Logarithms

Introduction to Logarithms

- If a number y can be written in the form a^x. Then the index x is called the Logarithm of y to the base of a

$$y = a^x \quad \text{then} \quad x = \text{Log}_a y$$

ex. $1000 = 10^3$

Then $3 = \text{Log}_{10} 1000$

- Logarithms having a base of 10 are called common Logarithm and Log_{10} is usully abbreviated to Lg

ex. $\text{Lg } 17.9 = 1.2528$

$\text{Lg } 4627 = 2.6652$

$\text{Lg } 0.0173 = -1.7619$

- Logarithms having a base of e (where e is a mathemical constant aproxmately equal to 2.7183) are called hyperbolic. Napierian or natural Logarithms and Log_e is usully abbreviated to Ln.

ex. $\text{Ln } 3.15 = 1.14714$

$\text{Ln } 362.7 = 5.8935$

$\text{Ln } 0.156 = -1.8578$

Law of Logarithms

(1) $\text{Log}(A \times B) = \text{Log}\,A + \text{Log}\,B$

$\text{Lg}\,10 = 1$

also $\text{Lg}5 + \text{Lg}2 = 0.69897 + 0.301029 = 1$

Hence $\text{Lg}(5 \times 2) = \text{Lg}\,10 = \text{Lg}5 + \text{Lg}2$

(2) $\text{Log}\left(\frac{A}{B}\right) = \text{Log}\,A - \text{Log}\,B$

$\text{Ln}\left(\frac{5}{2}\right) = \text{Ln}\,2.5 = 0.91629$

Also $\text{Ln}5 - \text{Ln}2 = 1.60943 - 0.69314 = 0.9629$

Hence $\text{Ln}\left(\frac{5}{2}\right) = \text{Ln}5 - \text{Ln}\,2$

(3) $\text{Lg}\,A^n = n\,\text{Log}\,A$

(4) $\text{Log}\,\frac{1}{A} = -\text{Log}\,A$

Natural Logarithms

$\text{Ln}(A \times B) = \text{Ln}\,A + \text{Ln}\,B$

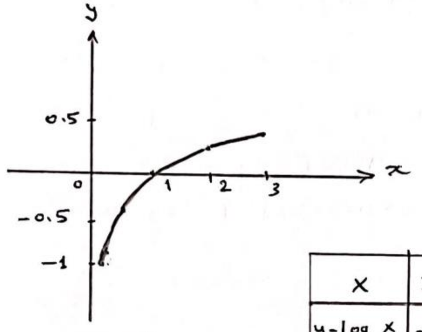
$\text{Ln}\left(\frac{A}{B}\right) = \text{Ln}\,A - \text{Ln}\,B$

$\text{Ln}\,A^n = n\,\text{Ln}\,A$

$\text{Ln}\,\frac{1}{A} = \text{Ln}\,A^{-1} = -\text{Ln}\,A$

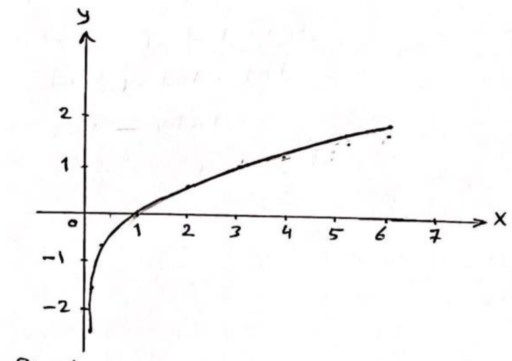
Graphs of Logarithmic functions

A graph of $y = \mathrm{Log}_{10} x$ is shown in fig(1) and a graph of $y = \mathrm{Log}_e x$ is shown in fig(2), Both are seen to be of similar shape, in fact the same general shape occurs for a Logarithm to any base.



fig(1)

X	3	2	1	0.5	0.2	0.1
$y = \mathrm{Log}_{10} x$	0.48	0.30	0	-0.30	-0.7	-1.0



fig(2)

	X	6	5	4	3	2	1	0.5	0.2	0.1
Ln=	$y = \mathrm{Log}_e x$	1.7[illegible]	1.61	1.39	1.10	0.69	0	-0.69	-1.69	-2.30

In general, with a Logarithm to any base a, it is noted that:

(1) $\log_a 1 = 0$

$\log_a 1 = x$, $a^x = 1$ $x = 0$ from the law of Logarithms

In above graphs its seen that $\log_{10} 1 = 0$

(2) $\log_a a = 1$

$\log_a a = x$, $a^x = a$

If $a^x = a$ then $x = 1$

$\log_a a = 1$

$\log_{10} 10 = 1$ and $\log_e e = 1$

(3) $\log_a 0 \rightarrow -\infty$

$\log_a 0 = x$ $a^x = 0$ and a is a positive real number, the x must approach minus infinity.

$x = -\infty$

$\frac{1}{a^{\infty}} = 0$

Problem 1 : Evaluate a) $\log_3 9$ b) $\log_{16} 8$

(a) $x = \log_3 9$ $3^x = 9$ $3^x = 3^2$ then $x = 2$

(b) $\log_{16} 8$

$x = \log_{16} 8$ $16^x = 8$ $16^x = 2^3$ $(2^4)^x = 2^3$

$4x = 3$ then $x = \frac{3}{4}$

Problem 2 : Evalute a) $\ln e$ b) $\log_3 \frac{1}{81}$

a) $\ln e$

$x = \ln e = \log_e e$

$e^x = e$, $e^x = e^1$

$\therefore x = 1$

$\therefore \ln e = 1$

b) $\log_3 \frac{1}{81}$

$x = \log_3 \frac{1}{81}$

$3^x = \frac{1}{81} = \frac{1}{3^4} = 3^{-4}$

$\therefore x = -4$

$\therefore \log_3 \frac{1}{81} = -4$

Problem 3 : Write Log30 in terms of Log 2, Log3 Log5 to any base

$\log 30 = \log(2 \times 15) = \log(2 \times 3 \times 5)$

$= \log 2 + \log 3 + \log 5$

by the first law of Logarithms

Problem 4 : Write Log450 in terms of log2, Log3 Log5 to any base.

$\log 450 = \log(2 \times 225) = \log(2 \times 3 \times 75)$

$= \log(2 \times 3 \times 3 \times 25)$

$= \log(2 \times 3 \times 3 \times 5 \times 5)$

$= \log 2 + \log 3^2 + \log 5^2$

$= \log 2 + 2\log 3 + 2\log 5$

by the third Law of Logarithms

Problem 5: Simplify

$$\log 64 - \log 128 + \log 32$$

$64 = 2^6$, $128 = 2^7$ and $32 = 2^5$

$$= \log 2^6 - \log 2^7 + \log 2^5$$

$$= 6\log 2 - 7\log 2 + 5\log 2$$

$$= 4\log 2$$

Problem 6: Evalute $\dfrac{\log 25 - \log 125 + \frac{1}{2}\log 625}{3\log 5}$

$$= \frac{\log 5^2 - \log 5^3 + \frac{1}{2}\log 5^4}{3\log 5}$$

$$= \frac{2\log 5 - 3\log 5 + \frac{\cancel{4}^{\,2}}{\cancel{2}}\log 5}{}$$

$$= \frac{1\log 5}{3\log 5} = \frac{1}{3}$$