3- $|1-2\sqrt{\alpha}|>1$

$-2\sqrt{\alpha}>1$ $\qquad$ $1-2\sqrt{\alpha}<-1$

$-2\sqrt{\alpha}>0$ $\qquad$ $-2\sqrt{\alpha}<-2$

$\sqrt{\alpha}=1$

$\sqrt{\alpha}<0$ $\qquad$ $\alpha>1$

$f(x)=\alpha-\frac{1}{4}-x^2$

$f^2(x)=\alpha-\frac{1}{4}-(\alpha-\frac{1}{4}-x^2)^2$

$=\alpha-\frac{1}{4}-[(\alpha-\frac{1}{4})^2-2(\alpha-\frac{1}{4})x^2+x^4]$

$=\alpha-\frac{1}{4}-[\alpha^2-\frac{1}{2}\alpha+\frac{1}{16}-2(x-\frac{1}{4})x^2+x^4]$

$=-(\frac{3}{2}\alpha-\frac{5}{16}+\alpha^2)+2(\alpha-\frac{1}{4})x^2-x^4$

$(x-x_1)(x-x_2)=f(x)$

$(x-x_1)(x-x_2)=\alpha-\frac{1}{4}-x^2$

⋮

نوجد ص

Def. let $\{f_\alpha\}$ be a family of map.

We say that $\{f_\alpha\}$ has

النوع الثالث   التشعب الحرج الجزئي

a Sub Critical pichfork bifurcation at $\alpha = \alpha_0$

1- If $\alpha < \alpha_0$ $(\alpha > \alpha_0)$ f_α has a repelling fixed point

2- If $\alpha = \alpha_0$ f_α has only one fixed Point.

3- If $\alpha > \alpha_0$ $(\alpha < \alpha_0)$ f_α has one attracting f.p and two repelling f.p.

Example 1- $f(x) = x^3 + \alpha x + x$ $\quad -2 < \alpha < 2$

solution $\quad f(x) = x$

$$x^3 + \alpha x + x = x$$

$$x^3 + \alpha x = 0$$

$$x[x^2 + \alpha] = 0$$

$$x = 0 \text{ or } x = \sqrt{-\alpha} \text{ or } x = -\sqrt{-\alpha}$$

$$f'(x) = 3x^2 + \alpha + 1$$

1- For $\alpha > 0$, Then $x = 0$ is only f.p for f

$-2 \quad \cdot \quad 2$

$|f'(0)| = |\alpha+1|$

$0 < \alpha < 2$

$+1 < \alpha+1 < 3$

$\therefore |\alpha+1| > 1$

$\therefore x = 0$ is a

repelling f.p.

2- For $\alpha = 0$ then f_α has only one f.p.

3- $\alpha < 0$ Then f_α has $x=0$, $x = \mp\sqrt{-\alpha}$

$|f'(0)| = |\alpha+1|$

$-2 < \alpha < 0$

$-1 < \alpha+1 < 1 \Rightarrow |\alpha+1| < 1$

$\therefore x = 0$ is attracting

$f'(x) = 3(\mp\sqrt{-\alpha})^2 + \alpha + 1$

$= 3(-\alpha) + \alpha + 1$

$= -2\alpha + 1$

$|f'(\pm\sqrt{\alpha})| > 1 \Rightarrow \mp\sqrt{-\alpha}$ are repelling f.p.

$-2 < \alpha < 0$

$4 > -2\alpha > 0$

$3 > 1 - 2\alpha > 1$

Example: $f_\lambda(x) = \lambda(x^3 - x)$

$\lambda(x^3 - x) = x$

$\lambda x^3 - \lambda x - x = 0$

$x[\lambda x^2 - \lambda - 1] = 0$

$x = 0, \quad \lambda x^2 - \lambda - 1 = 0$

$x = 0 \qquad x = \pm\sqrt{\frac{\lambda+1}{\lambda}}$

من التعريف نرى

1- $\lambda \lessgtr \lambda_0$ rep (1)

2- $\lambda = \lambda_0$ has (1) only

3 $\lambda \gtrless \lambda_0$ has one att.

has two repellin

$f'(x) = 3\lambda x^2 - \lambda$

1- $\lambda < -1$, Then $x=0$ is a f.p. for f_λ

and $f'(0) = |-\lambda|$

$\lambda < -1 \Rightarrow -\lambda > 1$

$\therefore f'(0) = |-\lambda| > 1$

$\therefore x=0$ is repelling f.p.

2- If $\lambda = -1$, Then f has only one fixed Point $x=0$.

3- $\lambda > -1$

$x_1 = 0 \qquad x_2 = +\sqrt{\frac{\lambda+1}{\lambda}} \qquad x_3 = -\sqrt{\frac{\lambda+1}{\lambda}}$

$f'(x) = 3\lambda x^2 - \lambda$

$0 > \lambda > -1$

$0 < -\lambda < 1$

$\therefore |-\lambda| < 1 \Rightarrow x=0$ is attracting.

$f'\left(\mp\sqrt{\frac{\lambda+1}{\lambda}}\right) = 3\lambda\left(\frac{\lambda+1}{\lambda}\right) - \lambda$ عوضنا في النقاط الثابتة

$= 3\lambda + 3 - \lambda \Rightarrow 2\lambda + 3$

اين هذه القيمة بالمطلق

تطلع اكبر من واحد

$|f'(x_{2,3})| = |2\lambda + 3|$

$\lambda > -1$

$2\lambda > -2$

$2\lambda + 3 > -2 + 3$

$|2\lambda + 3| > 1$

$\therefore x_2$ and x_3 are repelling

$\therefore$ 1, 2 and 3 implies f_λ has S.P.B.

at $\lambda = -1$