Stack & Queue

Two basic linear data structures

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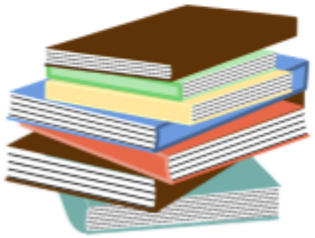
1-5 Stacks

Last-In-First-Out (LIFO)



stack: A collection based on the principle of adding elements and retrieving them in the opposite order.

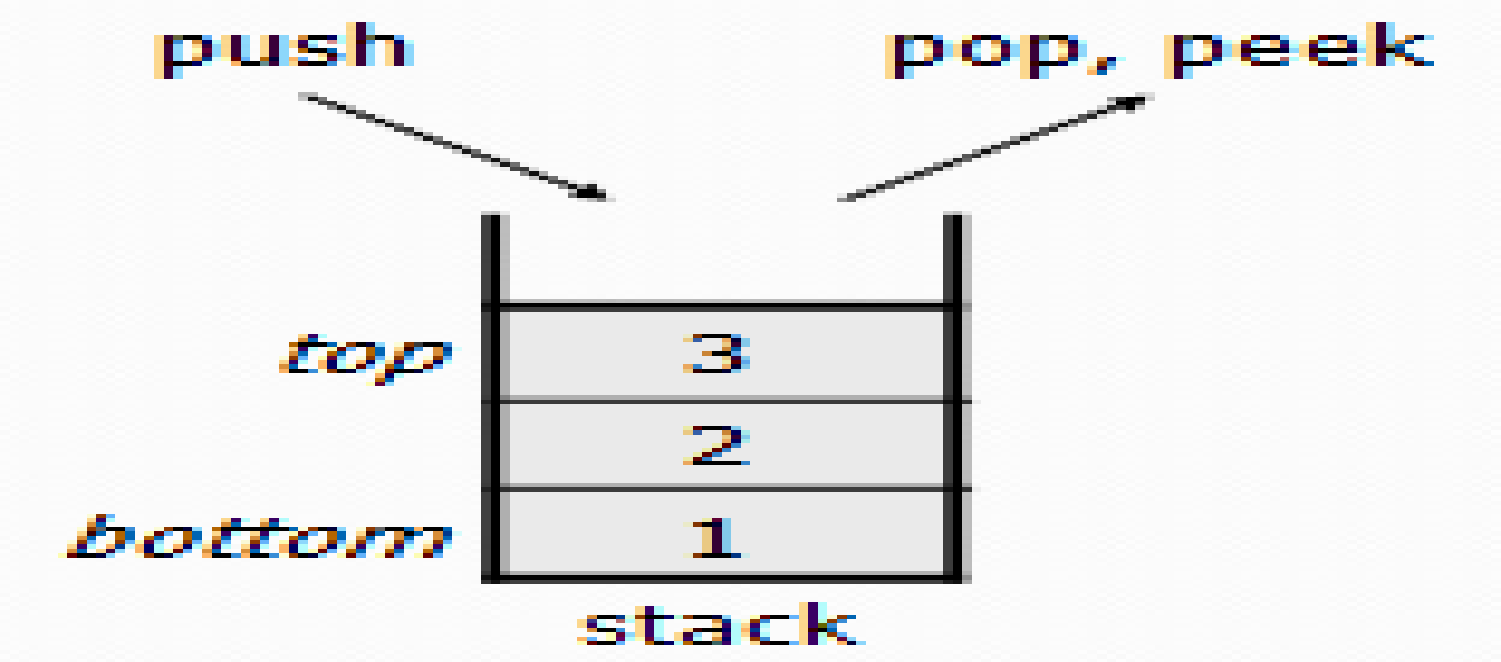
- Last-In, First-Out ("LIFO")
- Elements are stored in order of insertion.
- Client can only add/remove/examine the last element added (the "top").

basic stack operations:

push: Add an element to the top.

pop: Remove the top element.

peek: Examine the top element.

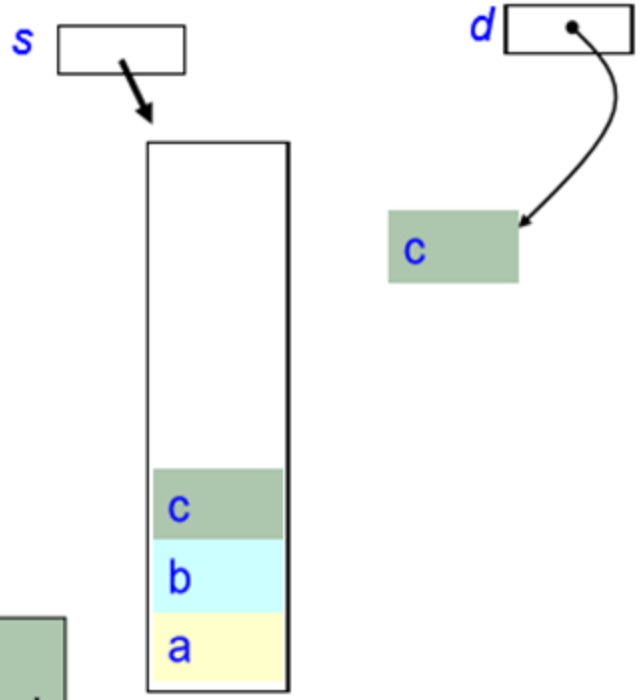

push
pop, peek
top
3
2
bottom
1
stack

1 Stack ADT: Uses

- Calling a function
  - Before the call, the state of computation is saved on the stack so that we will know where to resume
- Recursion
- Matching parentheses
- Evaluating arithmetic expressions (e.g. a + b – c) :
  - postfix calculation
  - Infix to postfix conversion

1 Stack: Usage

- Stack s = new Stack();
- s.push ("a");
- s.push ("b");
- s.push ("c");
- d = s.peek ();
- s.pop ();
- s.push ("e");
- s.pop ();



To be accurate, it is the references to "a", "b", "c", …, being pushed or popped.

Stack Algorithm using algorithm

An array can be used to implement a (bounded) stack, as follows. The first element (usually at the zero offset) is the bottom, resulting in array[0] being the first element pushed onto the stack and the last element popped off. The program must keep track of the size (length) of the stack, using a variable *top* that records the number of items pushed so far, therefore pointing to the place in the array where the next element is to be inserted (assuming a zero-based index convention). Thus, the stack itself can be effectively implemented as a three-element structure:

structure stack: maxsize : integer
top : integer
items : array of item

procedure initialize(stk : stack, size : integer):
stk.items ← new array of *size* items, initially empty
stk.maxsize ← size
stk.top ← 0

The *push* operation adds an element and increments the *top* index, after checking for overflow:

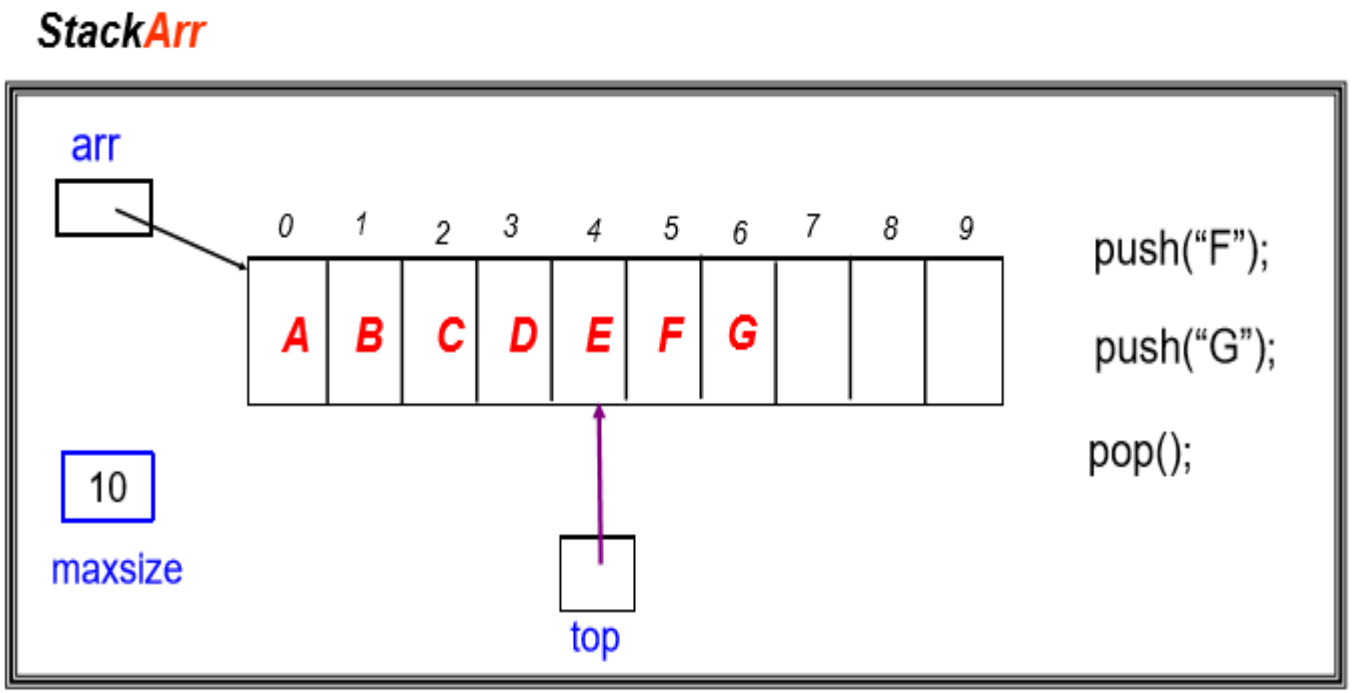
```
procedure push(stk : stack, x : item):
    if stk.top = stk.maxsize:
        report overflow error
    else:
        stk.items[stk.top] ← x
        stk.top ← stk.top + 1
```

Similarly, *pop* decrements the *top* index after checking for underflow, and returns the item that was previously the top one:

```
procedure pop(stk : stack):
    if stk.top = 0:
        report underflow error
    else:
        stk.top ← stk.top - 1
        r ← stk.items[stk.top]
```

2 Stack Implementation: Array (1/4)

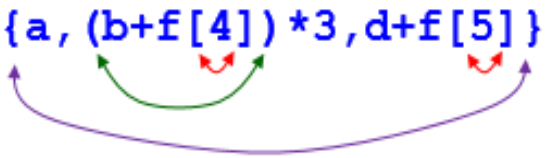
- Use an Array with a top index pointer



3- Application 1: Bracket Matching (1/2)

- Ensures that pairs of brackets are properly matched

An example:



Incorrect examples:

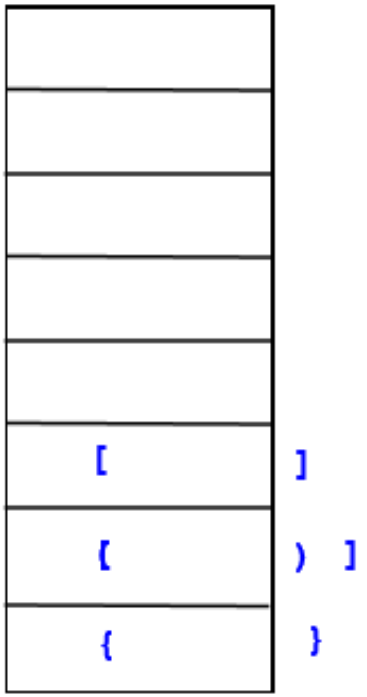
```
( . . ) . . )        // too many close brackets

( . . ( . . )        // too many open brackets

[ . . ( . . ] . . )  // mismatched brackets
```

3- Application 1: Bracket Matching (2/2)

```
create empty stack
for every char read
{
  if open bracket then
      push onto stack
  if close bracket, then
      pop from the stack
      if doesn't match or underflow then flag error
}
if stack is not empty then flag error
```

Q: What type of error does the last line test for?

A: too many closing brackets
B: too many opening brackets
C: bracket mismatch



Example

```
{ a - ( b + f [ 4 ] ) * 3 * d + f [ 5 ] }
```

Applicⁿ 2: Arithmetic Expression (1/7)

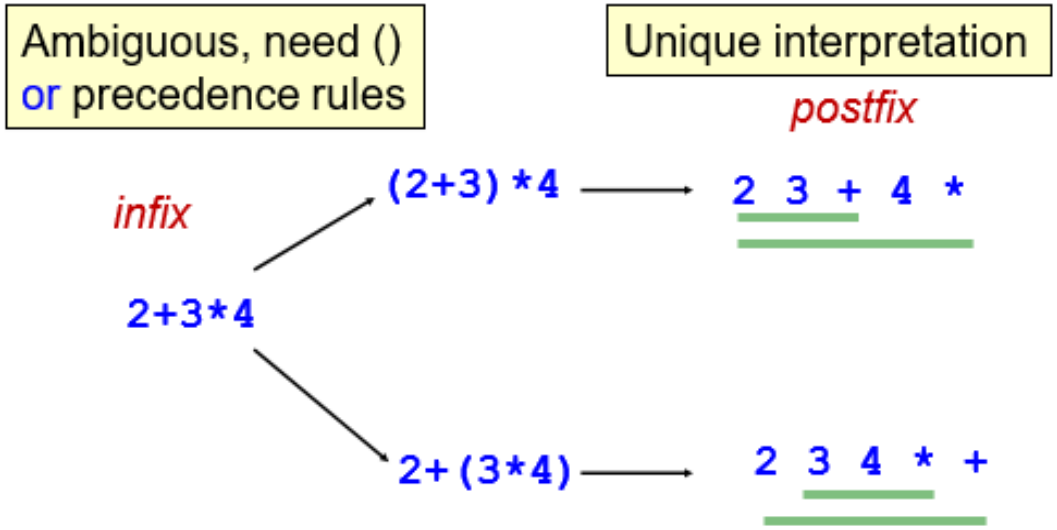
- Terms
  - Expression: a = b + c * d
  - Operands: a, b, c, d
  - Operators: =, +, –, *, /, %
- Precedence rules: Operators have priorities over one another as indicated in a table (which can be found in most books & our first few lectures)
  - Example: * and / have higher precedence over + and –.
  - For operators at the same precedence (such as * and /), we process them from left to right

Applicⁿ 2: Arithmetic Expression (2/7)

Infix : operand1 operator operand2

Prefix : operator operand1 operand2

Postfix : operand1 operand2 operator



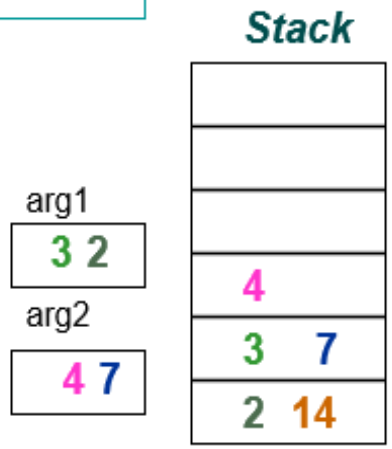
Applicⁿ 2: Arithmetic Expression (3/7)

Algorithm: Calculating Postfix expression with stack

Create an empty stack
for each item of the expression,
  if it is an operand,
    push it on the stack
  if it is an operator,
    pop arguments from stack;
    perform the operation;
    push the result onto the stack

Infix: 2 * (3 + 4) → postfix: 2 3 4 + *

2 s.push(2)
3 s.push(3)
4 s.push(4)
\+ arg2 = s.pop ()
arg1 = s.pop ()
s.push (arg1 + arg2)
* arg2 = s.pop ()
arg1 = s.pop ()
s.push (arg1 * arg2)



Problem

Find the result of arithmetic Exp. Using concept of stack

1- 7+6*3/2 convert to

2- (3+7) *2-6 convert to

3- ab*cde^/+ where a=5, b=6, c=8, d=2, e=2

Applicⁿ 2: Arithmetic Expression (4/7)

Brief steps for Infix to Postfix Conversion

1. Scan infix expression from left to right
2. If an operand is found, add it to the postfix expression.
3. If a "(" is found, push it onto the stack.
4. If a ")" is found
   a) repeatedly pop the stack and add the popped operator to the postfix expression until a "(" is found.
   b) remove the "(".
5. If an operator is found
   a) repeatedly pop the operator from stack which has higher or equal precedence than/to the operator found, and add the popped operator to the postfix expression.
   b) add the new operator to stack
6. If no more token in the infix expression, repeatedly pop the operator from stack and add it to the postfix expression.

Applicⁿ 2: Arithmetic Expression (6/7)

Algorithm: Converting Infix to an equivalent Postfix

ch	Stack (bottom to top)	postfixExp
a		a
–	–	a
(	– (	a
b	– (	a b
+	– (+	a b
c	– (+	a b c
*	– (+ *	a b c
d	– (+ *	a b c d
)	– (+	a b c d *
	– (	a b c d * +
	–	a b c d * +
/	– /	a b c d * +
e	– /	a b c d * + e
		a b c d * + e / –

Example: a – (b + c * d) / e

Move operators from stack to postfixExp until '('

Copy remaining operators from stack to postfixExp

Applicⁿ 2: Arithmetic Expression (7/7)

- How to code the above algorithm in **Python**
- How to do conversion of infix to prefix?
  - See Prefix.java

Call Function

- Begin {this is the main program}
- 100 Call A
- 102 -
- -
- 200 Call B -
- 202 -
- 300 Call
- 302 -
- -
- end

Priority of operation

Priority	Operation
1	^ , (unary -) , unary (+), Not
2	* , / , AND, DIV , MOD
3	+, - , OR
4	= , < ,> , <> , <= , >=

Problem

Convert the following infix exp. To Postfix Exp. Using the concept of Stack

1- a-b*(c+d)/(e-f)^g*h

2- y*m+(a^3/b-n)-d

3- m= x/6+(a-2*(b/3)^5+f)^2

4- (a>b) and ((e-c>a)or(g<f)

5- B-a+c and n^x-(p/M or f^2)

- Write an algorithm to read string end with (*) and print it in reverse order
- Data Structures and Algorithms in Python, Michael T. Goodrich, John Wiley & Sons (112, 115)
- Python Data Structures and Algorithms, Benjamin Baka Packt Publishing Ltd, 2017. (228)

6.1.1 The Stack Abstract Data Type

Stacks are the simplest of all data structures, yet they are also among the most important. They are used in a host of different applications, and as a tool for many more sophisticated data structures and algorithms. Formally, a stack is an abstract data type (ADT) such that an instance S supports the following two methods:

S.push(e): Add element e to the top of stack S.

S.pop( ): Remove and return the top element from the stack S; an error occurs if the stack is empty.

Additionally, let us define the following accessor methods for convenience:

S.top( ): Return a reference to the top element of stack S, without removing it; an error occurs if the stack is empty.

S.is_empty( ): Return True if stack S does not contain any elements.

len(S): Return the number of elements in stack S; in Python, we implement this with the special method __len__.

By convention, we assume that a newly created stack is empty, and that there is no a priori bound on the capacity of the stack. Elements added to the stack can have arbitrary type.

Example 6.3: *The following table shows a series of stack operations and their effects on an initially empty stack S of integers.*

Operation	Return Value	Stack Contents
S.push(5)	–	[5]
S.push(3)	–	[5, 3]
len(S)	2	[5, 3]
S.pop( )	3	[5]
S.is_empty( )	False	[5]
S.pop( )	5	[ ]
S.is_empty( )	True	[ ]
S.pop( )	"error"	[ ]
S.push(7)	–	[7]
S.push(9)	–	[7, 9]
S.top( )	9	[7, 9]
S.push(4)	–	[7, 9, 4]
len(S)	3	[7, 9, 4]
S.pop( )	4	[7, 9]
S.push(6)	–	[7, 9, 6]
S.push(8)	–	[7, 9, 6, 8]
S.pop( )	8	[7, 9, 6]