Def. :- The Difference set مجموعة الفرق

let A and B are sets. Then

$A-B=\{x : x\in A \text{ and } x\notin B\}$

$=\{x : x\in A \text{ and } x\in B^c\}$

$= A\cap B^c$

هذا الموضوع ما نفرض X تنتمي X∈

$B-A=\{x : x\in B \text{ and } x\notin A\}$

$=\{x : x\in B \text{ and } x\in A^c\}$

$= B\cap A^c$

Def. :- The symmetric Difference

الفرق التناظري يرمز له بالرمز (Δ)

let A and B are sets. Then

$A\Delta B=(A-B)\cup(B-A)$

$=(A\cap B^c)\cup(B\cap A^c)$

Ex.:- Show that $(A\cup C)\,\Delta\,(B\cup C)=(A\Delta B)-C$ (9)

Proof:- left side $(\overset{A}{\underline{A\cup C}})\,\Delta\,(\overset{B}{\underline{B\cup C}})$

$$=[(A\cup C)-(B\cup C)]\cup[(B\cup C)-(A\cup C)]$$

$$=[(A\cup C)\cap(B\cup C)^c]\cup[(B\cup C)\cap(A\cup C)^c]$$

$$=[(A\cup C)\cap(B^c\cap C^c)]\cup[(B\cup C)\cap(A^c\cap C^c)]$$

$$=[(A\cap B^c\cap C^c)\cup(C\cap B^c\cap C^c)]\cup[(B\cap A^c\cap C^c)\cup(C\cap A^c\cap C^c)]$$

نرتب $$=[(A\cap B^c\cap C^c)\cup(B^c\cap \underline{C\cap C^c})]\cup[(B\cap A^c\cap C^c)\cup(A^c\cap \underline{C\cap C^c})]$$

$$=[(A\cap B^c\cap C^c)\cup(B^c\cap\emptyset)]\cup[(B\cap A^c\cap C^c)\cup(A^c\cap\emptyset)]$$

$$=[(A\cap B^c\cap C^c)\cup\emptyset]\cup[(B\cap A^c\cap C^c)\cup\emptyset]$$

$$=(A\cap B^c\underline{\cap C^c})\cup(B\cap A^c\underline{\cap C^c})$$

$$=[(A\cap B^c)\cup(B\cap A^c)]\cap C^c$$

$$=[(A-B)\cup(B-A)]\cap C^c$$

$$=[(A-B)\cup(B-A)]-C$$

$$=(A\,\Delta B)-C$$

= right side.

Ex.:- Prove that $A-B=B^c-A^c$

Proof:- $A-B=A\cap B^c$

$$=B^c\cap A$$

$$=B^c\cap(A^c)^c$$

$$=B^c-A^c.$$

Ex.:- Show that $(A\cap B)\Delta(A\cap C)=A\cap(B\Delta C)$

Proof:- left side $=(A\cap B)\Delta(A\cap C)$

$$=[(A\cap B)-(A\cap C)]\cup[(A\cap C)-(A\cap B)]$$

$$=[(A\cap B)\cap(A\cap C)^c]\cup[(A\cap C)\cap(A\cap B)^c]$$

$$=[(A\cap B)\cap(A^c\cup C^c)]\cup[(A\cap C)\cap(A^c\cup B^c)]$$

$$=[(A\cap B\cap A^c)\cup(A\cap B\cap C^c)]\cup[(A\cap C\cap A^c)\cup(A\cap C\cap B^c)]$$

$$=[(A\cap A^c\cap B)\cup(A\cap B\cap C^c)]\cup[(A\cap A^c\cap C)\cup(A\cap C\cap B^c)]$$

$$=[(\phi\cap B)\cup(A\cap B\cap C^c)]\cup[(\phi\cap C)\cup(A\cap C\cap B^c)]$$

$$=\phi\cup(A\cap B\cap C^c)\cup(\phi\cup(A\cap C\cap B^c)$$

$$=(A\cap B\cap C^c)\cup(A\cap C\cap B^c)$$

$$=A\cap(B\cap C^c)\cup(C\cap B^c)$$

$$=A\cap(B-C)\cup(C-B)$$

$$=A\cap(B\Delta C)$$

= right side.

Th. If A and B are sets, then

① $A\cup(A\cap B)=A$ ② $A\cap(A\cup B)=A$ ③ $A\cap(A^c\cup B)=A\cap B$

④ $A\cup(A\cup B^c)=A\cup B$ ⑤ $A\subseteq B \Leftrightarrow A\cap B^c=\emptyset$

⑥ $A\subseteq B \Leftrightarrow A\cup B^c=U$ ⑦ $A\cap(B-C)=(A\cap B)-C$

⑧ $(A\cup B)-C=(A-C)\cup(B-C)$ ⑨ $A-(B\cup C)=(A-B)\cap(A-C)$

⑩ $A-(B\cap C)=(A-B)\cup(A-C)$

Proof(5):- $A\subseteq B \Leftrightarrow A\cap B^c=\emptyset$

suppose that $A\subseteq B$, if $A\cap B^c\neq\emptyset$

$\Rightarrow \exists x\in A\cap B^c \Rightarrow x\in A$ and $x\in B^c$

$\Rightarrow x\in A$ and $x\notin B$, تناقض (since $A\subseteq B$)

$\therefore A\cap B^c=\emptyset$

$\Leftarrow$ Conversly , suppose that $A\cap B^c=\emptyset$

let $x\in A$, $\because A\cap B^c=\emptyset$

$\Rightarrow x\in A$ and $x\notin B^c$

$\therefore x\in A$ and $x\in B$

$\Rightarrow A\subseteq B.$

⑦ $A\cap(B-C) = (A\cap B)-C$

Proof:- Left $= A\cap(B-C) = A\cap(B\cap C^c)$

$= (A\cap B)\cap C^c$

$= (A\cap B)-C$ = right

⑧ $(A\cup B)-C = (A-C)\cup(B-C)$

Proof:- Left $= (A\cup B)-C = (A\cup B)\cap C^c$

$= (A\cap C^c)\cup(B\cap C^c)$

$= (A-C)\cup(B-C)$ = right

⑨ $A-(B\cup C) = (A-B)\cap(A-C)$

Proof:- Left $= A-(B\cup C) = A\cap(B\cup C)^c$

$= A\cap(B^c\cap C^c)$

$= (A\cap B^c)\cap(A\cap C^c)$

$= (A-B)\cap(A-C)$ = right

⑩ $A-(B\cap C) = (A-B)\cup(A-C)$

left $= A-(B\cap C) = A\cap(B\cap C)^c$

$= A\cap(B^c\cup C^c)$

$= (A\cap B^c)\cup(A\cap C^c)$

$= (A-B)\cup(A-C)$ = right

⑥ H.W