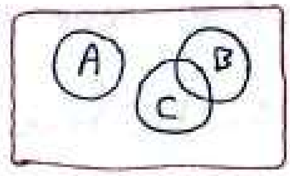
The Universal set

Definition:- The universal set U we mean the set of all elements.

U 

Definition:- The empty set we mean the set $\emptyset$ which has no elements at all.

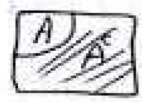
Theorem:- For any set A, the following hold

1- $\emptyset \subseteq A$ 2- $A \subseteq U$.

The complement set : A^c

Definition:- The complement of a set A is the set of all the elements which do not belong to A.

$A^c = \{x : x \notin A\}$



Thus $x \notin A$ iff $x \in A^c$

Ex ①:- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 5, 6\}$ $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 5, 7\}$

$A^c = \{3, 4, 7, 8, 9, 10\}$ $A^c = \{2, 4, 6\}$

Ex ②:- $U = Z = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

$A = Z_e$

$\Rightarrow A^c = Z_o$

Note:- $A \cap A^c = \emptyset$

$A \cup A^c = U$

$A \cap U = A$

Def. :- The Union : $\cup$ أو or الاتحاد

Let A and B are sets, the union of A and B is defined to be the set of all the elements which belong either to A or to B or to both A and B.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Thus $x \in A \cup B$ if and only if $x \in A$ or $x \in B$.

Def. :- The intersection : $\cap$ و and التقاطع

Let A and B are sets the intersection of A and B is defined to be the set of all elements which belong to both A and B

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Thus, $x \in A \cap B$ iff $x \in A$ and $x \in B$

Def. :- If two sets have no elements in شائعة Common they are said to be disjoint. منفصلة

A and B are disjoint iff $A \cap B = \emptyset$.

Ex. :- $A = \{a, b, c, d\}$, $B = \{a, c\}$

$A \cup B = \{a, b, c, d\}$, $A \cap B = \{a, c\}$

A and B are not disjoint.

Ex.:- $A = \{x : x \text{ is even integer}\} = E$

$B = \{x : x \text{ is odd integer}\} = O$

$\Rightarrow A = \{0, \mp 2, \mp 4, ---\}$

$B = \{\mp 1, \mp 3, \mp 5, ---\}$

$A \cup B = Z$, $A \cap B = \phi$

$\Rightarrow$ A and B are disjoint.

Theorem*:- If A and B are any sets, then

1- $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

2- $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Proof: 1) Let $x \in A \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

$\therefore A \subseteq A \cup B$

we shall prove $B \subseteq A \cup B$. H.W.

let $x \in B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

$\therefore B \subseteq A \cup B$

2) let $x \in A \cap B \Rightarrow x \in A$ and $x \in B$ by def. of $(\cap)$

$\Rightarrow x \in A$

$\therefore \underline{A \cap B \subseteq A}$.

prove $A \cap B \subseteq B$.

Theorem:-** If A and B are sets, then

1. $A \subseteq B$ if and only if $A \cup B = B$. (مع iff نستخدم suppose that assume that)
2. $A \subseteq B$ if and only if $A \cap B = A$.

Proof(1):- assume that $A \subseteq B$, that is $x \in A \Rightarrow x \in B$

let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\qquad \Rightarrow x \in B$ or $x \in B \Rightarrow x \in B$ ($A \subseteq B$ لان)

Thus $A \cup B \subseteq B$, since $B \subseteq A \cup B$ (by Th. *(1))

then $A \cup B = B$

Conversly :- assume that $A \cup B = B$, (by Th. *)

[illegible] $A \subseteq A \cup B$. Thus $A \subseteq A \cup B = B$

therefore $A \subseteq B$.

Proof(2):- Assume that $A \subseteq B$, we must prove that $A \cap B = A$

let $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$\qquad \Rightarrow A \cap B \subseteq A$ --- (a)

let $x \in A$. Since $A \subseteq B \Rightarrow x \in B$

$\qquad \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B$

$\qquad \Rightarrow A \subseteq A \cap B$ --- (b)

from (a) and (b) we get $A \cap B = A$.

conversely:

Assume that $A \cap B = A$, we must prove that $A \subseteq B$

let $y \in A$, since $A \cap B = A \Rightarrow y \in A \cap B$!

$\Rightarrow y \in A$ and $y \in B$ $\therefore A \subseteq B$.

Ex:- Show that $A \cap \emptyset = \emptyset$

Proof:- suppose that $A \cap \emptyset \neq \emptyset$

i.e. there exists $x \in A \cap \emptyset \Rightarrow x \in A$ and $x \in \emptyset$ C!

(controduction)

$\therefore A \cap \emptyset = \emptyset$.

Note:- $A \cup \emptyset = A$

$A \cup U = U$

Ex.: ① prove that: If $C \subseteq A$ and $C \subseteq B$, then $C \subseteq (A \cap B)$.

② prove that: If $A \subseteq X$ and $B \subseteq Y$, then $(A \cap B) \subseteq (X \cap Y)$.

Proof:- ① let $x \in C$, since $\left.\begin{array}{l} C \subseteq A \Rightarrow x \in A \\ C \subseteq B \Rightarrow x \in B \end{array}\right\} \Rightarrow x \in (A \cap B)$

$\therefore C \subseteq (A \cap B)$.

② let $z \in A \cap B$

$\Rightarrow z \in A$ and $z \in B$

$\left.\begin{array}{l} \because A \subseteq X \Rightarrow z \in X \\ \because B \subseteq Y \Rightarrow z \in Y \end{array}\right\} \Rightarrow z \in (X \cap Y)$

$\therefore (A \cap B) \subseteq (X \cap Y)$.