Mathematical Fonundation (1)

1. Sets and the operation of sets.

2. Logic.

3. cartesan product.

4. Relations.

5. Functions.

References:-

1. Charles C. Pinter, (1971) "Set Theory"

2- د. عادل غسان و د. باسل الهاشمي (2000)

"مقدمة في أسس الرياضيات"

1. Sets and the operations of sets

The objects in a set are called elements or members of the set. The set is denoted by capital letters $A, B, C, \dots$ and the elements is denoted by small letters $a, b, c, \dots$. The expression $x \in A$ "x is an element of A", $x \notin A$, "x is not an element of A"

The methods to described وصف the set:

1. $A = \{-1, 0, 1, 2\}$

2. $B = \{a, b, c\}$

3. $C = \{x : x \text{ is an integer less than } 2\}$

4. $D = \{y : y \text{ is an odd integer and } y \geq 3\}$
   $= \{3, 5, 7, \dots\}$

5. $A = \{x : x \in Z \text{ and } x^2 = x\}$
   $= \{0, 1\}$

   $x^2 = x$
   $\Rightarrow x^2 - x = 0$
   $\Rightarrow x(x-1) = 0$
   $\Rightarrow x = 0$
   $\Rightarrow x - 1 = 0 \Rightarrow x = 1$

6. $B = \{x : x \in N, x < 100\}$
   $= \{1, 2, \dots, 99\}$

Ex: $A = \{x : 2 < x < 3 ; x \in W\} = \emptyset$

7. The set of Rational numbers Q

$Q = \{\frac{a}{b} ; b \neq 0, a, b \in Z\}$

8. The set of Even integer numbers Z_e

$$Z_e = \{x : x \text{ is even number}\}$$
$$= \{x : x = 2n \,;\, n \in Z\}$$
$$= \{0, \mp 2, \mp 4, \dots\} = E$$

9. $Z_o = \{x : x \text{ is odd number}\}$
$$= \{x : x = 2n+1 \,;\, n \in Z\}$$
$$= \{\mp 1, \mp 3, \dots\} = O$$

10. Complex number $\mathbb{C}$

$$\mathbb{C} = \{a+ib : a, b \in Z,\ i = \sqrt{-1}\}$$

Definition (1) :- If A and B are sets we define $A=B$ to mean that every element of A is an element of B and vice versa.

وبالعكس

$$A = B \text{ iff } x \in A \Rightarrow x \in B \text{ and}$$
$$x \in B \Rightarrow x \in A.$$

EX. :- $\{1, 2, 3\} = \left\{\frac{4}{4}, \frac{6}{3}, \frac{9}{3}\right\}$

Note :- If $x = y$ and $x \in A$, then $y \in A$

Definition:- If A and B are sets, we define $A \subseteq B$ to mean that every element of A is an element of B.

$$A \subseteq B \text{ iff } x \in A \overset{\text{لازم}}{\Longrightarrow} x \in B$$ نعبر عنه رياضياً

If $A \subseteq B$, then we say that A is <u>asubset</u> of B

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If $A \subset B$, then we say that A is <u>proper subset</u> of B

Theorem:- For all sets A, B and C. The following hold:-

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1- $A = A$.

2- $A = B \Rightarrow B = A$.

3- $A = B$ and $B = C \Rightarrow A = C$. (تشبه علاقة التعدي)

4- $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$.

5- $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$.

Proof:- ① The statement $x \in A \Rightarrow x \in A$ and $x \in A \Rightarrow x \in A$ is obvious true. دائماً صحيحة

Thus by definition (1) $A = A$.

② suppose $A = B$, then $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$, thus $B = A$ (definition (1))

(3) suppose $A=B$ and $B=C$, then

$x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in C$ --- (a)

$x \in B \Rightarrow x \in A$ and $x \in C \Rightarrow x \in B$ — (b)

from (a) $\Rightarrow x \in A \Rightarrow x \in C$

from (b) $\Rightarrow x \in C \Rightarrow x \in A$

by def. (1) $A=C$.

(4) since $A \subseteq B$, then $x \in A \Rightarrow x \in B$

and $B \subseteq A$, then $x \in B \Rightarrow x \in A$

Hence $A=B$ by (def.(1))

(5) let $x \in A$

$\because A \subseteq B \Rightarrow x \in B$

$\because B \subseteq C \Rightarrow x \in C$

$\therefore A \subseteq C$.

by (def(2)) $\Rightarrow A \subseteq C$.