

Definitions: and Examples.

في مجموعة تعمل على الفضاء

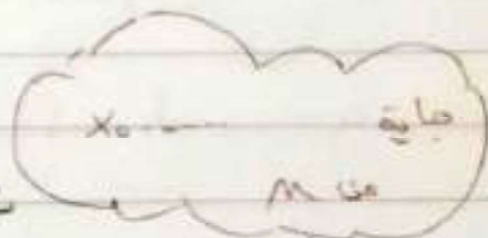
Definition: A dynamical system is a semi-group G .

acting on a space M i.e there is

$$T: G \times M \rightarrow M \text{ such that}$$

$$T_n \circ T_g = T_{ng}$$

نظام ديناميكي



Remark 1- (1) $G = \mathbb{Z}$ or $\mathbb{N}$ Then we have (Discrete dynamical system)

or $G = \mathbb{R}$ then we have (continuous dynamical system)

$$G \neq \emptyset$$

$$T: x_0 \rightarrow x_0$$

$$x_0 \in M$$

$$*: G \times G \rightarrow a + b \in G$$

$$\downarrow \quad \downarrow$$

$$a$$

$$b$$

x_0 في بداية الحركة

$$T x_0 \rightarrow x_1$$

$$T(T x_0) = T x_1 = x_2$$

$$T(T(T x_0)) = T x_2 = x_3$$

نموذج رياضي للنظام الديناميكي

(2) A mathematical model for the dynamical system

يمكن ان يكون غير قادر على ذلك على الإطلاق

(D.D.S) can be constructed by iterative

Def: Let $f: X \rightarrow X$ be a map.

Let $x_0 \in X$ نقطة في الفضاء
 $f(x_0) = f^1(x_0)$ is called the first iteration of x_0

$f \circ f(x_0) = f^2(x_0)$ is called the second iteration of x_0 .

$f \circ f \circ f(x_0) = f^3(x_0)$ is called the Third iteration of x_0

In general $f^n(x_0) = \underbrace{f \circ f \circ \dots \circ f}_{n\text{-times}}$ is called the n -th iteration of x_0 .

Definition: A set of points

$\{x_0, f(x_0), f^2(x_0), f^3(x_0), \dots\}$ is called the

orbit of x_0 . The Forward orbit نقطة في الفضاء
 f^i exists

Remark: if f is homeomorphism

$O^-(x_0) = \{x_0, f^{-1}(x_0), f^{-2}(x_0), \dots\}$ (backward orbit) نقطة في الفضاء

$O(x) = \{\dots, f^{-2}(x_0), f^{-1}(x_0), x_0, f(x_0), f^2(x_0), \dots\}$

the whole orbit

Example 1 - $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x$

$$O\left(\frac{1}{2}\right) = \left\{ \frac{1}{2}, 1, 2, 4, 8, \dots \right\} \quad \underline{\text{سلسلة}}$$

$$O(0) = \left\{ \underset{x_0}{0}, \underset{f}{0}, \underset{f^2}{0}, \dots \right\} \quad \text{نقطة ثابتة}$$

$$O\left(\frac{1}{4}\right) = \left\{ \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \dots \right\}$$

$$O(50) = \{ 50, 100, 200, 400, \dots \}$$

$$O(-1) = \{ -1, -2, -4, -8, \dots \}$$

Ex 2 Find the orbit is of

النقاط الثابتة

$f(p) = p$ ولذا القايض

$$x_0 = 3 \quad x_0 = -5 \quad x_0 = 10^{16} \quad f(x) = 2x$$

Solution

Defn- let $f: X \rightarrow X$ be a map

نقطة ثابتة

A point p is called fixed point

if $f(p) = p$

هذه النقطة لها تأثير كبير

نقطة دورية

A point $p \in X$ is called periodic point if $\exists n \in \mathbb{N}$

$$f^n(p) = p$$

$f_n = 1$, p is fixed point

نقطة ثابتة

Example 1- $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$

The fixed points of f are $x=0$ and $x=1$

$$f(0) = 0 \text{ and } f(1) = 1$$

Example 2- $f(x) = -x$ 0 is the fixed point of f

$$f(-1) = -(-1) = 1$$

$$-(-1)^2 = -1$$

$$f(-1) = f(1) = -1$$

دور 2

$\therefore$ a period point for f

$\forall p = -x \in \mathbb{R}$ is a p.p

$$P(P) = P \quad \text{النقطة الثابتة}$$

نقطة ثابتة

Q Find the L.P. for $P(x) = x^2$

$$P(x) = x \quad \text{نقطة}$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \text{ or } x = 1$$

Ex

$$P(x) = x^5 + 3x + 5$$

$$x^5 + 3x + 5 - x = 0$$

$$x^5 + 2x + 5 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

قانون المستورد