The bounded variables problem:

A liner programming problem may have in addition to the regular constraints, $\boldsymbol{Lowe}$
$\leq \boldsymbol{X_j}$
$\leq \boldsymbol{U_j}$ *where* $\boldsymbol{X_j}$ *is the* $\boldsymbol{j}$ *the variable of the problem and* $\boldsymbol{L_j}$ *and* $\boldsymbol{U_j}$ *are its lower and upp*

The lower bound constraint can be handled directly by substituting $\boldsymbol{X_j} = \boldsymbol{L_j} + \boldsymbol{\acute{X}_j}$ *where* $\boldsymbol{\acute{X}_j} \geq \boldsymbol{0}$*. For an* ***upper*** *bound constraint of the type* $\boldsymbol{X_j} \leq \boldsymbol{U_j}$*. The substituting* $\boldsymbol{X_j} = \boldsymbol{U_j} - \boldsymbol{\acute{X}_j}$, $\boldsymbol{\acute{X}_j} \geq \boldsymbol{0}$ *does not guarantee that* $\boldsymbol{X_j}$ *will remain non*
− negative. This difficulty is overcome by using a special technique called bounded v

1. *If* $\boldsymbol{R.H.S}$ *of any constraint is negative, make it positive by multiplying the con*
2. *Convert the inequalities of the constraints in to equations by the addition of su*
3. *If any variable is at a* ***positive lower*** *bound, it should be substituted at its low*
4. *Let* $\boldsymbol{X_j}$ *be a non −*
   basic variable at zero level which is selected to enter the solution. Let $(\boldsymbol{X_B})_i = (\boldsymbol{X_{\hat{B}}})_i$ *be the* $\boldsymbol{i}$ *the variable of the current basic solution* $\boldsymbol{X_B}$ *compute the quantit*

Q_1=min{ 〖(X_B ̂)〗 _i/(-a_ij) a_ij>0}

Q_2=min{ 〖U_i-(X_B ̂)〗 _i/(-a_ij) a_ij<0}

And **Q=min⁡(Q_1,Q_2,U_j)**

Where **U_j** is the **upper** bound for the variable **X_j** let 〖 〖(X〗 _B)〗 _r be the variable corresponding to **Q=min⁡(Q_1,Q_2,U_j)**. Then:

1. If **Q=Q_1** then 〖 〖(X〗 _B)〗 _r leave the solution and **X_j** enters by using the regular row operations of the simplex method.

2. If $Q=Q_2$, $(X_B)_r$ leave the solution and X_j enters then $(X_B)_r$ being non-basic at its **upper** bound must be substituted out by suing:

$$(X_B)_r = U_r - (X_B')_r \qquad 0 \le (X_{Br}') \le U_r$$

3. If $Q=U_j$ then X_j is substituted at is **upper** difference $U_j-(X_j')$ while remaining non-basic.

EXAMPLE: -

Solve the following **L.P.P** by the bounded variable simplex method:

Max $Z = 3x_1 + 5x_2 + 3x_3$

S.To

$x_1 + 2x_2 + 2x_3 \le 14$

$2x_1 + 4x_2 + 3x_3 \le 23$

$0 \le x_1 \le 4$, $0 \le x_2 \le 5$, $0 \le x_3 \le 3$

Sol:

$Z = 3x_1 + 5x_2 + 3x_3 + 0s_1 + 0s_2$

$Z = -3x_1 - 5x_2 - 3x_3 - 0s_1 - 0s_2$

$x_1 + 2x_2 + 2x_3 + s_1 = 14 \quad eq(s_1) \qquad s_1 \ge 0$

$2x_1 + 4x_2 + 3x_3 + s_2 = 23 \quad eq(s_2) \qquad s_2 \ge 0$

	x_1	x_2	x_3	s_1	s_2	x_B	$U_j-(X_B')_i$
Z	-3	-5	-3	0	0	0	
s_1	1	2	2	1	0	14	$\infty-14=\infty$
s_2	2	4	3	0	1	23	$\infty-23=\infty$

$Q_1 = \min(14/2, 23/4) = 23/4 \leftarrow$ **s_2** متغير داخل يقابل **x_2**

$Q_2=\infty$ Since all elements in **X_2** column is non-negative, **$U_2=5$**

$Q = \min(Q_1, Q_2, U_2)$

$Q=\min(23/4,\infty,5)$

Since **$Q_2=U_2$, x_2** substituted at its **upper** bound difference **i .e , $x_2=5-x_2'$** but it remains non-basic. The problems thus become:

$Z-3x_1-5(5-x_2')-3x_3-0s_1-0s_2=0$
$Z-3x_1+5x_2'-3x_3-0s_1-0s_2=25 \rightarrow (\quad)\ eq(Z)$

$x_1+2(5-x_2')+2x_3+s_1=14$
$x_1-2x_2'+2x_3+s_1=4 \rightarrow (\quad)\ eq(s_1)$

$2x_1+4(5-x_2')+3x_3+s_2=23$
$2x_1-4x_2'+3x_3+s_2=3 \rightarrow (\quad)\ eq(s_2)\quad s_1,s_2\geq 0$

	x_1	x_2'	x_3	s_1	s_2	x_B	$U_j-(X_B)_i$
Z	-3	5	-3	0	0	25	
s_1	1	-2	2	1	0	4	$\infty-4=\infty$
s_2	2	-4	3	0	1	3	$\infty-3=\infty$

$Q_1=\min(4/1\ ,3/2)=3/2 \leftarrow$ **s_2** متغير داخل يقابل **x_1**

$Q_2=\infty$ Since all elements in **X_1** columns are non-negative, **$U_1=4$**

$Q=\min(Q_1,Q_2,U_1)$

$=\min(3/2,\infty,4)=3/2=Q_1$ Introduce **X_1** and drop **s_2**

	x_1	x_2'	x_3	s_1	s_2		$U_j-(X_B)_i$
Z	0	-1	3/2	0	3/2	59/2	
s_1	0	0	1/2	1	(-1)/2	5/2	$\infty-5/2=\infty$
x_1	1	-2	3/2	0	1/2	3/2	$4-3/2=5/2$

x_2متغير داخل يقابل x_1

$Q_1=\min((5/2)/0,(3/2)/0)=\infty$

$Q_2=(4-3/2)/(-(-2))=5/4$ $\qquad$ x_1 يقابل $u_2=5$

$Q=\min(Q_1,Q_2,U_2)$

$Q=\min(\infty,5/4,5)=\frac{5}{4}=Q_2$

Since **$Q=Q_2$** introduce **x_2'** into the basic and drop **x_1** and the substitute it out at its **upper** bound **$4-x_1'$**, thus by removing **x_1** and introducing **x_2'** the table becomes:

	x_1	x_2'	x_3	s_1	s_2		$U_j-(X_B)_i$
Z	$\frac{-1}{2}$	0	3/4	0	5/4	115/4	
s_1	0	0	1/2	1	$\frac{-1}{2}$	5/2	$\infty-5/2=\infty$
x_2'	$\frac{-1}{2}$	1	$\frac{-3}{4}$	0	$\frac{-1}{4}$	$\frac{-3}{4}$	$4-3/2=5/2$

نعوض عن x_1 بـــــ $(4-x_2')$ كـالأتـي:

	x_1	x_2	x_3	s_1	s_2	x_B
Z	-6	0	3/4	0	5/4	123/4
s_1	2	4	1/2	1	$\frac{-1}{2}$	5/2
s_2	1	-2	$\frac{-3}{4}$	0	$\frac{-1}{4}$	5/4

The optimal solutions:

$x_1'=0$, $x_2'=5/4$, $x_3=0$

$x_1=4-x_1' \rightarrow x_1=4$

$x_2=5-5/4 \rightarrow x_2=15/4$

$x_3=0$

$\therefore \max Z=3(4)+5(15/4)+3(0)=$ 123/4

EXAMPLE:

Solve the following **L.P.P** by the bounded variable simplex method:

$\min Z=6x_1-2x_2-3x_3$

s.To

$2x_1+4x_2+2x_3\le 8$

$x_1-2x_2+3x_3\le 7$

$0\le x_1\le 2$

$0\le x_2\le 2$

$0\le x_3\le 1$

sol:

$Z-6x_1+2x_2+3x_3-0s_1-0s_2=0\rightarrow$ ( ) eq(Z)

$2x_1+4x_2+2x_3+s_1=8$

$x_1-2x_2+3x_3+s_2=7$

$s_1,s_2\ge 0$

	x_1	x_2'	x_3	s_1	s_2	x_B'	$U_j-(X_B')_i$
Z	-6	2	3	0	0	0	
s_1	1	4	2	1	0	8	$\infty-8=\infty$
s_2	2	-2	3	0	1	7	$\infty-7=\infty$

$Q_1=\min(8/2\ ,7/3)=7/3\ \leftarrow s_2$ متغير داخل يقابل x_3

$Q_2=\infty$ Since all elements in **X_3** columns are non-negative, **$U_3=1$**

$Q=\min(Q_1,Q_2,U_3)$

$=\min((7/3,\infty,1)=1)$

$Q=U_3\therefore U_3$ is substituted at its upper bound **i .e**: **$x_3=1-x_2'$** but it remains it non-basic.

The problem become:

$Z-6x_1+2x_2+3(1-x_3')-0s_1-0s_2=0$

$Z-6x_1+2x_2-3x_3'-0s_1-0s_2=-3\rightarrow$ ( ) eq(**Z**)

$2x_1+4x_2+2(1-x_3')+s_1=8$

〖2x〗 _1+ 〖4x〗 _2-2(x_3)́ +s_1=6→┬( ) eq(s_1)

x_1- 〖2x〗 _2+3(1-(x_3)́)+s_2=7

x_1- 〖2x〗 _2-3(x_3)́ +s_2=4→┬( ) eq(s_2)

	x_1	(x_2)́	(x_3)́	s_1	s_2		U_j- 〖(X_B ̂)〗 _i
Z	-6	2	-3	0	0	-3	
s_1	2	4	-2	1	0	6	∞-6=∞
s_2	1	-2	-3	0	1	4	∞-4=∞

Q_1=min⁡([redacted](@6)/4 ,[redacted](يُهمل@4)/(-2))=3/2 ← s_1 متغير داخل يقابل x_2

Q_2=(∞-4)/(-(-2))=∞ ,U_2=2

Q=min⁡(Q_1,Q_2,U_2)=min⁡(3/2,∞,2)=3/2

∴**Q=Q_1** is introduce **x_2** and drop **s_1**

	x_1	(x_2)́	(x_3)́	s_1	s_2	(x_B)́
Z	-7	0	-2	(-1)/2	0	-6
x_2	1/2	1	(-1)/2	1/4	0	3/2
s_2	2	0	-4	1/2	1	7

The optimal solution:

x_1=0 ,x_2=3/2

(x_3)̀ =0

x_3=1-(x_3)́ ⟹x_3=1

min⁡Z=-6

EXAMPLE:

max⁡Z= 〖3x〗 _1+ 〖5x〗 _2+ 〖2x〗 _3

s.To

x_1+x_2+ 〖2x〗 _3≤14

〖2x〗 _1+ 〖4x〗 _2+ 〖3x〗 _3≤43

$0 \le x_1 \le 4$

$7 \le x_2 \le 10$

$0 \le x_3 \le 3$

الحل: يوجد حد أدنى موجب للمتغير **x_2** لذلك يجب إحلاله بقيم تأخذ في الأعتبار حده الأدنى.

$x_2 = 7 + y_2$

$7 \le 7 + y_2 \le 10$

$7-7 \le 7 + y_2 - 7 \le 10-7$

$0 \le y_2 \le 3$

$Z - 3x_1 - 5(7+y_2) - 2x_3 - 0s_1 - 0s_2 = 0$

$Z - 3x_1 - 5y_2 - 2x_3 - 0s_1 - 0s_2 = 35 \rightarrow (\qquad)\ (Z)$

$x_1 + (7+y_2) + 2x_3 + s_1 = 14$

$x_1 + y_2 + 2x_3 + s_1 = 7 \rightarrow (\qquad)\ (s_1)$

$2x_1 + 4(7+y_2) + 3x_3 + s_2 = 43$

$2x_1 + 4y_2 + 3x_3 + s_2 = 15 \rightarrow (\qquad)\ (s_2)$

	x_1	y_2	x_3	s_1	s_2		$U_j - (\hat{X_B})_i$
Z	-3	-5	-2	0	0	35	
s_1	1	1	2	1	0	7	$\infty-7=\infty$
s_2	2	4	3	0	1	15	$\infty-15=\infty$

$Q_1 = \min(7/1,\ 15/4) = 15/4 \leftarrow s_2$ يقابل داخل متغير y_2

Q_2 $=\infty$, $U_2 = 3$

$Q = \min(15/4, \infty, 3) = 3$

$Q = U_2$ ∴ يبقى **y_2** متغير غير أساسي ولكن عند حده الاعلى

$y_2 = 3 - (y_2)'$

$Z - 3x_1 - 5(3-(y_2)') - 2x_3 - 0s_1 - 0s_2 = 35$

Z-〖3x〗_1+5(y_2)̂-〖2x〗_3-〖0s〗_1-〖0s〗_2=50 →┬(   ) (Z)

x_1+(3-(y_2)̂)+〖2x〗_3+s_1=7

x_1-(y_2)̂+〖2x〗_3+s_1=4 →┬(   ) (s_1)

〖2x〗_1+4(3-(y_2)̂)+〖3x〗_3+s_2=15

〖2x〗_1-4(y_2)̂+〖3x〗_3+s_2=3→┬(   ) (s_2)

	x_1	(y_2)̂	x_3	s_1	s_2		U_j-〖(X_B̂)〗_i
Z	-3	5	-2	0	0	50	
s_1	1	-1	2	1	0	4	∞-4=∞
s_2	2	-4	3	0	1	3	∞-3=∞

Q_1=min⁡(4/1,3/2)=3/2 ← s_2 يقابل داخل متغير x_1

Q_2=∞ , 〖U〗_1=4

Q=min⁡(3/2,∞,4)=3/2

Q=Q_1 s_2 ويخرج إلى الحل يدخل x_1∴

	x_1	(y_2)̂	x_3	s_1	s_2	(x_B)̂	U_j-〖(X_B̂)〗_i
Z	0	-1	1/2	0	3/2	109/2	
s_1	0	1	1/2	1	(-1)/2	5/2	∞-5/2=∞
x_1	1	-2	3/2	0	1/2	3/2	4-3/2=5/2

Q_1=min⁡((5/2)/1,(3/2)/(-2))=5/2 ← s_1 يقابل داخل متغير (y_2)̂

Q_2=(U_i-X_B̂)/(-(-a_ij))=(5/2)/(-(-2))=5/4 ← s_1 يقابل

U_2=3

Q=min⁡(5/2,5/4,3)=5/4

Q=Q_2 لذا فإن **(y_2)̂** يدخل ويخرج **x_1** ولإستبعاد المتغير **x_1** من الجدول نعوض عنه بقيمتة عند حده الأعلى **x_1=4-(x_1)̂** .

	x_1	(y_2)̂	x_3	s_1	s_2	

Z	(-1)/2	0	7/4	0	3/2	215/4
s_1	1/2	0	5/4	1	(-1)/4	13/4
(y_2)́	(-1)/2	1	(-3)/4	0	(-1)/4	(-3)/4

Z-1/2 (4-(x_1)́)+7/4 x_3+5/4 s_2=215/4

Z+1/2 (x_1)́ +7/4 x_3+5/4 s_2=223/4 →┬(   ) (Z)

1/2 (4-(x_1)́)+5/4 x_3+s_1-1/4 s_2=13/4

-(1)/(2) (x_1)́ +5/4 x_3+s_1-1/4 s_2=5/4 →┬(   ) (s_1)

-1/2 (4-(x_1)́)+(y_2)́ -3/4 x_3-1/4 s_2=(-3)/4

1/2 (x_1)́ +(y_2)́ -3/4 x_3-1/4 s_2=5/4 →┬(   ) ((y_2)́)

	(x_1)́	(y_2)́	x_3	s_1	s_2	
Z	1/2	0	7/4	0	5/4	223/4
s_1	(-1)/2	0	5/4	1	(-1)/4	5/4
(y_2)́	1/2	1	(-3)/4	0	(-1)/4	5/4

(x_1)́ =0→x_1=4-(x_1)́ →4-0 → ∴x_1=4

(y_2)́ =5/4→ y_2=3-(y_2)́ →3-5/4 → ∴y_2=7/4

x_2=7+y_2 →┬(   ) =7+7/4=45/4

∴x_3=0 , Z=223/4

EXAMPLE:

max⁡Z= 〖4x〗_1+ 〖3x〗_2

S.To

x_1+x_2≤6

〖2x〗_1+x_2≤8

x_1≥1 , 1≤x_2≤3

الحل: يوجد حل أدنى موجب للمتغيرين **x_1 , x_2** ولذلك يجب إحلال هذين المتغيرين بقيم تأخذ في الإعتبار حديهما الأدنى.

$x_1=1+y_1 \rightarrow y_1 \geq 0$

$x_2=1+y_2 \rightarrow 0 \leq y_2 \leq 2$

$Z-4(1+y_1)-3(1+y_2)=0$

$Z-4y_1-3y_2-0s_1-0s_2=7 \rightarrow$ ┬( ) (Z)

$(1+y_1)+(1+y_2)+0s_1=6$

$y_1+y_2+s_1=4 \rightarrow$ ┬( )(s_1)

$2(1+y_1)+(1+y_2)+s_2=8$

$2y_1+y_2+s_2=5 \rightarrow$ ┬( ) (s_2)

	y_1	y_2	s_1	s_2		$U_j-(X_{\hat{B}})_i$
Z	-4	-3	0	0	7	
s_1	1	1	1	0	4	$\infty-4=\infty$
s_2	2	1	0	1	5	$\infty-5=\infty$

$Q_1=min(4/1,5/2)=5/2 \leftarrow s_2$ متغير داخل يقابل y_1

$Q_2=\infty \quad ,U_1=\infty$

$Q=min(5/2,\infty,\infty)=5/2$

$Q=Q_1$ s_2 متغير خارج y_1 متغير داخل

	y_1	y_2	s_1	s_2		$U_j-〖(X_B^)〗_i$
Z	0	-1	0	2	17	
s_1	0	1/2	1	(-1)/2	3/2	∞-3/2=∞
y_1	1	1/2	0	1/2	5/2	∞-5/2=∞

Q=min⁡〖((3/2)/(1/2),(5/2)/(1/2))=min⁡(3,5)=3〗 ←s_1 متغير داخل يقابل y_2

$Q_2=\infty \quad , U_2=2$

$Q=min(3,\infty,2)=2$

$Q=U_2$ يبقى متغير غير أساسي ولكن عند حده الأعلى **〖y〗_2**

$y_2=2-(y_2)'$

$Z-(2-(y_2)')+2s_2=17$

$Z+(y_2)' + 2s_2 = 19 \rightarrow \top(\quad) \ (Z)$

$1/2\,(2-(y_2)') + s_1 - 1/2\, s_2 = 3/2$

$-1/2\,(y_2)' + s_1 - 1/2\, s_2 = 1/2 \rightarrow \top(\quad) \ (s_2)$

$y_1 + 1/2\,(2-(y_2)') + 1/2\, s_2 = 5/2$

$y_1 - 1/2\,(y_2)' + 1/2\, s_2 = 3/2 \rightarrow \top(\quad) \ (y_1)$

	y_1	y_2	s_1	s_2	
Z	0	1	0	2	19
s_1	0	$(-1)/2$	1	$(-1)/2$	1/2
y_1	1	$(-1)/2$	0	1/2	3/2

$y_1 = 3/2 \rightarrow x_1 = 1 + y_1 \rightarrow 1 + 3/2 = 5/2$

$(y_2)' = 0 \rightarrow y_2 = 2 - (y_2)' \rightarrow 2 - 0 \rightarrow y_2 = 2$

$x_2 = 1 + y_2 \rightarrow 1 + 2 = 3$

$\therefore x_1 = 5/2 \ , x_2 = 3 \ , Z = 19$