**Ex.:-** Let A, B and C are sets, then

① $A - B = A - (A \cap B)$

**Proof:-** right side $= A - (A \cap B)$

$= A \cap (A \cap B)^c$

$= A \cap (A^c \cup B^c)$

$= (A \cap A^c) \cup (A \cap B^c)$

$= \emptyset \cup (A \cap B^c) = A \cap B^c = A - B =$ left side

② $A - B = B^c - A^c$

**Proof:** right $= B^c - A^c = B^c \cap (A^c)^c = B^c \cap A$

$= A \cap B^c$ ابدالية

$= A - B =$ left

③ $(A - B) \cap (A \cap B) = \emptyset$

**Proof:-** left $= (A - B) \cap (A \cap B) = (A \cap B^c) \cap (A \cap B)$

$= A \cap A \cap (B^c \cap B)$

$= A \cap \emptyset = \emptyset =$ right.

④ $A = (A \cap B) \cup (A - B)$

**Proof:-** right $= (A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B^c)$

$= A \cap (B \cup B^c)$

$= A \cap U$

$= A =$ left.

# Power set.

**Definition:-** Let $A$ be a set, by the power set of $A$ we mean the set of all the subset of $A$.

$$P(A)=\{B : B \subseteq A\}$$ بشكل مجاميع جزئية

**EX.:-** If $A=\{a,b\}$, then $P(A)=\{\varnothing, \{a\}, \{b\}, \{a,b\}\}$

مثلاً $\{a\}$ جزء من $\{a,b\}$ وهكذا

$2^n = 2^2 = 4$ عدد المجاميع

**EX.:-** $A=\{1,2,3\}$ , $2^3=8$

$P(A)=\{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

جزئية من كل مجموعة

**Note:-** $\{a\} \in P(A)$ , $\{a\} \subseteq A$ لكن $a \in A$

or

If $A=\varnothing$ , $P(A)=\{\varnothing\}$ $(2^0=1)$

**EX.:-** Show that:-

① $A \subseteq B \Leftrightarrow P(A) \subseteq P(B)$

**Proof:-** suppose that $A \subseteq B$ , let $X \in P(A)$

$\Rightarrow X \subseteq A \Rightarrow X \subseteq B$ (since $A \subseteq B$)

$\Rightarrow X \in P(B)$

$\therefore P(A) \subseteq P(B)$.

Conversely $\Leftarrow$ Assume that $P(A) \subseteq P(B)$ and $Y \subseteq A$

$\Rightarrow Y \in P(A) \subseteq P(B)$

$\Rightarrow Y \in P(B)$

$\Rightarrow Y \subseteq B$

$\therefore A \subseteq B$.

② $P(A) \cap P(B) = P(A \cap B)$

Proof:- Let $Y \in P(A) \cap P(B) \Leftrightarrow Y \in P(A)$ and $Y \in P(B)$

$\Leftrightarrow Y \subseteq A$ and $Y \subseteq B \Leftrightarrow Y \subseteq (A \cap B)$

$\Leftrightarrow Y \in P(A \cap B)$.

$\therefore P(A) \cap P(B) \subseteq P(A \cap B)$ and $P(A \cap B) \subseteq P(A) \cap P(B)$

$\therefore P(A) \cap P(B) = P(A \cap B)$.

③ $P(A) \cup P(B) \subseteq P(A \cup B)$

Proof:- Let $Y \in P(A) \cup P(B)$

$\Rightarrow Y \in P(A)$ or $Y \in P(B)$

$\Rightarrow Y \subseteq A$ or $Y \subseteq B$

$\Rightarrow Y \subseteq (A \cup B)$

$\Rightarrow Y \in P(A \cup B)$.

$\therefore P(A) \cup P(B) \subseteq P(A \cup B)$

مهم

هل ان العكس صحيح :. الجواب كلا

EX.:- $A = \{a, b\}$, $B = \{c\}$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\emptyset, \{c\}\}$

$(A \cup B) = \{a, b, c\}$

$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c\}\}$

$\therefore P(A \cup B) \nsubseteq P(A) \cup P(B)$ مع التقاطع يتحقق الاتجاهين

④ $P(A) = P(B) \Leftrightarrow A = B$

proof:- let $X \subseteq A \Rightarrow X \in P(A)$

since $P(A) = P(B)$

$\Rightarrow X \in P(B) \Rightarrow X \subseteq B$

$\Rightarrow A \subseteq B$ ---- ①

let $Y \subseteq B \Rightarrow Y \in P(B)$

since $P(A) = P(B)$

$\Rightarrow Y \in P(A) \Rightarrow Y \subseteq A$

$\Rightarrow B \subseteq A$ ---- ②

from ① and ② we get $A = B$.

$\Leftarrow$ let $X \in P(A) \Rightarrow X \subseteq A$

since $A = B$

$\Rightarrow X \subseteq B \Rightarrow X \in P(B)$

$\Rightarrow P(A) \subseteq P(B)$ ---- ①

Now, let $Y \in P(B) \Rightarrow Y \subseteq B$

since $A = B \Rightarrow Y \subseteq A$

$\Rightarrow Y \in P(A)$

$\therefore P(B) \subseteq P(A)$ ---- ②

from ① and ② we get $P(A) = P(B)$.

5 $P(A) \cap P(B) = \emptyset$ iff $A \cap B = \emptyset$

Proof:- Let $A \cap B = \emptyset$ and assume that

$P(A) \cap P(B) \neq \emptyset$, then $\exists D \in P(A) \cap P(B)$

$\Rightarrow D \in P(A)$ and $D \in P(B)$

$\Rightarrow D \subseteq A$ and $D \subseteq B$

$\Rightarrow D \subseteq A \cap B$

since $A \cap B = \emptyset$

$\therefore D = \emptyset$

$\therefore P(A) \cap P(B) = \emptyset$

$\Longleftarrow$ suppose that $P(A) \cap P(B) = \emptyset$ and $A \cap B \neq \emptyset$

$\Rightarrow \exists Y \subseteq A \cap B \Rightarrow Y \subseteq A$ and $Y \subseteq B$

$\Rightarrow Y \in P(A)$ and $Y \in P(B)$

$\Rightarrow Y \in P(A) \cap P(B) = \emptyset$

$\therefore Y = \emptyset$

therefore $A \cap B = \emptyset$.