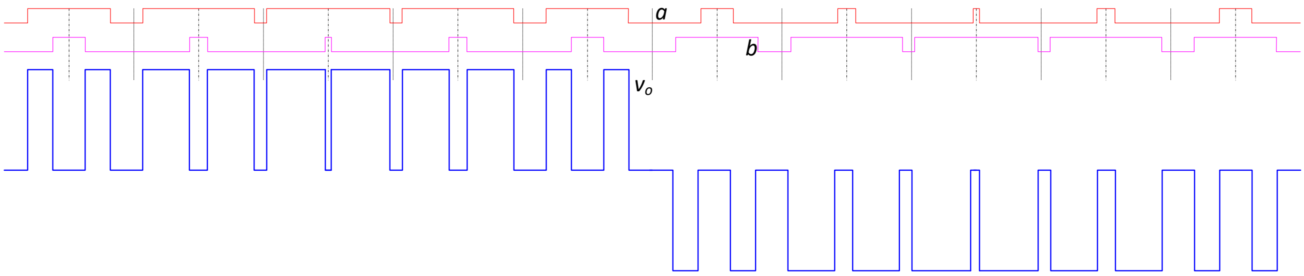
Example:

A single phase inverter is supplied from a 400V dc link and controlled with unipolar sinusoidal PWM with m_f=10 and $m=0.9$, determine the duty ratio of the switching functions *a and b* over the output voltage period and draw the switching functions and the output voltage waveforms.

Sol.

n (interval)	$\alpha_n = \frac{360}{m_f}(n-0.5)$	$d_{a,n}, d_{b,n}$	$\gamma_{a,n}, \gamma_{b,n}$ (°)
1	18	0.64, 0.36	11.5,6.5
2	54	0.864, 0.136	15.6, 2.4
3	90	0.95, 0.05	17.1, 0.9
4	126	0.864, 0.136	15.6, 2.4
5	162	0.64, 0.36	11.5,6.5
6	198	0.36, 0.64	6.5, 11.5
7	234	0.136, 0.864	2.4, 15.6
8	270	0.05, 0.95	0.9, 17.1
9	306	0.136, 0.864	2.4, 15.6
10	342	0.36, 0.64	6.5, 11.5

...



Chapter 6

Inverters

4-Three-phase VSI: Circuit, Switching States, and Square Wave Mode

Three phase VSI is the most common AC supplying converter. This lecture presents its circuit topology, defines the switching variables and the switching states. Relationships between the switching states and the output voltage are derived, also the dc side current is defined in terms of the switching state and the output currents. The square wave operation mode is presented and the corresponding analysis with RL load are given by example.

Circuit Topology

Most of the practical inverters, are of the three-phase type. The power circuit of a three-phase VSI in Fig. 15 is obtained by adding a third leg to the single-phase inverter.

Switching Variables and Switching States:

As before, of the two power switches in each leg of the inverter *one and only one* is always on. Neglecting the dead time, three binary switching variables, *a*, *b*, and *c*, can be assigned to the inverter. As for single phase inverter, the 0 is to turn off the upper switch and turn on the lower switch and the 1 value is to turn on the upper switch and turn off the lower switch

A state of an inverter is designated as abc_2, making for a total of eight states, from state 0, (when all output terminals are connected to the negative dc bus), to state 7, (when they are connected to the (+) side of Vi).

The Output Voltages

The three output points voltages with respect to the negative side of (Vi) can be represented as:

$$v_a = aV_i.$$
$$v_b = bV_i \qquad ..(6.19)$$
$$v_c = cV_i$$

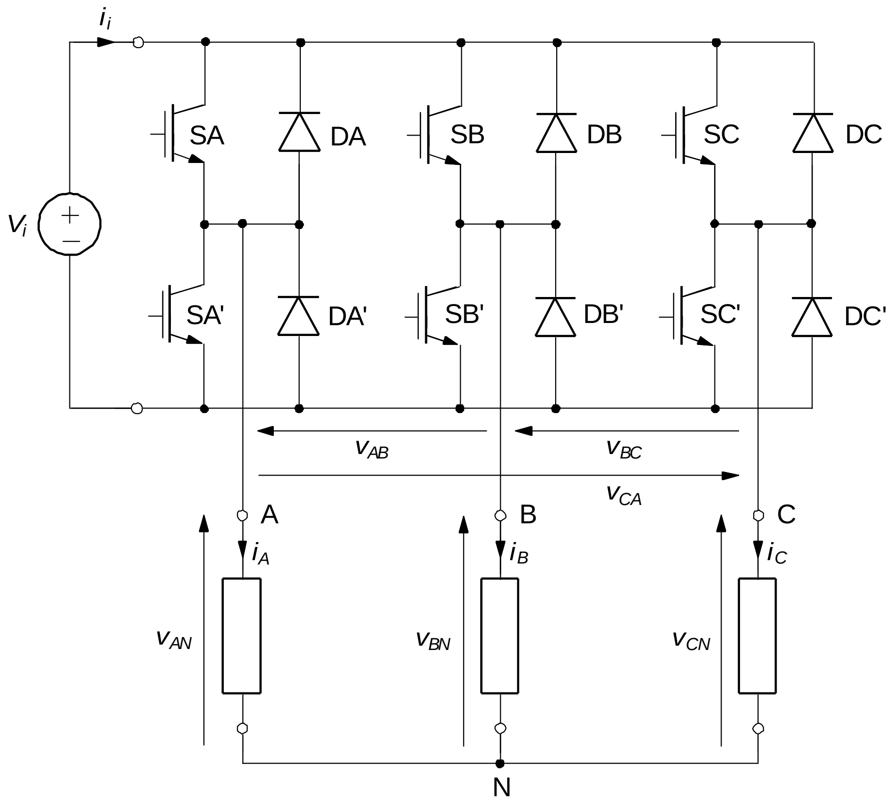


Fig. 15 Three-phase VSI

The line to line output voltages are given as follows:

$$v_{AB} = v_a - v_b = aV_i - bV_i = (a-b)V_i$$
$$v_{BC} = (b-c)V_i$$
$$v_{CA} = (c-a)V_i$$

In matrix form…

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} V_i \qquad ...(6.20)$$

In a balanced three-phase system:

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} v_{AB} - v_{CA} \\ v_{BC} - v_{AB} \\ v_{CA} - v_{BC} \end{bmatrix}$$

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{V_i}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \qquad (6.21)$$

Equations (6.20) and (6.21) allow the determination of the line-to-line and line to-neutral output voltages for all states of an inverter. The results are listed in Table .1. The line-to-line voltages can only take three values, 0 and ±Vi, while the line-to-neutral voltages can take five values, 0, ±Vi/3, and ±2Vi/3.

Table 1 States and output voltages and input current of three phase VSI

State	*abc₂*	v_{AB}	v_{BC}	v_{CA}	v_{AN}	v_{BN}	v_{CA}	i_i
0	000	0	0	0	0	0	0	0
1	001	0	$-V_i$	V_i	$-V_i/3$	$-V_i/3$	$2V_i/3$	i_C
2	010	$-V_i$	V_i	0	$-V_i/3$	$2V_i/3$	$-V_i/3$	i_B
3	011	$-V_i$	0	V_i	$-2V_i/3$	$V_i/3$	$V_i/3$	$-i_A$
4	100	V_i	0	$-V_i$	$2V_i/3$	$-V_i/3$	$-V_i/3$	i_A
5	101	V_i	$-V_i$	0	$V_i/3$	$-2V_i/3$	$V_i/3$	$-i_B$
6	110	0	V_i	$-V_i$	$V_i/3$	$V_i/3$	$-2V_i/3$	$-i_C$
7	111	0	0	0	0	0	0	0

DC Supply Current

Table 1 gives the inverter's input current as a function of output currents. Indeed the input current can be expressed as a function pf switching variables and output currents as follows:

$$i_i = ai_a + bi_b + ci_c \qquad ..(6.22)$$

Or alternatively:

$$i_i = (a-1)i_a + (b-1)i_b + (c-1)i_c \qquad ..(6.23)$$

Equation (6.22) expresses the input current as the sum of the current through the switches SA, SB and SC while Eq (6.23) expresses the input current as negative of the sum of the current through the switches SA', SB' and SC'.

Square-wave mode.

In square wave operation mode, the three switching variables have 50% duty cycle pulses i.e. equals 1 for T/2 followed by 0 for T/2 as shown in the top three waveforms in Fig 16. The three arms switching signals a, b and c are shifted from each other by one-third of a cycle (or 120°). The switching functions and the resultant line and phase voltages are shown in Fig.16. In this figure the time axis is represent in terms of angle ωt where ω=2πf; f is the desired output frequency and T=1/f is the operation cycle.

If the operation cycle starts by the instant at which *a* geos from 0-to-1, the corresponding switching sequence is: 5 – 4 – 6 – 2 – 3 – 1 –…, with each state lasting for T/6 (or 60°), the individual line-to-line and line to- neutral voltages have waveforms shown in Fig. 16. Notice that the voltages, although not sinusoidal, are balanced. This is the square-wave mode of operation, in which each switch of the inverter is turned on and off once within the cycle of output voltage. The peak value, $V_{LL,1,p}$, of the fundamental line-to-line output voltage equals approximately 1.1 Vi and that, $V_{LN,1,p}$, of the line-to-neutral voltage, 0.64 Vi. Both voltages have the same total harmonic distortion, THD, of 0.31. The ratio $V_{LL,1,p}$/Vi represents the dc supply utilization factor, which in this case is greater than unity. The shape of the phase voltage waveform (Fig. 16) is known as the six-step waveform, also the three-phase inverter has been known, previously, as the six step inverter.

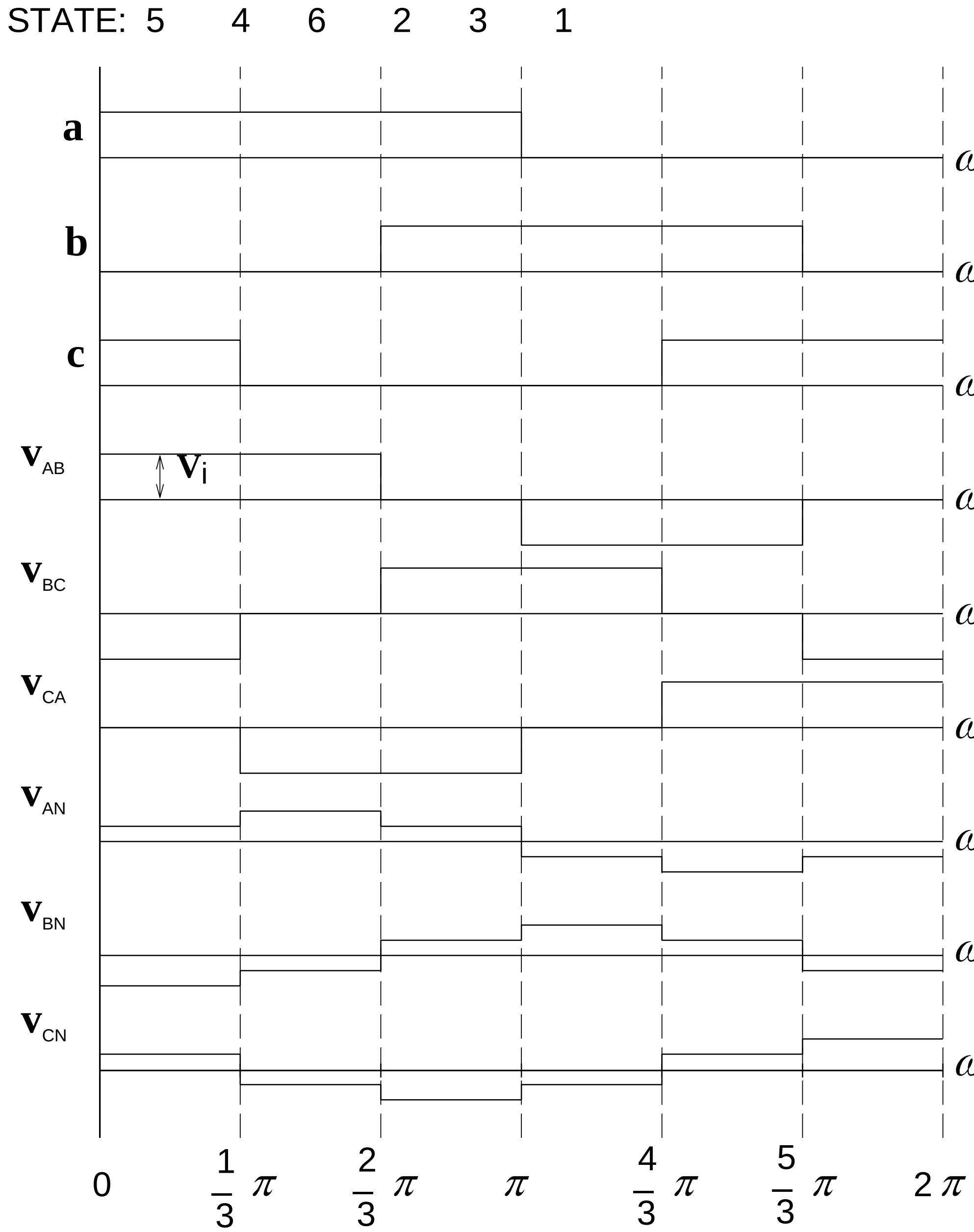


Fig. 16 switching signals and output voltage waveforms of the three-phase inverter in square wave mode.

Waveforms of the output voltage and current for one phase of the inverter are shown in Figure 16. They are, as in a single-phase inverter in the square wave mode, are highly distorted. But the three phase inverter has zero triple harmonics. and rich in low-order odd harmonics, except for the triple ones.

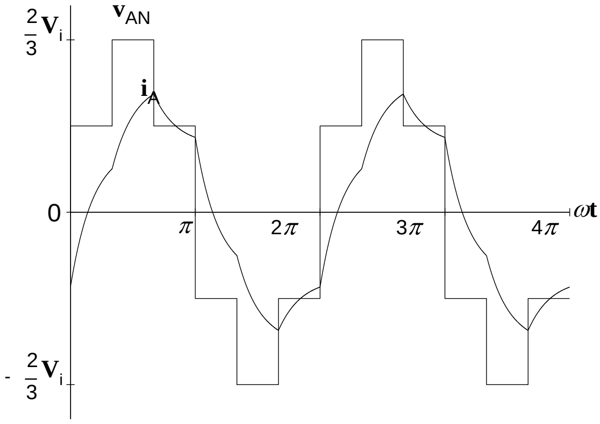


Fig. 17 Phase voltage and current for RL load

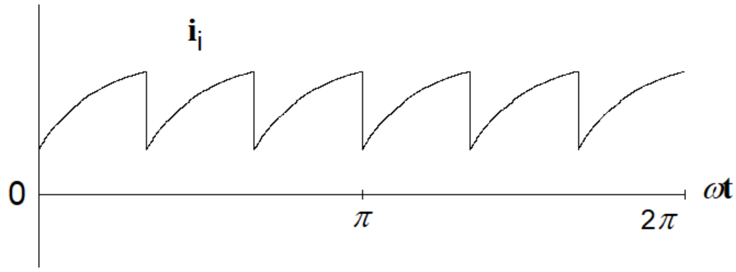


Fig. 18 the input current corresponding to the ac current shown in Fig. 17

Example 1 A VSI is supplied from a 620-V dc source and feeds a balanced wye-connected load. At a certain instant, the inverter is in state 3 and the output currents in phases A and B are −72 and 67 A, respectively. Neglect the voltage drops in the inverter, and determine all the output voltages (line-to-neutral and line-to-line) and the input current.

Drill exercise: The inverter in example 1 is operated with a Y-connected pure inductive load with 0.5H per phase, draw to scale the waveforms of the load currents, input (dc) current and the dc supply instantaneous power. Calculate the average power supplied by the DC source.

Example 2 : A three phase six-step inverter is producing an output voltage with $V_{LL,1}$=200V at 52Hz. The load is a Y-connected ac motor which is modeled by as 100mH inductor/phase in series with a back emf supply of fundamental frequency. Calculate the peak ripple current resultant in this case.

Chapter 6

Inverters

4-Three-phase VSI: Analysis of square wave mode voltages

This lecture considers the line and phase voltages of the six-step inverter. Fourier analysis are presented and discussed. Full analysis based on Fourier series are given by example.

Line voltages.

The line-to-line voltages have been obtained in Lect. 4 (last lecture Fig. 16) and redrawn in Fig. 19 after introducing a 30°shift. This phase shift is introduced to establish odd symmetry. If we examine Fig. 19 we will notice that the waveform is identical to the output voltage of the quasi square wave (Fig. 7) with α=30°(π/6). Therefore we may import the corresponding Fourier solution from Eq. (6.5) to perform the analysis as follows:

- The amplitude of the n[th] harmonic:

$$V_{o,n} = \frac{B_2}{\sqrt{2}} = \frac{2\sqrt{2}}{n\pi} V_i \cos(n\pi\,/6) \qquad ..(6.24)$$

$$n = 1,3,5,7,\ldots$$

- THD:

$$V_{LL,1} = \frac{\sqrt{6}}{\pi} V_i \qquad ..(6.25)$$

$$V_{LL} = V_i \sqrt{\frac{2}{3}} \qquad ..(6.26)$$

$$V_{LL,h} = \sqrt{V_o^2 - V_{o,1}^2} = V_i \sqrt{\frac{2}{3} - \frac{6}{\pi^2}} = 0.242 V_i \qquad ..(6.26)$$

$$THD_{vll} = \frac{V_{LL,h}}{V_{LL,1}} = 31\% \qquad ..(6.27)$$

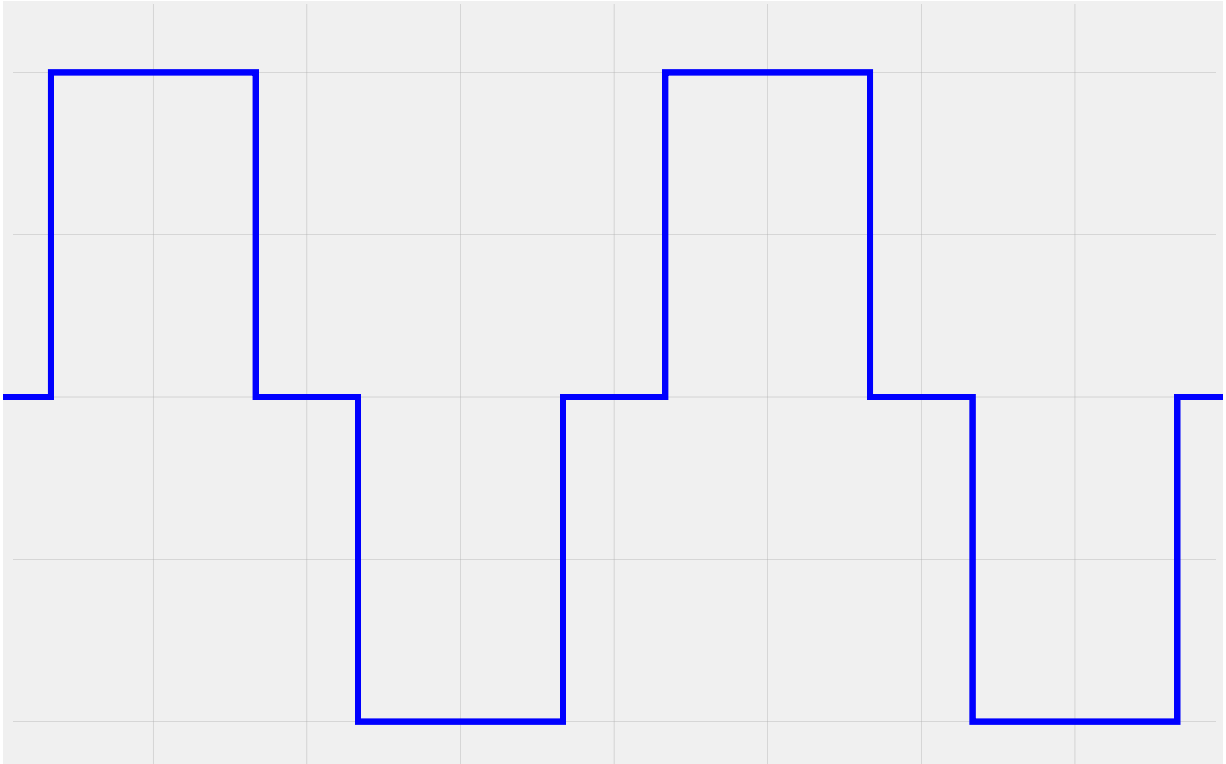


Fig. 19 The line voltage of a six-step inverter (Vi=100V)

- The spectrum of v_{LL}:

Based on Eq. (6.24), Fig. 20, is drawn to show the spectrum of the line voltage. It can be seen that besides even harmonics, all triple harmonics (3, 9, 15, 21…) are absent (zeros). The absence of the triple harmonics can be explained in two ways:

- ***One:*** From equation (6.24), the nth harmonic : $V_{o,n,p} = \frac{4}{n\pi} V_i \cos(n\pi/6)$ becomes zero every time the angle (nπ/6) becomes integer multiples of π/2, i.e. when *n* is multiples of 3 (3, 9, 15, …).

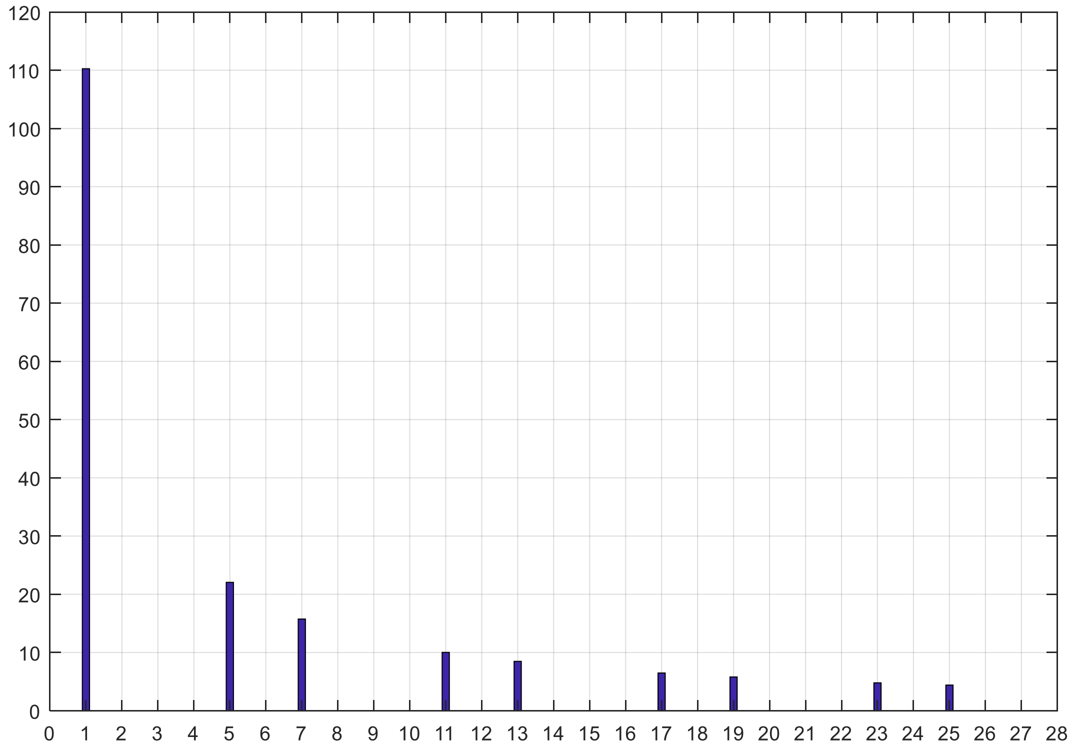


Fig. 20 Spectrum of the line voltage shown in Fig. 19

- ***Two:*** If we examine the n[th] component of a line voltage (say v_{AB}) as follows: $v_{AB,n} = v_{A,n} - v_{B,n}$, The third (and other triple) harmonic corresponding to the voltages are identical in amplitude and phase and therefore the difference between them is zero. This phenomenon is known as common mode cancellation. Figure 21 shows the total, fundamental and third harmonics in v_A, v_B and v_{AB}. It can be seen that the third harmonics in both output points voltages ($v_{A,3}$, $v_{B,3}$) are identical in amplitude and phase and therefore their difference is zero:

$$v_{AB,3} = v_{A,3} - v_{B,3} = 0$$

Notes: (1) The cancellation of triple harmonics is applicable to in any control method (not only square wave mode) in which the switching functions are identical and shifted by 120°.

(2) Like other three phase systems, the cancelation of the third harmonics can be disturbed by unbalance.

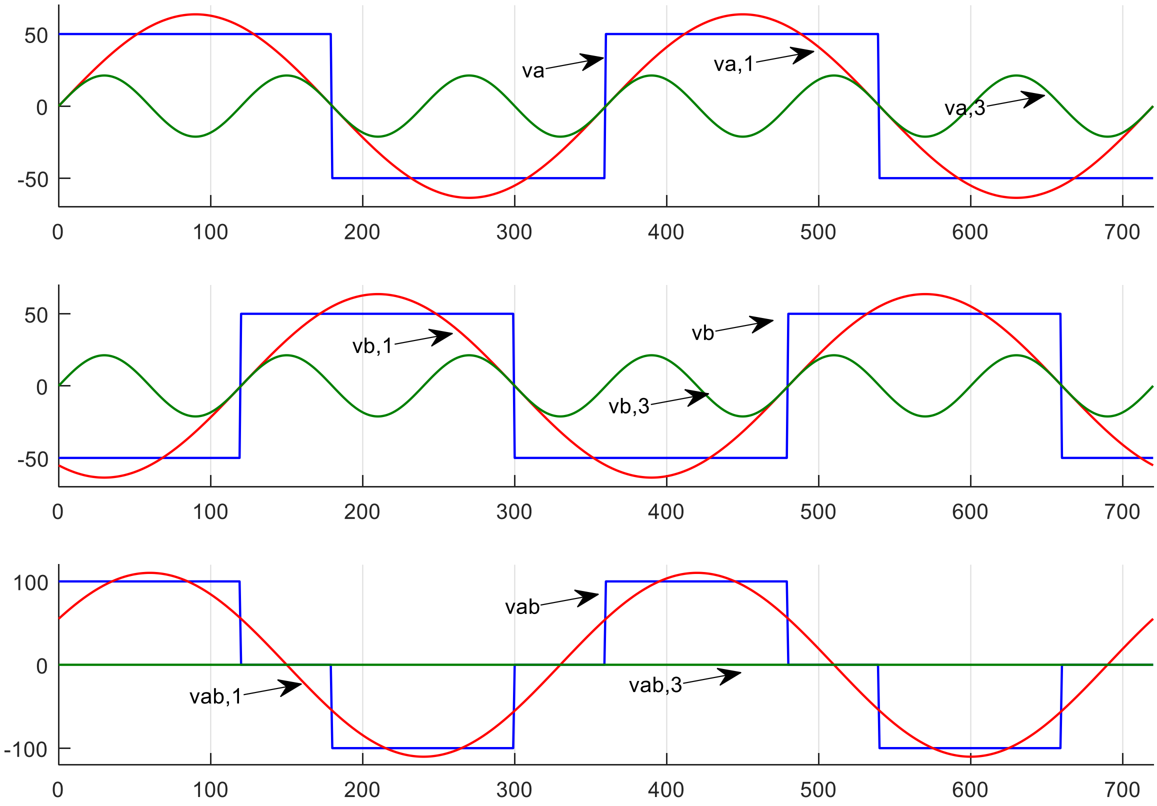


Fig. 21 Cancellation of third harmonic in three phase line voltage

Phase voltages

The phase voltage waveform v_{AN} corresponding to the inverter supplied by a 100V dc source and operated at 50Hz is shown in Fig. 22. This formation of this voltage has been described in the last session. In this part we want to obtain the attributes of this waveform

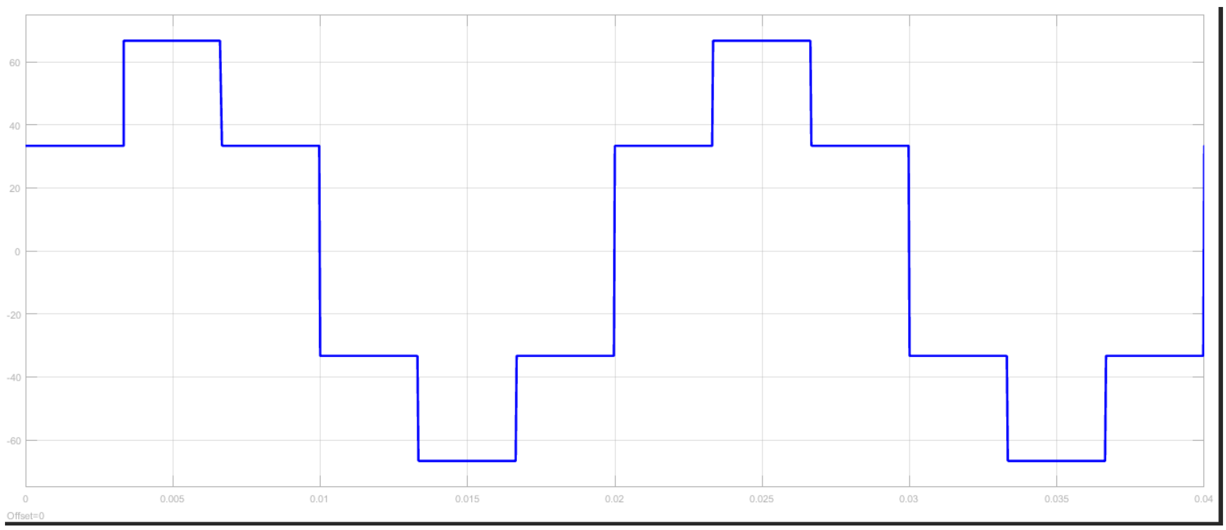


Fig. 22 Phase voltage of a three-phase VSI

- Amplitude of nth harmonic:

  The phase voltage is odd and half-wave symmetrical, therefore:

$$V_{LN,n,p} = B_n = \frac{4V_i}{\pi}\left[\int_0^{\frac{\pi}{3}} \frac{1}{3}\sin n\omega t\, d\omega t + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{2}{3}\sin n\omega t\, d\omega t\right]$$

$$V_{LN,n,p} = \frac{4V_i}{3n\pi}\left[1 + \cos n\frac{\pi}{3}\right] \qquad ..(6.28)$$

- Total Harmonic Distortion

  From 6.28

$$V_{LN,1} = \frac{2\sqrt{2}V_i}{3\pi}\left[1 + \cos\frac{\pi}{3}\right] = \frac{\sqrt{2}}{\pi}V_i \qquad ..(6.29)$$

The total RMS

$$V_{LN} = \sqrt{\frac{2}{9}}V_i \qquad ..(6.30)$$

$$V_{LN,h} = \sqrt{{V_{LN}}^2 - {V_{LN,1}}^2} = 0.14V_i \qquad ..(6.31)$$

$$THD_{VLN} = \frac{0.14}{\sqrt{2}/\pi} = 0.311 \qquad ..(6.32)$$

By comparing the attributes of the line and phase voltages we can find several interesting connections, for example:

The following line-to-phase values hold the essential $\sqrt{3}$ ratio:

Quantity	Line-to-line	Phase	Line-to-phase ratio
Fundamental voltage	$V_{LL,1} = \frac{\sqrt{6}}{\pi}V_i$	$V_{LN,1} = \frac{\sqrt{2}}{\pi}V_i$	$\sqrt{3}$
Total RMS voltage	$V_{LL} = \sqrt{\frac{2}{3}}V_i$	$V_{LN} = \sqrt{\frac{2}{9}}V_i$	$\sqrt{3}$
Harmonics voltage (rms)	$V_{LL,h} = 0.242V_i$	$V_{LN,h} = 0.14V_i$	$\sqrt{3}$

What make this result so interesting is that the two waveforms are different in shape. This also leads to equal THD of the two waveforms.

The following discussion of the spectrum will explain the reason for the above properties.

- The spectrum of v_{LN}:

Based on Eq. (6.28), Fig. 23, is drawn to show the spectrum of the phase voltage. Again even and triple harmonics are zeros. Figure 23 shows both line and phase voltage spectrums on a logarithmic scale to provide higher resolution. Where the line voltage components are represented by the thin red lines, while the phase voltages by the thick black bars. It can be seen that for all harmonics, the ratio of $\sqrt{3}$ is correct.

Now as the two waveforms equal similar ratio of all harmonics, what make the waveforms different in shapes? (This question is left for you to answer)

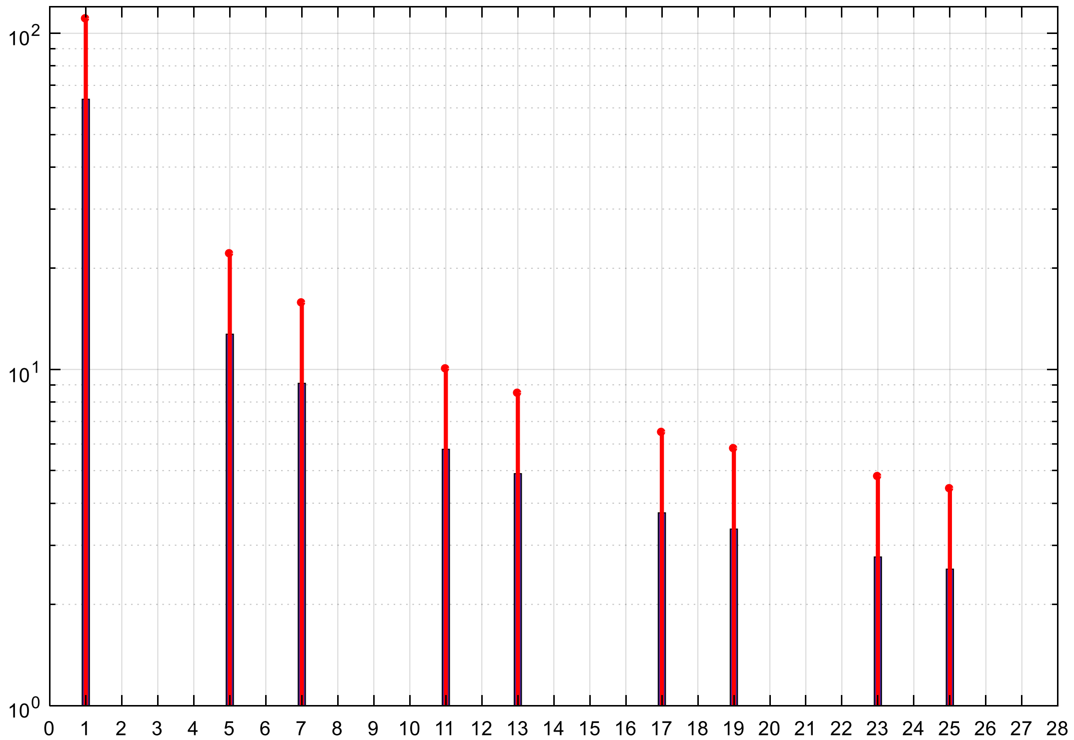


Fig. 23 Spectrum of line and Phase voltage of six step inverter.

Example 1 A VSI is supplied from a 630-V dc source and feeds a balanced wye-connected load of Z=42Ω ∠30° at fundamental frequency. The switching period is 18seconds. Determine:

$$I_{ph,1}, I_{L,1}, I_{ph}, I_L, THD_{Iph}, RF_{Ii}$$

Approximate the harmonic current to the first 6 nonzero harmonics.

Example 2 : A three phase six-step inverter is producing an output voltage with VLL,1=200V at 52Hz. The load is a Y-connected ac motor which is modeled by as 100mH inductor/phase in series with a back emf supply of fundamental frequency. Calculate the ripple current and the total harmonic distortions of the output current.

Chapter 6

Inverters

4-Three-phase VSI: Carrier Comparison PWM

The limitations of the square wave operation mode are discussed. The application of carrier comparison PWM in three phase VSI is presented and the features of sinusoidal PWM are studied. The performance of sinusoidal PWM is evaluated after listing the desirable characteristics of inverters.

Limitations of Square wave mode

The square wave mode operation studied in the last two lectures has two basic limitations, which are:

(1) The fundamental output (line and phase) voltages are constant if the input voltage is constant. **The lack of the ability to provide variable output voltage** is the main limitation of this method.

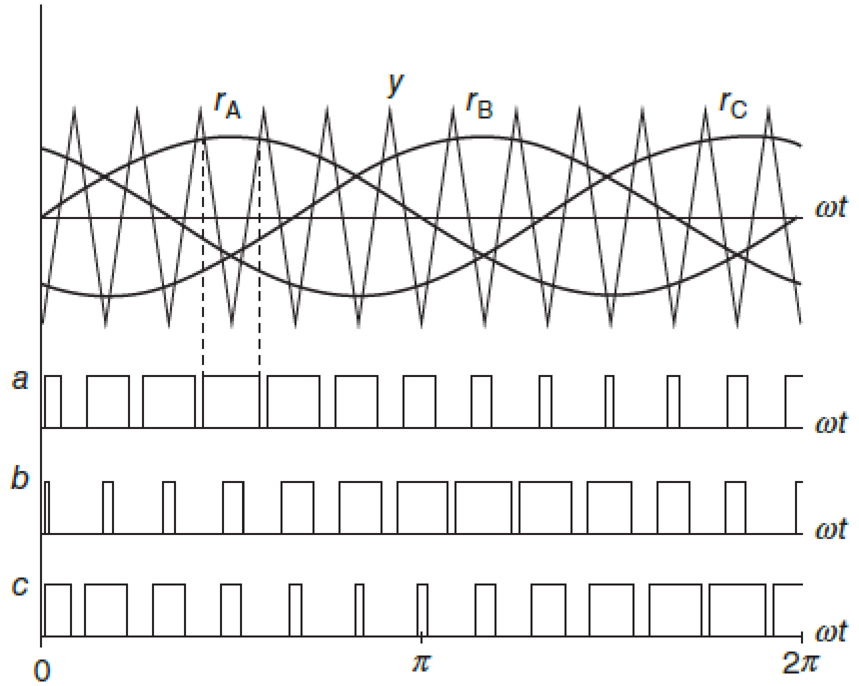
(2) The line and phase voltages spectrum show that the low order harmonics (odd-non-triple) are the most dominant harmonics. These harmonics can cause considerable distortion in the output current as its frequency is not too high. The **low order harmonic distortion** is the other basic drawback of the square wave mode.

Carrier-based PWM Control of Three Phase Inverter

As for the single phase inverter carrier comparison PWM can be implemented by comparing three reference sinusoidal phase voltages: r_A, r_B and r_C to the triangular carrier as shown in Fig. 19. Analogue to PWM in single phase inverter:

1- The reference signals have the desired output fundamental frequency.

2- Like the desired three output voltages, the three reference signals shifted from each other by 120°.
3- The three reference sinusoidal voltages have the same amplitude (as the desired three phase output voltages), and the output fundamental amplitude is proportional to the modulation index, *m*.
4- The triangular carrier have a frequency reflects the switching frequency, the ratio of the carrier frequency-to- the reference frequency is the frequency modulation index, m_f.



1 Illustration of the carrier-comparison PWM technique ($N = 12$, $m = 0.75$).

Fig. 19 Sinusoidal carrier comparison PWM of three phase inverter

The aim of the PWM is to produce a switching signal with a frequency, T_{sw} and with a duty ratio that is proportional to the reference amplitude, so the output voltage will change continuously according to the shape of the reference. This mechanism is explained in Fig. 20 where the reference is approximated to a constant value. This approximation is valid as the switching interval -the triangular signal interval- is small compared to the reference period ($T_{sw} \ll T$).

Figure 21 shows the relationship between the duty ratio of the switching variable (d) with the amplitude of the switching function during a switching interval.

In controlling the three-phase inverter it is desirable to keep a 120° displacement between the three-switching functions in order to keep the voltage balance. So, the reference waveforms satisfy the following form:

$$r_A = F(m, \omega t)$$

$$r_B = F\left(m, \omega t - \frac{2\pi}{3}\right)$$

$$r_c = F\left(m, \omega t - \frac{4\pi}{3}\right)$$

..(6.24)

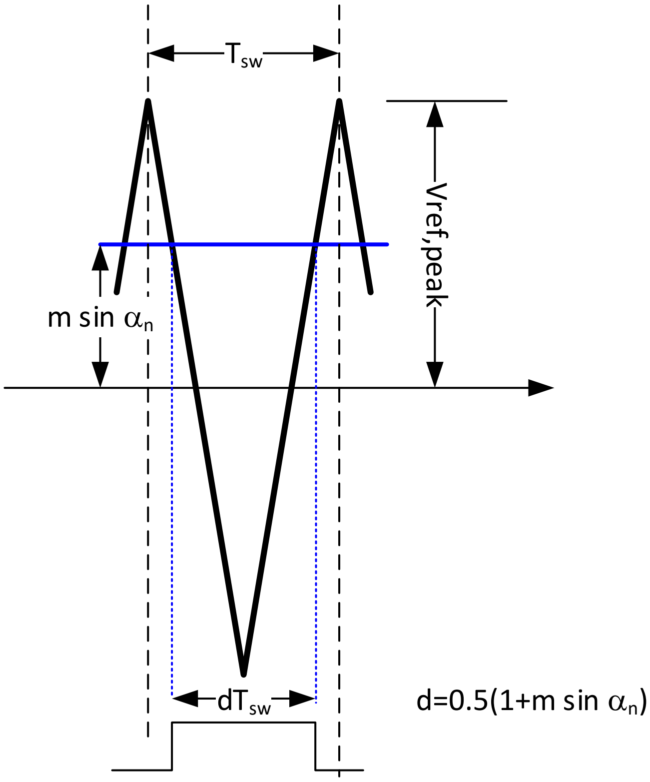


Fig. 20 The relationship between the switching variable duty ratio and the reference amplitude.

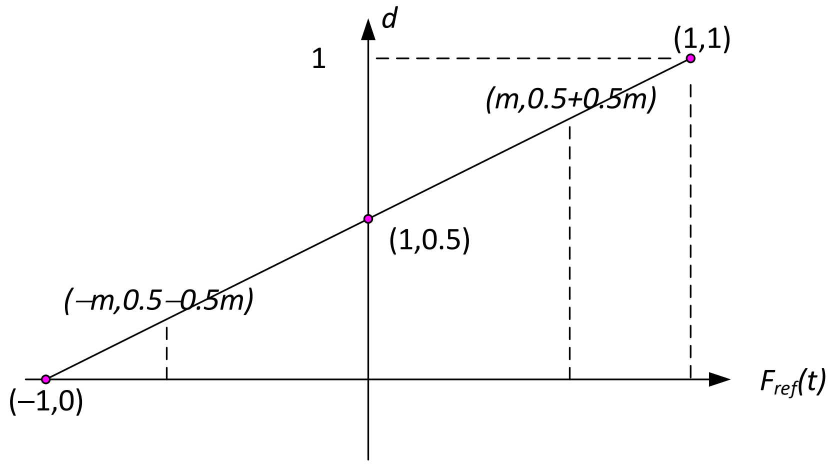


Fig. 21 the variation of the duty ratio with the amplitude of the modulation function.

Modulation Function

In Eq. (6.24) F represents the modulation function, the function that represent the desired output and therefore known here as the reference function. In Fig. 19 the modulation function is sinusoidal.

The relationship between the reference function and the duty ratio leads to the following specific equations for the duty ratios of the three switching functions:

$$d_a(n) = \frac{1}{2}[1 + m\sin(\alpha_n)\,]$$
$$d_b(n) = \frac{1}{2}\left[1 + m\sin\left(\alpha_n - \frac{2\pi}{3}\right)\right]$$
$$d_c(n) = \frac{1}{2}\left[1 + m\sin\left(\alpha_n - \frac{4\pi}{3}\right)\right]$$

(6.25)

Where n is the rank (position) of the switching interval, (n=1…m_f) and α_n is the center angle of the n[th] interval, $\alpha_n = \frac{2\pi}{m_f}\left(n - \frac{1}{2}\right)$.

The switching functions generated by the traditional sinusoidal carrier comparison shown in Fig. 19 have the duty ratios described in Eq. (6.25) providing that m_f>>1. Figure 22 shows that with m_f=12, each switching variable has m_f pulses per cycle. The line voltage has $2m_f$ pulses per cycle.

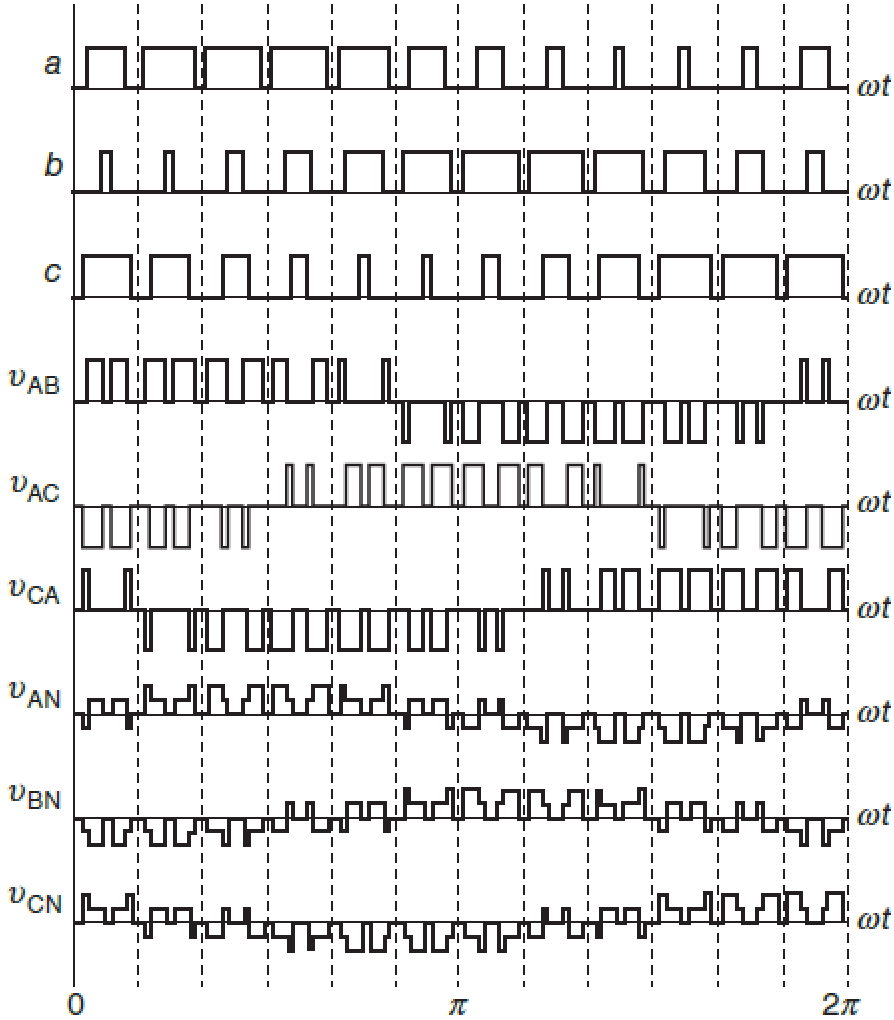


Fig. 22 Switching functions and output line and phase voltages of the three phase inverter.

Line voltage spectrum

In order to cancel low order harmonics, the symmetry between the three phase voltage besides the half-wave symmetry must be assured. This requires an ***m_f to be integer multiple of 3*** and **proper angle setting between the reference and the carrier**. The following discussion can be generalized if these two conditions are satisfied.

Fundamental component:

$$V_{LL,1,p} = \frac{\sqrt{3}}{2} m V_i \qquad ..(6.26)$$

where $(0 < m < 1)$

Dominant Harmonics Order:

The harmonics appear as side bands around m_f excluding even and triple.

With m_f=15 and m=0.8, the line voltage spectrum is shown in Fig. 23. It can be seen that the (m_f) and multiple harmonics are absent since m_f is multiple of 3 also no $m_f \pm 1$ as these numbers –in this case – are even.

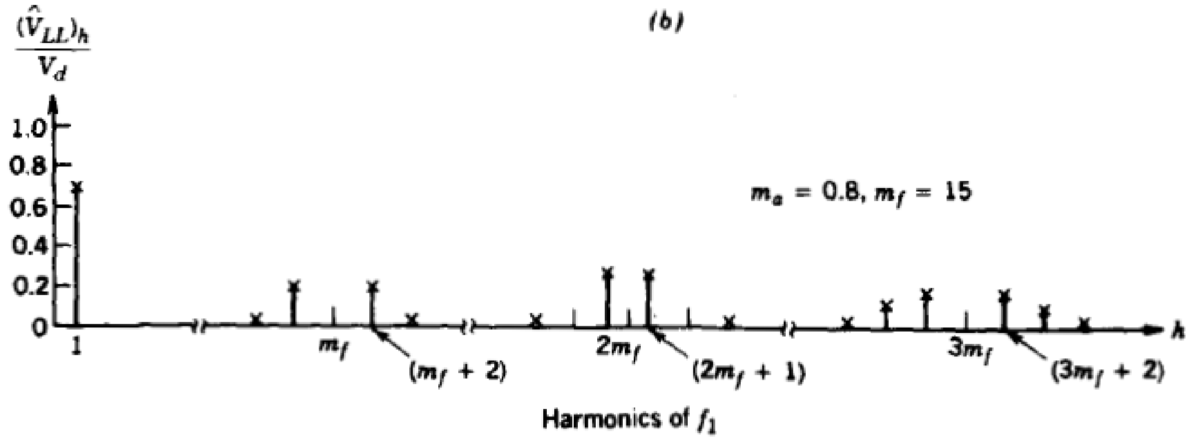


Fig. 23 Line-to-line voltage spectrum of the three phase PWM inverter

The table below shows that amplitude of different harmonics of a three-phase inverter with m_f which is odd and multiple of 3.

Table 8-2 Generalized Harmonics of v_{LL} for a Large and Odd m_f That Is a Multiple of 3.

h \ m_a	0.2	0.4	0.6	0.8	1.0
1	0.122	0.245	0.367	0.490	0.612
$m_f \pm 2$	0.010	0.037	0.080	0.135	0.195
$m_f \pm 4$				0.005	0.011
$2m_f \pm 1$	0.116	0.200	0.227	0.192	0.111
$2m_f \pm 5$				0.008	0.020
$3m_f \pm 2$	0.027	0.085	0.124	0.108	0.038
$3m_f \pm 4$		0.007	0.029	0.064	0.096
$4m_f \pm 1$	0.100	0.096	0.005	0.064	0.042
$4m_f \pm 5$			0.021	0.051	0.073
$4m_f \pm 7$				0.010	0.030

Note: $(V_{LL})_h/V_d$ are tabulated as a function of m_a where $(V_{LL})_h$ are the rms values of the harmonic voltages.

Example: A PWM controlled three phase inverter is operated with sinusoidal reference voltage. The modulation index, m=0.8, the frequency modulation index mf=39. The output voltage VLL,1 =300V at 60Hz. Determine (a) the DC link voltage (Vi).

Desirable characteristics

(1) Good utilization of the dc supply voltage, that is, a possibly high value of the maximum voltage gain, $K_{V(max)}$, defined here as: $K_{V(\max)} \equiv \frac{V_{LL,1,p(\max)}}{V_i}$

where $V_{LL,1,p(max)}$ denotes the maximum peak value of the fundamental line-to-line output voltage available using the technique under consideration.

(2) Linearity of the voltage control, that is,

$$V_{LL,1,p\,(m)} = mV_{LL,1,p(max)}$$

where m denotes the modulation index, which in inverters is identical to the magnitude control ratio. The magnitude control ratio is defined as the ratio of the actual output voltage (line-to-line or line-to-neutral, peak or rms value) to the maximum available value of this voltage.

(3) Low amplitudes of low-order harmonics of the output voltage to minimize the harmonic content of the output current.

(4) Low switching losses in the inverter switches.

(5) Sufficient time allowance for proper operation of the inverter switches and control system. (simple implementation).

Chapter 6

Inverters

PWM Techniques

The disadvantages of the basic sinusoidal PWM control is low supply voltage utilization and high switching losses. This lecture presents three alternative carrier comparison techniques to mitigate the above problems. The three carrier comparison techniques are: overmodulation, third harmonic and discontinuous PWM.

Overmodulation

Overmodulation indeed is not an individual control method! It is simply an extension of carrier comparison sinusoidal PWM (SPWM). By exceeding the limit of modulation index, *m*, for SPWM ($m>1$) the inverter enters overmodulation mode. Carrier and reference signals in overmodulation and switching signals and are shown in Fig. 24. The logic for producing switching signals is similar to that of SPWM (a=1 when $r_A>v_c$).

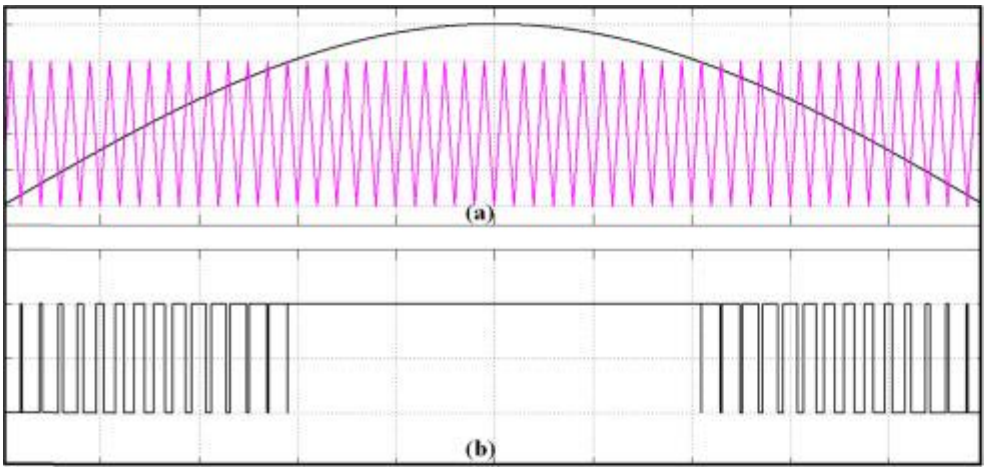


Fig. 24 (a) Reference and carrier waves and (b) the corresponding switching signal

Overmodulation is equally applicable to single phase and three phase inverters. The method is easy to apply and the extension of SPWM controller is straightforward and does not require any additional hardware. Overmodulation is normally used in inverters used to supply variable speed induction machines, where the inverter operate in linear mode to provide voltage up to 78% of the maximum voltage and then in overmodulation region to provide higher voltages.

The variation of the output voltage ($V_{o,1}$ for single phase or $V_{LL,1}$ for three phase) with *m*, is nonlinear in overmoduation region. Figure 25 shows that with large values of *m* (>3.24) the inverter reaches the square wave mode. This limit (*m* >3.24) changes slightly with m_f.

Operation in overmodulation region introduce low order harmonics as indicated in Fig. 26. As *m* increases from 1 to "3.24" the harmonics around m_f decreases and the low order harmonics (3, 5, 7 …) start to grow. This represent the frequency domain manifestation of the continuous transition from PWM to square wave.

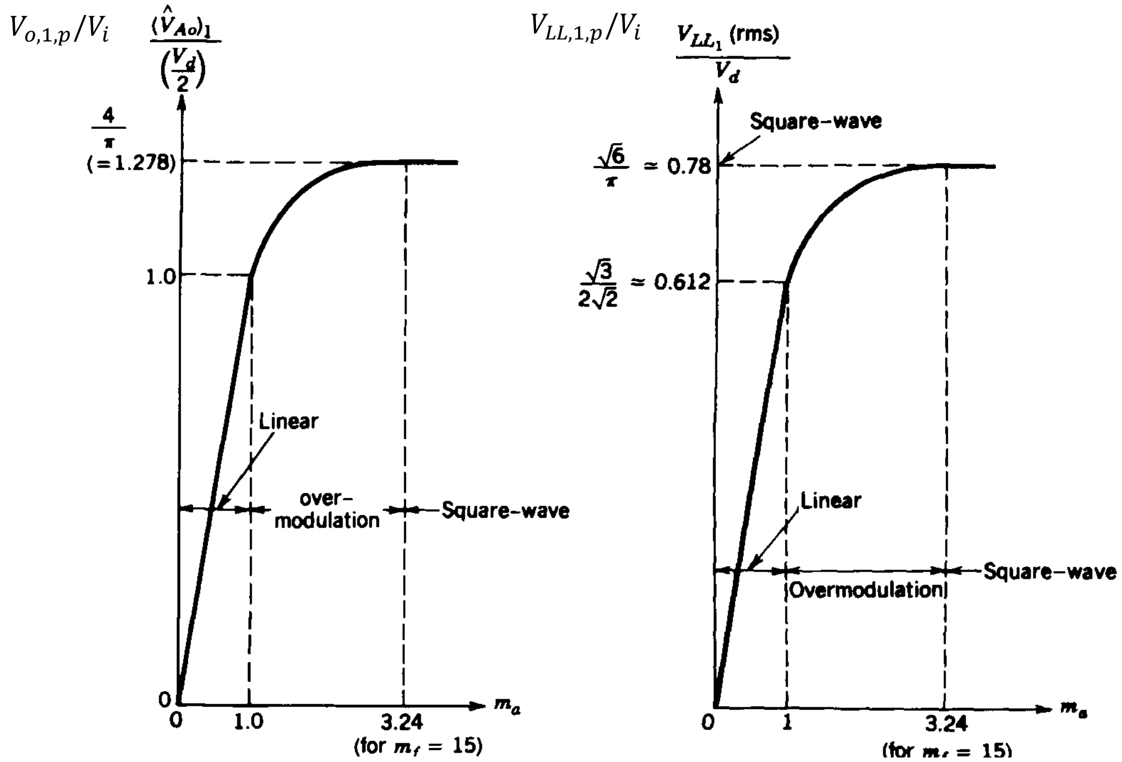


Fig. 25 The variation of the output voltage with modulation index in linear and overmodulation regions

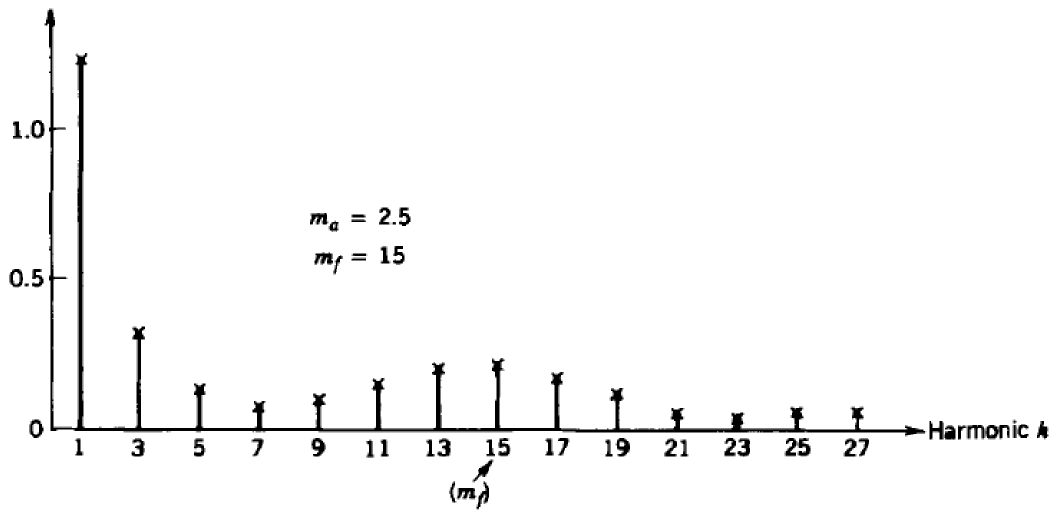


Fig. 26 Spectrum of the line voltage of three phase inverter in overmodulation mode

Third Harmonic PWM

The sinusoidal modulating function, $F(m,\omega t) = m\sin(\omega t)$, is simple, but the maximum output voltage (supply utilization) of the inverter can be significantly increased using special non-sinusoidal modulating functions. These functions are achieved by *third harmonic injection*, which means adding third harmonic component to the fundamental reference. In three phase inverters, the triple harmonics do not appear in output voltages and currents as shown earlier. The third-harmonic modulating function, is given by Eq. (6.27). Figure 27 shows the reference (r_A) for m = 1:

$$F(m,\omega t) = \frac{2}{\sqrt{3}} m \left[\sin(\omega t) + \frac{1}{6}\sin(3\omega t)\right] \quad ..(6.27)$$

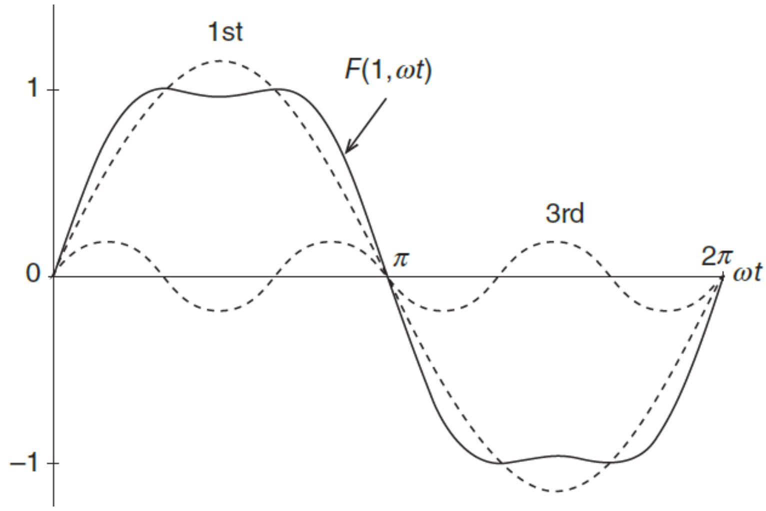


Fig. 27 Third harmonic modulation function.

The third harmonic component in the output lines are in common mode, this due to the 120° phase shift between the three voltages. Therefore, the output voltage (line or phase) having only the fundamental component, and third harmonics cancel each other.

At m = 1, the fundamental equals $2/\sqrt{3} \approx 1.15$, which represents a 15% increase in the maximum output voltage for the same V_i. The switching frequency, and hence the switching losses, are similar to its value in SPWM as there is one switching pulse per switching variable per switching interval (Ts) in both sinusoidal and third harmonic PWM.

The third harmonic PWM improves the supply utilization, but it does not affect the switching losses. The following method deals with the problem of switching losses.

Discontinuous PWM

The aim of discontinuous PWM is to save 1/3 of the switching losses!

This is done by adding a common mode (third harmonic) signal to the three reference signal r_A, r_B and r_C. The common mode has 120° period in such a way that each reference becomes 1 or –1 for 1/3 of each half cycle.

With m=1, the three sinusoidal reference signals and the injected third harmonics signal are shown in Fig. 28. For the switching variable a, the value of the new reference r_A' need to be 1 for $(60° \leq \omega t \leq 120°)$, therefore in this range the third harmonics injected function is calculated as:

$$cc = r_A' - r_A = 1 - m\sin\omega t \quad ((60° \leq \omega t \leq 120°),$$

Whereas the value of the new r_A' need to be –1 for $(240° \leq \omega t \leq 300°)$, gives the third harmonics injected function:

$$cc = r_A' - r_A = -1 - m\sin\omega t \quad ((240° \leq \omega t \leq 300°),$$

In similar way, cc is determined in other sectors as follows:

$$for\ 0° \leq \omega t \leq 60°\ r_B' = -1 \rightarrow cc = -1 - r_B$$

$$for\ 120° \leq \omega t \leq 180°\ r_C' = -1 \rightarrow cc = -1 - r_C$$

$$for\ 180° \le \omega t \le 240°\ r_B' = 1 \rightarrow cc = 1 - r_B$$

$$for\ 300° \le \omega t \le 360°\ r_C' = 1 \rightarrow cc = 1 - r_C$$

Where $r_A, r_B and\ r_C$ are the sinusoidal reference functions and $r_A', r_B' and\ r_C{}'$ are the discontinuous (modified) reference functions.

The resultant discontinuous reference are shown in Fig. 28 for m=1 and Fig. 29 for m=0.7.

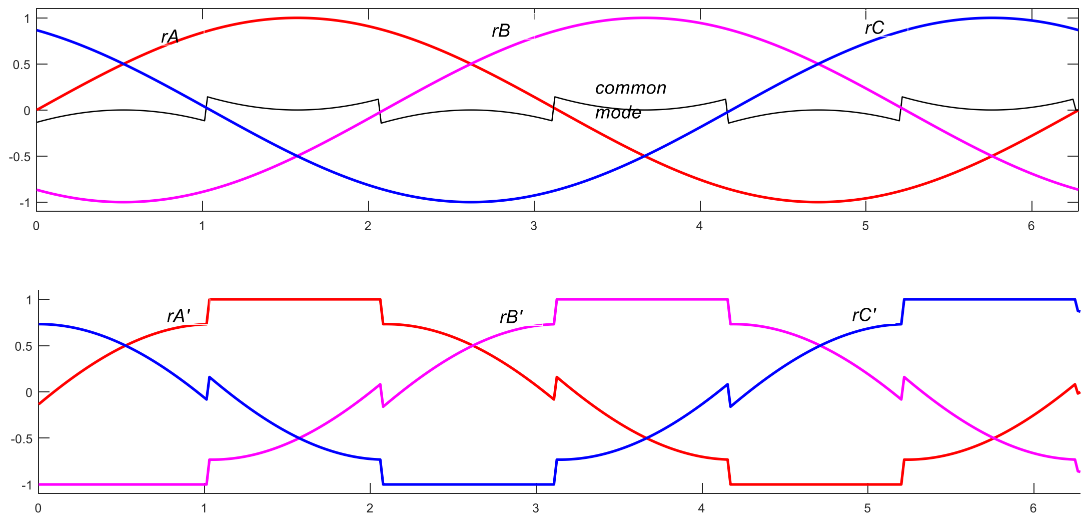


Fig. 28 discontinuous PWM (*m=1*) upper sinusoidal reference and common third harmonic voltage

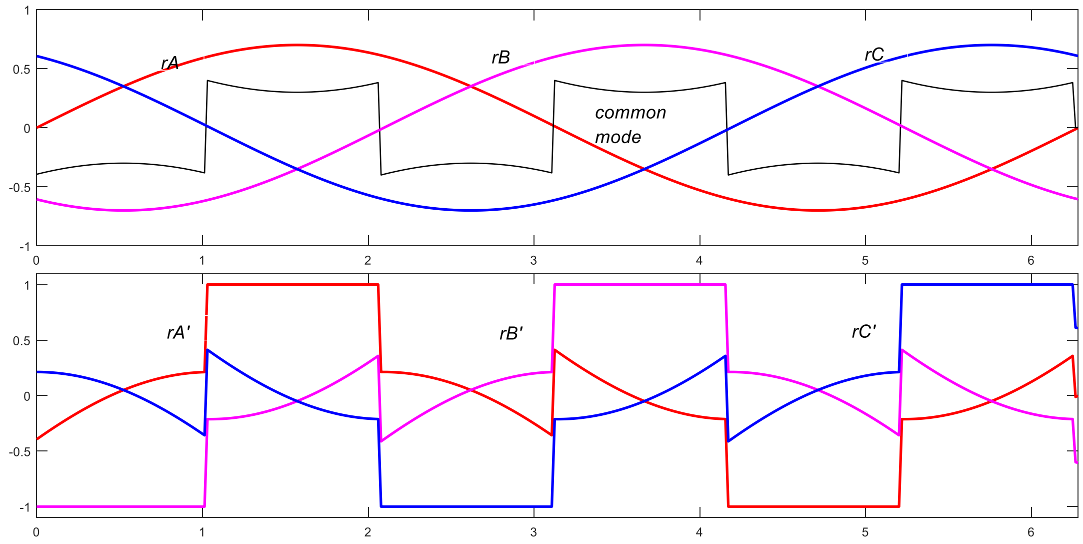


Fig. 29 discontinuous PWM (*m=1*) upper sinusoidal reference and common third harmonic voltage

Comparison between the different methods

Method	No. of switching cycles per switch per period	$V_{LL,1,p,Max}$ $/V_1$	Dominant Harmonics Order	Applicable to single-phase
Square wave	1	$\frac{2\sqrt{3}}{\pi}$	5, 7, 11, 13,..	✓
SPWM	m_f	$\frac{\sqrt{6}}{\pi}$	Around $m_f, 2m_f, ..$	✓
Overmodulation	$m_f \rightarrow 1$	$\frac{\sqrt{6}}{\pi} \rightarrow \frac{2\sqrt{3}}{\pi}$	Smoothly moves from SPWM to square wave mode	✓
Third harmonic PWM	m_f	$\frac{2\sqrt{2}}{\pi}$	Around $m_f, 2m_f, ..$	✕
Discontinuous PWM	$\frac{2}{3}m_f$	$\frac{\sqrt{6}}{\pi}$	Around $m_f, 2m_f, ..$	✕

`

Chapter 6

Programmed PWM Techniques

The best compromise between efficiency and quality of inverter operation is achieved in *programmed*, or *optimal switching* pattern, PWM techniques. The method is also known as *selected harmonic elimination (SHE)*. This lecture shows the application of SHE in single-phase and three-phase VSI control.

Programmable PWM in Single-phase inverter

Suppose that a single-phase inverter is controlled to produce the output voltage shown in Fig. 30. This waveform is quarter-wave symmetrical therefore the waveform, which has 12 switching events has only three independent switching angles. The independent switching angles are denoted by: α_1, α_2, and α_3. The independent angles are defined in the first quadrant as follows: $0 < \alpha_1 < \alpha_2 < \alpha_3 < \pi/2$.

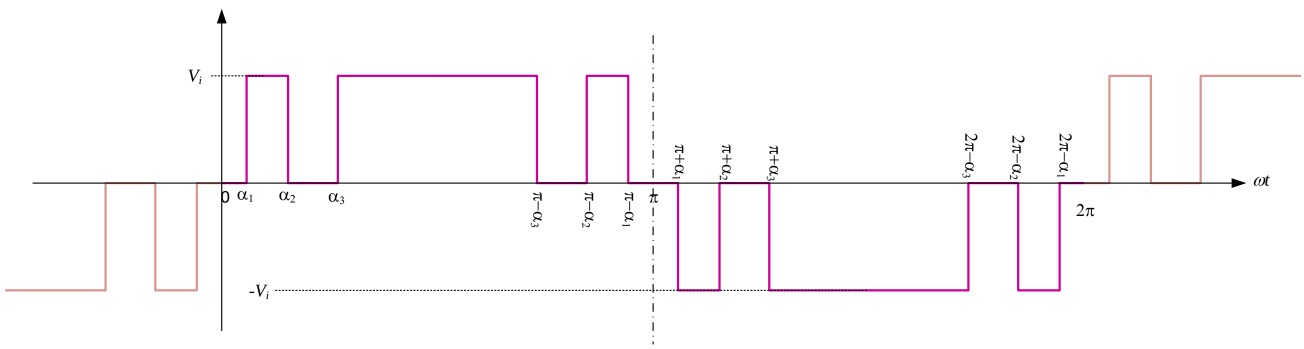


Fig. 30 Output voltage of single-phase inverter controlled with programmable PWM with three primary switching angles.

Consider Fourier analysis of the voltage waveform shown in Fig. 30. Since the waveform is half-wave and odd symmetrical:

$$B_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin n\omega t \, d\omega t = \frac{4V_i}{\pi}\left[\int_{\alpha_1}^{\alpha_2} \sin n\omega t \, d\omega t + \int_{\alpha_3}^{\frac{\pi}{2}} \sin n\omega t \, d\omega t\right]$$

$$= \frac{4V_i}{\pi n}[\cos n\alpha_1 - \cos n\alpha_2 + \cos n\alpha_3]$$

The amplitude of nth harmonic is given by:

$$V_{n,p} = \frac{4V_i}{\pi n}[\cos n\alpha_1 - \cos n\alpha_2 + \cos n\alpha_3] \qquad (6.28).$$

Equation (6.28) show that the amplitude of the n^{th} harmonic is a function of the independent switching angles (α_1, α_2, and α_3).

Inverter control by programmable PWM method with three primary switching angles is based on the idea that: *"since the output voltage depends on three control parameters (*α_1, α_2, and α_3); *we can determine the parameters that meet three specific requirements".*

From the inverter need point of view, the three most desirable needs are: (1) to set the amplitude of the fundamental output as desired. And (2) to eliminate the two most severe (lowest order) harmonics. The most severe harmonics are the third and fifth gives:

$$\frac{V_{1,p}{}^{*}}{V_i} = \frac{4}{\pi}[\cos\alpha_1 - \cos\alpha_2 + \cos\alpha_3]$$

$$V_{3,p} = 0 = \frac{4V_i}{3\pi}[\cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3]$$

$$V_{5,p} = 0 = \frac{4V_i}{5\pi}[\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3]$$

..(6.29)

The three equations (6.29) can be rearranged as follows:

$$\frac{V_{1,p}{}^{*}}{V_i} = m = \frac{4}{\pi}[\cos\alpha_1 - \cos\alpha_2 + \cos\alpha_3]$$

$$V_{3,p} = 0 = [\cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3]$$

$$V_{5,p} = 0 = [\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3]$$

..(6.29)

For a given reference output voltage Eq. (29) can be solved to obtain the primary switching angles that provide a fundamental output voltage as required and eliminate the 3^{rd} and 5^{th} harmonic.

Solving equations like those given in (6.26) is not straight forward. The equations are highly non-linear and require numerical iterations and the solution is beyond the scope of this course.

Example 1. A single-phase inverter is controlled by programmable PWM method with three primary switching angles. When, $V_{o,1}$=0.8 V_i, and the third and fifth harmonics are eliminated, (a) write the equations that used to determine the primary switching angles

(b) Show that the primary angles ($\alpha_1 = 31.43°, \alpha_2 = 54.52°\ and\ \alpha_3 = 69.24°$) meets the design conditions

Sol.

(a)

$$\frac{0.8\pi}{4} = [\cos\alpha_1 - \cos\alpha_2 + \cos\alpha_3] = 0.628$$

$$0 = [\cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3]$$

$$0 = [\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3]$$

(b) Substitute the given angles in the three equations give:

$$[\cos 31.43 - \cos 52.52 + \cos 69.24] = 0.6273\surd$$

$$[\cos(3\times 31.43) - \cos(3\times 54.52) + \cos(3\times 69.24)] = 0.0009\surd$$

$$[\cos(5\times 31.43) - \cos(5\times 54.52) + \cos(5\times 69.24)] = 0.004\surd$$

The switching signals corresponding to the SHE control shown in Fig. 31 show that these functions are fully defined as a function of the primary switching angles.

In general: if the inverter output voltage (or the switching signal) of the programmable PWM has *k* primary switching angles, we can use these angles to control the fundamental component ($V_{o,1}$) and eliminate *(k-1)* harmonics. Normally we will eliminate the most dominant (or lowest order) harmonics.

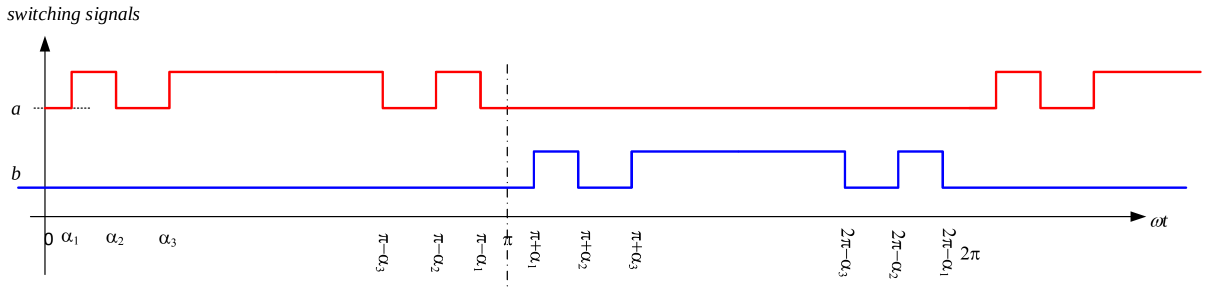


Fig. 30 Switching signals corresponding to programmable PWM with three primary angles

Example 2 The single-phase inverter is controlled with seven primary switching angles to eliminate the six lowest order harmonics. The seven switching angles are as follows: ($\alpha_1 = 17.249°, \alpha_2 = 24.023°, \alpha_3 = 34.995°, \alpha_4 = 48.227°, \alpha_5 = 53.933°, \alpha_6 = 73.777°$ and $\alpha_7 = 75.989°$).

(a) Show that the six lowest order harmonics has been eliminated.

(b) Determine the fundamental output voltage ($V_{o,1}/V_i$).

Sol.

(a) $V_{3,p} = \frac{4V_i}{3\pi}[\cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3 - \cos 3\alpha_4 + \cos 3\alpha_5 - \cos 3\alpha_6 + \cos 3\alpha_7]$

$$V_{3,p} = \frac{4V_i}{3\pi}(\dots) \approx 1 \times 10^{-5} V_i$$

In the same way, we can show that the 5th, 7th, 9th, 11st and 13th are all zeros. (left for students as drill)

(b)

$$V_{1,p} = \frac{4V_i}{\pi}[\cos \alpha_1 - \cos \alpha_2 + \cos \alpha_3 - \cos \alpha_4 + \cos \alpha_5 - \cos \alpha_6 + \cos \alpha_7]$$

$$V_{1,p} = 0.95\, V_i$$

In previous examples MATLAB function (fslove) has been used to obtain the switching angles used in the examples. You may try to find the switching angles for other conditions (like different fundamental amplitudes or different number of primary switching angles).

Programmable PWM in Three -phase inverter

As indicated earlier, the harmonics of the output voltages are functions of the switching angles. Therefore if we eliminate certain harmonic in the switching function, this component will not appear in the output voltage.

When the three-phase inverter is controlled with programmable PWM, the primary switching angles are the switching angle of the switching variable *a* as shown in Fig. 32. The switching signals (*b and c*) are obtained by shifting (*a*) by 120° and 240° respectively as shown in Fig. 32.

$$a(\omega t) = b\left(\omega t + \frac{2\pi}{3}\right) = c\left(\omega t + \frac{4\pi}{3}\right)$$

The half-wave symmetry of waveforms of switching variables results in the absence of even harmonics as well, while the quarter-wave symmetry allows expressing amplitude of the kth harmonic, $A_{k,p}$, of a(ωt) as follows:

-for three primary switching angles:

$$A_{k,p} = \frac{4}{k\pi}\left[\cos(k\alpha_1) - \cos(k\alpha_2) + \cos(k\alpha_3) - \frac{1}{2}\right]$$

-for K primary switching angles

$$A_{k,p} = \frac{4}{k\pi}\left[\sum_{i=1}^{k}(-1^{i+1}\cos(k\alpha_i)) - \frac{1}{2}\right]$$

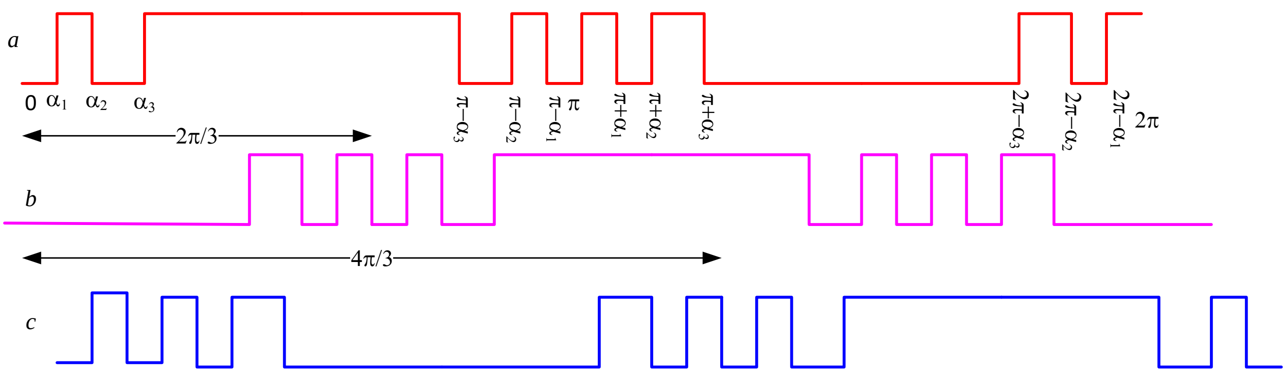


Fig. 32 switching angles of three phase inverter with programmable PWM and three primary switching angles.

As for single-phase inverter, optimal values of primary switching angles is the harmonic-elimination PWM technique are calculated to set $A_{1,p}$ to the desired output voltage. Then the *k-1* lowest order harmonics are set to zero. In three phase inverter, the triple harmonics are in common mode, therefore we eliminate the 5^{th} , 7^{th} 11^{st} … harmonics (odd non-triple).

For instance, if K = 5, the maximum available amplitude of the fundamental, $A_{1,p(max)}$, of a(ωt) is 0.583 [$V_{LL,1,p,max}/V_i = \sqrt{3} * 0.583 = 1.01$] and the 5th, 7th, 11th, and 13th harmonics can be eliminated, leaving the 17th as the lowest-order one.

The optimal angles in the example case of K = 5 are shown in Figure 33 as functions of the magnitude control ratio, M ($A_{1,p}/A_{i,p,max}$). A harmonic spectrum of the output line-to-neutral voltage in Figure 34, determined for M = 1, demonstrates the desired elimination of low-order, even, and triple harmonics.

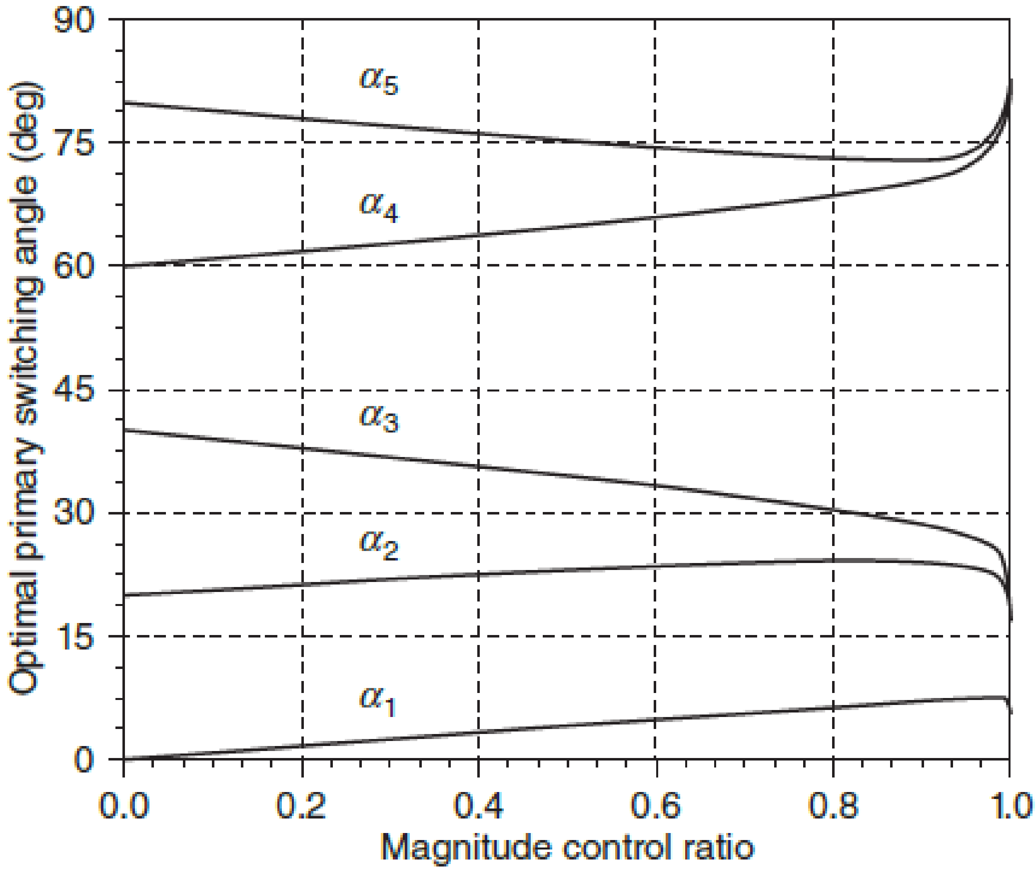


Fig. 33 Optimal primary switching angles as functions of the magnitude control ratio (K = 5).

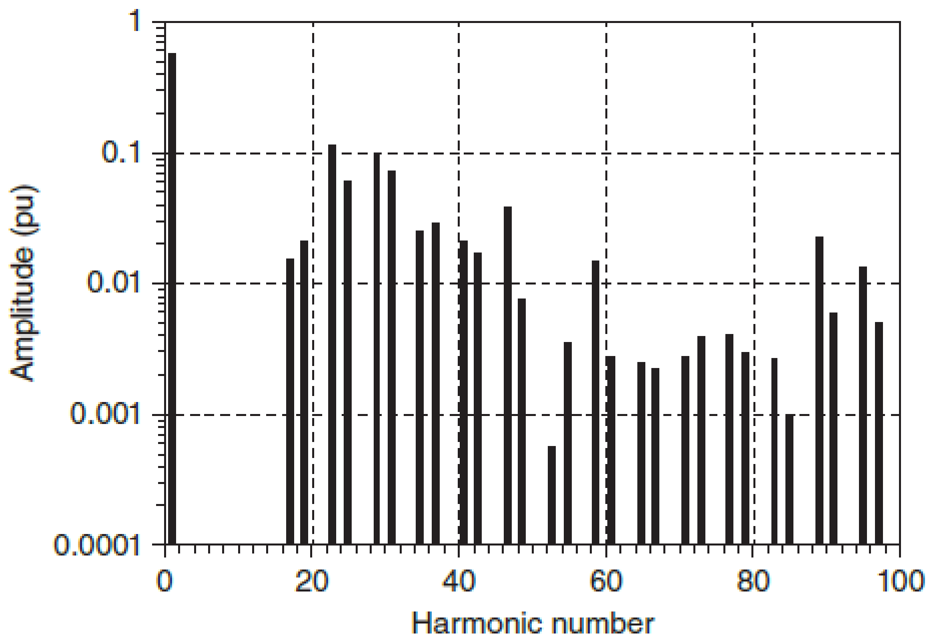


Fig. 34 Harmonic spectrum of the line-to-neutral voltage with the harmonic-elimination technique (K = 5, M = 1).

Comparison between programmable PWM and carrier comparison techniques.

The main advantages of programmable PWM are:

1- Bets compromise between current distortion and switching losses.
2- High DC voltage utility (slightly higher than 3rd harmonic PWM)

Disadvantages are

1- Not suitable for real time application. The switching angle must be calculated off-line and stored in look-up table to be played when operating the inverter.
2- Not suitable for closed loop current control.

Drill Question: Using Fig. 33 try to determine the five primary switching angles when m=0.8. Based on the switching angles found determine the fundamental output voltage and calculate the amplitudes of the harmonics of order below 20.

Chapter 7

DC Supply Circuits

1-Buck-Converter

This lecture presents the buck converter (the step-down converter): the lecture covers circuit operation, analysis and design.

Circuit Diagram and assumption

The buck converter is one of the basic DC supply circuits, the circuit can perform step-down conversion that is the output voltage is always less than the input voltage.

The circuit of the buck converter is shown in Fig.1. It is seen that the buck converter is nothing more, or less, than a first quadrant chopper (studied in chapter 5) with a low pass LC filter connected to the input side. Since the circuit is, generally, a low power circuit it is expected that the load is modeled as a resistor and we cannot count on the load inductance to filter out the ripple.

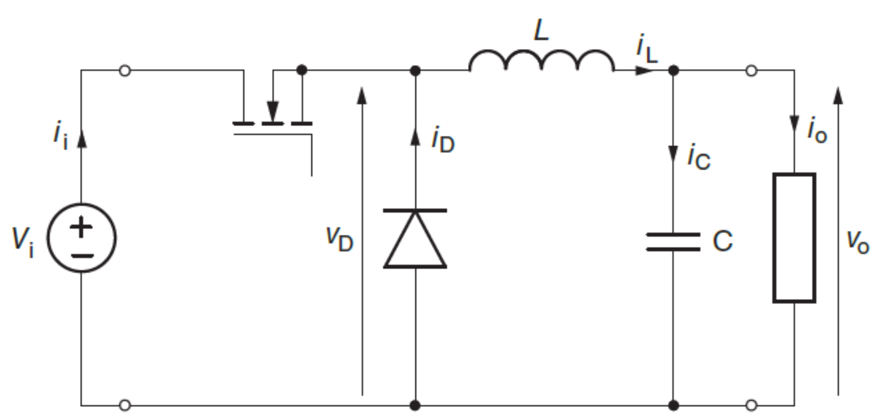


Fig. 1 Buck-Converter

The following analysis assumes the following:

- Constant capacitor voltage: initially we will assume that the capacitor is large enough to allow ripple free capacitor voltage.
- Continuous inductor current (i_L).

Switching function and circuit states

The switching is operated periodically using PWM control with constant duty ratio for a constant output voltage as shown in Fig.2.

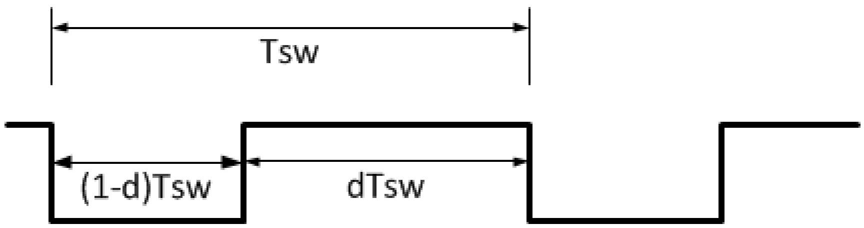


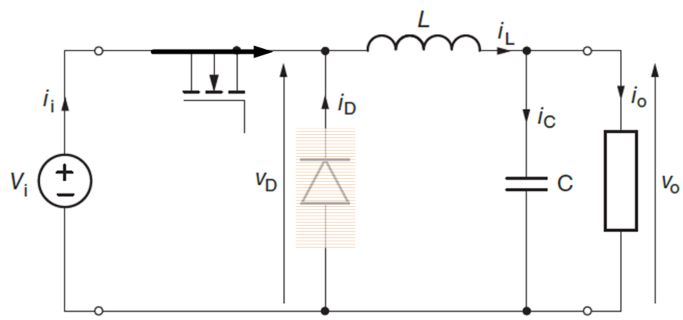
Fig. 2 Switching function

The equivalent circuits when the switch is on is shown in Fig. 3 a. As indicated by the faint line, the diode is reverse biased when the switch turns on. Based on the two above mentioned assumptions we can write the following equations for on state:

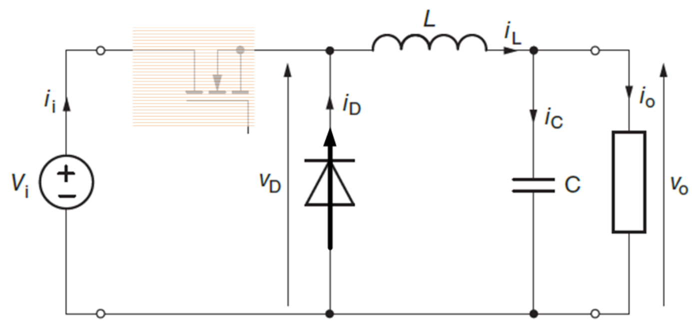
$$\begin{aligned} i_i &= i_L \\ i_D &= 0 \\ v_L &= V_o - V_i \end{aligned} \quad ..(1)$$

When the switch is off the equivalent circuit is shown in Fig. 3 b. The diode is forward biased because the inductor current is continuous. The corresponding equations for off state:

$$\begin{aligned} i_i &= 0 \\ i_D &= i_L \\ v_L &= -V_o \end{aligned} \quad ..(2)$$



(a)



(b)

Fig. 3 the equivalent circuits during (a) on-state, (b) off-state

To complete the analysis of the circuit, we must consider various voltages and currents in the circuit as discussed in the following paragraphs

Inductor voltage and output voltage

The inductor voltage variation with the switching signals-as defined in Equations (1) and (2)- is shown in Fig. 4. This voltage has zero average (in steady state). Therefore:

$$dT_{sw}(V_i - V_o) = (1-d)T_{sw}V_o$$

$$d\,\underbrace{T_{sw}}\,V_i\,\overbrace{-dT_{sw}V_o} = \underbrace{T_{sw}}\,V_o\,\overbrace{-dT_{sw}V_o}$$

$$dV_i = V_o$$

The magnitude control ratio:

$$\boxed{M = \frac{V_o}{V_i} = d \qquad \ldots(3)}$$

Inductor Current and Continuous Conduction Condition

Since the inductor voltage is constant- during ton and toff-, the inductor current changes linearly in both conditions. We will determine first the average inductor current, $I_{L,dc}$:

Since: $i_L = i_o + i_C$, and $v_C = V_o$ and $I_{C,dc} = 0$, which implies

$$I_{L,dc} = I_o = \frac{V_o}{R}$$

Where R is the load resistance.

To determine the inductor current ripple (ΔI_L), we use the basic inductor voltage equation:

$$v_L = L\frac{di}{dt}$$

During ton:

$$(V_i - V_o) = L\frac{\Delta I_L}{dT_{sw}},$$

Gives: $\Delta I_L = (V_i - V_o)\frac{dT_{sw}}{L} = \frac{V_i(1-d)d}{df_{sw}}$ ..(4)

You can reach the same expression by considering the variation during toff. (left to you as drill)

For continuous inductor current, the following condition must be satisfied:

$$\frac{\Delta I_L}{2} < I_{l,dc} = I_o$$

$$\frac{V_o}{R} > \frac{d(1-d)V_i}{2Lf_{sw}}$$

$$\boxed{Lf_{sw} > \frac{d(1-d)R}{2} \qquad ..(5)}$$

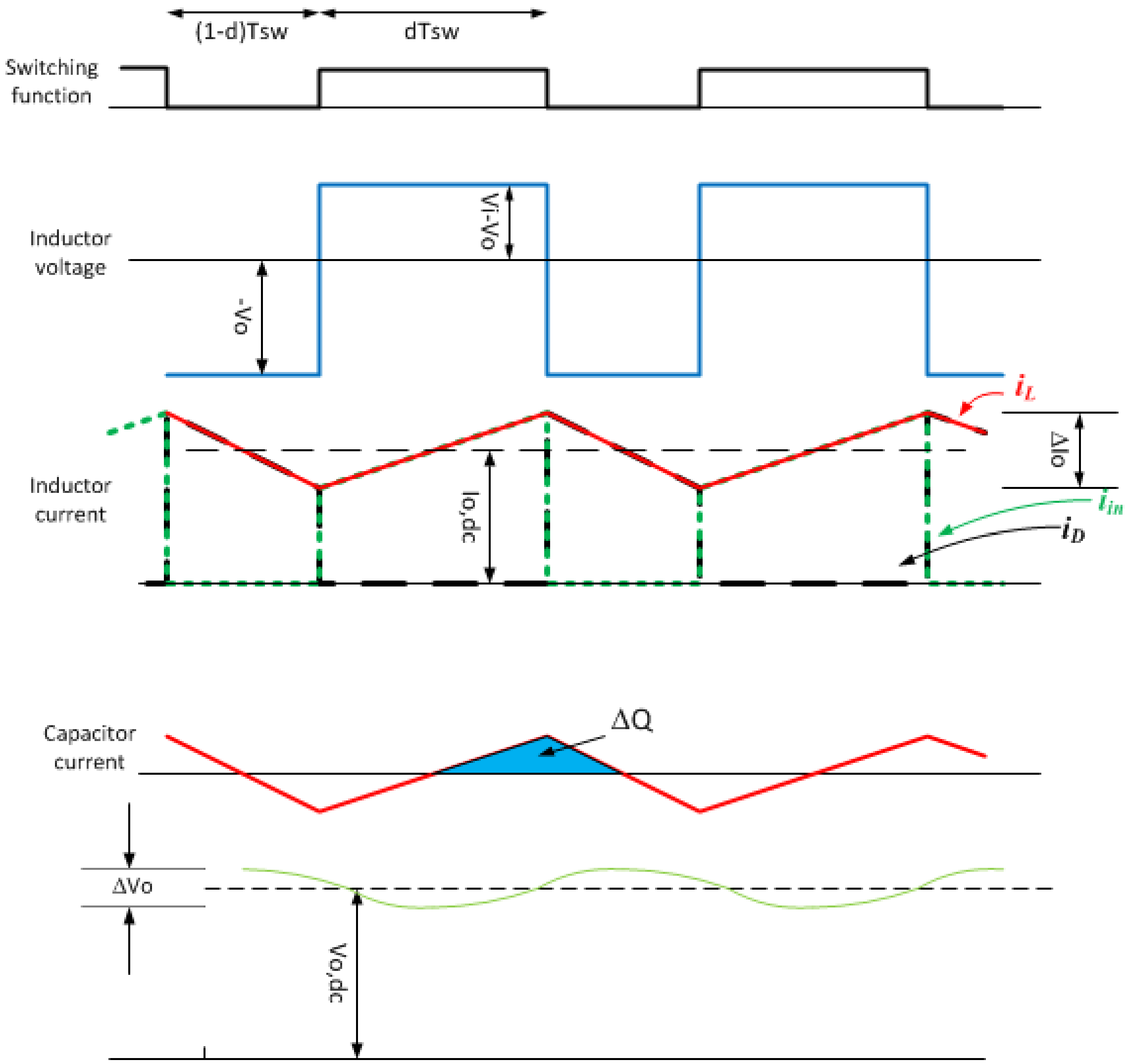


Fig. 4 Waveforms of voltages and currents for buck converter

Capacitor Current and Capacitor Voltage Ripple

The capacitor current is obtained by taking the ac component of the inductor current as indicated earlier. This current changes linearly, its peak value is $\Delta I_L/2$, where ΔI_L is given in Eq. (4). This current will change the capacitor voltage, the variation of the capacitor voltage (peak-to-peak) is determined by dividing the area ΔQ is Fig. 4 by the capacitance C. ΔQ represent the charge that has been added to the capacitor.

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{2}\frac{\Delta I_L}{2}\frac{T_{sw}}{2}\frac{1}{C}$$

$$\Delta I_L = \frac{1}{2}\frac{d(1-d)V_i}{2L\,f_{sw}}\frac{1}{2f_{sw}}\frac{1}{C}$$

$$\Delta V_o = \frac{(1-d)V_o}{8CLf_{sw}{}^2}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{1-d}{8CLf_{sw}{}^2}}$$

..(6)

Example

The buck dc-dc converter is supplied form a 50 V source, and operated with duty ratio, d, ranges between 0.4~0.6. The inverter uses inductance of L =400μH and capacitor the of (C =) 100 μF. The load has a variable resistance, R, of 20Ω minimum and 100Ω maximum. Assuming all components are ideal. Answer four of the following.

(a) Draw the circuit diagram and find the maximum and minimum values of the output voltage.

(b) Determine the minimum switching frequency that assure continuous inductor current.

(c) With switching frequency of 20kHz, d=0.4, and R=20Ω; draw to a scale the following quantities. (i) the switching signal, (ii) the inductor voltage, (iii) the inductor current. Use the same time scale, calculate and show all amplitudes at the switching points.

(d) Derive the expression of the output voltage-to- input voltage ratio at steady state using inductor voltage waveform.

(e) With switching frequency of 20kHz, calculate the maximum peak-to-peak output voltage ripple (ΔVo).

Chapter 7

DC Supply Circuits

2-Boost-Converter

This lecture presents the boost converter (the step-up converter): the lecture covers circuit operation, analysis and design.

Circuit Diagram and assumption

The boost converter is one of the basic DC supply circuits, the circuit can perform step-up conversion that is the output voltage is always higher than the input voltage.

The circuit of the boost converter is shown in Fig 5. It is seen that the inductor of the boost converter is no longer a part of the output filter. To enable the stepup function the inductor is connected in such a way to perform as energy storage element.

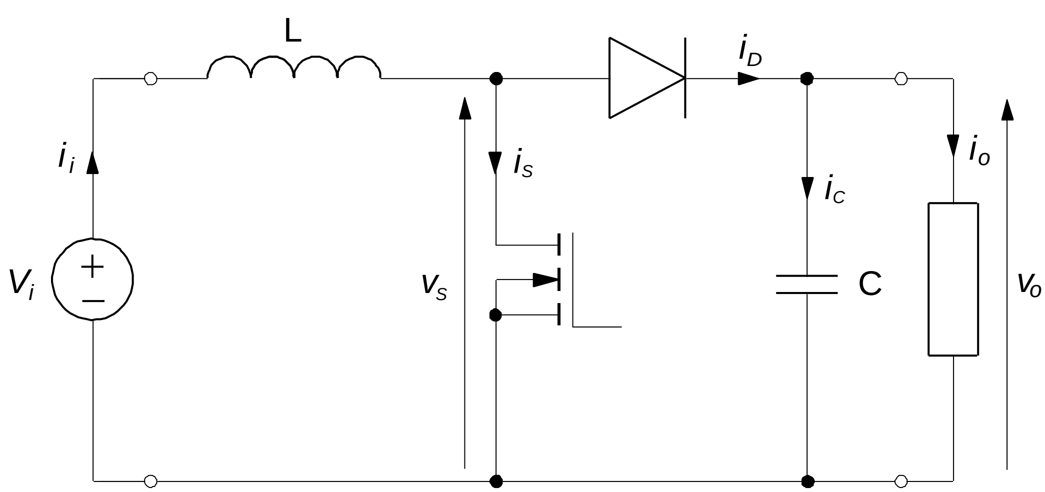


Fig. 5 Boost-Converter

The following analysis consider the assumptions:

- Constant capacitor voltage: initially we will assume that the capacitor is large enough to allow ripple free capacitor voltage.
- Continuous inductor current (i_L).
- Zero ON-state voltage drop, and zero inductor and capacitor parasitic resistor.

Switching function and circuit states

The switch is operates periodically using PWM control with constant duty ratio similar to that of buck converter shown in Fig. 2.

The equivalent circuits when the switch is ON is shown in Fig. 6a. As indicated by shading, the diode is reverse biased when the switch turns on. Therefore the corresponding ON-state equations

$$\begin{aligned} i_i &= i_L \\ i_D &= 0 \\ v_L &= V_i \end{aligned} \qquad ..(7)$$

When the switch is OFF the equivalent circuit is shown in Fig. 6 b. The diode is forward biased because the inductor current is continuous. The corresponding equations for off state:

$$\begin{aligned} i_i &= i_L \\ i_D &= i_L \\ v_L &= V_i - V_o \end{aligned} \qquad ..(8)$$

To complete the analysis of the circuit, we must consider various voltages and currents in the circuit as discussed in the following paragraphs

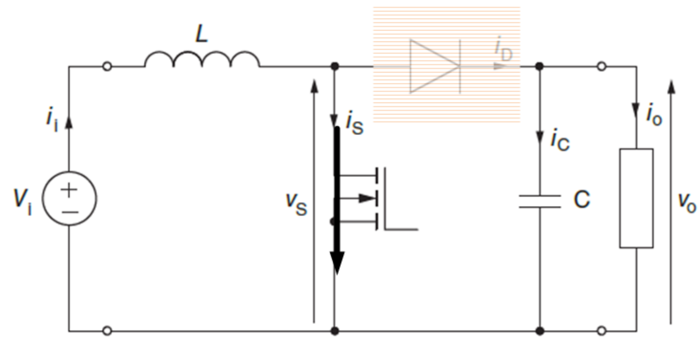
Inductor voltage and output voltage

The inductor voltage variation with the switching signals-as defined in Equations (7) and (8)- is shown in Fig. 7. This voltage has zero average (in steady state). Therefore:

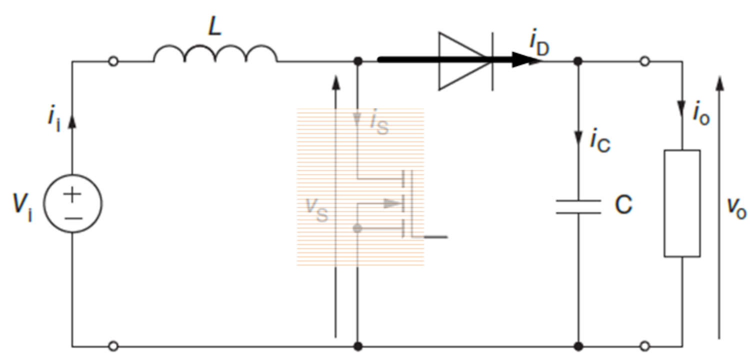
$$dT_{sw}V_i = (1-d)T_{sw}(V_o - V_i)$$
$$dV_i = (1-d)(V_o - V_i)$$
$$dV_i = V_o - dV_o - V_i + dV_i$$
$$0 = V_o - dV_o - V_i$$

Gives

$$\frac{V_o}{V_i} = \frac{1}{1-d} \qquad \ldots(9)$$



(a)



(b)

Fig. 6 the equivalent circuits during (a) on-state, (b) off-state

Inductor Current and Continuous Conduction Condition

Figure 7 shows the switching signals and the corresponding inductor voltage. Since the inductor voltage is constant- during ton and toff-, the inductor current changes linearly in both conditions. We will determine first the average inductor current, $I_{L,dc}$:

Since: $i_L = i_i$ and $I_{i,dc}V_i = I_oV_o$

$$I_{L,dc} = I_{i,dc} = \frac{V_o}{V_i}I_o = \frac{1}{1-d}\frac{V_o}{R} \quad ..(10)$$

Where R is the load resistance.

To determine the inductor current ripple (ΔI_L), we use the basic inductor voltage equation:

$$v_L = L\frac{di}{dt}$$

During ton:

$$V_i = L\frac{\Delta I_L}{dT_{sw}},$$

Gives: $\Delta I_L = V_i\frac{dT_{sw}}{L} = \frac{V_i d}{Lf_{sw}}$ ..(11)

You can reach the same expression by considering the variation during toff. (left to you as drill)

The previous analysis used to derive equation (10), are based on the assumption that the inductor current is always positive, when the inductor current tends to become negative a third state will be reached in which both the switch and the diode are off. Normally the converter is designed to operate with $i_L > 0$ all the time and that is known as the continuous conduction mode. To maintain continuous inductor current, the following condition must be satisfied:

$$\frac{\Delta I_L}{2} < I_{L,dc}$$

$$\frac{V_i d}{2Lf_{sw}} < \frac{1}{1-d}\frac{V_o}{R}$$

$$Lf_{sw} > \frac{(1-d)dV_iR}{2V_o}$$

By substituting for Vo from (10), the continuous conduction condition is given by:

$$Lf_{sw} > \frac{(1-d)^2 dR}{2} \qquad ..(12)$$

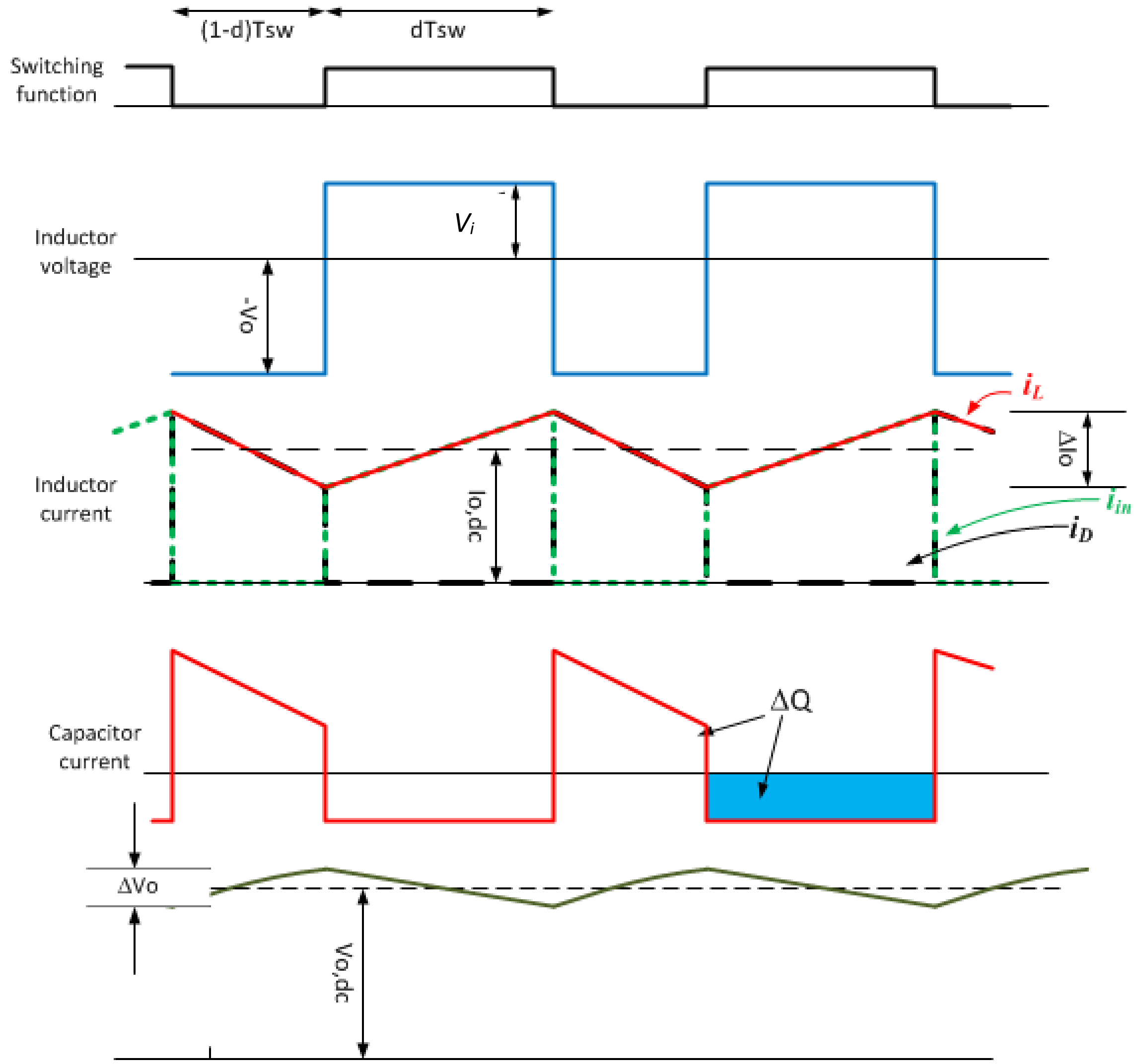


Fig. 7 Waveforms of voltages and currents for buck converter

Capacitor Current and Capacitor Voltage Ripple

The capacitor current is equivalent to the ac component of the diode current. As shown in Fig. 7, when the switch is on, the capacitor current is equals to ($i_{C,ON} = -I_o$). When the switch is off, the capacitor current ($i_{C,OFF} = i_L - I_o$); This current changes the capacitor voltage, the variation of the capacitor voltage (peak-to-peak) is determined by dividing the area ΔQ is Fig. 7 by the capacitance C. ΔQ represent the charge that has been added to the capacitor during t_{off} , or equivalently, the charge taken from the capacitor during t_{on}. The determination during t_{on} is simpler since the capacitor current is constant.

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{dI_o}{Cf_{sw}}$$

$$\Delta V_o = \frac{dV_o}{RCf_{sw}}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{d}{RCf_{sw}}}$$

..(13)

Example

A boost dc-dc converter is supplied form a 12 V source, and produces an output voltage in the range $15<V_o<40V$. The inverter uses inductance of L =0.5mH and capacitor the (C =) 100 μF. The load resistance, R= 60Ω. The switching frequency is 50kHz.

(a) Draw the circuit diagram and find the maximum and minimum values of the duty ratio.

(b) Show that the inductor current is continuous for the entire range of output voltage.

(c) With switching frequency of 20kHz, calculate the maximum peak-to-peak output voltage ripple (ΔVo).

(d) When Vo=20V; draw to a scale the following quantities. (i) the switching signal, (ii) the inductor voltage, (iii) the inductor current (iv) the diode current (v) the capacitor current and (vi) the capacitor voltage. Use the same time scale, calculate and show all amplitudes at the switching points.

Chapter 7

DC Supply Circuits

2-Boost-Converter

This lecture presents the boost converter (the step-up converter): the lecture covers circuit operation, analysis and design.

Circuit Diagram and assumption

The boost converter is one of the basic DC supply circuits, the circuit can perform step-up conversion that is the output voltage is always higher than the input voltage.

The circuit of the boost converter is shown in Fig 5. It is seen that the inductor of the boost converter is no longer a part of the output filter. To enable the stepup function the inductor is connected in such a way to perform as energy storage element.

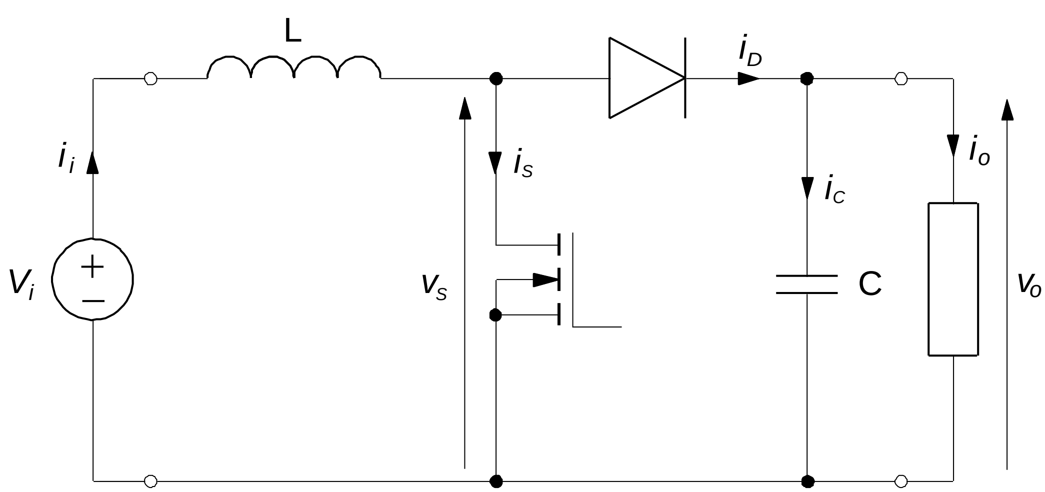


Fig. 5 Boost-Converter

The following analysis consider the assumptions:

- Constant capacitor voltage: initially we will assume that the capacitor is large enough to allow ripple free capacitor voltage.
- Continuous inductor current (i_L).
- Zero ON-state voltage drop, and zero inductor and capacitor parasitic resistor.

Switching function and circuit states

The switch is operates periodically using PWM control with constant duty ratio similar to that of buck converter shown in Fig. 2.

The equivalent circuits when the switch is ON is shown in Fig. 6a. As indicated by shading, the diode is reverse biased when the switch turns on. Therefore the corresponding ON-state equations

$$\begin{aligned} i_i &= i_L \\ i_D &= 0 \\ v_L &= V_i \end{aligned} \qquad ..(7)$$

When the switch is OFF the equivalent circuit is shown in Fig. 6 b. The diode is forward biased because the inductor current is continuous. The corresponding equations for off state:

$$\begin{aligned} i_i &= i_L \\ i_D &= i_L \\ v_L &= V_i - V_o \end{aligned} \qquad ..(8)$$

To complete the analysis of the circuit, we must consider various voltages and currents in the circuit as discussed in the following paragraphs

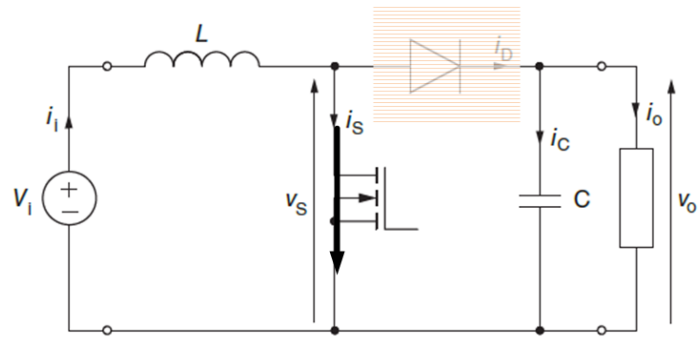
Inductor voltage and output voltage

The inductor voltage variation with the switching signals-as defined in Equations (7) and (8)- is shown in Fig. 7. This voltage has zero average (in steady state). Therefore:

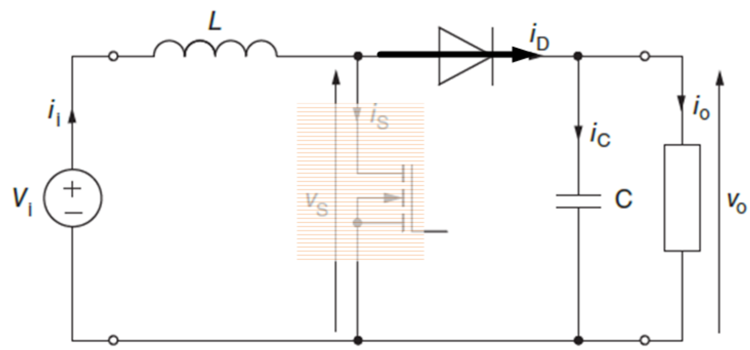
$$dT_{sw}V_i = (1-d)T_{sw}(V_o - V_i)$$
$$dV_i = (1-d)(V_o - V_i)$$
$$dV_i = V_o - dV_o - V_i + dV_i$$
$$0 = V_o - dV_o - V_i$$

Gives

$$\frac{V_o}{V_i} = \frac{1}{1-d} \quad \ldots(9)$$



(a)



(b)

Fig. 6 the equivalent circuits during (a) on-state, (b) off-state

Inductor Current and Continuous Conduction Condition

Figure 7 shows the switching signals and the corresponding inductor voltage. Since the inductor voltage is constant- during ton and toff-, the inductor current changes linearly in both conditions. We will determine first the average inductor current, $I_{L,dc}$:

Since: $i_L = i_i$ and $I_{i,dc}V_i = I_oV_o$

$$I_{L,dc} = I_{i,dc} = \frac{V_o}{V_i}I_o = \frac{1}{1-d}\frac{V_o}{R} \qquad ..(10)$$

Where R is the load resistance.

To determine the inductor current ripple (ΔI_L), we use the basic inductor voltage equation:

$$v_L = L\frac{di}{dt}$$

During ton:

$$V_i = L\frac{\Delta I_L}{dT_{sw}},$$

Gives: $\Delta I_L = V_i\frac{dT_{sw}}{L} = \frac{V_i d}{Lf_{sw}}$ ..(11)

You can reach the same expression by considering the variation during toff. (left to you as drill)

The previous analysis used to derive equation (10), are based on the assumption that the inductor current is always positive, when the inductor current tends to become negative a third state will be reached in which both the switch and the diode are off. Normally the converter is designed to operate with $i_L > 0$ all the time and that is known as the continuous conduction mode. To maintain continuous inductor current, the following condition must be satisfied:

$$\frac{\Delta I_L}{2} < I_{L,dc}$$

$$\frac{V_i d}{2Lf_{sw}} < \frac{1}{1-d}\frac{V_o}{R}$$

$$Lf_{sw} > \frac{(1-d)dV_iR}{2V_o}$$

By substituting for Vo from (10), the continuous conduction condition is given by:

$$Lf_{sw} > \frac{(1-d)^2 dR}{2} \qquad ..(12)$$

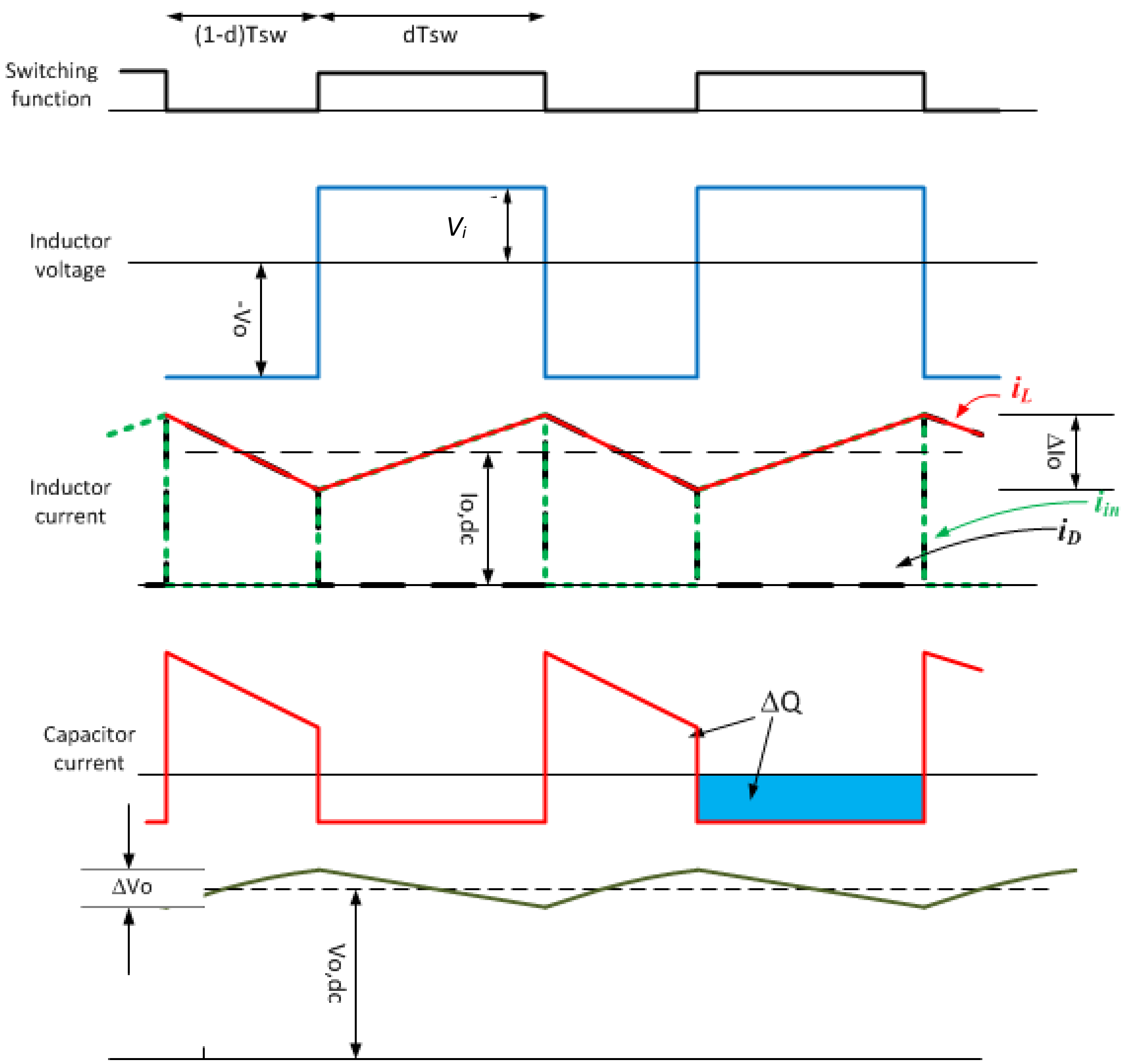


Fig. 7 Waveforms of voltages and currents for buck converter

Capacitor Current and Capacitor Voltage Ripple

The capacitor current is equivalent to the ac component of the diode current. As shown in Fig. 7, when the switch is on, the capacitor current is equals to ($i_{C,ON} = -I_o$). When the switch is off, the capacitor current ($i_{C,OFF} = i_L - I_o$); This current changes the capacitor voltage, the variation of the capacitor voltage (peak-to-peak) is determined by dividing the area ΔQ is Fig. 7 by the capacitance C. ΔQ represent the charge that has been added to the capacitor during t_{off} , or equivalently, the charge taken from the capacitor during t_{on}. The determination during t_{on} is simpler since the capacitor current is constant.

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{dI_o}{Cf_{sw}}$$

$$\Delta V_o = \frac{dV_o}{RCf_{sw}}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{d}{RCf_{sw}}}$$

..(13)

Example

A boost dc-dc converter is supplied form a 12 V source, and produces an output voltage in the range $15<V_o<40V$. The inverter uses inductance of L =0.5mH and capacitor the (C =) 100 μF. The load resistance, R= 60Ω. The switching frequency is 50kHz.

(a) Draw the circuit diagram and find the maximum and minimum values of the duty ratio.

(b) Show that the inductor current is continuous for the entire range of output voltage.

(c) With switching frequency of 20kHz, calculate the maximum peak-to-peak output voltage ripple (ΔVo).

(d) When Vo=20V; draw to a scale the following quantities. (i) the switching signal, (ii) the inductor voltage, (iii) the inductor current (iv) the diode current (v) the capacitor current and (vi) the capacitor voltage. Use the same time scale, calculate and show all amplitudes at the switching points.

Chapter 7

DC Supply Circuits

3-Buck-Boost Converter

Unlike buck (step-down) and boost (step-up), in the buck–boost converter, the output voltage can be made less than, equal to, or greater than the input voltage. This lecture presents the buck-boost converter (the step/down step-up converter): the lecture covers circuit operation, analysis and design.

Circuit Diagram and assumption

The buck-boost converter is the third basic DC supply circuits, the circuit can perform step-up or step-down conversion.

The circuit of the buck-boost converter is shown in Fig 8. It is seen that the inductor of the boost converter is no longer a part of the output filter.

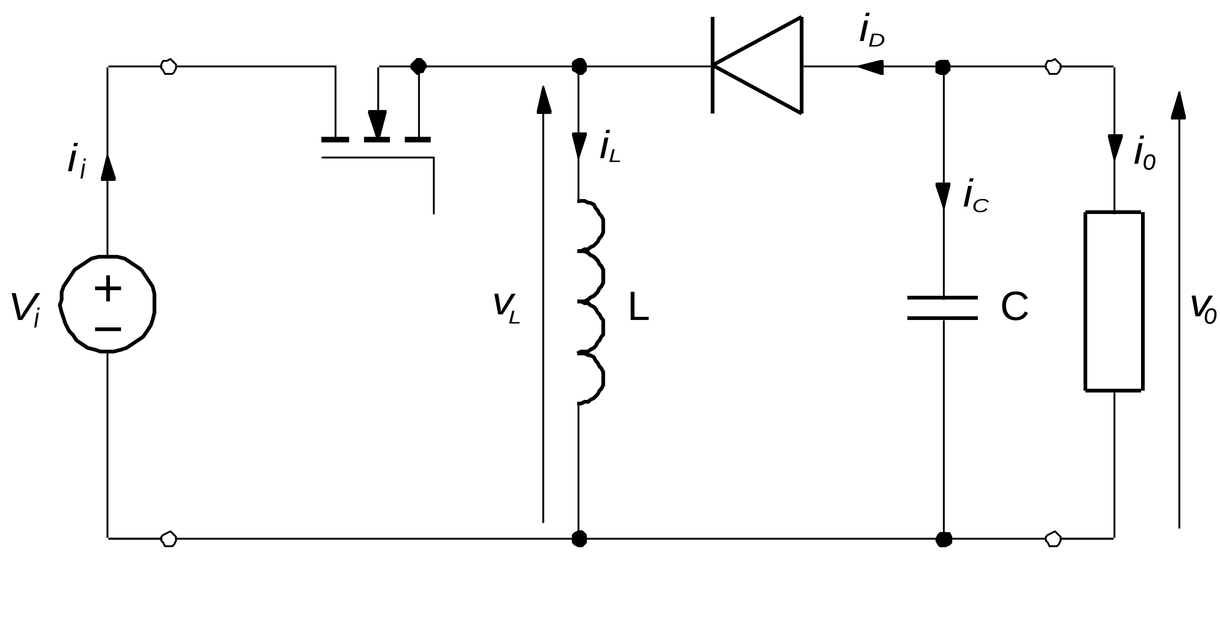


Fig. 8 Buck-Boost-Converter

As usual, the switch in operated periodically with adjustable duty cycle. Before starting the analysis, again the following assumptions are considered:

- Constant capacitor voltage: initially we will assume that the capacitor is large enough to allow ripple free capacitor voltage.
- Continuous inductor current (i_L) or ($i_L>0$).

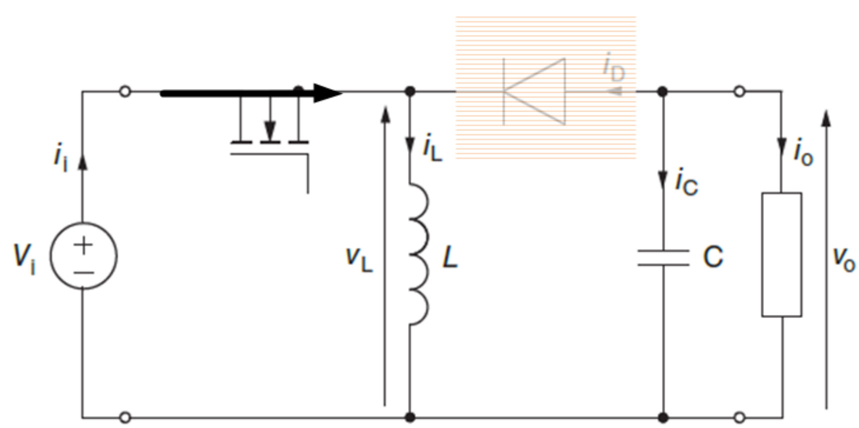
The circuit states

The equivalent circuits when the switch is ON is shown in Fig. 9a. As indicated, the diode is reverse biased when the switch turns on. Therefore the corresponding ON-state equations

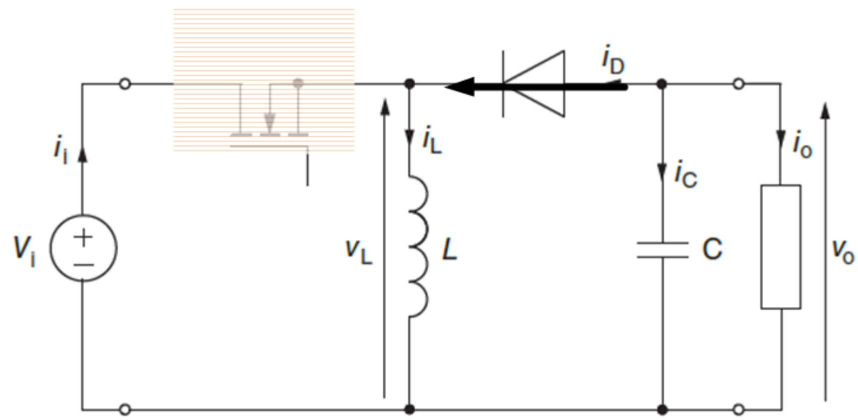
$$v_L = V_i$$
$$i_C = -i_o \qquad (14)$$

When the switch is OFF the equivalent circuit is shown in Fig. 9b. The diode is forward biased because the inductor current is continuous. The corresponding equations for off state:

$$v_L = V_o$$
$$i_C = i_o - i_L \qquad (15)$$



(a)



(b)

Fig. 9 the equivalent circuits during (a) on-state, (b) off-state

To complete the analysis of the circuit, we must consider various voltages and currents in the circuit as discussed in the following paragraphs

Inductor voltage and output voltage

The inductor voltage variation with the switching signals-as defined in Equations (14) and (15) and shown in Fig. 10. This voltage has zero average (in steady state). Therefore:

$$dT_{sw}V_i = -(1-d)T_{sw}V_o$$

Gives

$$\boxed{\frac{V_o}{V_i} = -\frac{d}{1-d} \qquad \ldots(16)}$$

The negative sign indicates that the output voltage is negative in the polarity shown in Fig. 8. This makes sense since when the switch turns off the inductor current follows to the RC section from the negative side, and therefore charges C to a negative voltage. Indeed the negative Vo also confirms that the diode will be reversed biased when the switch is ON even if |Vo|>Vi.

Important note:

If Vo is given to be positive, it means that it has been taken in opposite direction (compared to Fig. 8) in this case the (−) sign disappears from Eq. (9)

Inductor Current and Continuous Conduction Condition

Figure 10 shows the switching signals and the corresponding inductor voltage. Since the inductor voltage is constant- during ton and toff-, the inductor current changes linearly in both conditions. We will determine first the average inductor current, $I_{L,dc}$:

Since: $i_L = i_i + i_D$, gives

$$I_{L,dc} = I_{i,dc} + I_{D,dc} \quad ..(17)$$

The diode current:

$$i_D = -i_C - I_o$$

Since the capacitor current has zero average:

$$I_{D,dc} = -I_o \quad ..(18)$$

Substitute (18) into (17)

$$I_{L,dc} = I_{i,dc} - I_o \quad ..(19)$$

which implies (since $I_{i,dc}V_i = I_oV_o$)

$$I_{i,dc} = -\frac{d}{1-d}I_o \quad ..(20)$$

Where R is the load resistance.

Substitute (18) and (20) into (17)

$$I_{L,dc} = -\frac{d}{1-d}I_o - I_o = -\frac{d-1+d}{1-d}I_o$$

$$I_{L,dc} = -\frac{1}{1-d}I_o$$

$$I_{L,dc} = -\frac{1}{1-d}\frac{V_o}{R} \quad ..(21a)$$

$$I_{L,dc} = \frac{d}{(1-d)^2}\frac{V_i}{R} \quad ..(21b)$$

Note that V_o in Eq. (21a) is negative, which results a positive $I_{L,dc}$ in the direction indicated in Fig. 8. Once again if V_o (I_o) in (21a) is substituted by positive, the negative sign has to be removed.

To determine the inductor current ripple (ΔI_L), we use the basic inductor voltage equation:

$$v_L = L\frac{di}{dt}$$

During ton:

$$V_i = L\frac{\Delta I_L}{dT_{sw}},$$

Gives:

$$\Delta I_L = V_i\frac{dT_{sw}}{L} = \frac{V_i d}{Lf_{sw}} \qquad ..(22)$$

In this course, the converter is designed to operate with $i_L > 0$. For continuous inductor current, the following condition must be satisfied:

$$\frac{\Delta I_L}{2} < I_{L,dc}$$

$$\frac{V_i d}{2Lf_{sw}} < \frac{1}{1-d}\frac{|V_o|}{R}$$

$$Lf_{sw} > \frac{(1-d)dV_iR}{2|V_o|}$$

By substituting for Vo from (10), the continuous conduction condition is given by:

$$\boxed{Lf_{sw} > \frac{(1-d)^2R}{2} = \frac{(1-d)^2V_o}{2I_o} \qquad ..(23)}$$

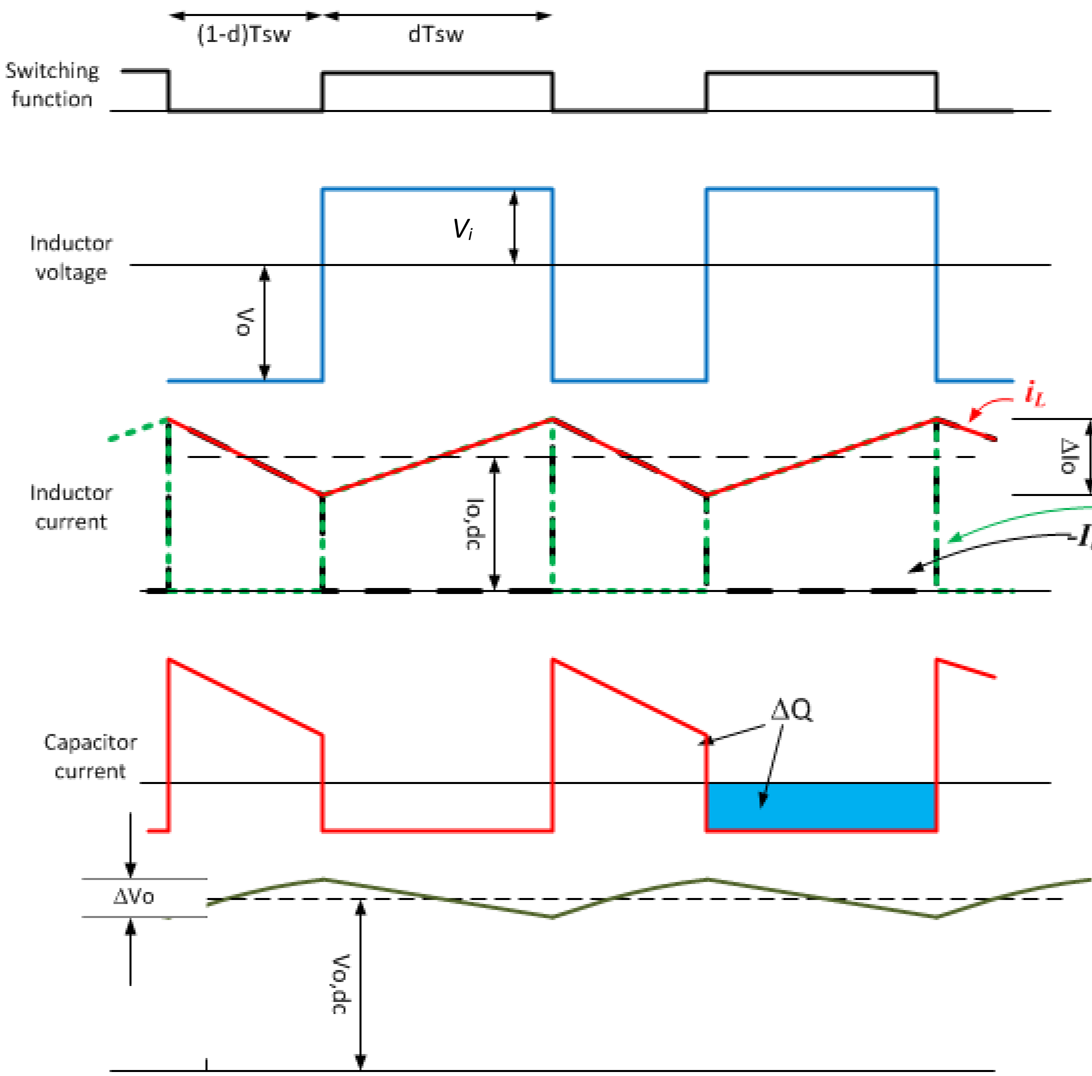


Fig. 10 Waveforms of voltages and currents for buck-boost converter

Capacitor Current and Capacitor Voltage Ripple

As in the case of the boost converter, the capacitor current $|i_C|$ represents the ac component of the diode current. This current, as shown in Fig. 10. The capacitor current is similar to that shown in Fig. 7 for the case of boost converter; Therefore Eq. 13 describes the capacitor voltage ripple for boost converter is perfectly applicable for buck boost converter.

$$\Delta \boldsymbol{V_o} = \frac{\Delta \boldsymbol{Q}}{\boldsymbol{C}} = \frac{\boldsymbol{dI_o}}{\boldsymbol{Cf_{sw}}}$$

$$\Delta V_o = \frac{dV_o}{RCf_{sw}}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{d}{RCf_{sw}}}$$

..(13)

Example

Design a buck-boost converter to supply a load of 75 W at a voltage ranges between 20- to- 80 V from a 40-V source. The output ripple must be no more than 1 percent. Assume a switching frequency of 50kHz

Specify rang of the duty ratio,

Calculate the inductor size, and capacitor size. The minimum inductor current is 20% of its average