- Dynamics -

* Introduction to Dynamics :-

- Dynamics: is that branch of mechanics which deals with the motion of bodies under the action of forces.

Dynamics has two distinct parts:-

- kinematics: which is the study of motion without reference to the forces which cause motion. which deals with four quantities only are time, displacement, velocity & acceleration.

- kinetics: which is the study of motion due to the action of forces on bodies. which deals with force, mass, weight, moment of inertia.

* Newton's Laws:-

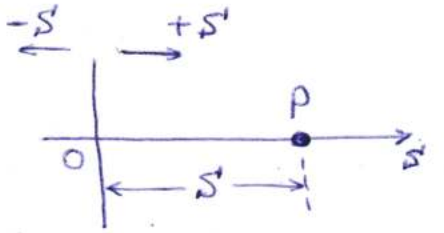
LawI : A particle remains at rest or continues to move in a straight line with a constant Velocity if there is no unbalanced force acting on it.

LawII : The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

Law III : The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

- Kinematics -

* Rectilinear motion of particles :-



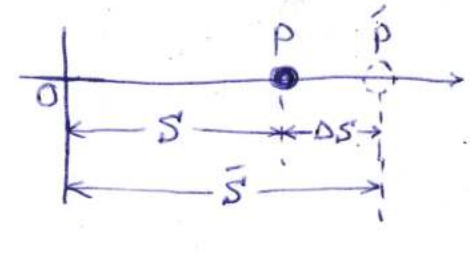
Position :

Displacement :

The displacement of a particle is defined as the change in its position.

$+\Delta S$, i.e $S' > S$

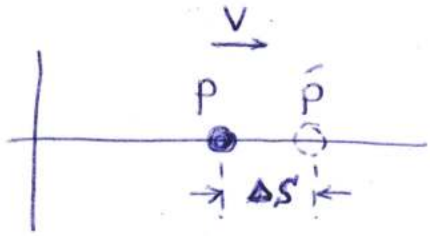
$-\Delta S$, i.e $S' < S$



S : displacement is a vector quantity

$S = f(t)$

Velocity :



The velocity of a particle is the time rate of change of its position.

The average velocity of the particle during the time interval Δt, is

$$V_{av.} = \frac{\Delta S}{\Delta t}$$

The instantaneous Velocity is

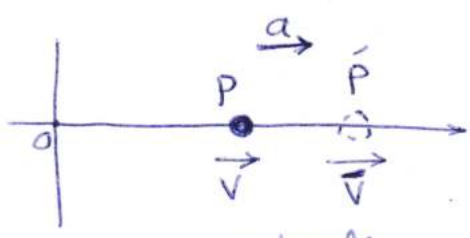
$$V = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \dot{s}$$

The velocity is a vector quantity. The sign used to define the direction of the velocity is the same as that of displacement. The magnitude of the velocity is known as the speed.

Acceleration:

The instantaneous velocities for the particle are known at the two points P and $\acute{P}$, the average acceleration for the particle during the time interval Δt is defined as:

$$a_{av.} = \frac{\Delta V}{\Delta t}$$

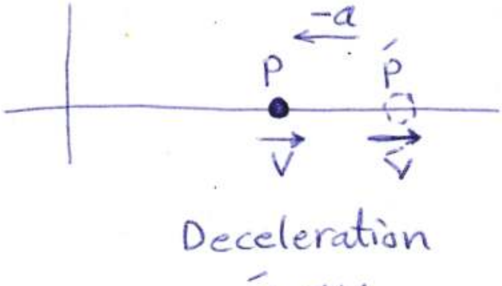


acceleration
$\acute{V} > V$

The instantaneous acceleration of time t is

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dv}{dt} = \dot{V}$$

also $$a = \dot{V} = \frac{d^2 s}{dt} = \ddot{s}$$



Deceleration
$\acute{V} < V$

Note:

- at acceleration, $\acute{V} > V$, $\Delta V = \acute{V} - V$ and a is positive.
- at deceleration, $\acute{V} < V$, $\Delta V = \acute{V} - V$ and a is negative.
- when the velocity is constant, $\acute{V} = V$, $\Delta V = 0$, the acceleration is zero.

Example(1):

The car moves in a straight line such that for a short time its velocity is defined by $V = (9t^2 + 2t)$ m/s, where t is measured in seconds. Determined its position and acceleration when t=3 sec.

Solution:

position: $V = \frac{ds}{dt}$

$(\xrightarrow{+})$

$$ds = V\,dt$$

$$\int_{S_o}^{S} ds = \int_{0}^{3} (9t^2 + 2t)\,dt$$

$$S\Big|_{S_o}^{S} = 9\,\frac{t^3}{3}\Big|_0^3 + \frac{2t^2}{2}\Big|_0^3$$

$$S - S_o = 3(3^3 - 0) + (3^2 - 0) \quad , \quad S_o = 0$$

$$S = 3 * 27 + 9$$

$\therefore S = 90$ m Ans.

acceleration:

$(\xrightarrow{+})$ $a = \frac{dv}{dt}$

$$a = \frac{dv}{dt} = \frac{d}{dt}(9t^2 + 2t) = 18t + 2$$

$$a\Big|_{t=3} = 18(3) + 2 = 56 \text{ m/s}^2$$ Ans.

Example(2):

A small projectile is fired vertically downward into a fluid medium with an initial velocity of 60 m/s. If fluid resistance causes a deceleration of the projectile which is equal to $a = -0.4V^3\ m/s^2$ where V is measured in m/s, determine both the velocity V and position S four seconds after the projectile is fired.

Solution:

Velocity: $(+\downarrow)$

$$a = \frac{dV}{dt} \Rightarrow dV = a\ dt$$

$$dV = -0.4V^3 dt$$

$$\frac{dV}{V^3} = -0.4\ dt$$

$$\int_{V_o}^{V} \frac{dV}{V^3} = -0.4 \int_{t_o}^{t} dt$$

$$\left.\frac{-1}{2V^2}\right|_{V_o}^{V} = -0.4 * t \Big|_{t_o}^{t}$$

$$-\frac{1}{2}\left(\frac{1}{V^2} - \frac{1}{V_o^2}\right) = -0.4(t - t_o) \quad --- (*)$$

at $t_o = 0$, $V_o = 60$

$$-\frac{1}{2}\left(\frac{1}{V^2} - \frac{1}{60^2}\right) = -0.4(4 - 0)$$

$$\frac{1}{V^2} = 0.8 * 4 + \frac{1}{60^2} \Rightarrow \frac{1}{V^2} = 3.2$$

$$\therefore V = \frac{1}{\sqrt{3.2}} = 0.559\ m/s \quad Ans.$$

position: $(+\downarrow)$

$$v = \frac{ds}{dt}$$

$$ds = v\,dt$$

v from eq. (*) $\Rightarrow -\frac{1}{2}\left(\frac{1}{v^2} - \frac{1}{v_0^2}\right) = -0.4\,(t - t_0)$

at $t_0 = 0$, $V_0 = 60$

$$\frac{1}{v^2} - \frac{1}{60^2} = 0.8\,t$$

$$\frac{1}{v^2} = 0.8\,t + \frac{1}{60^2} \Rightarrow V = \frac{1}{\sqrt{0.8t + \frac{1}{60^2}}}$$

$$\therefore V = \left[0.8t + \frac{1}{60^2}\right]^{-1/2}$$

$$\because ds = v\,dt$$

$$\therefore \int_{s_0}^{s} ds = \int_{t_0}^{t} \left[0.8t + \frac{1}{60^2}\right]^{-1/2} dt \quad * \frac{0.8}{0.8}$$

$$S\Big|_{s_0}^{s} = \frac{\left(0.8t + \frac{1}{60^2}\right)^{1/2}}{0.8 * 0.5}\Bigg|_{t_0}^{t}$$

$$S - S_0 = \frac{2}{0.8}\left[\left(0.8t + \frac{1}{60^2}\right)^{1/2} - \left(0.8t_0 + \frac{1}{60^2}\right)^{1/2}\right]$$

$$S_0 = 0,\ t_0 = 0,\ t = 4$$

$$\therefore S = \frac{2}{0.8}\left[\left(0.8 * 4 + \frac{1}{60^2}\right)^{1/2} - \left(\frac{1}{60^2}\right)^{1/2}\right]$$

$\therefore S = 4.43$ m   Ans.

Rectilinear Motion (Home Work)

Q1/ The motion of a Particle is defined by the relation $a = 2t$, where a is in m/s^2 and t is in seconds. It is known that $s = 4$ m and $v = 2$ m/s when $t = 1$ sec., find s and v at $t = 4$ sec.

Ans: $v = 17$ m/s , $s = 28$ m

Q2/ The rectilinear motion of a Particle is given by $s = v^2 - 9$, where s is in m and v in m/s. When $t = 0$, $s = 0$ and $v = 3$ m/s, determine the $s-t$, $v-t$ and $a-t$ relations.

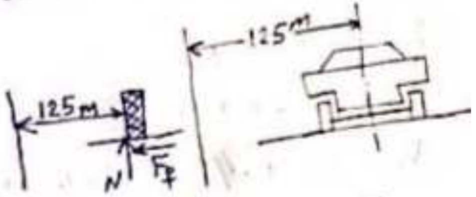
Ans: $s = 3t + \frac{1}{4}t^2$ (m)

Problems
- Kinetics -

Q1/ A Car is moving a bout a circular road with constant Velocity. The coefficient of static friction between the road and tires is 0.42 . find:

a/ velocity of the car without be its impending slip on the road A as shown in figure.

b/ velocity of the car without be its impending slip on the road B as shown in figure.



a/

$$F_n = m \cdot a_n \qquad ; F_n = F_f$$

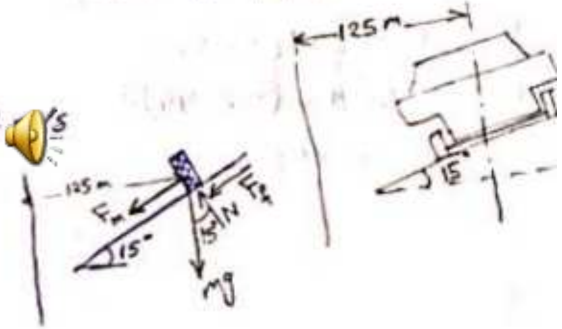
$$N\mu = m\frac{V^2}{S}$$

$$mg\mu = m\frac{V^2}{S}$$

$$V = \sqrt{Sg\mu}$$

$$= \sqrt{125 \times 9.81 \times 0.42} = 22.7 \ m/s$$

$$V = 22.7 \times \frac{3600}{1000} = 81.7 \ Km/h$$



b/

$$N = mg\cos 15$$

$$F_m = mg\sin 15$$

$$F_f = N\mu$$

$$\Sigma F = m a_n$$

$$N\mu + F_m = m a_n$$

$$mg\cos 15 \times \mu + mg\sin 15 = m\frac{V^2}{S}$$

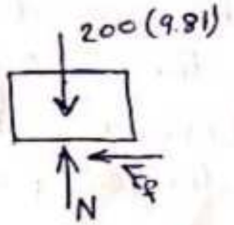
$$\mu g\cos 15 + g\sin 15 = \frac{V^2}{125}$$

$$\therefore V = 28.05 \ m/s$$

$$V = 28.05 \times \frac{3600}{1000} = 102.7 \ Km/h$$

Q2/ A vehicle is moving as shown in figure with velocity 30 Km/h. The driver used the brake to slow the vehicle with constant range to reach static state. Find minimume magnitude of total distance through the stoppage, by assuming no slip on the load box.

$m = 200\,kg \qquad \mu = 0.3$



$N = m \cdot g = 200 * 9.81$

$F_f = \mu N = 0.3 * 200 * 9.81 = 588\,N$

$\Sigma F = m \cdot a$

$-588 = 200 \cdot a \Rightarrow a = -2.94\ m/s^2$

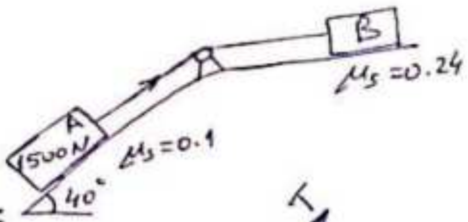
$V_o = 30\ Km/h = 8.33\ m/s$

$V^2 = V_o^2 + 2a(S - S_o)$

$0 = 8.33^2 + 2(-2.94)S$

$S = S_{min} = 11.8\ m$

Q3/ Find magnitude of the mass for the body B as shown in figure. If sistem acceleration was 2.9 m/s²

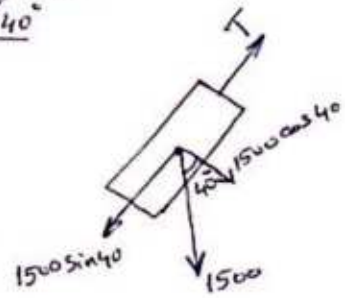


$$\Sigma F_A = m \cdot a$$

$$1500 \sin 40 - F_{f_A} - T = m \cdot a$$

$$1500 \sin 40 - 0.1 * 1500 \cos 40 - T = \frac{1500}{9.81} \cdot (2.9)$$

$$\therefore T = 405.85 \text{ N}$$



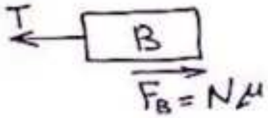
$$\Sigma F_B = m \cdot a$$

$$T - F_B = m \cdot a$$

$$T - N\mu = m \cdot a$$

$$T - mg\mu = m \cdot a$$

$$T = m(a + g\mu)$$



$$\therefore m = \frac{T}{a + g\mu} = \frac{405.85}{2.9 + 9.81 * 0.24} =$$

$$m = 77.23 \text{ Kg}$$

* Constant acceleration: $a = a_c$

$S = f(t)$, $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$, $a\, ds = v\, dv$ Kinematic equations

Velocity as a function of time:

Integrate $a = \frac{dv}{dt}$, assuming that initially $V = V_0$ at $t = 0$

$$dv = a\, dt \quad , \quad a = a_c$$

$$\int_{V_0}^{V} dv = \int_0^t a_c\, dt$$

$$\int_{V_0}^{V} dv = a_c \int_0^t dt$$

$$V\Big|_{V_0}^{V} = a_c t\Big|_0^t$$

$$(V - V_0) = a_c (t - 0)$$

$$\therefore V = V_0 + a_c t \quad \text{----} \; ①$$

Position as a function of time:

Integrate $V = \frac{ds}{dt} = V_0 + a_c t$, assuming that initially $s = s_0$ at $t = 0$

$$ds = V\, dt$$

$$\int_{s_0}^{s} ds = \int_0^t V\, dt$$

$$\int_{s_0}^{s} ds = \int_0^t (V_0 + a_c t)\, dt$$

$$\int_{s_0}^{s} ds = V_0 \int_0^t dt + a_c \int_0^t t\, dt$$

$$s\Big|_{s_0}^{s} = v_0\, t\Big|_0^t + a_c \frac{t^2}{2}\Big|_0^t$$

$$s - s_0 = v_0 t + \frac{1}{2} a_c t^2$$

$$\therefore s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \quad \text{----} \; (2)$$

Velocity as a function of position:

either solve for t in eq. 1 and substitute into eq. 2, or integral $v\,dv = a\,ds$, assuming that initially $v = v_0$ when $s = s_0$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$\frac{v^2}{2}\Big|_{v_0}^{v} = a_c\, s\Big|_{s_0}^{s}$$

$$\frac{1}{2}(v^2 - v_0^2) = a_c (s - s_0)$$

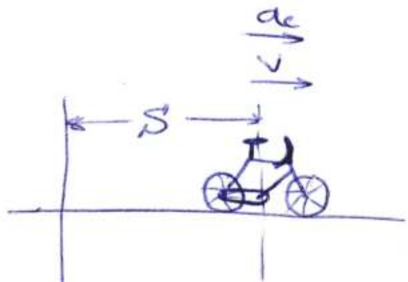
$$v^2 - v_0^2 = 2a_c (s - s_0)$$

$$\therefore v^2 = v_0^2 + 2a_c (s - s_0) \quad \text{---} \; (3)$$

The above equations are useful only when the acceleration is constant. A common example of this motion occurs when a body falls freely toward the earth, if air resistance is neglected and the distance of fall is short, the constant acceleration of the body is 9.81 m/s².

Example (3):

The bicyclist has an acceleration of 0.6 m/s^2. If he starts from rest, Determine his velocity and position at $t = 5$ sec.



Solution: $a_c = 0.6 \ m/s^2$, $V_0 = 0$

Velocity: at $t = 5$ sec.

$(\xrightarrow{+})$ $V = V_0 + a_c t$

$V = 0 + 0.6 * 5 = 3 \ m/s$ Ans.

position: at $t = 5$ sec.

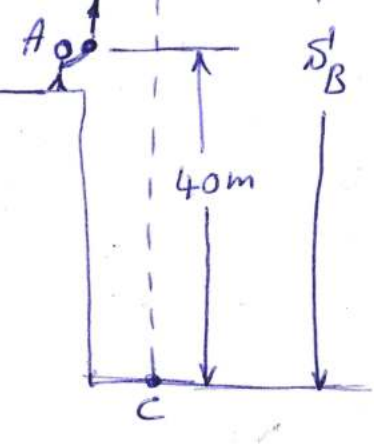
$(\xrightarrow{+})$ $S = S_0 + V_0 t + \frac{1}{2} a_c t^2$

$S = 0 + 0 * t + \frac{1}{2} * 0.6 * (5)^2$

$\therefore S = 7.5 \ m$ Ans.

Example (4):

A boy tosses a ball in the vertical direction off the side of a cliff, as shown in figure. If the initial velocity of the ball is 15 m/s upward, and the ball is released 40 m from the bottom of the cliff, determine the maximum height S_B reached by the ball and the speed of the ball just before it hits the ground. Neglect the effect of air resistance.



Solution:

maximum height: S_B

$(+\uparrow)$ $$V_B^2 = V_A^2 + 2a_c(S_B - S_A)$$

$V_B = 0$, $V_A = 15$ m/s, $a_c = -g = -9.81$ m/s^2, $S_A = 40$ m

$$\therefore \quad 0 = 15^2 + 2(-9.81)(S_B - 40)$$

$$\therefore S_B = 51.5 \text{ m}$$

speed at C:

$(+\uparrow)$ $V_c^2 = V_B^2 + 2a_c(S_c - S_B)$, or $\left[V_c^2 = V_A^2 + 2a_c(S_c - S_A)\right]$

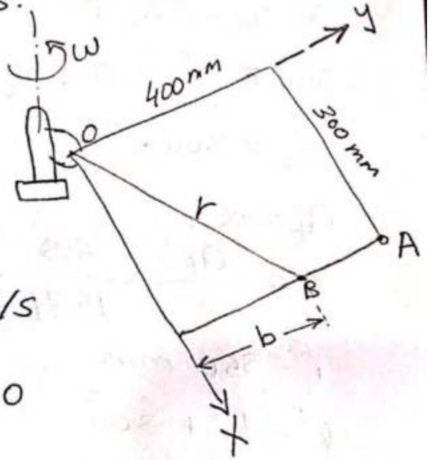
$V_c^2 = (0) + 2(-9.81)(0 - 51.5)$

$\therefore V_c = -31.8$ m/s (negative sign represent opposite direction of the velocity)

or $V_c = 31.8$ m/s $\downarrow$ (downward)

Rotation about a fixed Axis

Q1/ The rectangular plate is rotating about its corner axis through o with a constant angular velocity $\omega = 10$ rad/s. Determine the magnitudes of the Velocity (V) and acceleration (a) of the corner A by using the scalar relations.



$$V = \omega . r$$

$$r = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$= 0.5 \text{ m}$$

$$V = \omega . r = \frac{10}{2\pi} * 0.5 = 0.796 \text{ m/s}$$

$$a_t = \alpha . r \quad , \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}(10) = 0$$

$$\therefore a_t = 0 \Rightarrow a = a_n$$

$$a_n = \omega^2 . r = 0.5 * \left(\frac{\omega}{2\pi}\right)^2 = 1.266 \text{ m/s}^2$$

Q2/ If the rectangular plate of prob(1) starts from rest and point B has an initial acceleration of $5.5\ m/s^2$. determine the distance b if the plate reaches an angular speed of 300 rev/min in 2 seconds with a constant angular acceleration.

$$\omega = \omega_0 + \alpha_c \cdot t$$

$$300 * \frac{2\pi}{60} = 0 + \alpha_c \cdot (2)$$

$$\therefore \alpha_c = 300 * \frac{2\pi}{60} \cdot \frac{1}{2} = 15.71\ rad/s^2$$

$$a_t = \alpha \cdot r$$

$$r = \frac{a_t}{\alpha} = \frac{5.5}{15.71} = 0.350\ m$$

$$r = 350\ mm$$

$$r^2 = b^2 + 300^2$$

$$\therefore b = \sqrt{r^2 - 300^2}$$

$$b = \sqrt{350^2 - 300^2}$$

$$\therefore b = 180.277\ mm$$

Q3/ The angular position of a radial line in a rotating disk is given by the clockwise angle $\theta = 2t^3 - 3t^2 + 4$, where θ is in radians and t is in seconds. Calculate the angular displacement $\Delta\theta$ of the disk during the interval in which its angular acceleration increases from $42 \ rad/s^2$ to $66 \ rad/s^2$.

$$\theta = 2t^3 - 3t^2 + 4 \quad \text{----} \ (1)$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 6t \quad \text{----} \ (2)$$

$$\alpha = \frac{d\omega}{dt} = 12t - 6 \quad \text{----} \ (3)$$

- where $\alpha_1 = 42 \ rad/s^2$

$42 = 12t - 6$

$48 = 12t \Rightarrow t_1 = 4 \ sec$

subs. $t_1 = 4$ into (1) :-

$\theta_1 = 2(4)^3 - 3(4)^2 + 4$

$\theta_1 = 84 \ rad.$

- where $\alpha_2 = 66 \ rad/s^2$

$66 = 12t - 6$

$72 = 12t \Rightarrow t_2 = 6 \ sec$

sub. $t_2 = 6$ into (1) :-

$\theta_2 = 2(6)^3 - 3(6)^2 + 4$

$\theta_2 = 328 \ rad.$

$\therefore \Delta\theta = \theta_2 - \theta_1$

$= 328 - 84 = 244 \ rad.$

Problem
Curvilinear motion
Normal and tangential components

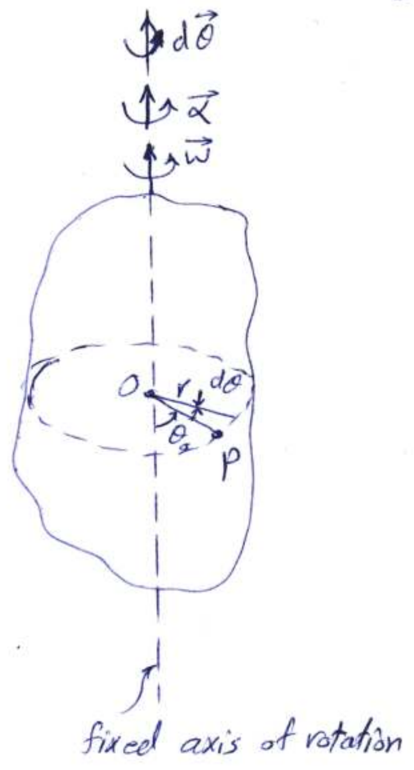
Q 1/ A boat is traveling along a circular path having a radius of 20m .Determine the magnitude of the boat's acceleration if at a given instant the boat 's speed is $V = 5m/s$ and the rate of increase in speed is $\dot{V} = 2m/s^2$. Ans: $a = 2.36\ m/s^2$

Q2/ A train travels along a horizontal circular curv ,that has a radius of 200 m .If the Speed of the train is uniformly increased from 30 km/h to 45 km/h in 5 sec., Determine the magnitude of the acceleration at the instant the speed of the train is 40 km/h . Ans: $a = 1.037\ m/s^2$.

Q3/ an automobile is traveling with a constant speed along a horizontal circular curv that has a radius of $\rho = 750$ m if the magnitude of acceleration is $a = 8m/s^2$,determine the speed at which the automobile istraveling . Ans: $V = 77.46$ m/s .

Q4/ starting from rest ,the motorboat travels around the circular path , $\rho = 60$ m ,at a speed of $V = 0.8t$ m/s, where t is measured in seconds. Determine the magnitudes of the boat 's velocity and acceleration just after it has traveled 20 m .

* Rotation about a fixed axis:



- Angular position: θ_o
- Angular displacement: $d\vec{\theta}$ or $\vec{\theta}$
- Angular velocity: (ω)

$$\omega = \frac{d\theta}{dt} \quad \text{---} \; (1)$$

- Angular acceleration: (α)

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \quad \text{---} \; (2)$$

also $\alpha = \ddot{\theta} = \dfrac{d^2\theta}{dt^2}$

by eliminating dt from eq(1) and eq(2), we obtain:

$$\alpha \, d\theta = \omega \, d\omega$$

The similarity between the differential relations for angular motion and for rectilinear motion of a particle ($v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$, $a\,ds = v\,dv$).

* Constant angular acceleration: $\alpha = \alpha_c$

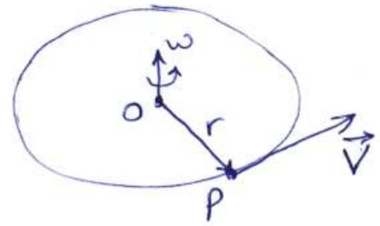
we derive equations for constant angular acceleration same way those equations derived for constant rectilinear acceleration, therfore we'get:

$$\omega = \omega_o + \alpha_c t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha_c t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha_c(\theta - \theta_o)$$

* Motion of point P:



Position: It is defined by the position vector $\vec{r}$

Velocity:

$$v = \omega r$$

acceleration:

the acceleration of P will be expressed in terms of its normal and tangential components.

$$\therefore \quad a_n = \frac{v^2}{r} = \omega^2 r$$

$$a_t = \dot{v} = \frac{dv}{dt}$$

$$\because \quad v = \omega r$$

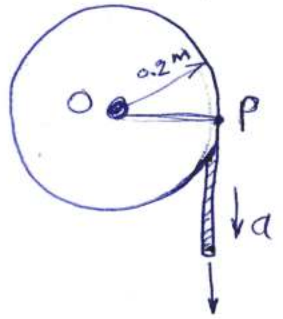
$$\frac{dv}{dt} = \frac{d\omega}{dt} r = \alpha r$$

$$\therefore \quad a_t = \alpha r$$

also may be use polar coordinates to find a_r and a_θ.

Example (1):

A cord is wrapped around a wheel which is initially at rest as shown in figure. If a force is applied to the cord and gives it an acceleration of $a = 4t$ m/s^2, where t is in seconds, determine as a function of time (a) the angular velocity of the wheel, and (b) the angular position in radians.



Solution:

(a) $\circlearrowright +$ $\quad a_t = \alpha r$

$4t = \alpha (0.2) \Rightarrow \alpha = 20t$ rad/s^2

$\circlearrowright +$ $\quad \alpha = \frac{d\omega}{dt} = 20t$

$\int_0^{\omega} d\omega = \int_0^{t} 20t \, dt$ $\quad$ where: $\omega_0 = 0$ at $t_0 = 0$

$\omega = 10t^2$ rad/s

(b) $\circlearrowright +$ $\quad \omega = \frac{d\theta}{dt}$

$d\theta = \omega \, dt$

$\int_0^{\theta} d\theta = \int_0^{t} 10t^2 \, dt$ $\quad$ where: $\theta_0 = 0$ at $t_0 = 0$

$\theta = \frac{10}{3} t^3$ rad

Example (2):

A wheel's angular speed is increased uniformly from zero to 1200 rpm in 3 sec. Determine (a) angular acceleration of the wheel, and (b) the number of revolutions the wheel executes in 3 s.

Solution:

(a) $\omega = \omega_o + \alpha_c t$

$1200 * \frac{2\pi}{60} = 0 + \alpha_c (3)$

$\therefore \alpha_c = 41.9 \ rad/s^2$

(b) $\omega^2 = \omega_o^2 + 2\alpha_c(\theta - \theta_o)$

or, $\theta = \cancel{\theta_o}^{0} + \cancel{\omega_o t}^{0} + \frac{1}{2}\alpha_c t^2$

$\therefore \theta = \frac{1}{2}\alpha_c t^2 = \frac{1}{2}(41.9)(3)^2$

$\therefore \theta = 188.55 \ rad * \frac{1}{2\pi} = 30 \ rev.$