and since, for large samples, X_1 and X_2, and hence also their difference, can be approximated with normal distributions, it follows that

$$Z = \frac{(\hat{\Theta}_1 - \hat{\Theta}_2) - (\theta_1 - \theta_2)}{\sqrt{\dfrac{\theta_1(1-\theta_1)}{n_1} + \dfrac{\theta_2(1-\theta_2)}{n_2}}}$$

is a random variable having approximately the standard normal distribution. Substituting this expression for Z into $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1-\alpha$, we arrive at the following result.

THEOREM 8. If X_1 is a binomial random variable with the parameters n_1 and θ_1, X_2 is a binomial random variable with the parameters n_2 and θ_2, n_1 and n_2 are large, and $\hat{\theta}_1 = \dfrac{x_1}{n_1}$ and $\hat{\theta}_2 = \dfrac{x_2}{n_2}$, then

$$(\hat{\theta}_1 - \hat{\theta}_2) - z_{\alpha/2} \cdot \sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}} < \theta_1 - \theta_2$$

$$< (\hat{\theta}_1 - \hat{\theta}_2) + z_{\alpha/2} \cdot \sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$$

is an approximate $(1-\alpha)100\%$ confidence interval for $\theta_1 - \theta_2$.

EXAMPLE 9

If 132 of 200 male voters and 90 of 150 female voters favor a certain candidate running for governor of Illinois, find a 99% confidence interval for the difference between the actual proportions of male and female voters who favor the candidate.

Solution

Substituting $\hat{\theta}_1 = \frac{132}{200} = 0.66$, $\hat{\theta}_2 = \frac{90}{150} = 0.60$, and $z_{0.005} = 2.575$ into the confidence-interval formula of Theorem 8, we get

$$(0.66 - 0.60) - 2.575\sqrt{\frac{(0.66)(0.34)}{200} + \frac{(0.60)(0.40)}{150}} < \theta_1 - \theta_2$$

$$< (0.66 - 0.60) + 2.575\sqrt{\frac{(0.66)(0.34)}{200} + \frac{(0.60)(0.40)}{150}}$$

which reduces to

$$-0.074 < \theta_1 - \theta_2 < 0.194$$

Thus, we are 99% confident that the interval from -0.074 to 0.194 contains the difference between the actual proportions of male and female voters who favor the candidate. Observe that this includes the possibility of a zero difference between the two proportions.

Exercises

11. By solving

$$-z_{\alpha/2} = \frac{x - n\theta}{\sqrt{n\theta(1-\theta)}} \quad \text{and} \quad \frac{x - n\theta}{\sqrt{n\theta(1-\theta)}} = z_{\alpha/2}$$

for θ, show that

$$\frac{x + \frac{1}{2} \cdot z^2_{\alpha/2} \pm z_{\alpha/2}\sqrt{\frac{x(n-x)}{n} + \frac{1}{4} \cdot z^2_{\alpha/2}}}{n + z^2_{\alpha/2}}$$

are $(1-\alpha)100\%$ confidence limits for θ.

12. Use the formula of Theorem 7 to demonstrate that we can be at least $(1-\alpha)100\%$ confident that the error we make is less than e when we use a sample proportion $\hat{\theta} = \frac{x}{n}$ with

$$n = \frac{z^2_{\alpha/2}}{4e^2}$$

as an estimate of θ.

13. Find a formula for n analogous to that of Exercise 12 when it is known that θ must lie on the interval from θ' to θ''.

14. Fill in the details that led from the Z statistic on the previous page, substituted into $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1-\alpha$, to the confidence-interval formula of Theorem 8.

15. Find a formula for the maximum error analogous to that of Theorem 7 when we use $\hat{\theta}_1 - \hat{\theta}_2$ as an estimate of $\theta_1 - \theta_2$.

16. Use the result of Exercise 15 to show that when $n_1 = n_2 = n$, we can be at least $(1-\alpha)100\%$ confident that the error that we make when using $\hat{\theta}_1 - \hat{\theta}_2$ as an estimate of $\theta_1 - \theta_2$ is less than e when

$$n = \frac{z^2_{\alpha/2}}{2e^2}$$

6 The Estimation of Variances

Given a random sample of size n from a normal population, we can obtain a $(1-\alpha)100\%$ confidence interval for σ^2 by making use of the theorem referred under Section 3, according to which

$$\frac{(n-1)S^2}{\sigma^2}$$

is a random variable having a chi-square distribution with $n-1$ degrees of freedom. Thus,

$$P\left[\chi^2_{1-\alpha/2,\,n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2,\,n-1}\right] = 1-\alpha$$

$$P\left[\frac{(n-1)S^2}{\chi^2_{\alpha/2,\,n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,\,n-1}}\right] = 1-\alpha$$

Thus, we obtain the following theorem.

Theorem 9. If s^2 is the value of the variance of a random sample of size n from a normal population, then

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,\,n-1}}$$

is a $(1-\alpha)100\%$ confidence interval for σ^2.

Corresponding $(1-\alpha)100\%$ confidence limits for σ can be obtained by taking the square roots of the confidence limits for σ^2.

EXAMPLE 10

In 16 test runs the gasoline consumption of an experimental engine had a standard deviation of 2.2 gallons. Construct a 99% confidence interval for σ^2, which measures the true variability of the gasoline consumption of the engine.

Solution

Assuming that the observed data can be looked upon as a random sample from a normal population, we substitute $n = 16$ and $s = 2.2$, along with $\chi^2_{0.005,15} = 32.801$ and $\chi^2_{0.995,15} = 4.601$, obtained from Table V of "Statistical Tables", into the confidence-interval formula of Theorem 9, and we get

$$\frac{15(2.2)^2}{32.801} < \sigma^2 < \frac{15(2.2)^2}{4.601}$$

or

$$2.21 < \sigma^2 < 15.78$$

To get a corresponding 99% confidence interval for σ, we take square roots and get $1.49 < \sigma < 3.97$.

7 The Estimation of the Ratio of Two Variances

If S_1^2 and S_2^2 are the variances of independent random samples of sizes n_1 and n_2 from normal populations, then, according to the theorem, "If S_1^2 and S_2^2 are the variances of independent random samples of sizes n_1 and n_2 from normal populations with the variances σ_1^2 and σ_2^2, then $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$ is a random variable having an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom",

$$F = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$$

is a random variable having an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom. Thus, we can write

$$P\left(f_{1-\alpha/2,\,n_1-1,\,n_2-1} < \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2} < f_{\alpha/2,\,n_1-1,\,n_2-1}\right) = 1-\alpha$$

Since

$$f_{1-\alpha/2,\,n_1-1,\,n_2-1} = \frac{1}{f_{\alpha/2,\,n_2-1,\,n_1-1}}$$

we have the following result.

THEOREM 10. If s_1^2 and s_2^2 are the values of the variances of independent random samples of sizes n_1 and n_2 from normal populations, then

$$\frac{s_1^2}{s_2^2} \cdot \frac{1}{f_{\alpha/2, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} \cdot f_{\alpha/2, n_2-1, n_1-1}$$

is a $(1-\alpha)100\%$ confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$.

Corresponding $(1-\alpha)100\%$ confidence limits for $\frac{\sigma_1}{\sigma_2}$ can be obtained by taking the square roots of the confidence limits for $\frac{\sigma_1^2}{\sigma_2^2}$.

EXAMPLE 11

With reference to Example 6, find a 98% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$.

Solution

Substituting $n_1 = 10, n_2 = 8, s_1 = 0.5, s_2 = 0.7$, and $f_{0.01,9,7} = 6.72$ and $f_{0.01,7,9} = 5.61$ from Table VI of "Statistical Tables", we get

$$\frac{0.25}{0.49} \cdot \frac{1}{6.72} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{0.25}{0.49} \cdot 5.61$$

or

$$0.076 < \frac{\sigma_1^2}{\sigma_2^2} < 2.862$$

Since the interval obtained here includes the possibility that the ratio is 1, there is no real evidence against the assumption of equal population variances in Example 6.

Exercises

17. If it can be assumed that the binomial parameter θ assumes a value close to zero, upper confidence limits of the form $\theta < C$ are often useful. For a random sample of size n, the one-sided interval

$$\theta < \frac{1}{2n} \chi^2_{\alpha, 2(x+1)}$$

has a confidence level closely approximating $(1-\alpha)$ 100%. Use this formula to find a 99% upper confidence limit for the proportion of defectives produced by a process if a sample of 200 units contains three defectives.

18. Fill in the details that led from the probability in Section 6 to the confidence-interval formula of Theorem 10.

19. For large n, the sampling distribution of S is sometimes approximated with a normal distribution having the mean σ and the variance $\frac{\sigma^2}{2n}$. Show that this approximation leads to the following $(1-\alpha)100\%$ large-sample confidence interval for σ:

$$\frac{s}{1 + \frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1 - \frac{z_{\alpha/2}}{\sqrt{2n}}}$$

8 The Theory in Practice

In the examples of this chapter we showed a number of details about substitutions into the various formulas and subsequent calculations. In practice, none of this is really necessary, because there is an abundance of software that requires only that we enter the original **raw** (untreated) **data** into our computer together with the appropriate commands. To illustrate, consider the following example.

EXAMPLE 12

To study the durability of a new paint for white center lines, a highway department painted test strips across heavily traveled roads in eight different locations, and electronic counters showed that they deteriorated after having been crossed by (to the nearest hundred) 142,600, 167,800, 136,500, 108,300, 126,400, 133,700, 162,000, and 149,400 cars. Construct a 95% confidence interval for the average amount of traffic (car crossings) that this paint can withstand before it deteriorates.

Solution

The computer printout of Figure 1 shows that the desired confidence interval is

$$124{,}758 < \mu < 156{,}917$$

car crossings. It also shows the sample size, the mean of the data, their standard deviation, and the estimated standard error of the mean, SE MEAN, which is given by $\frac{s}{\sqrt{n}}$.

```
DATA> 142600 167800 136500 108300 126400 133700 162000 149400
DATA> tint 95 c1

One-Sample T: C1

Variable          N      Mean     StDev   SE Mean        95.0% CI
C1                8    140838     19228      6798   ( 124751,  156924)
MTB >
```

Figure 1. Computer printout for Example 12.

As used in this example, computers enable us to do more efficiently—faster, more cheaply, and almost automatically—what was done previously by means of desk calculators, hand-held calculators, or even by hand. However, dealing with a sample of size $n = 8$, the example cannot very well do justice to the power of computers to handle enormous sets of data and perform calculations not even deemed possible until recent years. Also, our example does not show how computers can summarize the output as well the input and the results as well as the original data in various kinds of graphs and charts, which allow for methods of analysis that were not available in the past.

All this is important, but it does not do justice to the phenomenal impact that computers have had on statistics. Among other things, computers can be used to tabulate or graph functions (say, the t, F, or χ^2 distributions) and thus give the investigator a clear understanding of underlying models and make it possible to study the effects of violations of assumptions. Also important is the use of computers in simulating values of random variables (that is, sampling all kinds of populations) when a formal mathematical approach is not feasible. This provides an important tool when we study the appropriateness of statistical models.

In the applied exercises that follow, the reader is encouraged to use a statistical computer program as much as possible.

Applied Exercises SECS. 1–3

20. A district official intends to use the mean of a random sample of 150 sixth graders from a very large school district to estimate the mean score that all the sixth graders in the district would get if they took a certain arithmetic achievement test. If, based on experience, the official knows that $\sigma = 9.4$ for such data, what can she assert with probability 0.95 about the maximum error?

21. With reference to Exercise 20, suppose that the district official takes her sample and gets $\bar{x} = 61.8$. Use all the given information to construct a 99% confidence interval for the mean score of all the sixth graders in the district.

22. A medical research worker intends to use the mean of a random sample of size $n = 120$ to estimate the mean blood pressure of women in their fifties. If, based on experience, he knows that $\sigma = 10.5$ mm of mercury, what can he assert with probability 0.99 about the maximum error?

23. With reference to Exercise 22, suppose that the research worker takes his sample and gets $\bar{x} = 141.8$ mm of mercury. Construct a 98% confidence interval for the mean blood pressure of women in their fifties.

24. A study of the annual growth of certain cacti showed that 64 of them, selected at random in a desert region, grew on the average 52.80 mm with a standard deviation of 4.5 mm. Construct a 99% confidence interval for the true average annual growth of the given kind of cactus.

25. To estimate the average time required for certain repairs, an automobile manufacturer had 40 mechanics, a random sample, timed in the performance of this task. If it took them on the average 24.05 minutes with a standard deviation of 2.68 minutes, what can the manufacturer assert with 95% confidence about the maximum error if he uses $\bar{x} = 24.05$ minutes as an estimate of the actual mean time required to perform the given repairs?

26. This question has been intentionally omitted for this edition.

27. Use the modification suggested in Exercise 26 to rework Exercise 21, given that there are 900 sixth graders in the school district.

28. An efficiency expert wants to determine the average amount of time it takes a pit crew to change a set of four tires on a race car. Use the formula for n in Exercise 6 to determine the sample size that is needed so that the efficiency expert can assert with probability 0.95 that the sample mean will differ from μ, the quantity to be estimated, by less than 2.5 seconds. It is known from previous studies that $\sigma = 12.2$ seconds.

29. In a study of television viewing habits, it is desired to estimate the average number of hours that teenagers spend watching per week. If it is reasonable to assume that $\sigma = 3.2$ hours, how large a sample is needed so that it will be possible to assert with 95% confidence that the sample mean is off by less than 20 minutes. (*Hint*: Refer to Exercise 6.)

30. The length of the skulls of 10 fossil skeletons of an extinct species of bird has a mean of 5.68 cm and a standard deviation of 0.29 cm. Assuming that such measurements are normally distributed, find a 95% confidence interval for the mean length of the skulls of this species of bird.

31. A major truck stop has kept extensive records on various transactions with its customers. If a random sample of 18 of these records shows average sales of 63.84 gallons of diesel fuel with a standard deviation of 2.75 gallons, construct a 99% confidence interval for the mean of the population sampled.

32. A food inspector, examining 12 jars of a certain brand of peanut butter, obtained the following percentages of impurities: 2.3, 1.9, 2.1, 2.8, 2.3, 3.6, 1.4, 1.8, 2.1, 3.2, 2.0, and 1.9. Based on the modification of Theorem 1 of Exercise 7, what can she assert with 95% confidence about the maximum error if she uses the mean of this sample as an estimate of the average percentage of impurities in this brand of peanut butter?

33. Independent random samples of sizes $n_1 = 16$ and $n_2 = 25$ from normal populations with $\sigma_1 = 4.8$ and $\sigma_2 = 3.5$ have the means $\bar{x}_1 = 18.2$ and $\bar{x}_2 = 23.4$. Find a 90% confidence interval for $\mu_1 - \mu_2$.

34. A study of two kinds of photocopying equipment shows that 61 failures of the first kind of equipment took on the average 80.7 minutes to repair with a standard deviation of 19.4 minutes, whereas 61 failures of the second kind of equipment took on the average 88.1 minutes to repair with a standard deviation of 18.8 minutes. Find a 99% confidence interval for the difference between the true average amounts of time it takes to repair failures of the two kinds of photocopying equipment.

35. Twelve randomly selected mature citrus trees of one variety have a mean height of 13.8 feet with a standard deviation of 1.2 feet, and 15 randomly selected mature citrus trees of another variety have a mean height of 12.9 feet with a standard deviation of 1.5 feet. Assuming that the random samples were selected from normal populations with equal variances, construct a 95% confidence interval for the difference between the true average heights of the two kinds of citrus trees.

36. The following are the heat-producing capacities of coal from two mines (in millions of calories per ton):

Mine A:	8,500,	8,330,	8,480,	7,960,	8,030
Mine B:	7,710,	7,890,	7,920,	8,270,	7,860

Assuming that the data constitute independent random samples from normal populations with equal variances, construct a 99% confidence interval for the difference between the true average heat-producing capacities of coal from the two mines.

37. To study the effect of alloying on the resistance of electric wires, an engineer plans to measure the resistance of $n_1 = 35$ standard wires and $n_2 = 45$ alloyed wires. If it can be assumed that $\sigma_1 = 0.004$ ohm and $\sigma_2 = 0.005$ ohm for such data, what can she assert with 98% confidence about the maximum error if she uses $\bar{x}_1 - \bar{x}_2$ as an estimate of $\mu_1 - \mu_2$? (*Hint*: Use the result of Exercise 8.)

SECS. 4–5

38. A sample survey at a supermarket showed that 204 of 300 shoppers regularly use coupons. Use the large-sample confidence-interval formula of Theorem 6 to construct a 95% confidence interval for the corresponding true proportion.

39. With reference to Exercise 38, what can we say with 99% confidence about the maximum error if we use the observed sample proportion as an estimate of the proportion of all shoppers in the population sampled who use coupons?

40. In a random sample of 250 television viewers in a large city, 190 had seen a certain controversial program. Construct a 99% confidence interval for the corresponding true proportion using

(a) the large-sample confidence-interval formula of Theorem 6;

(b) the confidence limits of Exercise 11.

41. With reference to Exercise 40, what can we say with 95% confidence about the maximum error if we use the observed sample proportion as an estimate of the corresponding true proportion?

42. Among 100 fish caught in a certain lake, 18 were inedible as a result of chemical pollution. Construct a 99% confidence interval for the corresponding true proportion.

43. In a random sample of 120 cheerleaders, 54 had suffered moderate to severe damage to their voices. With 90% confidence, what can we say about the maximum error if we use the sample proportion $\frac{54}{120} = 0.45$ as an estimate of the true proportion of cheerleaders who are afflicted in this way?

44. In a random sample of 300 persons eating lunch at a department store cafeteria, only 102 had dessert. If we use $\frac{102}{300} = 0.34$ as an estimate of the corresponding true proportion, with what confidence can we assert that our error is less than 0.05?

45. A private opinion poll is engaged by a politician to estimate what proportion of her constituents favor the decriminalization of certain minor narcotics violations. Use the formula of Exercise 12 to determine how large a sample the poll will have to take to be at least 95% confident that the sample proportion is off by less than 0.02.

46. Use the result of Exercise 13 to rework Exercise 45, given that the poll has reason to believe that the true proportion does not exceed 0.30.

47. Suppose that we want to estimate what proportions of all drivers exceed the legal speed limit on a certain stretch of road between Los Angeles and Bakersfield. Use the formula of Exercise 12 to determine how large a sample we will need to be at least 99% confident that the resulting estimate, the sample proportion, is off by less than 0.04.

48. Use the result of Exercise 13 to rework Exercise 47, given that we have good reason to believe that the proportion we are trying to estimate is at least 0.65.

49. In a random sample of visitors to a famous tourist attraction, 84 of 250 men and 156 of 250 women bought souvenirs. Construct a 95% confidence interval for the difference between the true proportions of men and women who buy souvenirs at this tourist attraction.

50. Among 500 marriage license applications chosen at random in a given year, there were 48 in which the woman was at least one year older than the man, and among 400 marriage license applications chosen at random six years later, there were 68 in which the woman was at least one year older than the man. Construct a 99% confidence interval for the difference between the corresponding true proportions of marriage license applications in which the woman was at least one year older than the man.

51. With reference to Exercise 50, what can we say with 98% confidence about the maximum error if we use the difference between the observed sample proportions as an estimate of the difference between the corresponding true proportions? (*Hint*: Use the result of Exercise 15.)

52. Suppose that we want to determine the difference between the proportions of the customers of a donut chain in North Carolina and Vermont who prefer the chain's donuts to those of all its competitors. Use the formula of Exercise 16 to determine the size of the samples that are needed to be at least 95% confident that the difference between the two sample proportions is off by less than 0.05.

SECS. 6–7

53. With reference to Exercise 30, construct a 95% confidence interval for the true variance of the skull length of the given species of bird.

54. With reference to Exercise 32, construct a 90% confidence interval for the standard deviation of the population sampled, that is, for the percentage of impurities in the given brand of peanut butter.

55. With reference to Exercise 24, use the large-sample confidence-interval formula of Exercise 19 to construct a 99% confidence interval for the standard deviation of the annual growth of the given kind of cactus.

56. With reference to Exercise 25, use the large-sample confidence-interval formula of Exercise 19 to construct a 98% confidence interval for the standard deviation of the time it takes a mechanic to perform the given task.

57. With reference to Exercise 34, construct a 98% confidence interval for the ratio of the variances of the two populations sampled.

58. With reference to Exercise 35, construct a 98% confidence interval for the ratio of the variances of the two populations sampled.

59. With reference to Exercise 36, construct a 90% confidence interval for the ratio of the variances of the two populations sampled.

SEC. 8

60. Twenty pilots were tested in a flight simulator, and the time for each to complete a certain corrective action was measured in seconds, with the following results:

5.2	5.6	7.6	6.8	4.8	5.7	9.0	6.0	4.9	7.4
6.5	7.9	6.8	4.3	8.5	3.6	6.1	5.8	6.4	4.0

Use a computer program to find a 95% confidence interval for the mean time to take corrective action.

61. The following are the compressive strengths (given to the nearest 10 psi) of 30 concrete samples.

4890	4830	5490	4820	5230	4960	5040	5060	4500	5260
4600	4630	5330	5160	4950	4480	5310	4730	4710	4390
4820	4550	4970	4740	4840	4910	4880	5200	5150	4890

Use a computer program to find a 90% confidence interval for the standard deviation of these compressive strengths.

References

A general method for obtaining confidence intervals is given in

Mood, A. M., Graybill, F. A., and Boes, D. C., *Introduction to the Theory of Statistics*, 3rd ed. New York: McGraw-Hill Book Company, 1974,

and further criteria for judging the relative merits of confidence intervals may be found in

Lehmann, E. L., *Testing Statistical Hypotheses*. New York: John Wiley & Sons, Inc., 1959,

and in other advanced texts on mathematical statistics. Special tables for constructing 95% and 99% confidence intervals for proportions are given in the *Biometrika Tables*. For a proof of the independence of the random variables Z and Y in Section 3, see

Brunk, H. D., *An Introduction to Mathematical Statistics*, 3rd ed. Lexington, Mass.: Xerox Publishing Co., 1975.

Answers to Odd-Numbered Exercises

1 $k = \dfrac{-1}{\ln(1-\alpha)}$.

3 $c = \dfrac{1 \pm \sqrt{1-\alpha}}{\alpha}$.

7 Substitute $t_{\alpha/2,n-1}\dfrac{s}{\sqrt{n}}$ for $z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$.

9 $\dfrac{2\sigma^4}{(n_1+n_2-\lambda)}$.

13 $n = \theta^*(1-\theta^*)\dfrac{z_{\alpha/2}^2}{e^2}$, where θ^* is the value on the interval from θ' to θ^n closest to $\frac{1}{2}$.

15 $E < z_{\alpha/2}\sqrt{\dfrac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \dfrac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$.

17 0.050.

21 $59.82 < \mu < 63.78$.

23 $139.57 < \mu < 144.03$.

25 0.83 minute.

27 $59.99 < \mu < 63.61$.

29 355.

31 $61.96 < \mu < 65.72$ gallons.

33 $-7.485 < \mu_1 - \mu_2 < -2.915$.

35 $-1.198 < \mu_1 - \mu_2 < 1.998$ feet.

37 0.0023 ohm.

39 0.069.

41 0.053.

43 0.075.

45 $n = 2,401$.

47 $n = 1,037$.

49 $-0.372 < \theta_1 - \theta_2 < -0.204$.

51 0.053.

53 $0.04 < \sigma^2 < 0.28$.

55 $3.67 < \sigma < 5.83$.

57 $0.58 < \dfrac{\sigma_1^2}{\sigma_2^2} < 1.96$.

59 $0.233 < \dfrac{\sigma_1^2}{\sigma_2^2} < 9.506$.

61 $227.7 < \sigma < 352.3$.