قابلة للاشتقاق

Example:- $f: R \to R$ f is differentiable

Consider $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $n = 0, 1, 2, \ldots$ طريقة نيوتن

Let $g(x) = x - \frac{f(x)}{f'(x)}$, صيغة نيوتن

$x_{n+1} = g(x_n) \Rightarrow x_1 = g(x_0)$. $x_2 = g(x_1)$ ----

$o(x_0) = \{x_0, x_1, x_2 \ldots\}$

$= \{x_0, g(x_0), g'(x_0) \ldots\}$

نيوتن رابسون

The formula (*) is called Newton-Raphson method for finding the approximated zeros of f. This for gives ar dynamical system discrete

مجزأ

Example:- $f: I \to R$, $I = [0,1]$, $f(x) = \lambda x(1-x)$

(logistic map)

Find The Fixed Points for f.

$f(x) = x$

$\lambda x(1-x) = x \Rightarrow \lambda x - \lambda x^2 - x = 0 \Rightarrow \lambda x - x - \lambda x^2 = 0$

$x(\lambda - 1) - \lambda x^2 = 0$

$x((\lambda-1)-\lambda x)=0$

او $x=0$

او $(\lambda-1)-\lambda x=0 \Rightarrow \lambda x=\lambda-1$

$\Rightarrow x=\frac{\lambda-1}{\lambda} \Rightarrow x=1-\frac{1}{\lambda}$

(2) Examine The typas of $x=0$

$f(x)=\lambda x-\lambda x^2$

$\bar{f}(x)=\lambda-2\lambda x$

نعوض الصفر في م.

$\bar{f}(0)=\lambda$

جاذبة

There fore if $|\lambda|<1$ Then $x=0$ is attracting fixed point

طاردة

if $|\lambda|>1$ Then $x=0$ is repelling fixed point

3- Examina The Tyes of $x=1-\frac{1}{\lambda}=\frac{\lambda-1}{\lambda}$

$\bar{f}\left(\frac{\lambda-1}{\lambda}\right)=\lambda-2\lambda\left(\frac{\lambda-1}{\lambda}\right)$

$=\lambda-2\lambda+2=2-\lambda$

دنكون طاردة او جاذبة على حسابنة

if $|2-\lambda|<1 \Rightarrow -1<2-\lambda<1$ بالطرفين

$-3 < -\lambda < -1$

$3 > \lambda > 1$

$\therefore x = \frac{\lambda - 1}{\lambda}$ is attracting whenever $3 > \lambda > 1$

$\lambda \in (1) \Rightarrow \frac{2-1}{2} = \left(\frac{1}{2}\right)$ orbit

if $|2-\lambda| > 1 \Rightarrow -1 > 2-\lambda$, $2-\lambda > 1$

$\Rightarrow -3 > -\lambda$, $-\lambda > -1$

$\Rightarrow \lambda > 3$, $\lambda < 1$ $\Rightarrow$

$x = \frac{\lambda - 1}{\lambda}$ is repelling whenever $\lambda > 3$

How Find the orbit of (0.2) $f(x) = \lambda x(1-x)$

for $\lambda = 2$, $\lambda = 3$, $\lambda = 4$

Graphical analysis of orbits :- تحليل الهندسي

1) chose a point on the line $y=x$, say (P,P)
اختيار نقطة على المستقيم — احداثيات نقطة

2) A vertical line from (P,P) meets the graph of f at $(P, f(P))$
خط رأسي — المستقيم الرأسي يلتقي بالرسم البياني

3) The horizontal line from $(P, f(P))$ meets the line $y=x$ at $(f(P), f(P))$

4) Repeat the procedure
تكرار الإجراءات

$\forall \varepsilon > 0 \; \exists K \in \mathbb{N}$ s.t
$|x_n - x_0| < \varepsilon \quad \forall n > K$

تعريف متتابعة تقترب من النقطة p

الاستمرارية لدالة وتابعة لمشتقات

Proposition:- Let $f: \mathbb{R} \to \mathbb{R}$ be a map, $f \in C[\mathbb{R}]$

الدالة مستمرة على مجموعة

$f(p) = p$. $|f'(p)| < 1$.

فإن الدالة

او مشتقتها الأولى

then there exists open interval U, $p \in U$

and $f^n(x) \to p \quad \forall x \in U$

نبدأ الإثبات برياضيات

proof:- let $t \in C^1[\mathbb{R}]$

let $\varepsilon > 0$

since f is continuous at p . $|f'(p)| < 1$

$\therefore \exists$ an interval U such that

$|f'(x)| < 1 \quad \forall x \in U$ let $x \in U$

نعتمد على برهنة النتيجة

المتوسطة تعريفها

f is continuous $[a,b]$

f is diff (a,b)

$\exists c \in (a,b)$

$|f'(c)| = \dfrac{|f(b) - f(a)|}{|b-a|}$

$|f(x) - p| = |f(x) - f(p)|$

$= f'(c) \, |x - p|$

for some $c \in (p-\varepsilon, p+\varepsilon)$

$< 1 \cdot |x-p| < |x-p| < \varepsilon \quad \forall x \in (p-\varepsilon, p+\varepsilon)$

similarly, we can show that

$|\tilde{f}(x) - p| < \epsilon \quad \forall x \in (p-\epsilon, p+\epsilon)$

$\therefore \tilde{f}(x) \longrightarrow p \quad \forall x \in (p-\epsilon, p+\epsilon)$

Example: Def:- Let $f: X \to X$ A subset

$B \subset X$ is called the Basin of attraction of p.

denoted be $B(p)$ if $\forall x \in B(p)$

$\tilde{f}^n(x) \longrightarrow p$ that is

$B(p) = \{ x \in X : \tilde{f}(x) \longrightarrow p \}$

Example:- $f: R \to R$, $f(x) = x^2$

(1) $f(x) = x \Rightarrow x^2 = x \Rightarrow x^2 - x = 0$

$x(x-1) = 0 \Rightarrow \boxed{x=0}, \boxed{x=1}$

(2) $f(x) = ex \Rightarrow |f'(0)| = 0 < 1 \Rightarrow x=0$ is attracting

$|f'(1)| = 2 > 1 \Rightarrow 2$ is repelling

$|f'(p)| < 1$ اذا كانت جاذبة

$B(p) = \{ x \; f^n(x) \to p \}$ تنجذب نحو النقطة

$\forall \epsilon > 0 \; , \; \exists n > k \; , \; |f^n(x) - p| < \epsilon$

حتى نجد هو هل الكلام يلزم يدخل في الحساب

$\forall$ تابعة $f^n(x) \to 0 \; , \; x^{2^n} \to 0$

$|x^{2^n} - 0| < \epsilon$

3. Let $\epsilon > 0$ then $\exists k \; , \; \frac{1}{k} < \epsilon$

$|x^{2^n} - 0| = |x^{2^n}| = |x|^{2^n}$

if $|x| < 1$, Then $|x|^{2^n} < \frac{1}{k} < \epsilon$

$\therefore |x^{2^n} - 0| < \epsilon$

$\because f^n(x) = x^{2^n} \to 0$

$\therefore B(0) = \{ x : -1 < x < 1 \}$

$= (-1, 1)$

اجب النقاط التالية

س٦٧

$f(x) = 2x(1-x)$