المادة / الرياضيات - ٢

المحاضرة الاولى / حسابات التكامل

Integral Calculus

Standard Integration التكاملات الاساسية

The Proces of integration

The proces of integration reverses the process of differention. In differentiation if

$f(x) = 2x^2$ then $f'(x) = 4x$

Thus the integral of $4x$ is $2x^2$

Thus $\int 4x\,.dx$ means the integral of $4x$ with respect to x

and $\int 2t\,.dt$ means the integral of $2t$ with respect to t

Whenever the process of integration is performed a constant 'c' is added to the result

Thus $\int 4x\,dx = 2x^2 + c$ and $\int 2t\,dt = t^2 + c$

'c' is called the arbitrary constant of integration.

The general solution of integrals of the form ax^n

$\int ax^n.dx$, where a and n are constant is given by

$$\int ax^n.dx = \frac{ax^{n+1}}{n+1} + c$$ صيغة التكامل لهذا الشكل

Standard integrals

التكاملات الاساسية

(1) $\int ax^n \cdot dx = \frac{ax^{n+1}}{n+1} + c$ ماذا عندما $n=-1$

(2) $\int dx = x + c$

(3) $\int a \cdot dx = ax + c$

(4) $\int \cos ax \, dx = \frac{1}{a} \sin ax + c$

(5) $\int \sin ax \, dx = -\frac{1}{a} \cos ax + c$

(6) $\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + c$

(7) $\int \operatorname{cosec}^2 ax \, dx = -\frac{1}{a} \cot ax + c$

(8) $\int \operatorname{cosec} ax \cot ax \, dx = -\frac{1}{a} \operatorname{cosec} ax + c$

(9) $\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + c$

(10) $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$

(11) $\int \frac{1}{x} dx = \ln x + c$

Problem 1 Determine $\int 5x^2 \cdot dx$

$$\int 5x^2 \cdot dx = \frac{5x^{2+1}}{2+1} + c = \frac{5}{3} x^3 + c$$

Problem 2: Using this rule gives:

(a) $$\int \frac{2}{x^2}\cdot dx = \int 2x^{-2}\,dx = \frac{2x^{-2+1}}{-2+1} + c$$

$$= \frac{2x^{-1}}{-1} + c = \frac{-2}{x} + c$$

(b) $$\int \sqrt{x}\cdot dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$$

* The integral of constant k is $kx + c$, for example:

$$\int 8x\,dx = 8x + c$$

* when a sum of several terms is integrated the result is the sum of the integrals of separate terms for example:

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$$\int (3x + 2x^2 - 5)\,dx$$

$$= \int 3x\,dx + \int 2x^2\,dx - \int 5\,dx$$

$$= \frac{3x^2}{2} + \frac{2}{3}x^3 - 5x - c$$

Problem 3: Determine

(a) $$\int \frac{2x^3 - 3x}{4x}\cdot dx = \int \frac{2x^3}{4x} - \frac{3x}{4x}\cdot dx$$

$$= \int \frac{x^2}{2} - \frac{3}{4}\,dx = \frac{1}{2}\,\frac{x^{2+1}}{2+1} - \frac{3}{4}x + c$$

$$= \frac{1}{2}\,\frac{x^3}{3} - \frac{3}{4}x + c = \frac{1}{6}x^3 - \frac{3}{4}x + c$$

(b) $\int (1-x)^2 \cdot dx = \int (1-2x+x^2)\, dx$

$$= \int dx - \int 2x\, dx + \int x^2 dx$$

$$= x - 2\frac{x^{1+1}}{1+1} + \frac{x^{2+1}}{2+1} + c$$

$$= x - \frac{2x^2}{2} + \frac{x^3}{3} + c$$

$$= x - x^2 + \frac{1}{3}x^3 + c$$

<u>Problem 4</u>: Determine $\int 3\sqrt{x} \cdot dx$

$$\int 3\sqrt{x}\, dx = \int 3x^{\frac{1}{2}} dx = \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c = 2x^{\frac{3}{2}} + c = 2\sqrt{x^3} + c$$

<u>Problem 5</u>: Determine $\int \frac{-5}{9\sqrt[4]{x^3}} \cdot dx$

$$\int \frac{-5}{9\sqrt[4]{x^3}} \cdot dx = \int \frac{-5}{9x^{\frac{3}{4}}} \cdot dx = \int \left(-\frac{5}{9}\right) x^{-\frac{3}{4}} \cdot dx$$

$$= \left(-\frac{5}{9}\right) \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c = \left(-\frac{5}{9}\right) \frac{x^{\frac{1}{4}}}{\frac{1}{4}} + c$$

$$= \left(-\frac{5}{9}\right)\left(\frac{4}{1}\right) x^{\frac{1}{4}} + c$$

$$= -\frac{20}{9} \sqrt[4]{x} + c$$

Problem 6: Determine

(a) $$\int 4\cos 3x \, dx = 4\,\frac{1}{3}\sin 3x + c = \frac{4}{3}\sin 3x + c$$

(b) $$\int 5\sin 2x \, dx = 5\left(-\frac{1}{2}\right)\cos 2x + c$$
$$= -\frac{5}{2}\cos 2x + c$$

Problem 7:

$$\int 7\sec^2 4x \, dx$$

$$\int 7\sec^2 4x \, dx = 7\,\frac{1}{4}\tan 4x + c = \frac{7}{4}\tan 4x + c$$

Problem 8: Determine (a) $\int 5e^{3x} \cdot dx$ (b) $\int \frac{2}{3e^{4x}} \cdot dx$

a) $$\int 5e^{3x} \cdot dx = 5\,\frac{1}{3}e^{3x} + c = \frac{5}{3}e^{3x} + c$$

b) $$\int \frac{2}{3e^{4x}} \cdot dx = \int \frac{2}{3}e^{-4x} \cdot dx = \frac{2}{3}\left(-\frac{1}{4}\right)e^{-4x} + c$$
$$= \frac{2}{3}\left(-\frac{1}{4}\right)\frac{1}{e^{4x}} + c = -\frac{2}{12e^{4x}} + c$$
$$= -\frac{1}{6e^{4x}} + c$$

Problem 9: Determine a) $\int \frac{3}{5x} \cdot dx$ b) $\int \frac{2x^2+1}{x} \cdot dx$

a) $$\int \frac{3}{5x} \cdot dx = \frac{3}{5}\ln x + c$$

b) $$\int \frac{2x^2+1}{x} \cdot dx = \int \left(\frac{2x^2}{x} + \frac{1}{x}\right) \cdot dx$$
$$= \int \left(2x + \frac{1}{x}\right) \cdot dx = \frac{\cancel{2}x^2}{\cancel{2}} + \ln x + c$$
$$= x^2 + \ln x + c$$