التحليل الرياضي (1)

Mathematical Analysis (I)

(1) Real Numbers

(2) Sequences of Real Numbers

(3) Series of Real Numbers

(4) Metric Spaces

(5) Continuity

(6) Sequences and Series of Functions

References:

(1) Apostol, T.M., “Mathematical Analysis”, 2nd, 1974, London.

(2) Ash, R.B., “Real Analysis and Probability”, 1972, New York.

(3) Royden. H.L., “Real Analysis”, 3rd, 1988, London.

(4) Rudin, W., “Principles of Mathematical Analysis”, 3rd, 1976, McGraw-Hill, Inc., New York.

(5) عادل غسان نعوم "مقدمة في التحليل الرياضي"، جامعة بغداد– العراق 1986 الطبعة الأولى.

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Chapter One
الاعــداد الــحــقــيــقــيــة
The Real Numbers

Definition (1.1): (Field) الــحــقــل

A **field** F is a non-empty set together with two binary operations addition "+" and multiplication "." satisfying:

(1) $(F,+)$ is an abelian group.

(2) $(F,\cdot)$ is an abelian group.

(3) The distribution law: $u.(v+w)=u.v+u.w$, $\forall\, u.v.w$ in F

Example (1.1):

(1) $(N,+,.)$ is not a field

(2) $(Z,+,.)$ is not a field

(3) $(Q,+,.)$ is a field

(4) $(R,+,.)$ is a field

Definition (1.2): (Ordered Field) حــقــل مــرتــب

Any field $(F,+,.)$ which has ordered relation "≤" and satisfy:

(1) $a \le b$ and $c \le d \Rightarrow a+c \le b+d$, $\forall\, a,b,c,d \in F$

(2) If $0 < a < b$ and $c \le d \Rightarrow a.c \le b.d$

is called **ordered field.**

Example (1.2):

(1) $(Q,+,.)$ is an ordered field

(2) $(R,+,.)$ is an ordered field

Definition (1.3): (Upper Bound) قيد أعلى

A real number u is called an **upper bound** of the non-empty set $S \subseteq R$ if $u \geq x$, $\forall\, x \in S$.

Definition (1.4): (Least Upper Bound) أصغر قيد أعلى

A real number x is called the **least upper bound (l.u.b)** of the S or **supremum** of S if $x \leq u$, $\forall\, u$ upper bound, denoted by

$$x = sup(S) = l.u.b(S)$$

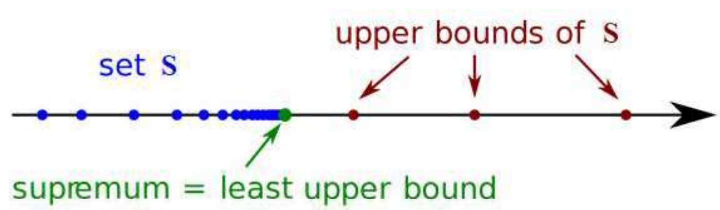

Figure (1.1) Upper and least upper bounds

Example (1.3):

(1) $sup[0,4) = sup[0,4] = 4$

(2) $sup\{3,5,8,9,10\} = 10$

(3) $sup\left\{\frac{(-1)^n}{n} : n \in N\right\} = \frac{1}{2}$

(4) $sup\{x^2 : -1 < x < 2 , x \in R\} = 4$

Definition (1.5): (Lower Bound) قيد أدنى

A real number ℓ is called a **lower bound** of the non-empty set $S \subseteq R$ if $\ell \leq x$, $\forall\, x \in S$.

Definition (1.6): (Greatest Lower Bound) أكبر قيد أدنى

A real number y is called the **greatest lower bound (g.l.b)** of the S or **infimum** of S if $y \geq \ell$, $\forall\, \ell$ lower bound, denoted by

$$y = inf(S) = g.l.b(S)$$

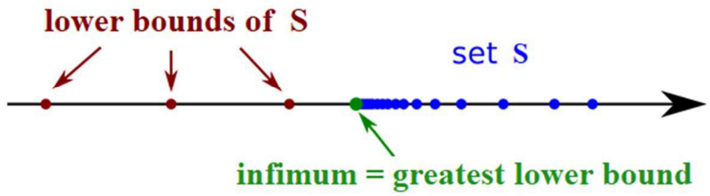

Figure (1.2) Lower and greatest lower bounds

Example (1.4):

(1) inf[0,4)= inf[0,4]= 0

(2) $inf\{3,5,8,9,10\} = 3$

(3) $inf\left\{\frac{(-1)^n}{n} : n \in N\right\} = -1$

(4) $inf\{x^2 \ : -1 < x < 2 \ , x \in R\} = 0$

Exercises (1.1): (Homework)

For each subset of R below, determine if it is bounded above, bounded below, or both. If it is bounded above (below) find the supremum (infimum).

(1) $\{1,6,15\}$

(2) [0,8)

(3) $\left\{1 + \frac{(-1)^n}{n} : n \in N\right\}$

(4) (-2,∞)

(5) $\{x^2 - 3x + 2 = 0 : x \in R\}$

(6) $\{1 \le |x| < 3 : x \in R\}$

Definition (1.7): (Equivalnce Property) خـاصـيــة الـتـكــافــو

Every non-empty subset of R which bounded below has a greatest lower bound.

Definition (1.8): (Completeness Axiom) بـديــهـيــة الـكـمــال

Every non-empty subset of R which bounded above has a least upper bound.
