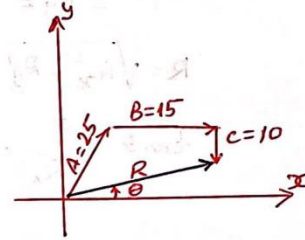


- Force System -  
(Problems Solutions)

Q<sub>1</sub>/  
6-1

(A) Graphically

Scale 1 cm = 5 km



(B) Analytically

1. Vector analysis

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{A}_x = 25 \cos 45 = 17.677 i$$

$$\vec{A}_y = 25 \sin 45 = 17.677 j$$

$$\therefore \vec{A} = \vec{A}_x + \vec{A}_y = 17.677 i + 17.677 j$$

$$\vec{B} = 15 i$$

$$\vec{C} = -10 j$$

$$\therefore \vec{R} = 17.677 i + 17.677 j + 15 i - 10 j$$

$$\vec{R} = 32.677 i + 7.677 j$$

$$|\vec{R}| = \sqrt{32.677^2 + 7.677^2} = 33.566 \text{ km}$$

$$\tan \theta = \frac{R_y}{R_x}$$

$$\therefore \theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{7.677}{32.677} = 13.2^\circ$$

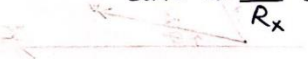
2. Scalar analysis

$$\rightarrow \Sigma F_x = R_x = 25 \cos 45 + 15 = 32.677$$

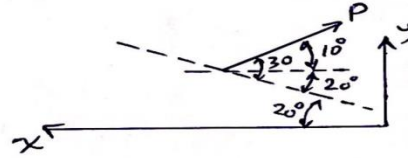
$$\uparrow \Sigma F_y = R_y = 25 \sin 45 - 10 = 7.677$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{32.677^2 + 7.677^2} = 33.566 \text{ km}$$

$$\tan \theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1} \frac{32.677}{7.677} = 13.2^\circ$$



Q2/  
2-1

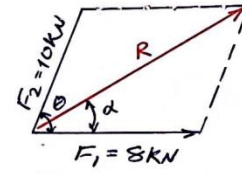


$$P_x = -P \cos 10 = -20 \cos 10 = -19.7 \text{ kN}$$

$$P_y = P \sin 10 = 20 \sin 10 = 3.47 \text{ kN}$$

Q3/  
2-2

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$



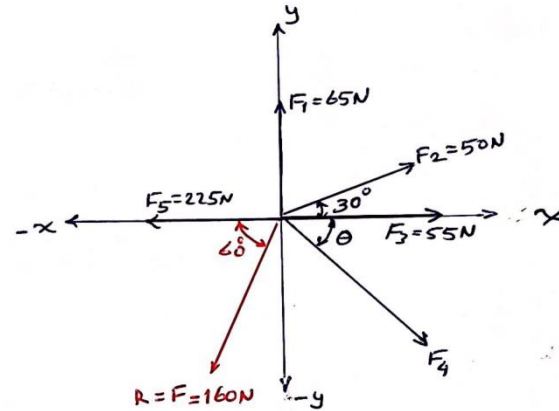
$$\therefore R = \sqrt{8^2 + 10^2 + 2 \times 8 \times 10 \cos 60}$$

$$= 15.62 \text{ kN}$$

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \frac{10 \sin 60}{8 + 10 \cos 60} = 33.67^\circ$$

Q.4/  
2-6



$$\rightarrow \sum F_x = R_x$$

$$F_4 \cos \theta + F_3 + F_2 \cos 30 - F_5 = -160 \cos 60$$

$$F_4 \cos \theta + 55 + 50 \cos 30 - 225 = -160 \cos 60$$

$$F_4 \cos \theta = 46.7 \quad \dots (1)$$

$$+\uparrow \sum F_y = R_y$$

$$F_1 + F_2 \sin 30 - F_4 \sin \theta = -160 \sin 60$$

$$65 + 50 \sin 30 - F_4 \sin \theta = -160 \sin 60$$

$$F_4 \sin \theta = 228.564 \quad \dots (2)$$

$$\frac{F_4 \sin \theta}{F_4 \cos \theta} = \frac{228.564}{46.7} \Rightarrow \tan \theta = \frac{228.564}{46.7} \Rightarrow$$

$$\theta = 78.4^\circ$$

Substitute  $\theta$  into (2) or (1)

$$\therefore F_4 = 233.29 \text{ N}$$

Q5/  
2-7

$$\rightarrow \Sigma F_x = R_x = 0$$

$$F_4 \cos \theta + F_3 + F_2 \cos 30 - F_5 = 0$$

$$F_4 \cos \theta + 55 + 50 \cos 30 - 225 = 0$$

$$\therefore F_4 \cos \theta = 126.7 \quad \dots (1)$$

$$\uparrow \Sigma F_y = R_y = 0$$

$$F_1 + F_2 \sin 30 - F_4 \sin \theta = 0$$

$$65 + 50 \sin 30 - F_4 \sin \theta = 0$$

$$\therefore F_4 \sin \theta = 90$$

$$F_4 = \frac{90}{\sin \theta} \quad \dots (2)$$

Substitute ② into ①

$$\frac{90}{\sin \theta} \cos \theta = 126.7$$

$$\therefore \tan \theta = \frac{90}{126.7} \Rightarrow \theta = 35.4^\circ$$

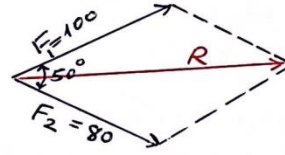
$$F_4 = 155.5 \text{ N}$$

Q6/

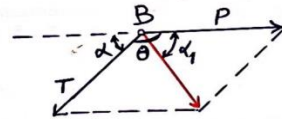
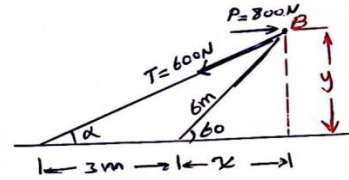
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

$$= \sqrt{100^2 + 80^2 + 2 \times 100 \times 80 \cos 50}$$

$$= 163.4 \text{ N}$$



Q7/



$$\tan \alpha = \frac{y}{3+x} = \frac{6 \sin 60}{3+6 \cos 60} \quad \begin{aligned} x &= 6 \cos 60 \\ y &= 6 \sin 60 \end{aligned}$$

$$\therefore \alpha = 40.893^\circ$$

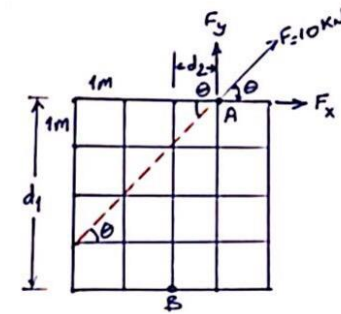
$$\therefore \theta = 180^\circ - 40.893^\circ = 139.1^\circ$$

$$\begin{aligned} \therefore R &= \sqrt{P^2 + T^2 + 2PT \cos \theta} \\ &= \sqrt{800^2 + 600^2 + 2(800)(600) \cos 139.1} \\ &= 523.8 \text{ N} \end{aligned}$$



Moments and Couples  
- Solution of Problems -

Q1/  
(3-3)



$$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.7^\circ$$

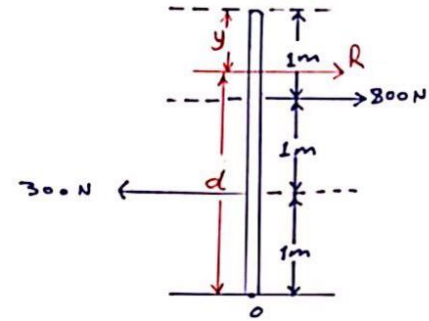
$$F_x = 10 \cos \theta = 10 \cos 33.7^\circ = 8.319 \text{ kN}$$

$$F_y = 10 \sin \theta = 10 \sin 33.7^\circ = 5.548 \text{ kN}$$

$$\begin{aligned} \text{*) } M_B &= F_x \cdot d_1 + F_y \cdot d_2 \\ &= 8.319 \times 4 - 5.548 \times 1 \end{aligned}$$

$$\therefore M_B = 27.728 \text{ N}\cdot\text{m}$$

Q<sub>2</sub>/  
(3-4)



$$\rightarrow \sum F_x = R_x = R = 800 - 300 = 500 \text{ N}$$

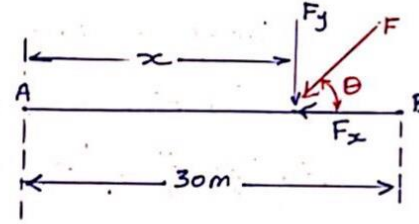
$$\uparrow \curvearrow M_o = \sum M_i = \sum F_i d_i$$

$$\uparrow \curvearrow 500 \times d = 800 \times 2 - 300 \times 1$$

$$\therefore d = 2.6 \text{ m}$$

$$\therefore y = 3 - d = 3 - 2.6 = 0.4 \text{ m}$$

Q3/  
(3-5)



$$M_A = 100\sqrt{3} \text{ N}\cdot\text{m}, M_B = -50\sqrt{3} \text{ N}\cdot\text{m}$$

$$F_x = 5 \text{ N}, F_y = ?, x = ?, \theta = ?$$

$$F_x = F \cos \theta \quad \dots \textcircled{1}$$

$$M_A = F_y \cdot x \quad \dots \textcircled{2}$$

$$M_B = F_y \times (30 - x) \quad \dots \textcircled{3}$$

$$F_y = \frac{M_B}{(30 - x)}$$

Substitute  $\textcircled{3}$  into  $\textcircled{2}$

$$M_A = \frac{M_B}{(30 - x)} \cdot x$$

$$(30 - x) = \frac{M_B}{M_A} \cdot x$$

$$30 = \frac{M_B}{M_A} \cdot x + x \quad /: x$$

$$\frac{30}{x} = \frac{M_B}{M_A} + 1$$

$$30 = \left( \frac{M_B}{M_A} + 1 \right) x$$

$$\therefore x = \frac{30}{\left( \frac{M_B}{M_A} + 1 \right)}$$

$$\therefore x = 20 \text{ m}$$

Substitute  $x$  into ②

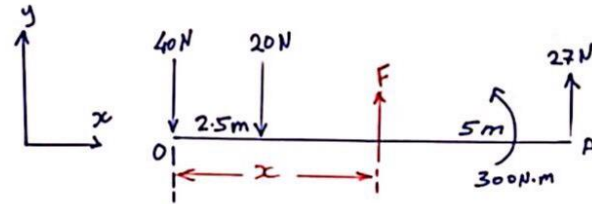
$$100\sqrt{3} = F_y \cdot 20 \Rightarrow F_y = 5\sqrt{3} \text{ N}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{5^2 + (5\sqrt{3})^2} = 10 \text{ N}$$

from ①

$$5 = 10 \cos \theta \Rightarrow \theta = 60^\circ$$

Q4/  
(3-7)



$$\uparrow M_o = \sum M_i = \sum F_i d_i$$

$$-300 = 20 \times 2.5 - F \cdot x - 27 \times 7.5$$

$$F \cdot x = 20 \times 2.5 - 27 \times 7.5 + 300$$

$$\therefore F \cdot x = 147.5 \Rightarrow F = \frac{147.5}{x} \dots \textcircled{1}$$

$$\uparrow M_A = \sum M_i = \sum F_i d_i$$

$$-300 = -40 \times 7.5 - 20 \times 5 + F(7.5 - x)$$

$$\therefore F(7.5 - x) = 100 \dots \textcircled{2}$$

Substitute  $\textcircled{1}$  into  $\textcircled{2}$

$$\frac{147.5}{x}(7.5 - x) = 100$$

$$\frac{147.5 \times 7.5}{x} - \frac{147.5 \cancel{x}}{\cancel{x}} = 100$$

$$\therefore x = 4.46 \text{ m} \text{ substitute into } \textcircled{1}$$

$$\therefore F = \frac{147.5}{4.46} = 33 \text{ N}$$

Q5)

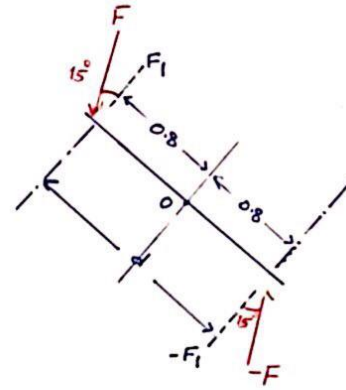
$$M_0 = F_1 \times d$$

$$15 = F_1 \times 1.6$$

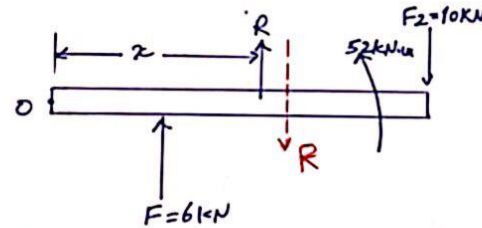
$$\therefore F_1 = F \cos 15$$

$$15 = F \cos 15 \times 1.6$$

$$\therefore F = 9.7 \text{ N}$$



Q6)



$$\uparrow \sum F_y = R_y = R = 6 - 10 = -4 \text{ kN}$$

$$\therefore R = 4 \text{ kN in down direction}$$

$$M_0 = \sum M_i = \sum F_i d_i$$

$$R \times x = -52 - 6 \times 3 + 10 \times 9$$

$$R x = 20$$

$$\therefore x = \frac{20}{4} = 5 \text{ m}$$

Q5/

$$\sum M_B = 0$$

$$5 \times 1 + N_A \times 4 - 1.25 - 10 \cos 36.87 \times 2 = 0$$

$$N_A = 3.06 \text{ kN}$$

$$\sum F_x = 0$$

$$N_{Bx} - 10 \sin \theta = 0$$

$$N_{Bx} = 10 \sin 36.87 = 6 \text{ kN}$$

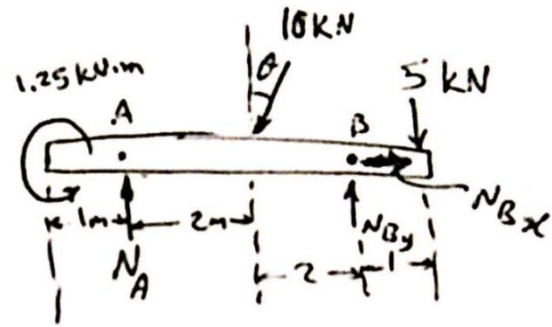
$$\sum F_y = 0$$

$$N_A + N_{By} - 10 \cos \theta - 5 = 0$$

$$3.06 + N_{By} - 10 \cos 36.87 - 5 = 0$$

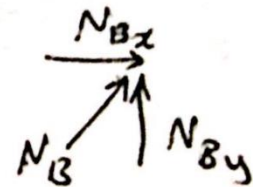
$$N_{By} = 9.94 \text{ kN}$$

$$N_B = \sqrt{(N_{Bx})^2 + (N_{By})^2} = \sqrt{(6)^2 + (9.94)^2} = 11.61 \text{ kN}$$



$$\tan \theta = \frac{3}{4}$$

$$\theta = \cancel{36.87} \\ = 36.87$$



$$Q6/ F_L = \frac{1}{2} (900 \times 1.2) = 540 \text{ N}$$

$$W = mg = 350 \times 9.81 = 3433.5 \text{ N}$$

$$\sum F_x = 0, \therefore N_{Ax} = 0$$

$$\curvearrowright M_A = 0$$

$$500 \times 2.4 + W \times 1.2 + F_L \times 0.8 - N_B \times 1.8 = 0$$

$$500 \times 2.4 + 3433.5 \times 1.2 + 540 \times 0.8 - N_B \times 1.8 = 0$$

$$\therefore N_B = 3195.66 \text{ N}$$

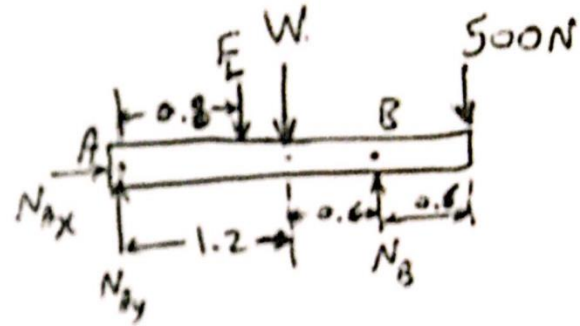
$$\uparrow \sum F_y = 0$$

$$N_{Ay} + N_B - F_L - W - 500 = 0$$

$$N_{Ay} + 3195.66 - 540 - 3433.5 - 500 = 0 \Rightarrow$$

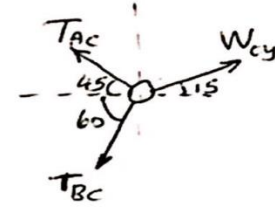
$$N_{Ay} = 1277.84 \text{ N}$$

$$= N_A$$





$$Q7) W_{cy} = mg = 30 \times 9.81 = 294.3 \text{ N}$$



$$(\rightarrow) R_x = \sum F_x = 0$$

$$W_{cy} \cos 15 - T_{AC} \cos 45 - T_{BC} \cos 60 = 0 \quad \text{--- (1)}$$

$$(\uparrow) R_y = \sum F_y = 0$$

$$W_{cy} \sin 15 + T_{AC} \sin 45 - T_{BC} \sin 60 = 0 \quad \text{--- (2)}$$

$$T_{BC} = \frac{W_{cy} \sin 15 + T_{AC} \sin 45}{\sin 60} \quad \text{--- (2) substitute into eq. (1)}$$

$$W_{cy} \cos 15 - T_{AC} \cos 45 - \left[ \frac{W_{cy} \sin 15 + T_{AC} \sin 45}{\sin 60} \right] \cos 60 = 0$$

$$W_{cy} \cos 15 - T_{AC} \cos 45 - \frac{W_{cy} \sin 15}{\tan 60} - \frac{T_{AC} \sin 45}{\tan 60} = 0$$

$$W_{cy} \cos 15 - T_{AC} \left[ \cos 45 + \frac{\sin 45}{\tan 60} \right] - \frac{W_{cy} \sin 15}{\tan 60} = 0$$

$$T_{AC} \left[ \cos 45 + \frac{\sin 45}{\tan 60} \right] = W_{cy} \cos 15 - \frac{W_{cy} \sin 15}{\tan 60}$$

$$T_{AC} (1.115) = 294.3 \cos 15 - \frac{294.3 \sin 15}{\tan 60}$$

$$T_{AC} = 215.5 \text{ N} \quad \text{--- substitute into eq. (2)}$$

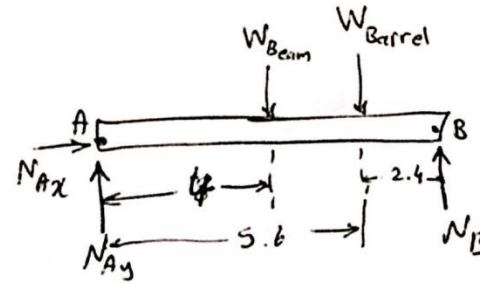
$$T_{BC} = \frac{294.3 \sin 15 + 215.5 \sin 45}{\sin 60}$$

$$T_{BC} = 263.9 \text{ N}$$

Q8/

$$\begin{aligned}W_{\text{Barrel}} &= mg \\ &= 220 \times 9.81 \\ &= 2158.2 \text{ N}\end{aligned}$$

$$\begin{aligned}W_{\text{Beam}} &= mg = 450 \times 9.81 \\ &= 4414.5 \text{ N}\end{aligned}$$



$$R_x = \sum F_x = 0, \therefore N_{Ax} = 0$$

$$\sum M_A = 0$$

$$W_{\text{Barrel}} \times 5.6 + W_{\text{Beam}} \times 4 - N_B \times 8 = 0$$

$$2158.2 \times 5.6 + 4414.5 \times 4 - N_B \times 8 = 0$$

$$N_B = 3718 \text{ N}$$

$$R_y = \sum F_y = 0$$

$$N_{Ay} + N_B - W_{\text{Beam}} - W_{\text{Barrel}} = 0$$

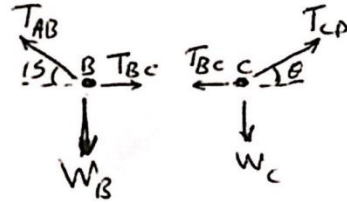
$$\begin{aligned}N_{Ay} &= W_{\text{Beam}} + W_{\text{Barrel}} - N_B \\ &= 4414.5 + 2158.2 - 3718 \\ &= 2854.7 \text{ N} = N_A\end{aligned}$$

Q9/  
for point B

$$(\uparrow) R_y = \sum F_y = 0$$

$$T_{AB} \sin 15 - W_B = 0$$

$$\therefore T_{AB} = \frac{W_B}{\sin 15} = \frac{mg}{\sin 15} = \frac{10 \times 9.81}{\sin 15} = \boxed{379 \text{ N}}$$



$$(\rightarrow) R_x = \sum F_x = 0$$

$$T_{BC} - T_{AB} \cos 15 = 0$$

$$T_{BC} = 379 \cos 15 = \boxed{366 \text{ N}}$$

for point C

$$(\rightarrow) R_x = \sum F_x = 0$$

$$-T_{BC} + T_{CD} \cos \theta = 0$$

$$T_{CD} \cos \theta = T_{BC} = 366 \text{ N} \quad \text{--- (1)}$$

$$(\uparrow) R_y = \sum F_y = 0$$

$$T_{CD} \sin \theta - W_C = 0$$

$$T_{CD} \sin \theta = W_C = m_c g = 15 \times 9.81 = 147.15 \text{ N}$$

$$T_{CD} = \frac{147.15}{\sin \theta} \quad \text{--- (2) substitute into eq. (1)}$$

$$\frac{147.15}{\sin \theta} \cos \theta = 366 \Rightarrow \tan \theta = \frac{147.15}{366} \Rightarrow \theta = 21.9^\circ$$

$$\therefore T_{CD} = \frac{147.15}{\sin 21.9} = \boxed{394.5 \text{ N}}$$