Theorem :- If A and B are sets, then

1. $A \subseteq B$ if and only if $A \cup B = B$.
2. $A \subseteq B$ if and only if $A \cap B = A$.

(مع iff نستخدم
suppose that
assume that)

Proof (1) :- assume that $A \subseteq B$, that is $x \in A \Rightarrow x \in B$

let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

لأن $A \subseteq B$

$\Rightarrow x \in B$ or $x \in B \Rightarrow x \in B$

Thus $A \cup B \subseteq B$, since $B \subseteq A \cup B$ (by Th. * (1))

then $A \cup B = B$

Conversly :- assume that $A \cup B = B$, (by Th. *)

$A \subseteq A \cup B$. Thus $A \subseteq A \cup B = B$

therefore $A \subseteq B$.

Proof (2) :- Assume that $A \subseteq B$, we must prove that $A \cap B = A$

let $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$\Rightarrow A \cap B \subseteq A$ --- (a)

let $x \in A$. Since $A \subseteq B \Rightarrow x \in B$

$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B$

$\Rightarrow A \subseteq A \cap B$ --- (b)

from (a) and (b) we get $A \cap B = A$.

conversely :

Assume that $A \cap B = A$, we must prove that $A \subseteq B$

let $y \in A$, since $A \cap B = A \Rightarrow y \in A \cap B$!

$\Rightarrow y \in A$ and $y \in B$ $\therefore A \subseteq B$.

Theorem:- If A and B are sets, then

1. $A \cup (A \cap B) = A$ 2. $A \cap (A \cup B) = A$

Proof:- 1. By Th. * $A \cap B \subseteq A$, therefore by Th. ** we get

$A \cup (A \cap B) = A$.

2. By Th. * $A \subseteq A \cup B$, therefore by Th. ** we get

$A \cap (A \cup B) = A$.

Ex.:- Show that $(A^c)^c = A$, for every set A.

proof:- let $x \in (A^c)^c \Rightarrow x \notin A^c \Rightarrow x \in A$

$\therefore (A^c)^c \subseteq A$ --- (1)

Assume that $x \in A \Rightarrow x \notin A^c \Rightarrow x \in (A^c)^c$

$\therefore A \subseteq (A^c)^c$ ---- (2)

from (1) and (2) we get $(A^c)^c = A$.

Ex.:- $A = \{1,2,3,5\}$, $B = \{4,10\}$

$U = \{1,2,3,4,5,10,11\}$

$A^c = \{4,10,11\}$, $(A^c)^c = \{1,2,3,5\} = A$

$A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$

Ex.:- prove that $A \cap (A^c \cup B) = A \cap B$

proof:- $A \cap (A^c \cup B) = (A \cap A^c) \cup (A \cap B)$

$= \emptyset \cup (A \cap B)$

$= A \cap B$.

Prove that $(A\cup B)^c = A^c \cap B^c$

$x \notin A \Leftrightarrow x \in A^c$ (7)

قوانين دي مورگان Demorgan's laws

① $(A\cap B)^c = A^c \cup B^c$

② $(A\cup B)^c = A^c \cap B^c$

Proof:- let $x \in (A\cup B)^c \Rightarrow x \notin A\cup B$

$\Rightarrow x \notin A$ <u>and</u> $x \notin B$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

$\Rightarrow (A\cup B)^c \subseteq A^c \cap B^c$ ---- (a)

$A = \{1,2,3\}$

$B = \{3,4,5\}$

$A\cup B = \{1,2,3,4,5\}$

$7 \notin A\cup B$

Now, let $y \in A^c \cap B^c \Rightarrow y \in A^c$ and $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin A\cup B$

$\Rightarrow y \in (A\cup B)^c$

$\Rightarrow A^c \cap B^c \subseteq (A\cup B)^c$ ---- (b)

from (a) and (b) we get $(A\cup B)^c = A^c \cap B^c$

$(A\cap B)^c = A^c \cup B^c$

<u>مثال</u>

$A = \{1,2,3\}$

$B = \{3,4,5\}$

$A\cap B = \{3\}$

$4 \notin A\cap B$

$4 \notin A$

$4 \in B$

Proof:- let $x \in (A\cap B)^c$

$\Rightarrow x \notin A\cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$\therefore (A\cap B)^c \subseteq A^c \cup B^c$ --- ①

Now, let $y \in A^c \cup B^c$

$\Rightarrow y \in A^c$ or $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin A\cap B$

$\Rightarrow y \in (A\cap B)^2$

$\Rightarrow A^c \cup B^c \subseteq (A\cap B)^c$ --- ②

from (1) and (2) we get

$(A\cap B)^c = A^c \cup B^c$.

Ex.:- Prove that $A \subseteq B$ iff $B^c \subseteq A^c$

Proof:- suppose that $A \subseteq B$, let $y \in B^c \Rightarrow y \notin B$

since $A \subseteq B \Rightarrow y \notin A \Rightarrow y \in A^c$

$\therefore B^c \subseteq A^c$.

$\Leftarrow$ Assume that $B^c \subseteq A^c$, let $x \in A \Rightarrow x \notin A^c$

since $B^c \subseteq A^c \Rightarrow x \notin B^c \Rightarrow x \in B$

$\therefore A \subseteq B$.

$\emptyset^c = U$, $U^c = \emptyset$

Th.:- let A, B and C are sets then

① $A \cap A = A$, $A \cup A = A$

② $A \cap B = B \cap A$

③ $A \cap (B \cap C) = (A \cap B) \cap C$

④ $A \cup B = B \cup A$

⑤ $A \cup (B \cup C) = (A \cup B) \cup C$

⑥ $A \cap B \subseteq A$, $A \cap B \subseteq B$

⑦ $A \subseteq A \cup B$, $B \subseteq A \cup B$

⑧ $A \cap \emptyset = \emptyset$, $A \cup \emptyset = A$

⑨ $A \cup U = U$, $A \cap U = A$.

proof(8):- let $A \cap \emptyset \neq \emptyset$

$\Rightarrow \exists x \in A \cap \emptyset$

$\Rightarrow x \in A$ and $x \in \emptyset$ C! (controoduction) تناقض

$\therefore A \cap \emptyset = \emptyset$

Ex.:- Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:- let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (A \cup C)$

$\Rightarrow x \in A$ and $(x \in A$ or $x \in C)$

تضعيف $x \in A$

$\Rightarrow (x \in A$ and $x \in A)$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ ----- (1)

Now,

let $y \in [(A \cap B) \cup (A \cap C)]$

$\Rightarrow y \in (A \cap B)$ or $y \in (A \cap C)$

$\Rightarrow (y \in A$ and $y \in B)$ or $(y \in A$ and $y \in C)$

عامل مشترك

$\Rightarrow y \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow y \in A \cap (y \in B \cup y \in C)$

$\Rightarrow y \in A \cap (y \in B \cup C)$

$\Rightarrow y \in A \cap (B \cup C)$

$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ --- (2)

From (1) and (2) we get

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$