

Sorting Algorithms

Sorting

- ***Sorting*** : is a process that organizes a collection of data into either ascending or descending order.
- **There are two type of sorting:**
 - 1- *internal sort*** requires that the collection of data fit entirely in the computer's main memory.
 - 2- *external sort*** when the collection of data cannot fit in the computer's main memory all at once but must reside in secondary storage such as on a disk.
- **There are many sorting algorithms, such as:**
 - Selection Sort
 - Insertion Sort
 - Bubble Sort
 - Merge Sort
 - Quick Sort

Selection Sort

- The list is divided into two sublists, *sorted* and *unsorted*, which are divided by an imaginary wall.
- We find the smallest element from the unsorted sublist and swap it with the element at the beginning of the unsorted data.
- After each selection and swapping, the imaginary wall between the two sublists move one element ahead, increasing the number of sorted elements and decreasing the number of unsorted ones.
- Each time we move one element from the unsorted sublist to the sorted sublist, we say that we have completed a sort pass.
- A list of n elements requires $n-1$ passes to completely rearrange the data.

Sorted

Unsorted

23	78	45	8	32	56
----	----	----	---	----	----

Original List

8	78	45	23	32	56
---	----	----	----	----	----

After pass 1

8	23	45	78	32	56
---	----	----	----	----	----

After pass 2

8	23	32	78	45	56
---	----	----	----	----	----

After pass 3

8	23	32	45	78	56
---	----	----	----	----	----

After pass 4

8	23	32	45	56	78
---	----	----	----	----	----

After pass 5

Selection Sort (cont.)

```
template <class Item>
void selectionSort( Item a[], int n) {
    for (int i = 0; i < n-1; i++) {
        int min = i;
        for (int j = i+1; j < n; j++)
            if (a[j] < a[min]) min = j;
        swap(a[i], a[min]);
    }
}
```

```
template < class Object>
void swap( Object &lhs, Object &rhs )
{
    Object tmp = lhs;
    lhs = rhs;
    rhs = tmp;
}
```

Insertion Sort

- Insertion sort is a simple sorting algorithm that is appropriate for small inputs.
- The list is divided into two parts: sorted and unsorted.
- In each pass, the first element of the unsorted part is picked up, transferred to the sorted sublist, and inserted at the appropriate place.
- A list of n elements will take at most $n-1$ passes to sort the data.

Sorted

Unsorted

23	78	45	8	32	56
----	----	----	---	----	----

Original List

23	78	45	8	32	56
----	----	----	---	----	----

After pass 1

23	45	78	8	32	56
----	----	----	---	----	----

After pass 2

8	23	45	78	32	56
---	----	----	----	----	----

After pass 3

8	23	32	45	78	56
---	----	----	----	----	----

After pass 4

8	23	32	45	56	78
---	----	----	----	----	----

After pass 5

Insertion Sort Algorithm

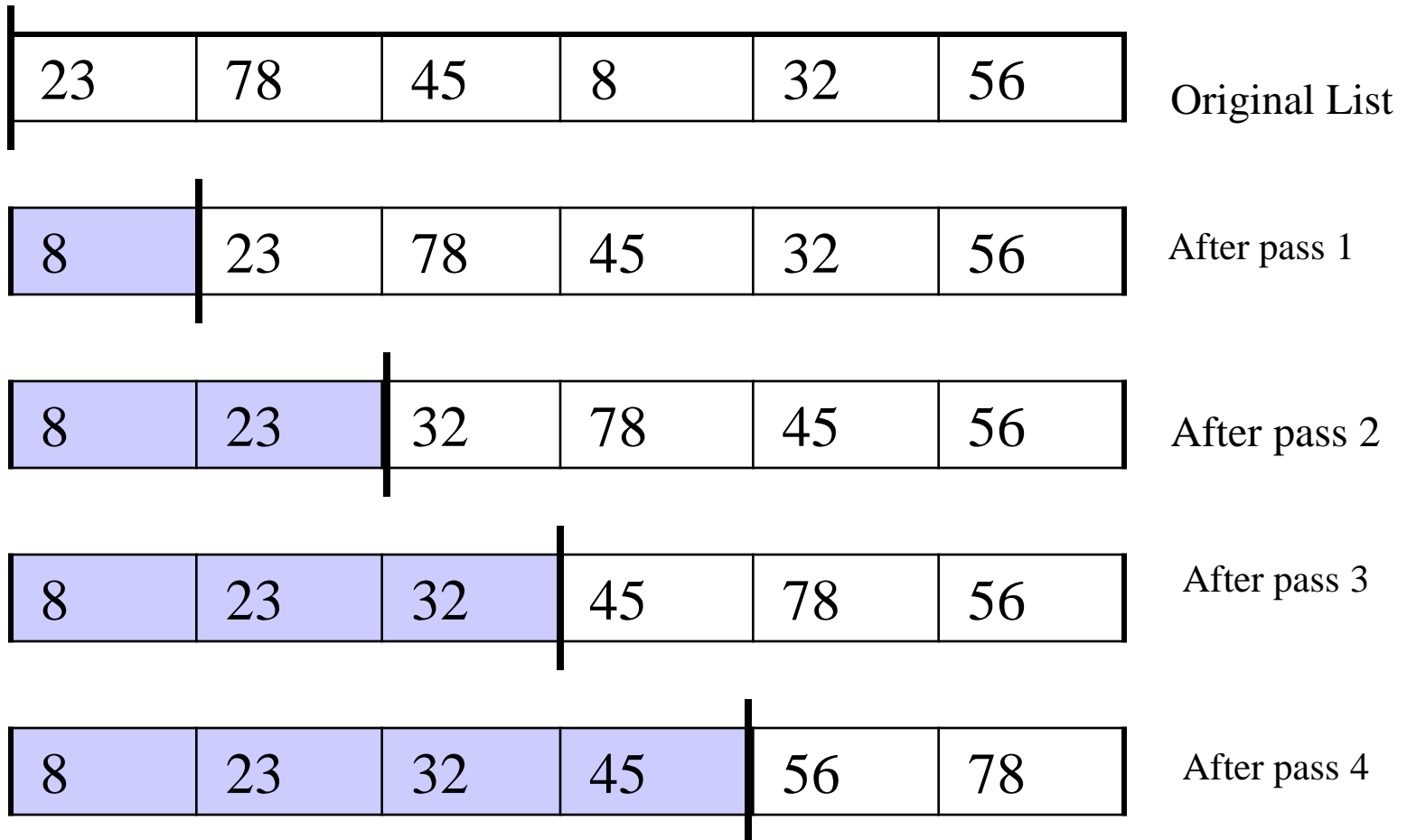
```
template <class Item>
void insertionSort(Item a[], int n)
{
    for (int i = 1; i < n; i++)
    {
        Item tmp = a[i];

        for (int j=i; j>0 && tmp < a[j-1]; j--)
            a[j] = a[j-1];
        a[j] = tmp;
    }
}
```


Bubble Sort

- The list is divided into two sublists: sorted and unsorted.
- The smallest element is bubbled from the unsorted list and moved to the sorted sublist.
- After that, the wall moves one element ahead, increasing the number of sorted elements and decreasing the number of unsorted ones.
- Each time an element moves from the unsorted part to the sorted part one sort pass is completed.
- Given a list of n elements, bubble sort requires up to $n-1$ passes to sort the data.

Bubble Sort



Bubble Sort Algorithm

```
template <class Item>
void bubbleSort(Item a[], int n)
{
    bool sorted = false;
    int last = n-1;

    for (int i = 0; (i < last) && !sorted; i++){
        sorted = true;
        for (int j=last; j > i; j--){
            if (a[j-1] > a[j]){
                swap(a[j],a[j-1]);
                sorted = false; // signal exchange
            }
        }
    }
}
```

Mergesort

- Mergesort algorithm is one of two important divide-and-conquer sorting algorithms (the other one is quicksort).
- It is a recursive algorithm.
 - Divides the list into halves,
 - Sort each halve separately, and
 - Then merge the sorted halves into one sorted array.

Mergesort - Example

theArray:

8	1	4	3	2
---	---	---	---	---

Divide the array in half

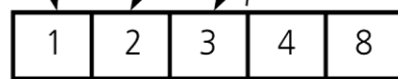


Sort the halves

Merge the halves:

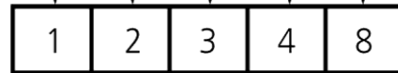
- a. $1 < 2$, so move 1 from left half to tempArray
- b. $4 > 2$, so move 2 from right half to tempArray
- c. $4 > 3$, so move 3 from right half to tempArray
- d. Right half is finished, so move rest of left half to tempArray

Temporary array
tempArray:

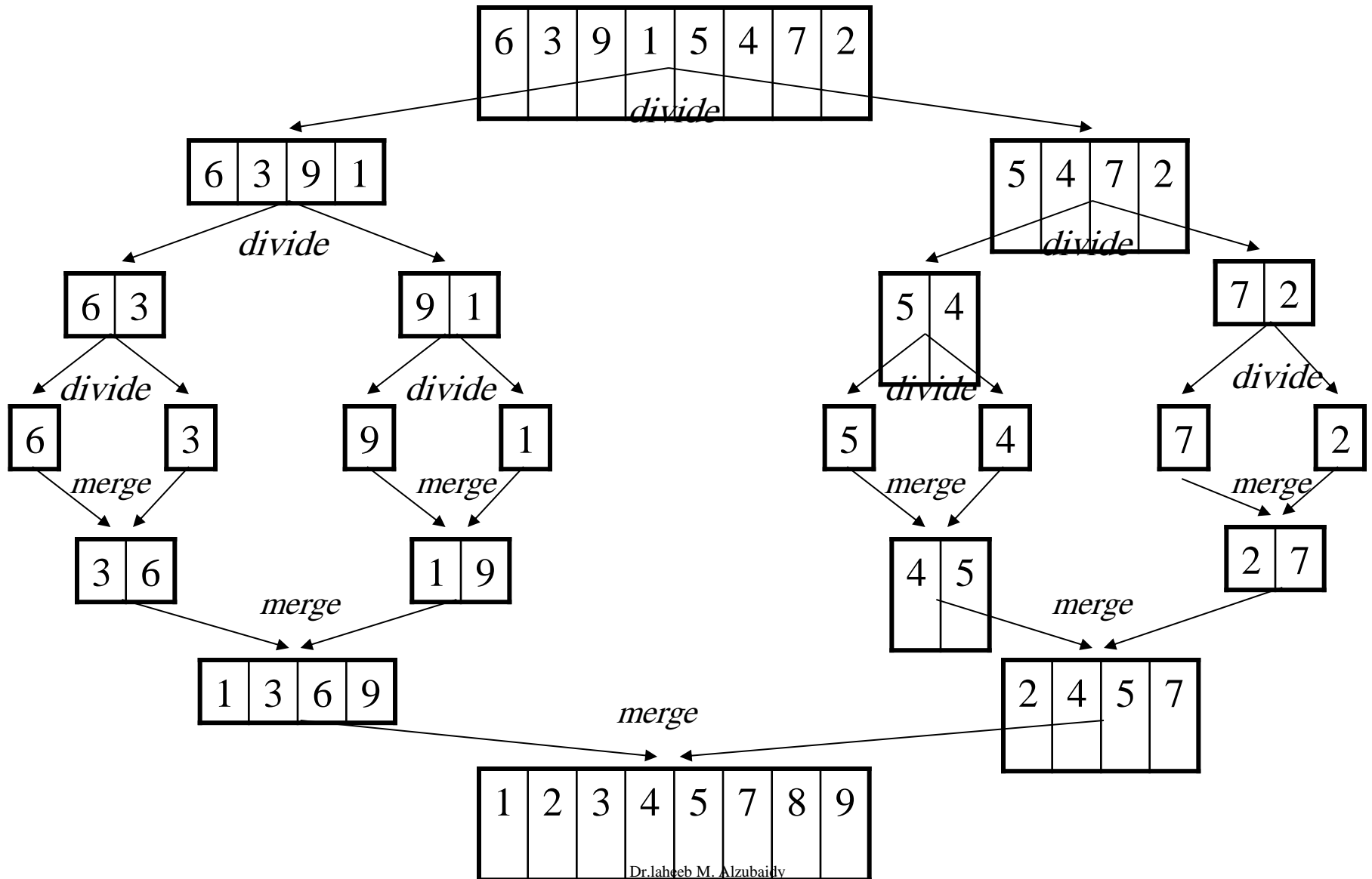


Copy temporary array back into original array

theArray:



Mergesort - Example



Static Method mergeSort()

```
Public static void mergeSort(Comparable []a, int left,
    int right)
{
    // sort a[left:right]
    if (left < right)
    { // at least two elements
        int mid = (left+right)/2;    //midpoint
        mergeSort(a, left, mid);
        mergeSort(a, mid + 1, right);
        merge(a, b, left, mid, right); //merge from a to b
        copy(b, a, left, right); //copy result back to a
    }
}
```

Quicksort Algorithm

Given an array of n elements (e.g., integers):

- If array only contains one element, return
- Else
 - pick one element to use as *pivot*.
 - Partition elements into two sub-arrays:
 - Elements less than or equal to pivot
 - Elements greater than pivot
 - Quicksort two sub-arrays
 - Return results

Example

We are given array of n integers to sort:

40	20	10	80	60	50	7	30	100
----	----	----	----	----	----	---	----	-----

Pick Pivot Element

There are a number of ways to pick the pivot element. In this example, we will use the first element in the array:

40	20	10	80	60	50	7	30	100
----	----	----	----	----	----	---	----	-----

Partitioning Array

Given a pivot, partition the elements of the array such that the resulting array consists of:

1. One sub-array that contains elements \geq pivot
2. Another sub-array that contains elements $<$ pivot

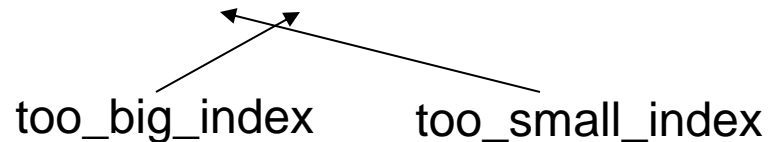
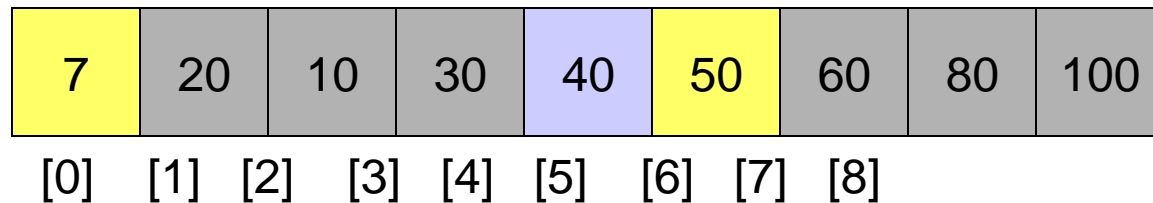
The sub-arrays are stored in the original data array.

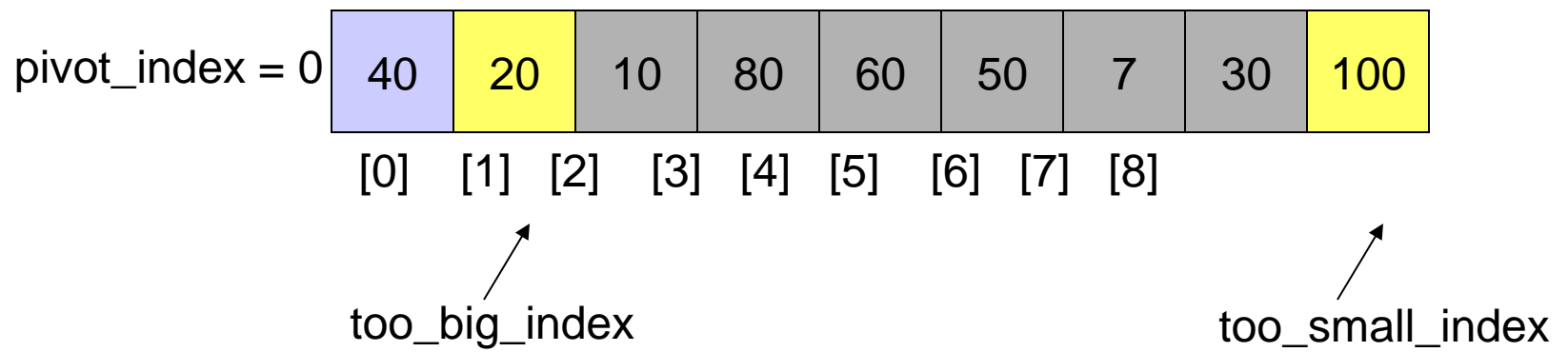
Partitioning loops through, swapping elements below/above pivot.

1. While $\text{data}[\text{too_big_index}] \leq \text{data}[\text{pivot}]$
 $++\text{too_big_index}$
2. While $\text{data}[\text{too_small_index}] > \text{data}[\text{pivot}]$
 $--\text{too_small_index}$
3. If $\text{too_big_index} < \text{too_small_index}$
 swap $\text{data}[\text{too_big_index}]$ and $\text{data}[\text{too_small_index}]$
4. While $\text{too_small_index} > \text{too_big_index}$, go to 1.
5. Swap $\text{data}[\text{too_small_index}]$ and $\text{data}[\text{pivot_index}]$

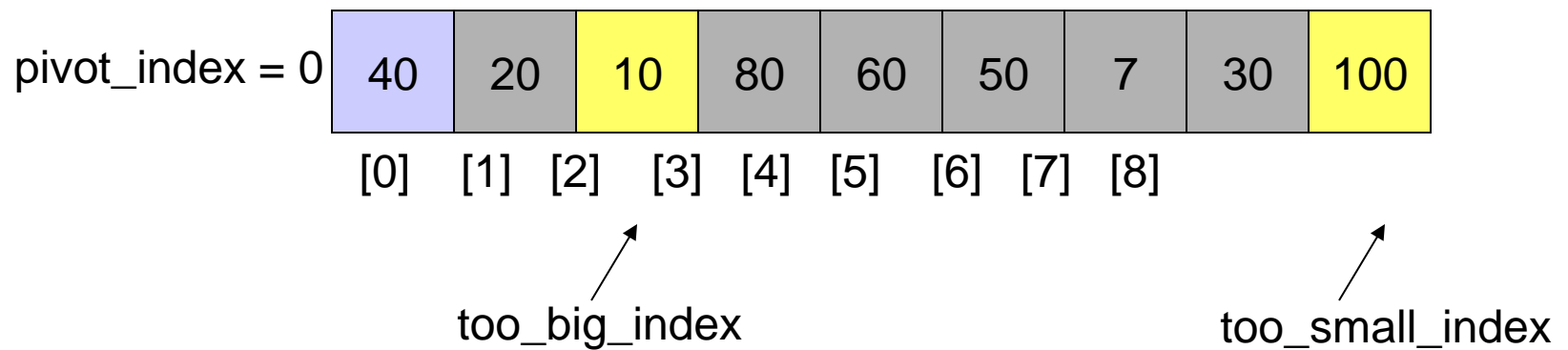


pivot_index = 4

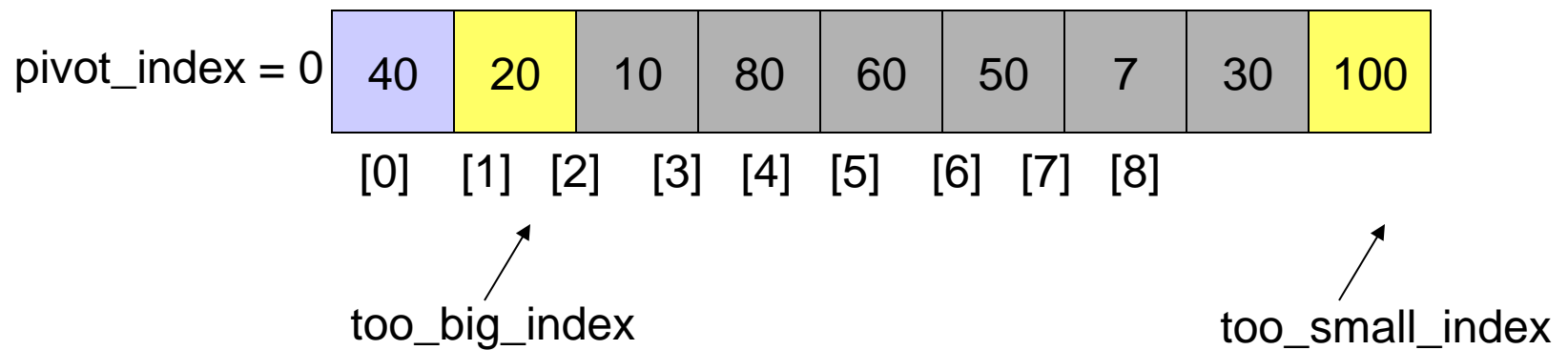




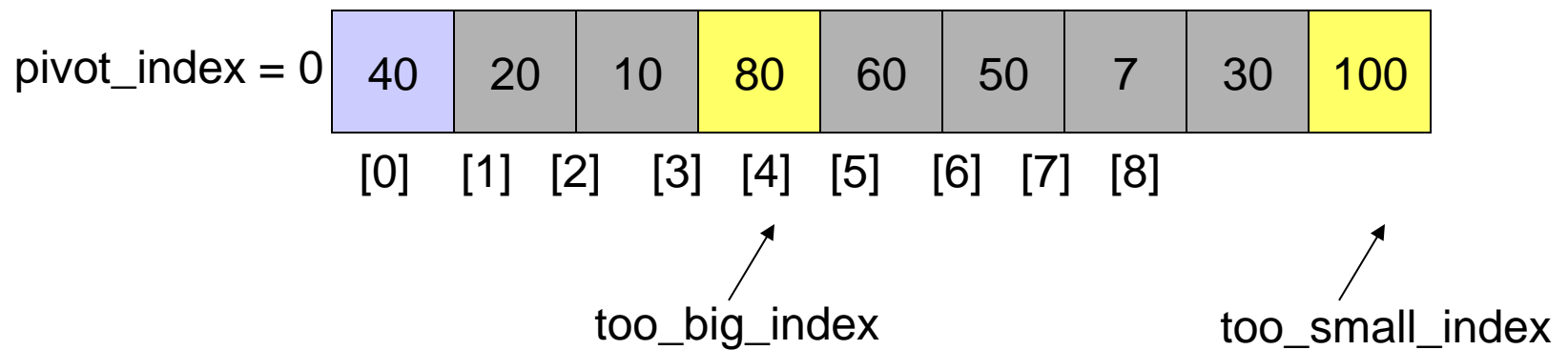
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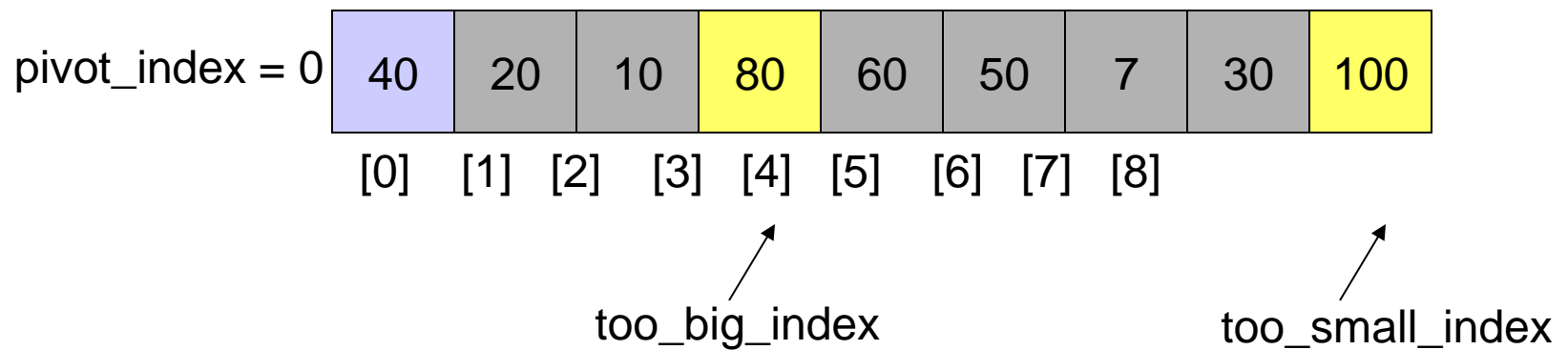
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 `++too_big_index`



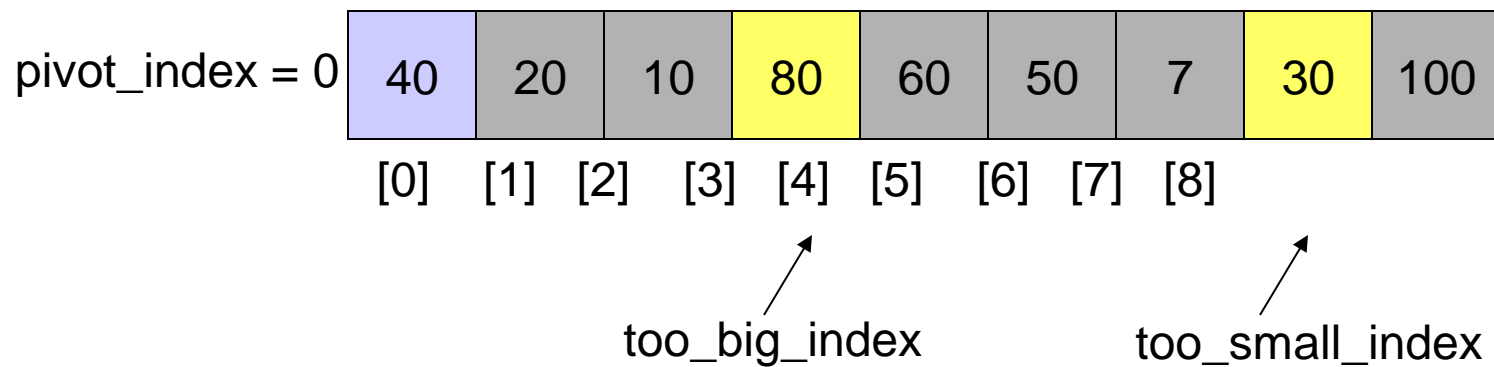
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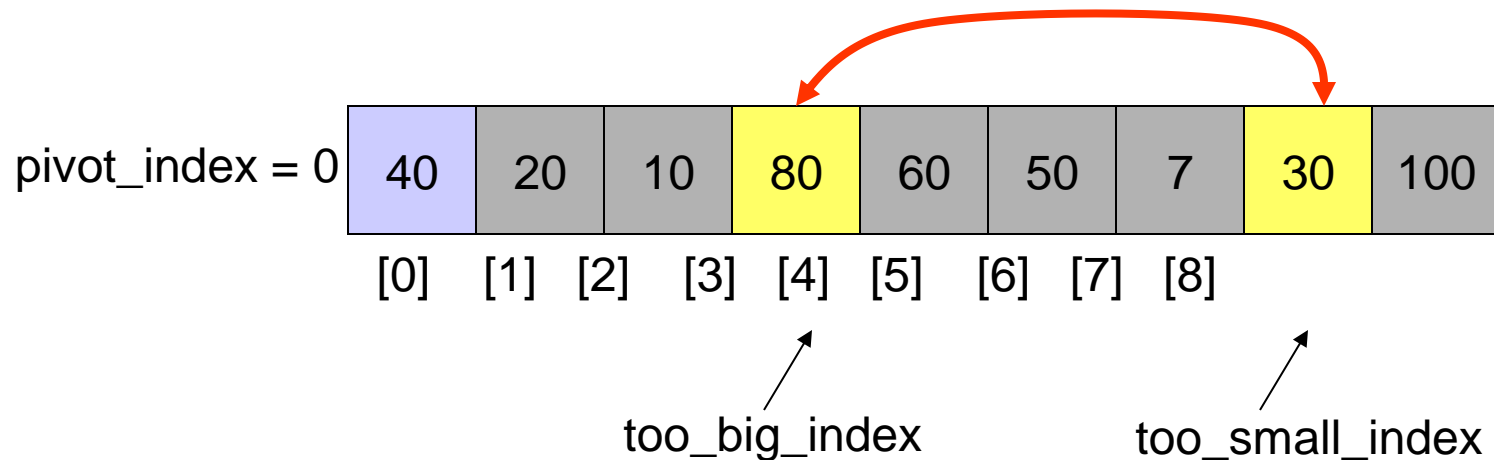
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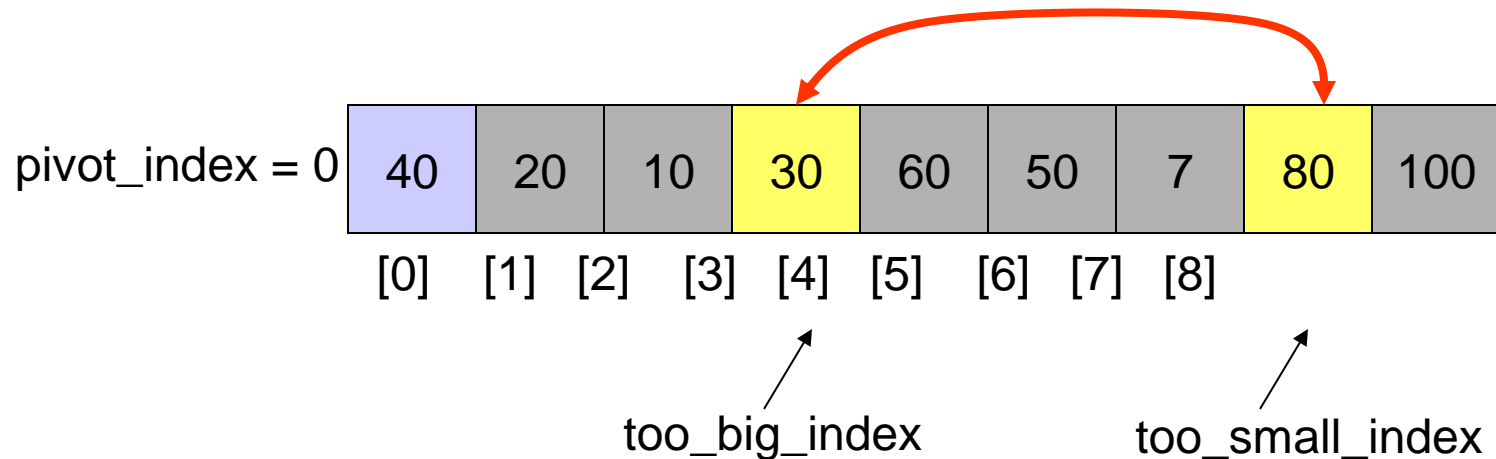
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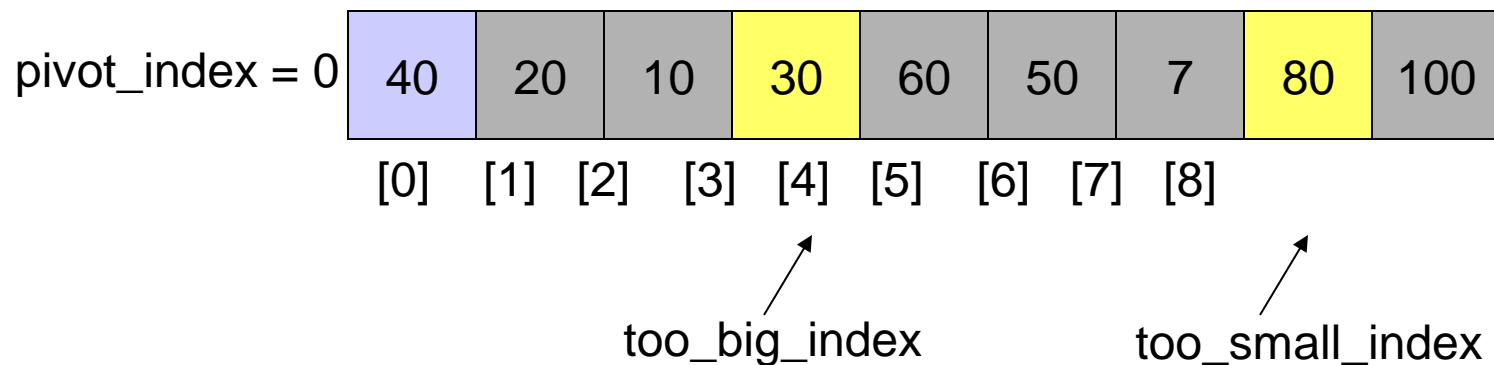
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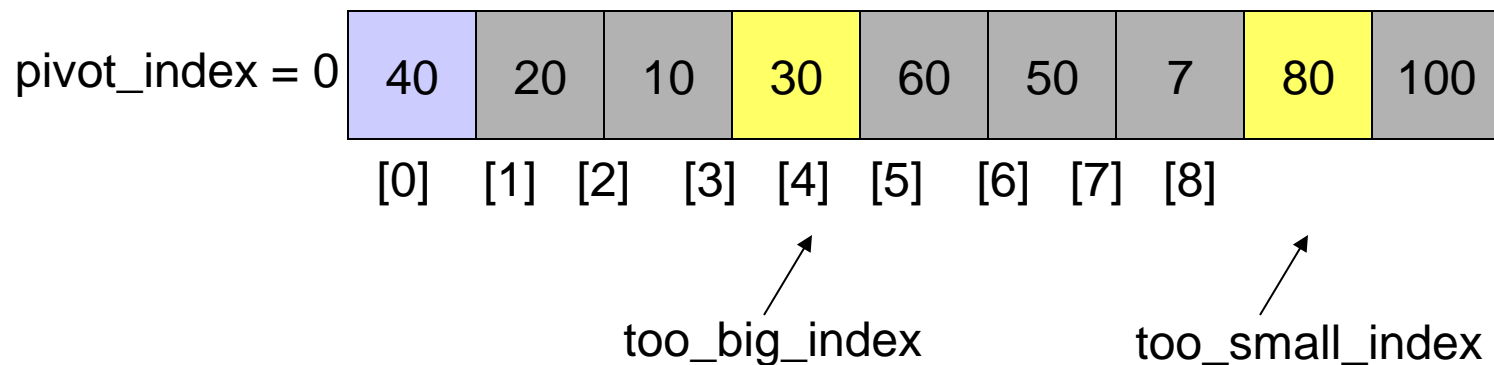
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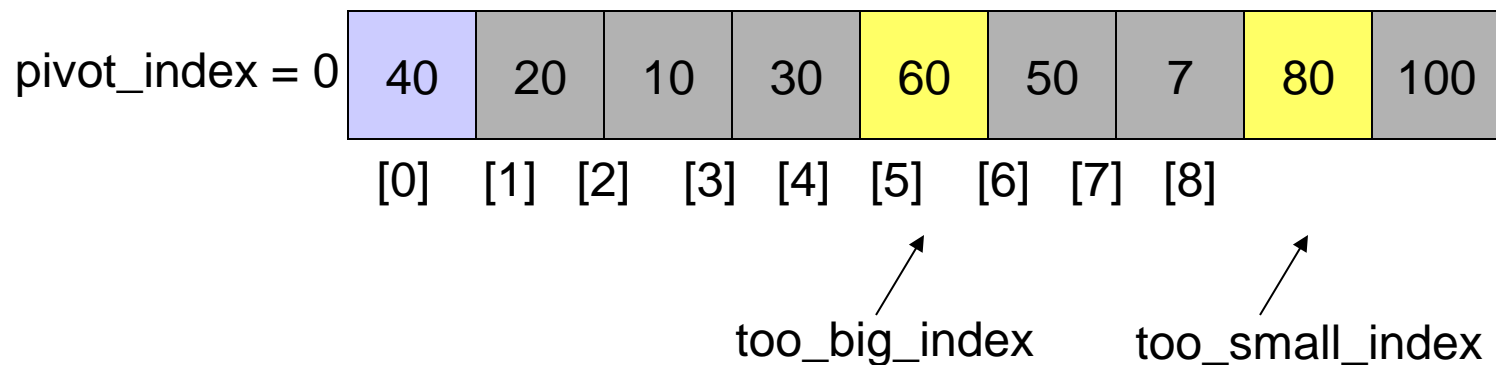
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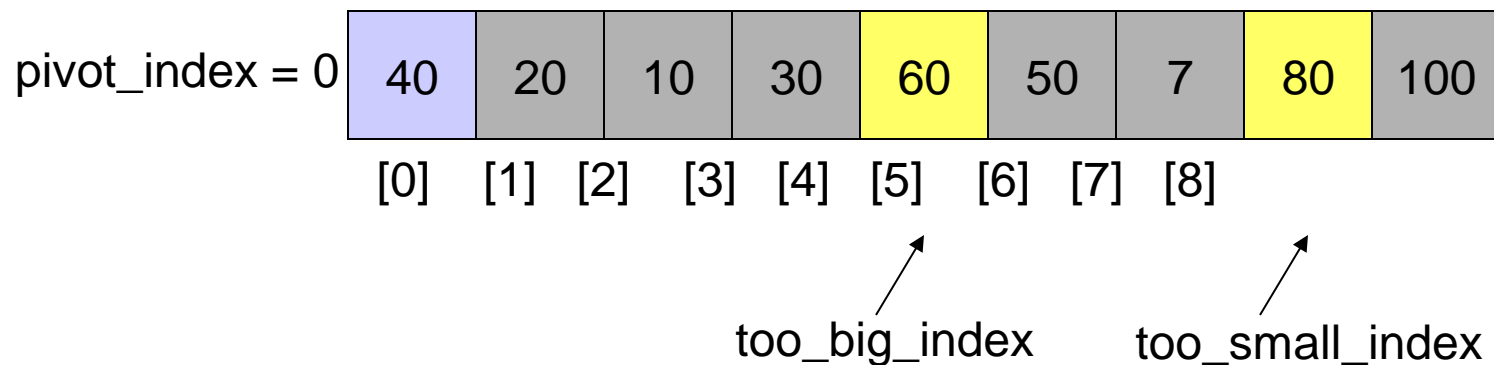
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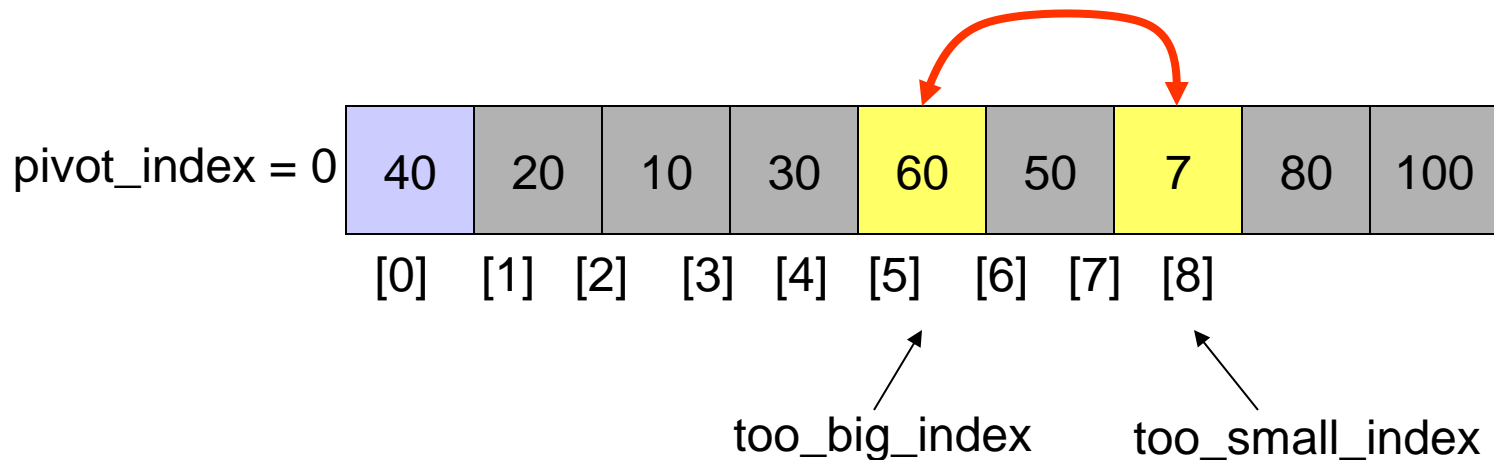
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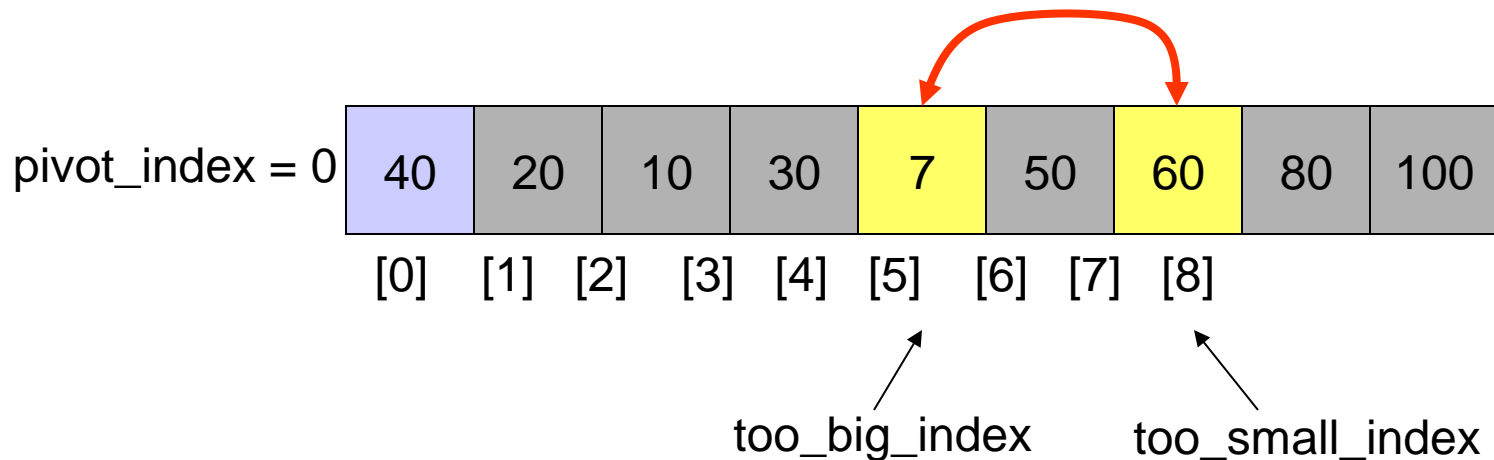
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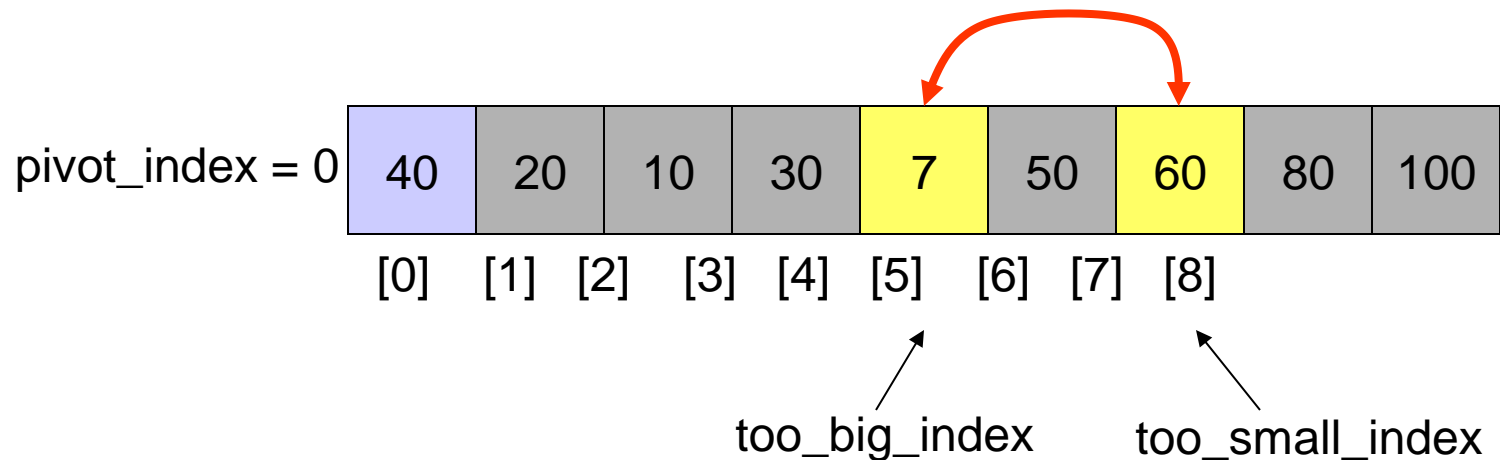
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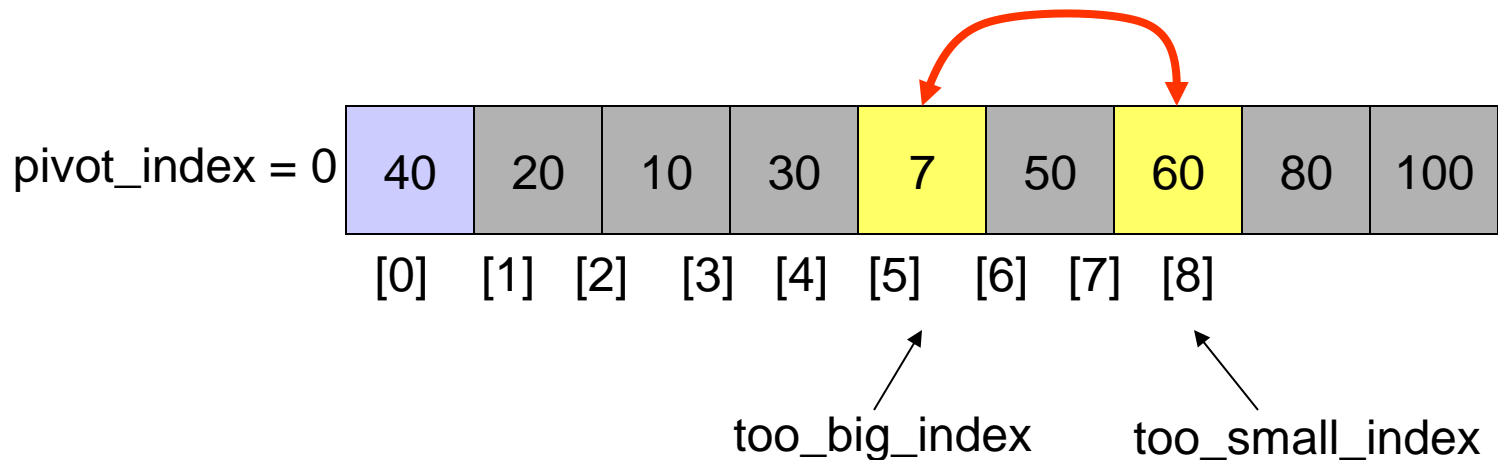
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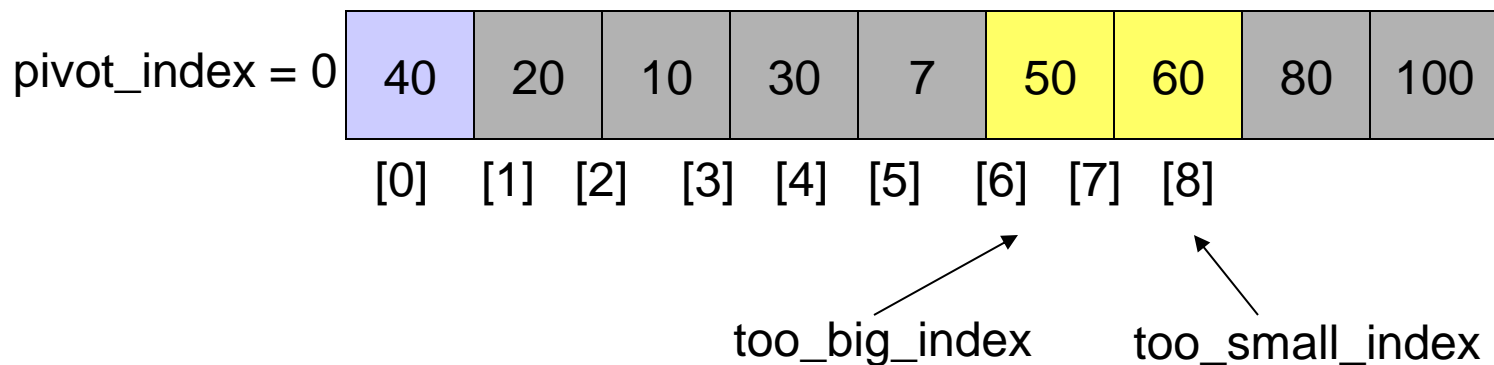
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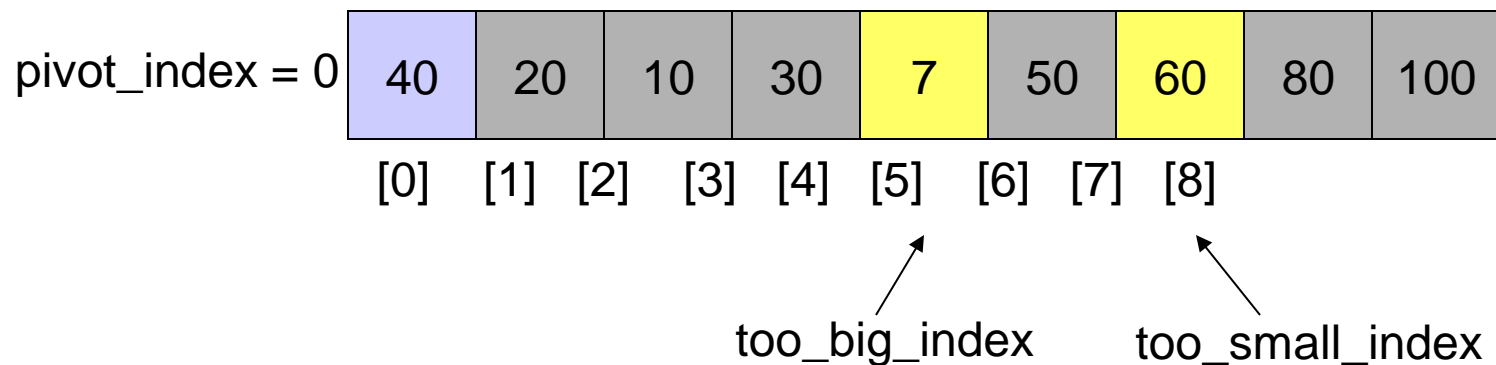
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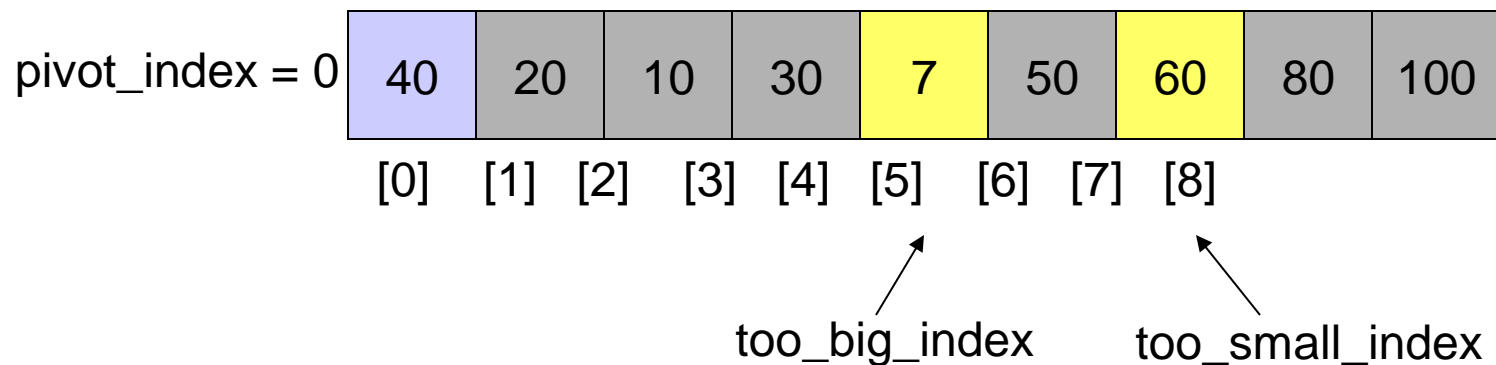
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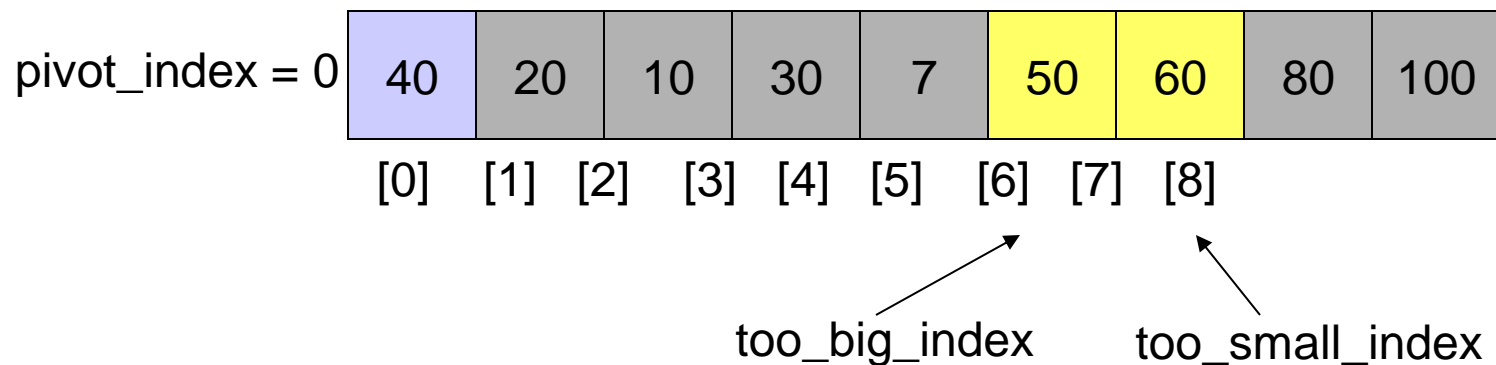
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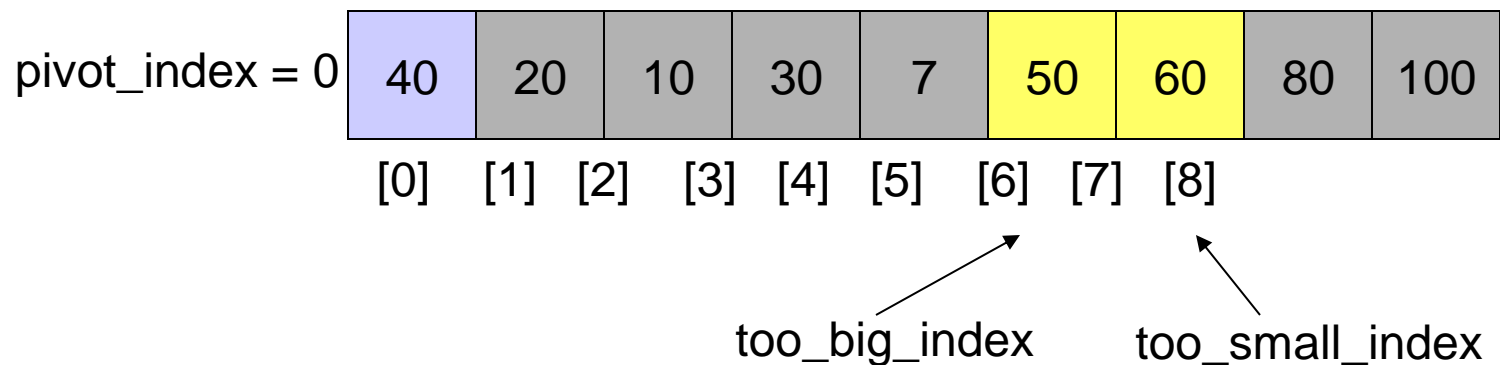
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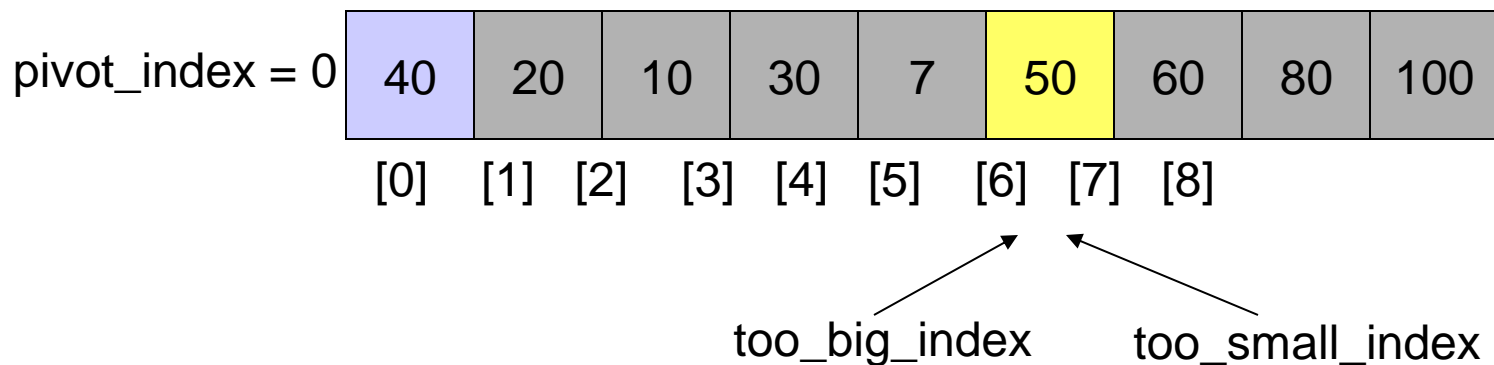
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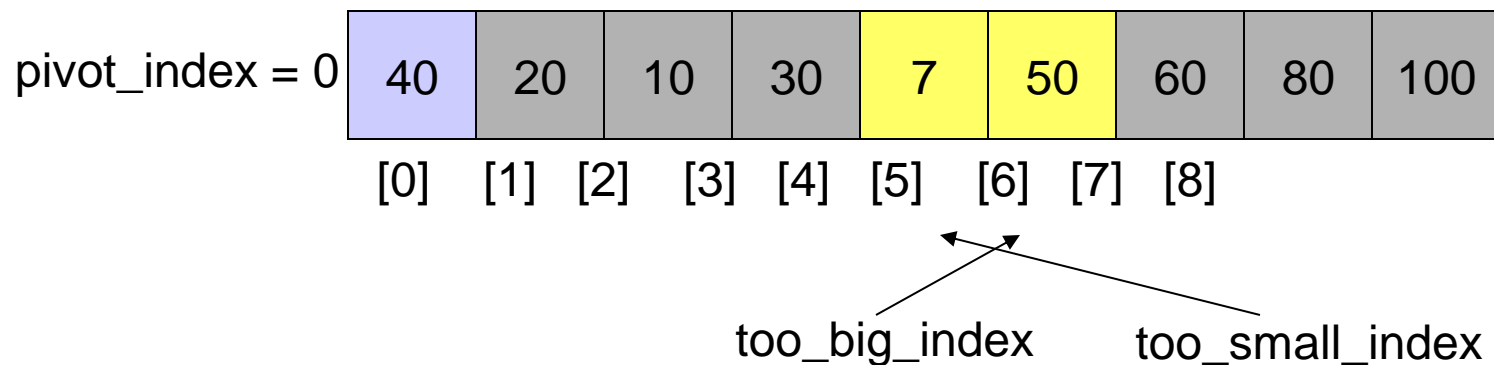
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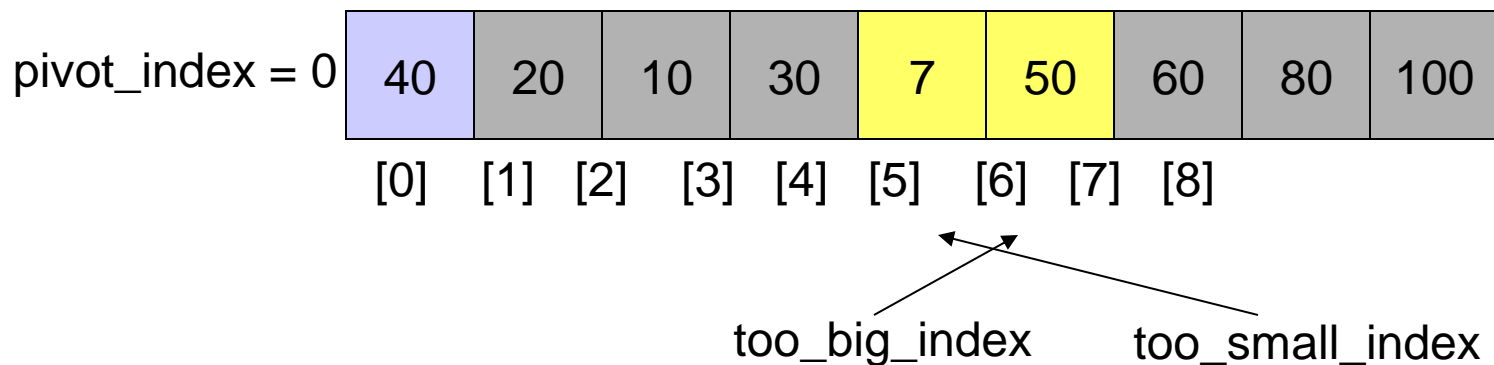
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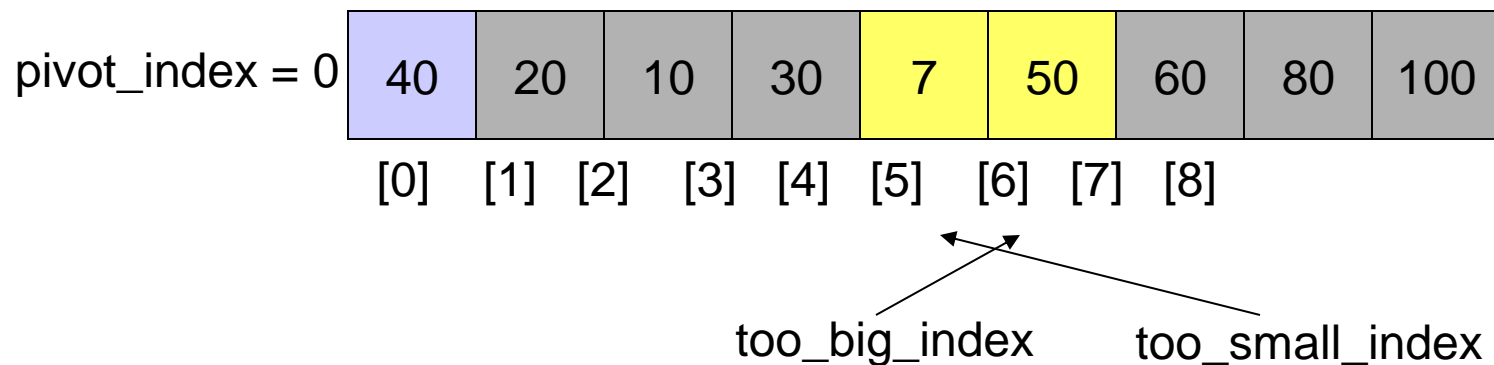
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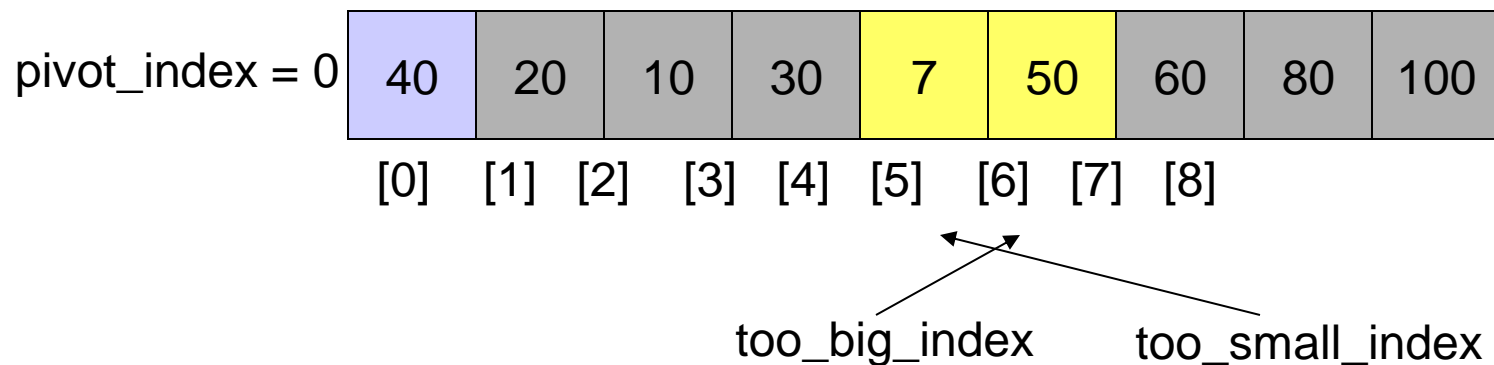
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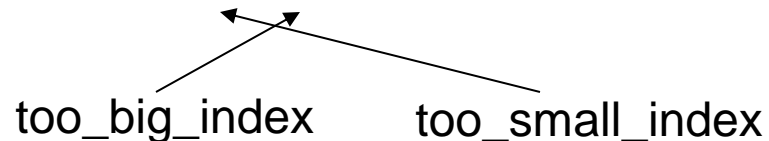
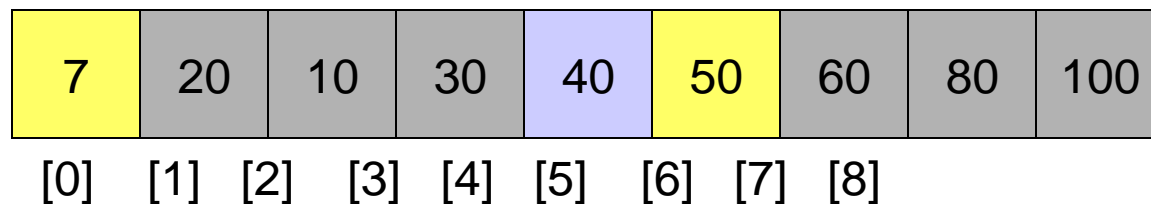
1. While $\text{data}[\text{too_big_index}] \leq \text{data}[\text{pivot}]$
 $++\text{too_big_index}$
2. While $\text{data}[\text{too_small_index}] > \text{data}[\text{pivot}]$
 $--\text{too_small_index}$
3. If $\text{too_big_index} < \text{too_small_index}$
 swap $\text{data}[\text{too_big_index}]$ and $\text{data}[\text{too_small_index}]$
4. While $\text{too_small_index} > \text{too_big_index}$, go to 1.
5. Swap $\text{data}[\text{too_small_index}]$ and $\text{data}[\text{pivot_index}]$



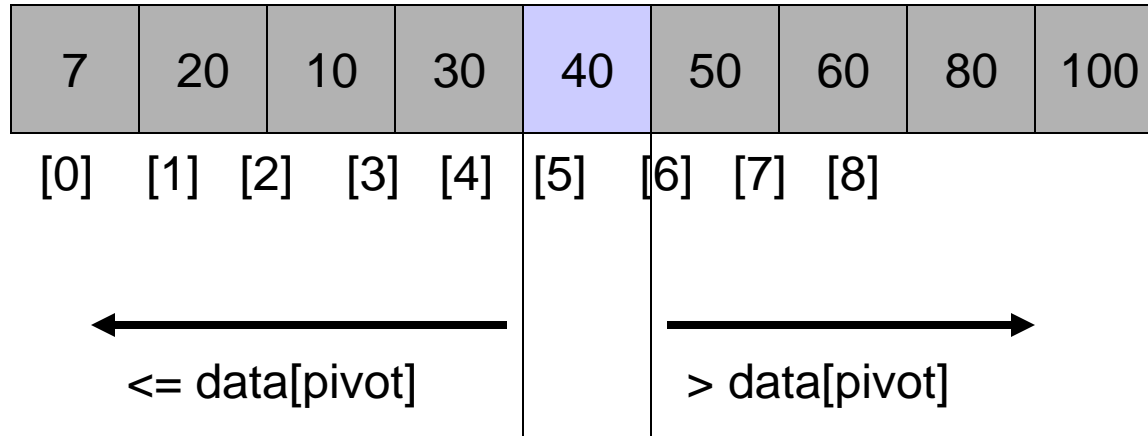
1. While $\text{data}[\text{too_big_index}] \leq \text{data}[\text{pivot}]$
 ++too_big_index
2. While $\text{data}[\text{too_small_index}] > \text{data}[\text{pivot}]$
 --too_small_index
3. If $\text{too_big_index} < \text{too_small_index}$
 swap $\text{data}[\text{too_big_index}]$ and $\text{data}[\text{too_small_index}]$
4. While $\text{too_small_index} > \text{too_big_index}$, go to 1.
5. Swap $\text{data}[\text{too_small_index}]$ and $\text{data}[\text{pivot_index}]$



pivot_index = 4



Partition Result



Recursion: Quicksort Sub-arrays

