- Mechanics -

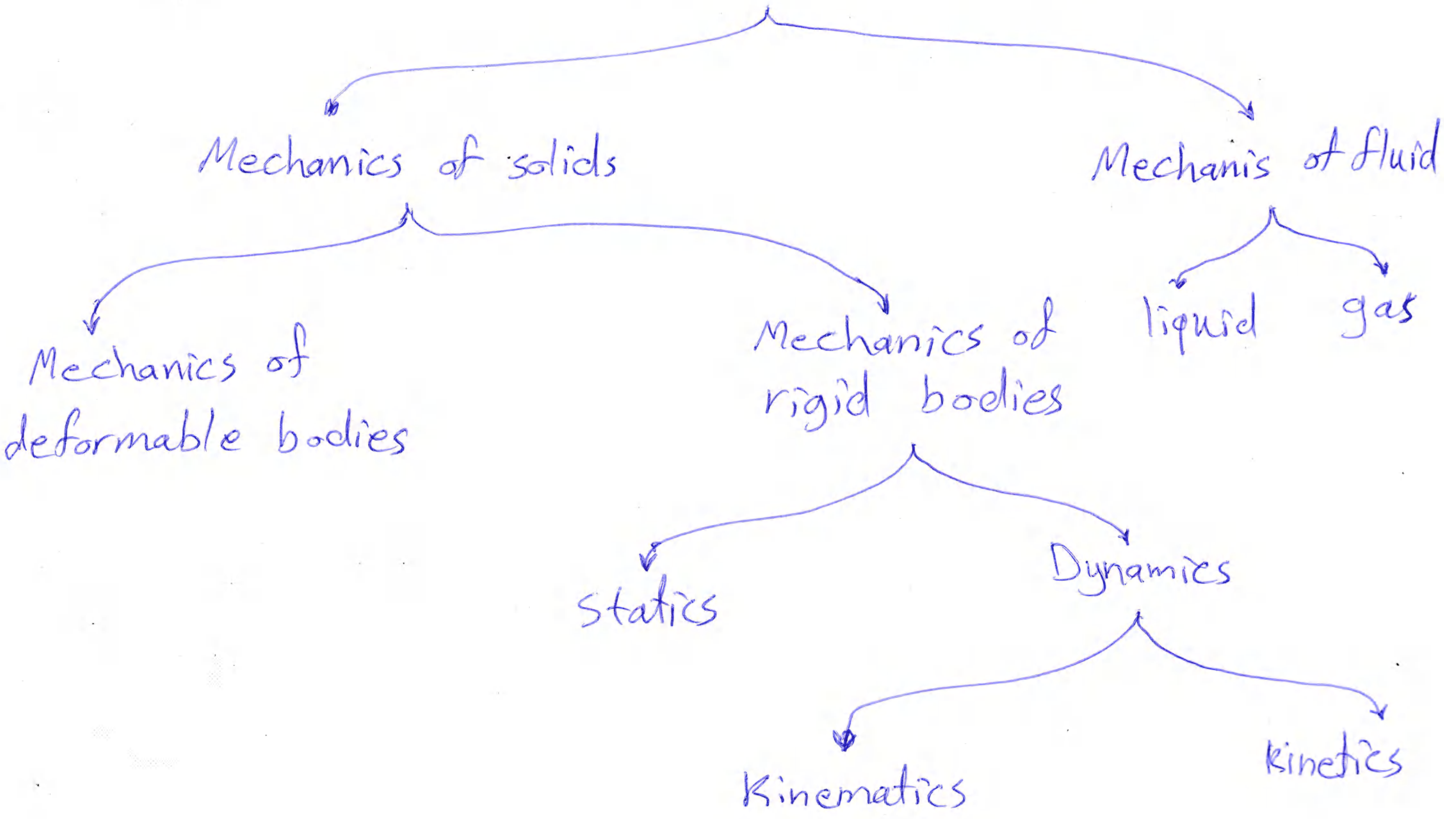
* Introduction:

Mechanics is that branch of physical science which deals with the state of rest or motion of bodies under the action of forces.

The subject of mechanics is logically divided into two parts:

1. Statics, which concerns the equilibrium of bodies under the action of forces.
2. Dynamics, which concerns the motion of bodies.

- Engineering Mechanics -



* Basic concepts:

- Space: is the geometric region occupied by bodies whose positions described by linear and angular measurments relative to a coordinate system. for two dimentional problems only two coordinate will be required.

- Time: is the measure of succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

- Mass: is a measure of the inertia of a body, which is its resistance to a change of Velocity.

- Force: is the action of one body on another. A force tends to move a body in the direction of its action. the action of a force is characterized by its magnitude, by the direction of its action, and by its point of application. force is a vector quantity.

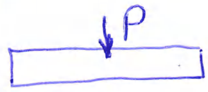
The action of a force on a body can be seperated into two effects:

1. external effects 2. Internal effects

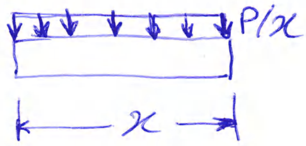
forces are classified as either contact or body forces. Contact forces are generated through direct physical contact between two bodies. Body forces are those applied by remote action such as gravitational and magnetic forces.

Also, contact forces are classified into two Parts:

1. concentrated forces.



2. distributed forces.

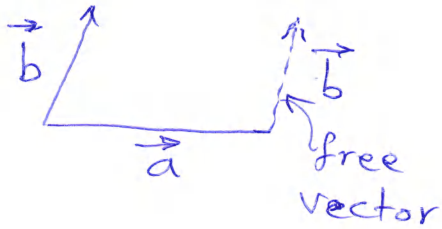


- Particle: A body of negligible dimensions is called a particle.

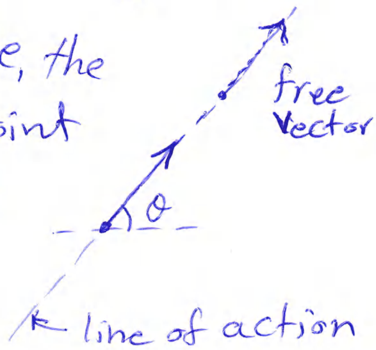
- Rigid body: A body is considered rigid when the relative movements between its parts are negligible.

* Types of vectors:

1. Free vector: is one whose action is not confined or associated with a unique line in space. for example, represent the displacement of a body by a free vector.



2. Sliding vector: is one for which a unique line in space must be maintained along which the quantity acts, for example, the force may be applied at any point along its line of action without changing its effect on the body

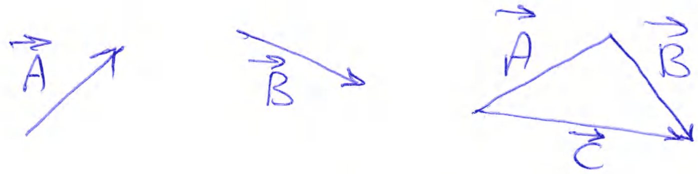


3. Fixed vector: is one for which a unique point of application is specified, therefore, a vector occupies a particular position in space. for example, the action of a force on a deformable or nonrigid body must be specified by a fixed Vector at the point of application of the forces.

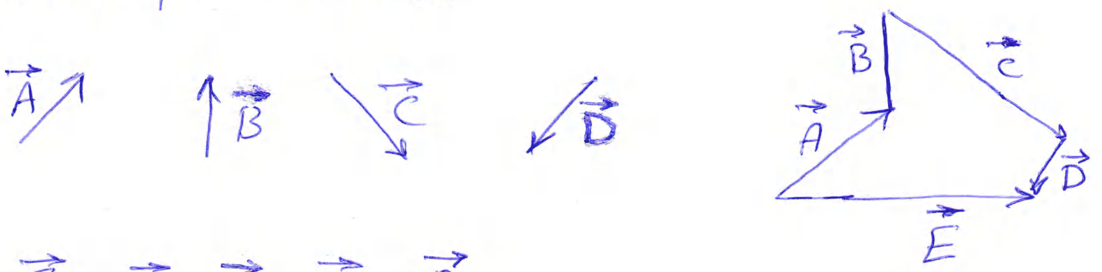
* Addition of vectors:

assume $\vec{A}$ and $\vec{B}$, two vectors and $\vec{C}$ is addition of $\vec{A}$ and $\vec{B}$

$\therefore \vec{C} = \vec{A} + \vec{B}$



$\vec{C}$ equivalent vector (Resultant) of $\vec{A}$ and $\vec{B}$,
If we have more of two vectors, sum it in the same procedure.



$\therefore \vec{E} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

we use several methods to find the addition of vectors

1. Graphical Method:

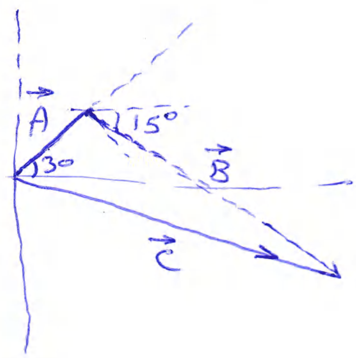
Use scale, such as 1 cm = 10 N and measuring the length of the $\vec{C}$

example: find $\vec{C} = \vec{A} + \vec{B}$

use scale: 1 cm = 10 N

$\vec{A} = 0.5$ cm

$\vec{B} = 1.5$ cm



calculate the lengthe of $\vec{C}$

on graph, $\vec{C} =$ cm $\Rightarrow \vec{C} =$ N

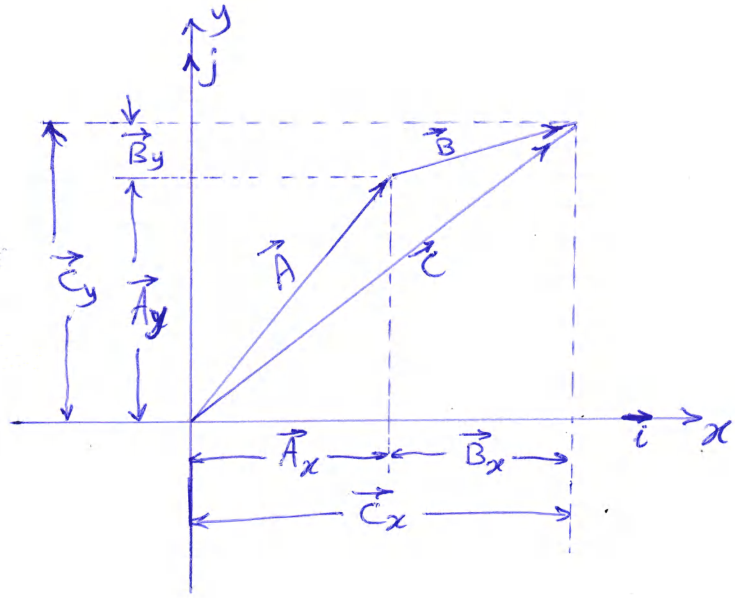
2. Analytical Method:

1. Use Vectors:

$\vec{C}$ is resultant of adding $\vec{A}$ and $\vec{B}$

noted the resultant by $\vec{R}$

$\therefore \vec{R} = \vec{C}$



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

for any vector $\vec{V}$ has magnitude $|\vec{V}|$ and direction $\vec{U}_{\vec{V}}$

$$\therefore \vec{V} = |\vec{V}|\vec{U}_{\vec{V}}$$

$$\therefore \vec{A} = |\vec{A}_x|\, i + |\vec{A}_y|\, j$$

$$= A_x i + A_y j$$

also, $\vec{B} = \vec{B}_x + \vec{B}_y \Rightarrow \vec{B} = B_x i + B_y j$

Now, $\vec{R}_x = \vec{C}_x = \vec{A}_x + \vec{B}_x$

$$= A_x i + B_x i$$

$$= (A_x + B_x) i = R_x i$$

$$\vec{R}_y = \vec{C}_y = \vec{A}_y + \vec{B}_y$$

$$= A_y j + B_y j$$

$$= (A_y + B_y) j$$

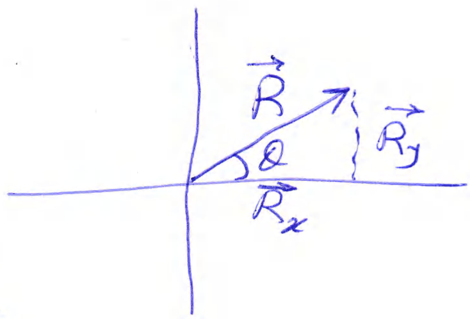
$$= R_y j$$

$$\therefore \vec{R} = \vec{R}_x + \vec{R}_y = R_x i + R_y j$$

Also, the magnitude of

$$|\vec{R}| = R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta = \frac{|\vec{R}_y|}{|\vec{R}_x|} = \frac{R_y}{R_x}$$



$$R_x = R\cos\theta \quad , \quad R_y = R\sin\theta$$

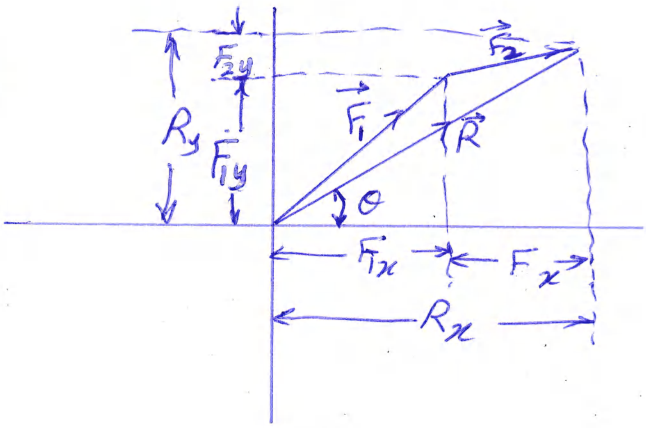
2. Algebrical components summation:

$$R_x = \Sigma F_x = F_{1x} + F_{2x} + \cdots$$

$$R_y = \Sigma F_y = F_{1y} + F_{2y} + \cdots$$

$$R = \sqrt{R_x^2 + R_y^2}$$

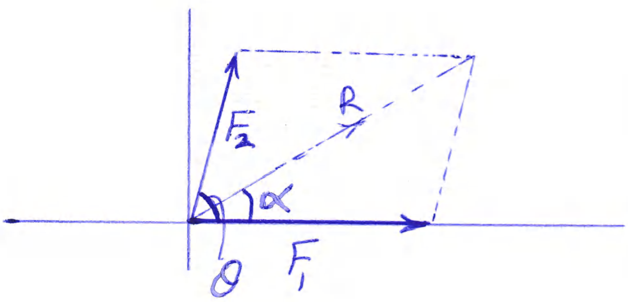
$$\theta = \tan^{-1}\frac{R_y}{R_x}$$



3. Parallelogram law:

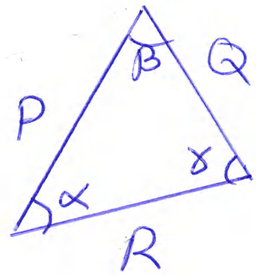
$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \quad \text{---(cosine Law)}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

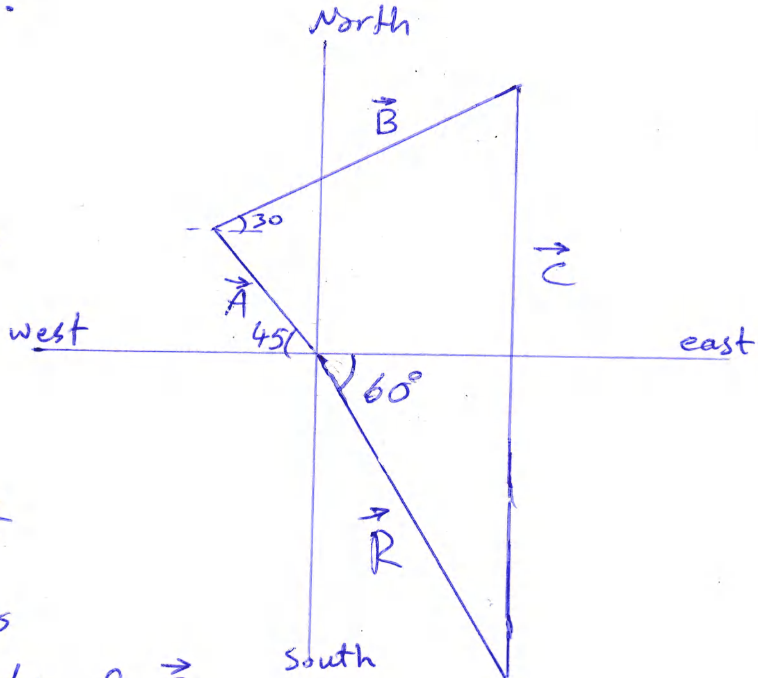


4. Triangle law :

$$\frac{P}{\sin \gamma} = \frac{Q}{\sin \alpha} = \frac{R}{\sin \beta} \quad \text{---(sine law)}$$



Example: Find equivalent displacement by use (a) Graphical Method, (b) Analytical Method for $\vec{A} = 10\ m$ in west northern direction and $\vec{B} = 20m$ in east northern direction with $30°$ and $\vec{C} = 35\ m$ in the south direction.



(a) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

Use scale $1\ cm = 5\ m$

$\vec{A} = 2\ cm$

$\vec{B} = 4\ cm$

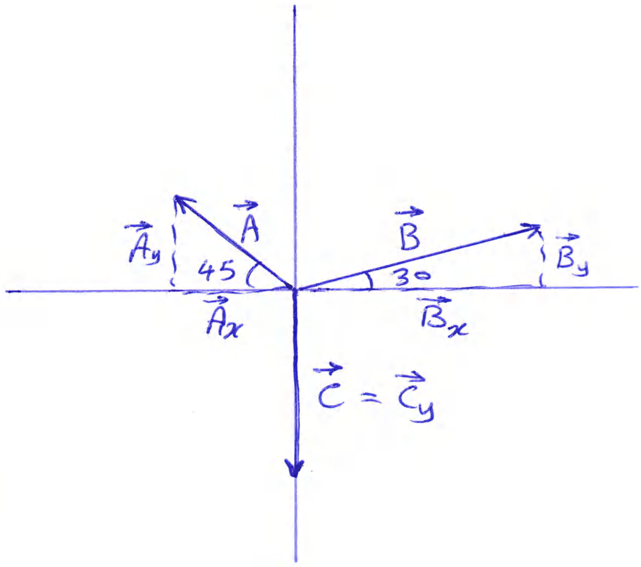
$\vec{C} = 7\ cm$

measuring equivalent displacement $\vec{R}$ in Vectors Polygon, we find magnitude of $\vec{R}$ equal $4.1\ cm$

$\therefore$ equivalent displacement $= 5 \times 4.1 = 20.5\ m$ and in direction $60°$ east southern

(b) Analytical Method

Use vectors



$\vec{A} = A_x \hat{i} + A_y \hat{j}$

$= -10 \cos 45 \hat{i} + 10 \sin 45 \hat{j}$

$\vec{B} = B_x \hat{i} + B_y \hat{j}$

$= 20 \cos 30 \hat{i} + 20 \sin 30 \hat{j}$

$\vec{C} = -35 \hat{j}$

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$= -10 \cos 45 \hat{i} + 10 \sin 45 \hat{j} + 20 \cos 30 \hat{i} + 20 \sin 30 \hat{j} - 35 \hat{j}$

$= (-10 \cos 45 + 20 \cos 30) \hat{i} + (10 \sin 45 + 20 \sin 30 - 35) \hat{j}$

$= 10.25 \hat{i} - 17.93 \hat{j}$

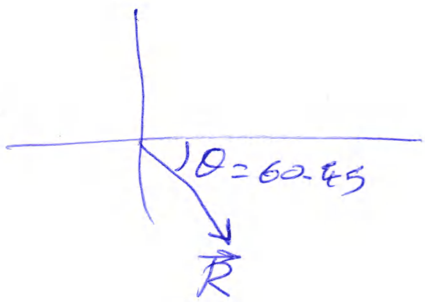
$\vec{R} = R_x \hat{i} + R_y \hat{j}$

$R_x = 10.25$, $R_y = -17.93$

$R = |\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(10.25)^2 + (-17.93)^2} = 20.65$ cm

and the direction of $\vec{R}$ is:

$\theta^\circ = \tan^{-1}\left(\frac{R_y}{R_x}\right) \Rightarrow \theta = \tan^{-1} \frac{-17.93}{10.25} = -60.45^\circ$



~~or~~ Also use algebrical components summation

$$R_x = A_x + B_x + C_x$$

$$= -10 \cos 45 + 20 \cos 30$$

$$= 10.25$$

$$R_y = A_y + B_y + C_y$$

$$= 10 \sin 45 + 20 \sin 30 - 35$$

$$= -17.93$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

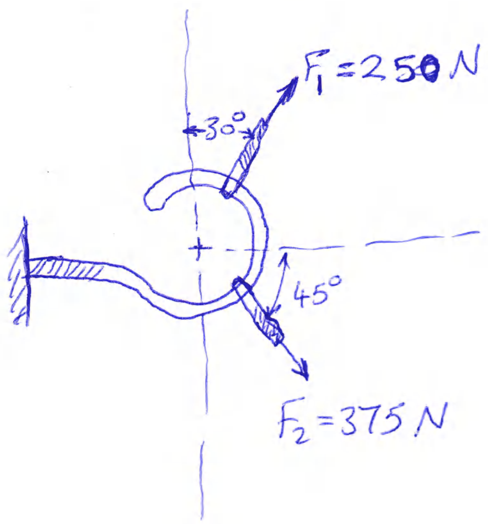
$$= \sqrt{(10.25)^2 + (-17.93)^2}$$

$$= 20.65 \text{ cm}$$

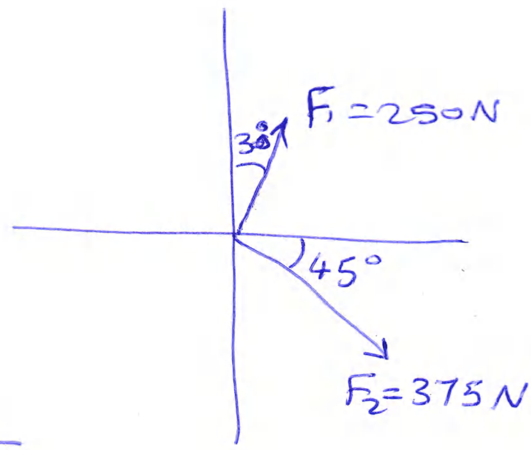
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1} \frac{-17.93}{10.25} = -60.45^\circ$$

Example:

Determine the magnitude of the resultant force and its direction



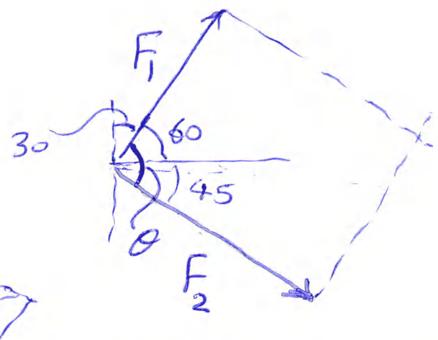
Use Parallelogram law:



$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$$

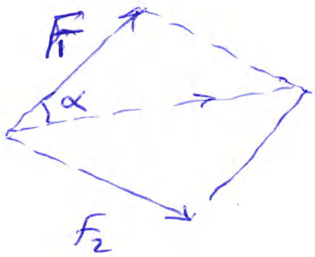
$$\therefore R = \sqrt{(250)^2 + (375)^2 + 2(250)(375)\cos(105)}$$

$$= 393.18 \text{ N}$$



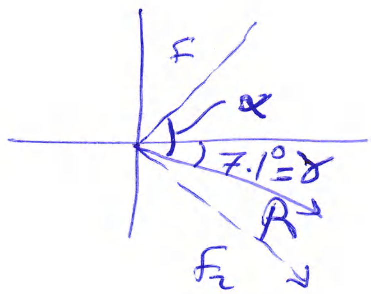
$\theta = 60 + 45 = 105°$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$



$$= \frac{375\sin(105)}{250 + 375\cos(105)}$$

$$\alpha = 67.105°$$



$$\gamma = \alpha - 60 = 67.105 - 60 = 7.105°$$