$f: R \longrightarrow R \quad , \quad f(x) = 2x(1-x)$

① عند النقاط الثابتة

[illegible]

[illegible]

مثلاً $f(x) = 3x(1-x)$

$3x - 3x^2 = x$

$x(2-3x) = 0$

$x = 0$ or $x = \frac{2}{3}$

$f'(x) = 3 - 6x$

$|f'(0)| = 3 > 1$ repel

$|f'(\frac{2}{3})| = 1$ [illegible]

sol:

(1) $2x - 2x^2 = x$

$x - 2x^2 = 0$

$x[1-2x] = 0$

$\boxed{x = 0}$ or $\boxed{x = \frac{1}{2}}$

② $f'(x) = 2 - 4x$

$|f'(0)| = 2 > 1$ is repelling.

$|f'(\frac{1}{2})| = 0 < 1$ is attracting

[illegible]

[illegible]

[illegible]

To proof: $B(\frac{1}{2}) = (0,1)$

let $\epsilon > 0$, such that $|x - \frac{1}{2}| < \epsilon$

$2x - 2x^2 = 0 \Rightarrow 2x[1-x] = 0$

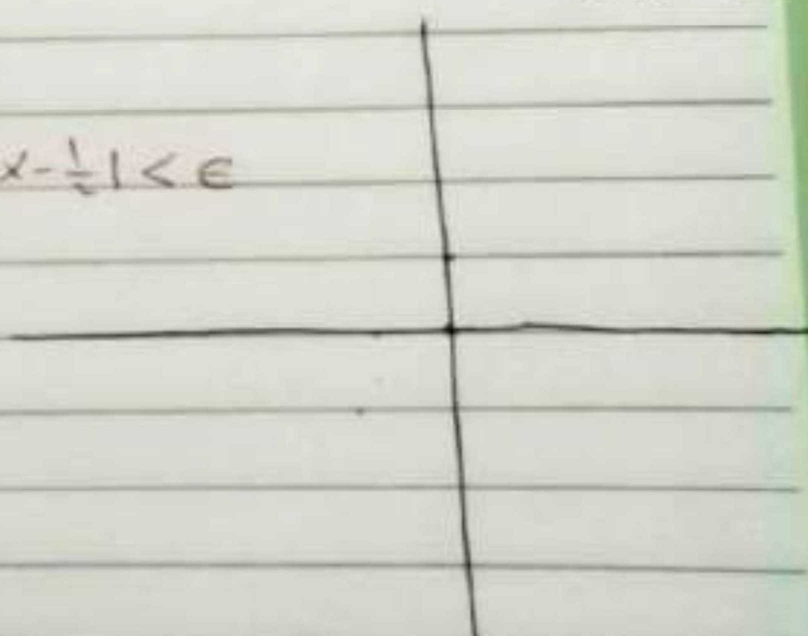
$x = 0, x = 1$

$2 - 4x = 0 \Rightarrow \frac{2}{4} = \frac{4x}{4}$ $\boxed{x = \frac{1}{2}}$

$|f(x) - \frac{1}{2}| = |2x - 2x^2 - \frac{1}{2}|$

$= 2|x^2 - x + \frac{1}{4}|$

$= 2|(x - \frac{1}{2})^2|$ $\quad |x - \frac{1}{2}| < \epsilon$



if $0 < x < 1$

$-\frac{1}{2} < x - \frac{1}{2} < \frac{1}{2}$

$|x - \frac{1}{2}| < \frac{1}{2}$

$2|x - \frac{1}{2}| < 1$ .... (*)

$\therefore |f(x) - \frac{1}{2}| = 2\,|x - \frac{1}{2}|\,|x - \frac{1}{2}|$

$< 1 .\, |x - \frac{1}{2}|$ (By ①)

$< \epsilon$

$\therefore |\hat{f}(x) - \frac{1}{2}| < \epsilon$

$\therefore \hat{f}(x) \longrightarrow \frac{1}{2} \quad \forall\, x \in (0,1)$

$\therefore B\left(\frac{1}{2}\right) = (0,1)$

HW $f(x) = x^2$

Find $B(0)$ graphically

$f(x) = \frac{1}{2}x$

$f(x) = x^2 - 1$

<u>**Proposition**</u> Let $f: X \longrightarrow X$ be a map

$f \in C^1[X]$, and $|f'(p)| > 1$, $p \in X$

Then $\exists$ an open set $U \subseteq X$ s.t

$\forall x \in U$, $\exists k \in \mathbb{N}$ s.t

$f^n(x) \notin U$

ليس كل الـ orbit
موجود بـ U الفترة
يخرج خارج الفترة

chaos

دالة الخيمة   دالة لوجستية

Example:- $f : [0,1] \to \mathbb{R}$ Tent map

$$f_2(x) = \begin{cases} 2x & 0 \le x \le \frac{1}{2} \\ 2(1-x) & \frac{1}{2} \le x \le 1 \end{cases}$$

Proposition

① $2x = x \Rightarrow x = 0$

② $2 - 2x = x \quad x = \frac{2}{3}$

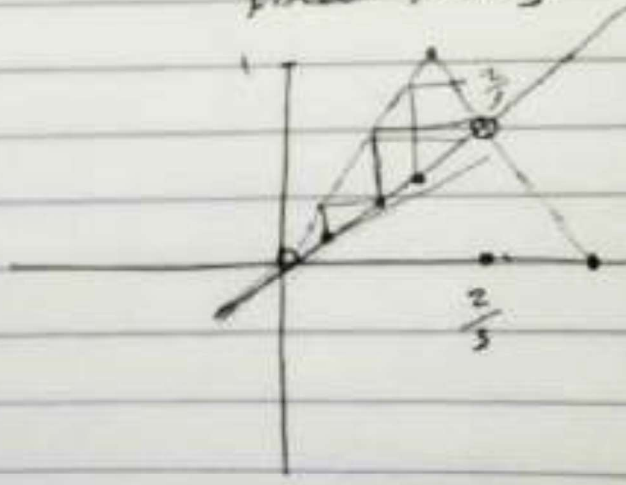
fixed points for f

② $f'(x) = \begin{cases} 2 & 0 \le x \le \frac{1}{2} \\ -2 & \frac{1}{2} \le x \le 1 \end{cases}$

$\therefore |f'(0)| = 2 > 1$

$|f'(\frac{2}{3})| = 2 > 1$

$x = 0$, $x = \frac{2}{3}$ are repelling fixed points



11.2 $x=0$ $x=\frac{\pi}{3}$ اختبركون

graphically .

كايوس

Chaos   الفوضى   دراسة سلوك الانظمة

هو قوة من نشأة الدالة عن الاستقرار في الحد

Def.- $f: R \to R$, $U \subseteq R$

$f(U) = \{ f(x) : x \in U \}$

$f^k(U) = \{ f^k(x) : x \in U \}$

ان تكون الدالة حساسة للقيم الابتدائية

Def. SDIC

let $f: J \to J$ we say f is sensetive dependence on

اذا اخذ جوار كامل   يوجد جوار اخر

intial condition if $\exists\ \delta > 0$ s.t $\forall x \in J\ \exists N_x \subseteq J$

N s.t $\forall y \in N\ \exists k \in N$ s.t

$$| f^k(y) - f^k(x) | > \delta$$

$|f(x) - f(y)| = 2$

$\vdots \qquad \vdots$

$|f^m(x) - f^m(x)| > 2$

~~$f(x)$~~ $y\ x$

N

$f(y)$ $f(x)$

$f^2 y$ $f^2 x$

$f^n y$ $f^n x$

$f: R \to R$, $f(x) = 2x$

$f(x) = 2x$

$f^n(x) = 2^n x$

$x = 1$
$y = 2$ $|f(1) - f(2)| = |2 - 4| = 2 > 1$

$|f^n(1) - f^n(2)| = |2^n - 2^{n+1}| >>> 1$

$f: R \to R$ $f(x) = 2x$, $f^n(x) = 2^n x$

$x = 1$ $\delta = 500$

$f^n(1) = 2^n$

$f^n(2) = 2^n \cdot 2 \Rightarrow f^n(2) = 2^{n+1}$

$|f^n(2) - f^n(1)| =$

لازم نفرضها تطلع اكبر من 500 $|f^{10}(2) - f^{10}(1)| =$

$|2^{10} - 2^{11}| > 500$

$f(x) = e^x$ $\delta = 20$, $x_0 = 2$ $y = 10$

$f(1) = e^2$ $f^2(2) = e^{e^2}$ $f^3 = (e^{e^{e^2}})$

$f : R \longrightarrow R$

$f(x) = 2x$ let $\delta > 0$ let $x \in R$

$f(x) = 2x$

$f^2(x) = 2(2x) = 2^2 x$

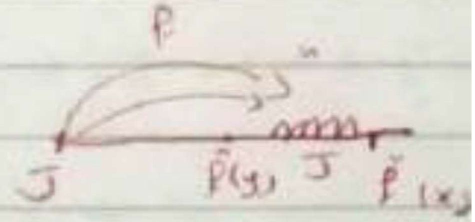
$f^3(x) = 2^3 x$

$\vdots$

$f^n(x) = 2^n x$

let $y \in R$

Assume $|x - y| = r$



$|f^n(x) - f^n(y)| = |2^n x - 2^n y|$

$= 2^n |x - y|$

$= 2^n r$

Chose $k \in N$ s.t $2^k > \delta$

لأنه d n اعتبرناها فرض k

$|f^k(x) - f^k(y)| =$

$2^k |x - y| =$

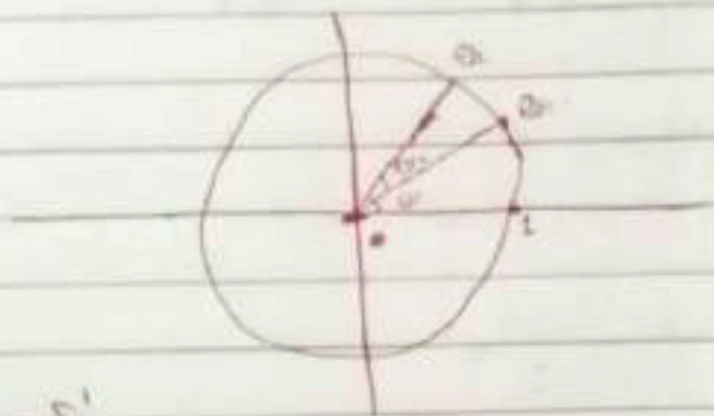
$2^k r > \delta r > \delta$ $\therefore$ f is SDIC

$f^n(U) \cap V \neq \emptyset$

Topological transtive (المتعدية تبولوجياً)

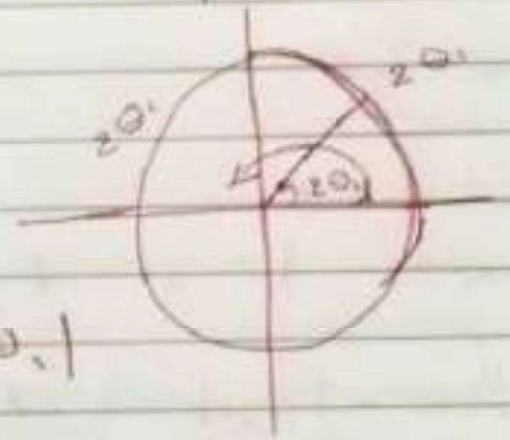
$f : S^1 \longrightarrow S^1$

$f(\theta) = 2\theta$



$f^n(\theta) = 2^n\theta$

let $\delta > 0$ let $\theta_1 \in S^1$
$\theta_2 \in S^1$



$|\theta_1 - \theta_2| = \alpha$

$|f^n(\theta_1) - f^n(\theta_2)| = |2^n\theta_1 - 2^n\theta_2|$

$= 2^n|\theta_1 - \theta_2|$

$= 2^n\alpha$

chose $k \in N$, $2^k\alpha > \delta$

$|f^k(\theta_1) - f^k(\theta_2)| = 2^k\alpha > \delta$