The MAPLE BOOK

The MAPLE BOOK

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1. GETTING STARTED

1.1 Starting a MAPLE session

On most systems a MAPLE session is started by double clicking on the MAPLE icon  . In the UNIX X Windows version, MAPLE is started by entering the command `xmaple`. In the command-line (tty) version, the Maple logo appears followed by the > prompt.

In most versions a window with menus will appear. See Figure 1.1 below. At the top are the menus File, Edit, View, Insert, Format, Spreadsheet, Options, Window and Help. Beneath are two rows of buttons. The first row of buttons is called the *tool bar* and contains 24 buttons:

 Create a new worksheet.

 Open an existing worksheet.

 Open a specified URL.

 Save the active worksheet.

 Print the active worksheet.

 Cut the selection to the clipboard.

 Copy the selection to the clipboard.

 Paste the clipboard contents into the current worksheet.

 Undo the last operation.

 Undo the last "undo."

 Insert MAPLE commands.

 Insert text.

 Insert a new MAPLE input region after the cursor.

 Remove any section enclosing the selection.


Enclose the selection in a subsection.


Go back in the hyperlink history.


Go forward in the hyperlink history.


Interrupt the current computation.


Set magnification to 100%.


Set magnification to 150%.


Set magnification to 200%.


Display nonprinting characters.


Resize the active window to fill the available space.


Restart.

The next row is called the *context bar* and contains five buttons:


Toggle the expression display between mathematical and MAPLE notation.


Toggle the expression display between inert text and executable MAPLE command.


Auto-correct the expression for syntax.


Execute the current expression.


Execute the worksheet.

The > prompt will be in the *worksheet* window. Don't worry about the buttons too much at this stage.

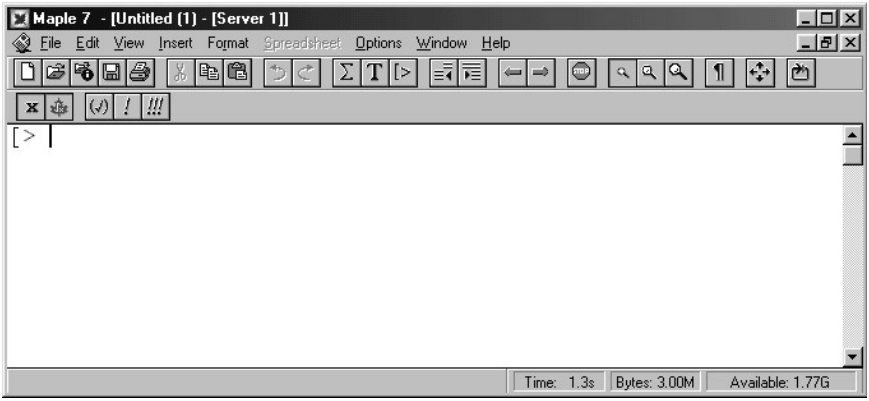


Figure 1.1 MAPLE worksheet window.

1.2 Different versions of MAPLE

The current version of MAPLE is MAPLE 7  . The previous version was MAPLE 6  . Before that, there was MAPLE V Release 5  , MAPLE V Release 4, and way back in 1994, we had MAPLE V Release 3. This book covers MAPLE 7. The change from MAPLE 6 to MAPLE 7 was not a big one so most of the book applies to MAPLE 6. Occasionally we will point out differences between the earlier versions.

1.3 Basic syntax

In MAPLE the prompt is the symbol >. MAPLE commands are entered to the right of the prompt. Each command ends with either ":" or ";". If the colon is used, the command is executed but the output is not printed. When the semicolon is used, the output is printed. Try typing `105/25:` followed by a `Return` (or `Enter`).

```
> 105/25:
```

Observe that the output was not printed. Now type `105/25;`

```
> 105/25;
```

$$\frac{21}{5}$$

Below in Figure 1.2 is a rendering of how this looks in the worksheet window.

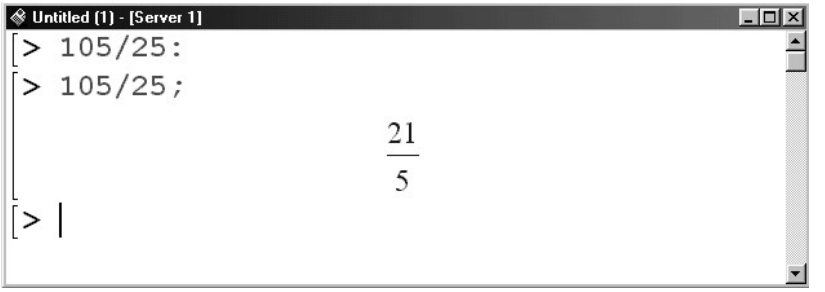


Figure 1.2 MAPLE commands with output.

Try these

```
> 105/25-1/5;
```

$$4$$

```
> %+1/5;
```

$$\frac{21}{5}$$

```
> %%;
```

$$4$$

Observe that MAPLE uses exact arithmetic. The percent sign `%` refers to the previous result. The double percent `%%` refers to the result before the previous result. It is possible to refer back 3 lines using `%%%`, but one cannot refer back any further. The percent sign `%` is called the ditto operator.

Warning: In MAPLE V Release 4 (and earlier versions), the ditto operator was the double quote character `"`. The two double quotes `""` were used to refer to the result before the previous result, and to refer back 3 lines one used triple double quotes `"""`.

One of the most common mistakes is to omit the semicolon or colon.

```
> 105/25
Warning, incomplete statement or missing semicolon
> 105/25;
syntax error, unexpected number
```

Don't panic! MAPLE has interpreted this to mean 105/25 105/25, hence the syntax error. MAPLE also gave a warning about the missing semicolon! If you forget the semicolon, simply type it on the next line.

```
> 105/25
> ;
```

$$21/5$$

See Section 1.3 for a method for editing mistakes.

Results can be assigned to variables using the colon-equals "`:=`".

```
> f:=%;
```

$$f := 21/5$$

```
> G:= -1/5;
```

$$G := -1/5$$

```
> f+g;
```

$$21/5 + g$$

```
> #Observe that Maple is case sensitive.
> f+G;
```

$$4$$

Note that comment lines begin with `#`. In the first line of our session we used the ditto operator `%`. Remember, if you are using MAPLE V Release 4 (or an earlier version), use `"` as the ditto operator.

1.4 Editing mistakes

MAPLE has built-in editing facilities. On most platforms, lines of input can be edited using the arrow keys and the mouse. Cutting and pasting is also possible with the mouse. In the Windows version, you can select input by highlighting with the mouse, then you can copy, cut, and paste by using `Control C`, `x`, and `v` as usual. In the command-line (or tty) version, MAPLE has two built-in editors: *emacs* and *vi*. Use the help command `?editing` for more information.

```
> 105/25
> 105/25;
syntax error, unexpected number
```

Just click the mouse after `105/25`, enter a semicolon, and press enter.

```
> 105/25;
```

$$21/5$$

The *vi* editor can be invoked using the `Esc` key.

1.5 Help

Ever since MAPLE V (Release 4) came out, MAPLE has had a fabulous

interactive help facility. Click on Help and a menu should appear:

Introduction	
Help on Context	Ctrl+F1
New User's Tour	
What's New	
Using Help	
Glossary	
Topic Search ...	
Full Text Search ...	
History ...	
Save to Database ...	
Remove Topic ...	
Balloon Help ...	
Register Maple 7 ...	
About Maple 7 ...	

Select Full Text Search. A little window should appear. In the Word(s) box, type `floating point arithmetic` then click on Search. A search is then made of the interactive help manual. A list of topics should appear in the Matching Topics box. See Figure 1.3.

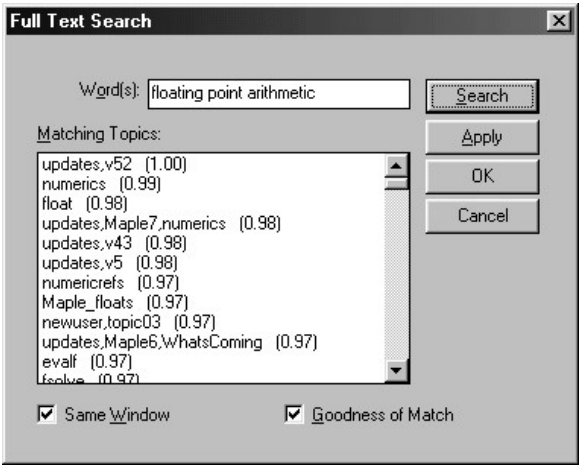


Figure 1.3 Full Text Search window.

Select `evalf` with the mouse, then click on Apply. A help window should appear with information on the `evalf` command. Click on OK.

Now go back to the Help menu and select Introduction. A new window should appear offering you a list of topics to explore.

If you know the name of a command, then you can select Topic Search in the Help menu.

To return to our original worksheet window, click on Window and select Untitled(1)-Server(1).

Help can also be accessed directly from the worksheet. Try

```
> ?evalf
```

The `evalf` help window should appear. In the command-line version, this information will appear below the cursor.

Now try selecting Balloon Help in the Help menu. Next move the cursor onto a button and a little bubble should appear, giving a brief description. Keep this option until you are familiar with the buttons and menus.

The command `?index` provides a list of categories: expression, function, misc, module, etc. For instance, `?index[function]` gives a list of MAPLE's standard library functions. For more information on navigating through the worksheet environment, see `?worksheet`.

1.6 A sample session and context menus

Open a new worksheet by pressing  . Enter the following into the worksheet:

```
> Int(x/sqrt(1+x^4),x);
```

and hit return after you type ";". You should have something like this:

```
> Int(x/sqrt(1+x^4),x);
```

$$\int \frac{x}{\sqrt{1+x^4}}\,dx$$

The `Int` function is for calculating integrals. More information can be found in Section 5.7. Now click on the integral (above) with the right mouse button. A menu should appear:

Copy	
Differentiate	▶
Integrate	▶
Evaluate	
Complex Maps	▶
Integer Functions	▶
Simplications	▶
Conversions	▶
Plots	▶

This menu is called a *context menu*. When you click on MAPLE output, such a menu will appear. It won't always be the same menu. The menu depends on the type of object you click, hence the name context menu. Now select

`Differentiate` and click on x. Magically MAPLE has taken the derivative with respect to x:

```
> Int(x/sqrt(1+x^4),x);
```

$$\int \frac{x}{\sqrt{1+x^4}}dx$$

```
> R0 := diff(Int(x/sqrt(1+x^4),x),x);
```

$$R0 := \frac{x}{\sqrt{1+x^4}}$$

Naturally, MAPLE found that

$$\frac{d}{dx}\int \frac{x}{\sqrt{1+x^4}}dx = \frac{x}{\sqrt{1+x^4}}.$$

Now, click on the integral again and this time select `Evaluate` in the contex menu. This time MAPLE evaluates the integral:

```
> Int(x/sqrt(1+x^4),x);
```

$$\int \frac{x}{\sqrt{1+x^4}}dx$$

```
> R1 := value(Int(x/sqrt(1+x^4),x));
```

$$R1 := \frac{1}{2}\, arcsinh(x^2)$$

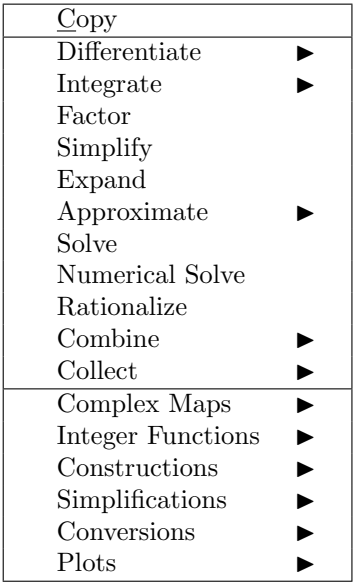
```
> R0 := diff(Int(x/sqrt(1+x^4),x),x);
```

$$R0 := \frac{x}{\sqrt{1+x^4}}$$

MAPLE found that

$$\int \frac{x}{\sqrt{1+x^4}}dx = \frac{1}{2}\sinh^{-1} x^2.$$

Click on the output with name `R0`, and a different context menu will appear:



Select `Plots` and press 2-D Plot. MAPLE produces a graph of the function $y = \frac{x}{\sqrt{1+x^4}}$. See Figure 1.4.

```
> smartplot(R0);
```

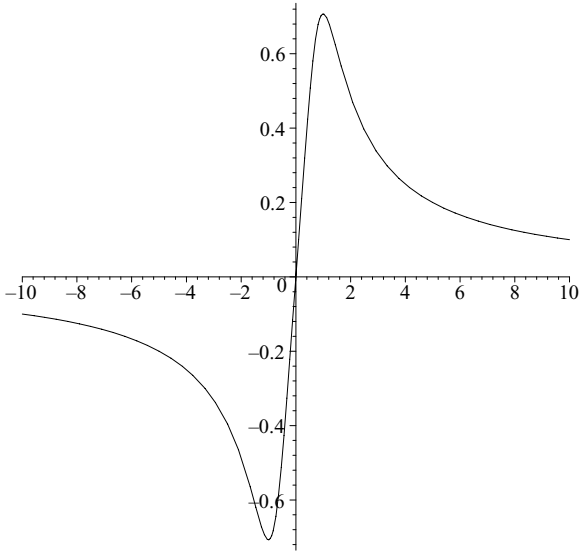


Figure 1.4 A smartplot.

We will learn a lot more about plotting in Chapter 6.

Before going on we should save our work. Click on File and a menu appears:

	New	Ctrl+o
	Open ...	
	Save	Ctrl+s
	Save As ...	
	Export As	▶
	Close	Ctrl+F4
	Save Settings	
✓	AutoSave Settings	
	Print ...	Ctrl+P
	Print Preview ...	
	Print Setup ...	
	Exit	Alt+F4

Click on Save. A **Save As** window appears. In the File name box type `ch1a.mws`. Then click on OK. The worksheet has been saved as the file *ch1a.mws*. Here *mws* is a file type which stands for MAPLE worksheet.

1.7 Palettes

So far we have seen how to enter MAPLE commands by typing after the MAPLE prompt >, and by using a context menu. Another method is to use palettes. Open a new worksheet by pressing  . Now click on View and a menu appears:

✓	Toolbar	
✓	Context Bar	
✓	Status Bar	
	Palettes	▶
	Zoom Factor	▶
	Bookmarks	▶
	Back	
	Forward	
	Hide Content	▶
	Show Invisible Characters	
✓	Show Section Ranges	Shift+F9
✓	Show Group Ranges	F9
	Show Object type	
	Expand All Sections	
	Collapse All Sections	

Select Palettes, slide to the right, and another menu appears:

Symbol Palette
Expression Palette
Matrix Palette
Vector Palette
Show All Palettes
Hide All Palettes

In MAPLE 7 there are four palettes: the **Symbol** palette, the **Expression** palette, the **Matrix** palette, and the **Vector** palette. In Chapter 9 we will use the **Matrix** and **Vector** palettes. For the time being let's select `Expression Palette`. A window should appear. See Figure 1.5.

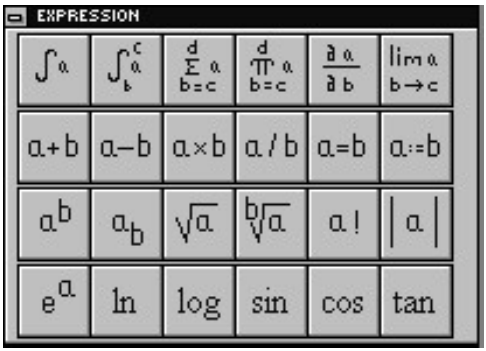


Figure 1.5 The **Expression** palette.

Let's start with something simple. In the **Expression** palette press a−b.

```
> (%? - %?);
```

MAPLE has produced a template for an expression of the form $(a - b)$. Notice %?. Now type `105/25`.

```
> (105/25 - %?);
```

Notice that 105/25 has been entered where %? was. Now hit the `Tab` key.

```
> (105/25 - %?);
```

MAPLE is now waiting for us to type the second number. We type `1/5`.

```
> (105/25 - 1/5);
```

We hit `Return` (or `Enter`):

```
> (105/25 - 1/5);
```

4

Do you see how the **Expression** palette works? Many other types of expressions can be entered in this way. You should be able to figure out the possible expressions by looking at the buttons in the palette. Try each button and experiment.

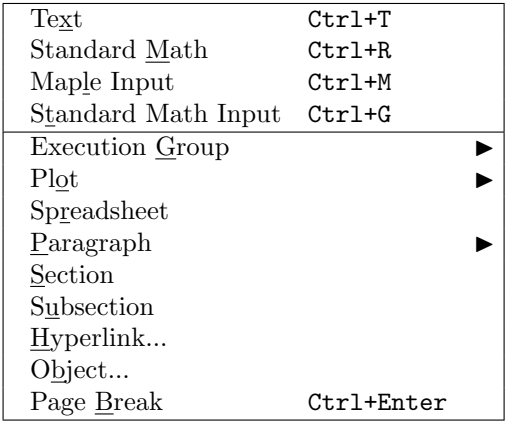
To open the **Symbol** palette, click on View, select Palettes, slide right, and select Symbol Palette. See Figure 1.6.

Figure 1.6 The **Symbol** palette.

The **Symbol** palette is used for entering Greek letters and some mathematical constants such as e and π. Try out some of the buttons.

1.8 Spreadsheets

Click on Insert. A menu should appear:



Select Spreadsheet. A spreadsheet appears in the worksheet:

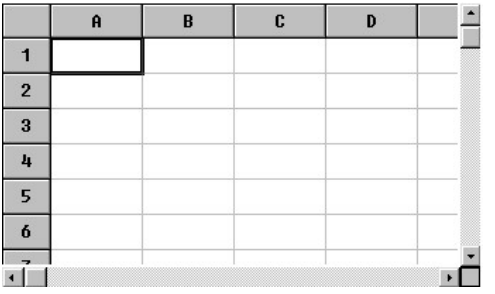


Figure 1.7 A MAPLE spreadsheet.

Notice that the upper left-most cell (A1) is highlighted. There are four new buttons in the context bar:


Fill a range of cells.


Evaluate all stale cells.


Accept the input and evaluate it.


Restore input to the previous value.

Type `n` and press enter. The symbol n should appear in cell A1. In cell A2 type `1` and press enter. Now click on cell A2 and select the first column of cells up to cell A9 by holding the mouse button down. Now click on  . A **Fill** window should appear. Enter `1` for Step Size and press OK. The numbers 2, 3, ... , 8 should appear in cells A3, A4, ... , A9. Type `x^n-1` in cell B1. We now have $x^n - 1$ in cell B1. This is good, but we want to change it. Click on cell B1. Notice that `x^n-1` is in the edit field (the box to the right of the new buttons). Backspace over it and type `x^(~A1) - 1`. We still get $x^n - 1$ in cell B1. What is going on? Here `~A1` refers to value in cell A1 which is n, so that the value of cell B1 is linked to that of A1. We want to put $x^n - 1$ with $n = 1, 2, \ldots 8$ in the second column. Click on Spreadsheet. A menu should appear:

	Evaluate Selection	
	Evaluate Spreadsheet	
	Row	▶
	Column	▶
	Fill	▶
	Import Data	▶
	Export Data	▶
	Properties...	
✓	Show Border	
	Resize To Grid	

Select Fill, slide right, and select Down. Did you get the polynomials $x-1$, x^2-1, ... , $x^8 - 1$ in column B? You will probably want to resize the spreadsheet. Click in the bottom right corner, hold the mouse button down, and stretch the spreadsheet so you can see all the entries. Now we want to factor the polynomials in column B. Enter `factored polynomial` in cell C1. In cell C2 enter `factor(~B2)`. Select the column of cells C2, C3, ... , C9. From the Sreadsheet menu select Fill and then Down. Did you get the desired effect? You

should now have a table:

n	$x^n - 1$	*factored polynomial*
1	$x - 1$	$x - 1$
2	$x^2 - 1$	$(x-1)(x+1)$
3	$x^3 - 1$	$(x-1)(x^2+x+1)$
4	$x^4 - 1$	$(x-1)(x+1)(x^2+1)$
5	$x^5 - 1$	$(x-1)(x^4+x^3+x^2+x+1)$
6	$x^6 - 1$	$(x-1)(x+1)(x^2+x+1)(x^2-x+1)$
7	$x^7 - 1$	$(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)$
8	$x^8 - 1$	$(x-1)(x+1)(x^2+1)(x^4+1)$

For more information on MAPLE spreadsheets see `?worksheet,spreadsheet`. For programmers there is a spreadsheet package called *Spread*. See Section 17.7.19.

1.9 Quitting MAPLE

If you are done with your MAPLE session, click on  . The **Save As** window should appear. In the File name box type `ch1.mws` and click on OK. Your worksheet has now been saved. To quit MAPLE, go to the File menu and select Exit. Later you can reopen your worksheet by clicking on  .

In the command-line version, the easiest way to quit a Maple session is to use `quit`.

```
> quit
```

2. MAPLE AS A CALCULATOR

2.1 Exact arithmetic and basic functions

As we noted in Section 1.3, MAPLE does exact arithmetic. Also, MAPLE does integer arithmetic to infinite precision. Try the following examples:

```
> 2/3 + 3/5;
```

$$\frac{19}{15}$$

```
> 7 - 11/15;
```

$$\frac{94}{15}$$

```
> 12^20;
```

$$383375999244747512217 6$$

The basic arithmetic operations in MAPLE are

$+$	addition
$-$	subtraction
$*$	multiplication
^or $**$	exponentiation
$/$	division

MAPLE also has the basic mathematical functions (and much more) that are available on a scientific calculator.

`abs(x)`	absolute value	$\|x\|$
`sqrt(x)`	square root	$\sqrt{x}$
`n!`	factorial	
`sin(x)`	sine	
`cos(x)`	cosine	
`tan(x)`	tangent	
`sec(x)`	secant	
`csc(x)`	cosecant	
`cot(x)`	cotangent	
`log(x)`	natural logarithm	
also `ln(x)`		
`exp(x)`	exponential function	e^x
`sinh(x)`	hyperbolic sine	
`cosh(x)`	hyperbolic cosine	
`tanh(x)`	hyperbolic tan	

MAPLE has many other built-in mathematical functions. For instance, it has the inverse trig functions (`arcsin`, `arccos`, etc.), the Bessel functions (`BesselI`),

the Riemann zeta function (`Zeta`), the gamma function (`GAMMA`), and the complete and incomplete elliptic integrals (`EllipticE`). For a complete listing, see `?index[functions]` or Section 15.1.

2.2 Floating-point arithmetic

MAPLE can do floating-point calculation to any required precision. This is done using `evalf`.

```
> tan(Pi/5);
```

$$\sqrt{5-2\sqrt{5}}$$

```
> evalf(%);
```

$$0.7265425273$$

Notice that `evalf` found $\tan(\pi/5)$ to 10 decimal places, which is the default. Also, note that in MAPLE, π is represented by `Pi`. There are two ways to change the default and increase the number of decimal places.

```
> E := exp(1);
```

$$E := \mathbf{e}$$

```
> evalf(E,20);
```

$$2.7182818284590452354$$

```
> Digits := 30;
```

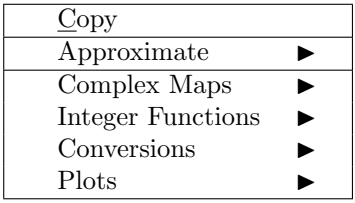
$$30$$

```
> evalf(E);
```

$$2.71828182845904523536028747135$$

Here `E` is the mathematical constant e, which is represented in MAPLE by `exp(1)`. We found e to 20 digits using `evalf(E,20)`. The other method is to use the global variable `Digits` (whose default value is 10). After assigning `Digits := 30`, we found e correct to 30 digits simply by calling `evalf(E)`.

We can also find an approximation using a context menu (see Section 1.6). Right-click on **e** which is the output of `E := exp(1)`. A context menu appears:



Select `Approximate` and press `20`.

```
> E := exp(1);
```

$$E := \mathbf{e}$$

```
> R0 := evalf(E,20);
```

$$R0 := 2.7182818284590452354$$

Under this context menu the number of digits can be 5, 10, 20, 50, or 100.

We reset the default and calculate $\sin(\pi/6)$.

```
> Digits := 10:
> evalf(sin(Pi/6));
```

$$0.5000000000$$

```
> convert(%,rational);
```

$$\frac{1}{2}$$

Notice that after we found the decimal approximation, we were able to convert it into an exact rational using `convert(%,rational)`. The `convert` function is used to convert expressions from one type to another. More on the `convert` function is to be found in section 4.6. The interested reader can find more using `?convert`. Below is a table of MAPLE's built-in mathematical constants.

`Catalan`	Catalan's constant (about .9159655942)
`gamma`	Euler's constant (about 0.5772156649)
`I`	complex number i ($i^2 = -1$)
`Pi`	π (about 3.141592654)