The logic المنطق

$F \vee F = F$, $T \wedge T = T$

let p and q be two statements. Then

① $\vee$ (or أو) ② $\wedge$ (and و) ③ $\sim$ negation نفي

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	$\sim p$
T	F
F	T

Ex.:- Find the truth table of these statements
(truth: الصحيح, table: جدول, statements: تعبيرات)

① $(p \vee q) \wedge \sim p$ ② $(p \wedge q) \vee \sim p$ H.w

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

عدد الاحتمالات هو 2^n (حيث n هو عدد المتغيرات)

④ If p then q ($p \Rightarrow q$)

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

من يتشابهون T ومن يختلفون يعتمد على النهاية الطرف الذي يذهب اليه السهم

⑤ p iff q ($p \Leftrightarrow q$)

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

من يتشابهون T
ومن يختلفون F

EX.:- Write out the truth tables for each of the staments :- (14)

H.W

① $p \vee q \Rightarrow \sim p$ ② $(p \wedge \sim q) \Rightarrow (q \vee r)$.

③ $(p \wedge q \wedge r) \Rightarrow (p \vee q) \wedge r$ ④ $\sim(\sim p) \Leftrightarrow p$.

⑤ $\sim(p \Rightarrow q) \Leftrightarrow (p \wedge \sim q)$ ⑥ $(p \wedge (p \Rightarrow q)) \Rightarrow q$.

⑦ $[p \Rightarrow (p \wedge r)] \Leftrightarrow [(p \Rightarrow q) \wedge (p \Rightarrow r)]$.

⑧ $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$.

⑨ $[(q \Rightarrow r) \wedge (p \Rightarrow q)] \Rightarrow (p \Rightarrow r) \equiv I$.

⑩ $p \rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$.

⑪ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

⑫ $p \Longrightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$. ⑬ $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

تطابق
Equivalent
$\equiv$

EX.:- Show that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$ *	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$ *
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Ex. :- Show that

$$[(q \Rightarrow r) \wedge (p \Rightarrow q)] \Rightarrow (p \Rightarrow r) \equiv I$$ (معناها الكل T)

p	q	r	① $q \Rightarrow r$	② $p \Rightarrow q$	$p \Rightarrow r$	① $\wedge$ ②	① $\wedge$ ② $\Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	F	T
T	F	F	T	F	F	F	T $\equiv I$
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Ordered pairs cartesian products:-

Definition:- The order pair الزوج المرتب

let a and b elements, the ordered pair (a,b) is defined to the set:-

$$\left.\begin{array}{l}(a,b)=\{\{a\},\{a,b\}\}\\(b,a)=\{\{b\},\{a,b\}\}\end{array}\right\}\ (a,b)\neq(b,a)$$

Theorem: If $(a,b)=(c,d)$, then $a=c$ and $b=d$.

$$(2,3)=\left(\frac{4}{2},\frac{9}{3}\right)\quad,\quad(5,3)=\left(\frac{10}{2},\frac{9}{3}\right)$$

Def.:- The Cartesian Products الضرب الديكارتي

The cartesian product of two sets A and B, is the set of all ordered pairs (x,y), where $x\in A$, $y\in B$

$$A\times B=\{(x,y): x\in A \text{ and } y\in B\}$$
$$B\times A=\{(y,x): y\in B \text{ and } x\in A\}$$

EX.:- let $A=\{1,2,3\}$, $B=\{c,d\}$

$$A\times B=\{(1,c),(1,d),(2,c),(2,d),(3,c),(3,d)\}$$
$$B\times A=\{(c,1),(c,2),(c,3),(d,1),(d,2),(d,3)\}$$
$$\Rightarrow A\times B=B\times A$$

EX:- $A=\{a,b,c\}$, $B=\{\}=\emptyset$

$$A\times B=\emptyset=\{\}$$

Th. For all sets A, B and C, then

① $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

② $A\times(B\cup C)=(A\times B)\cup(A\times C)$. H.W

③ $(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$.

④ $(A\times A)\cap(B\times C)=(A\cap B)\times(A\cap C)$. H.W

⑤ $(A\times B)-(C\times C)=[(A-C)\times B]\cup[A\times(B-C)]$. H.W

⑥ $(A\times A)-(B\times C)=[(A-B)\times A]\cup[A\times(A-C)]$.

Proof (1):- let $(x,y)\in A\times(B\cap C)$

$\Leftrightarrow x\in A$ and $y\in(B\cap C)$

$\Leftrightarrow x\in A$ and $y\in B$ and $y\in C$

$\Leftrightarrow x\in A$ and $x\in A$ and $y\in B$ and $y\in C$

$\Leftrightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y\in C)$

$\Leftrightarrow (x,y)\in A\times B$ and $(x,y)\in A\times C$

$\Leftrightarrow (x,y)\in(A\times B)\cap(A\times C)$

$\therefore A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ --- (a)

$(A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$ --- (b)

From (a) and (b) we get

$A\times(B\cap C)=(A\times B)\cap(A\times C)$.