

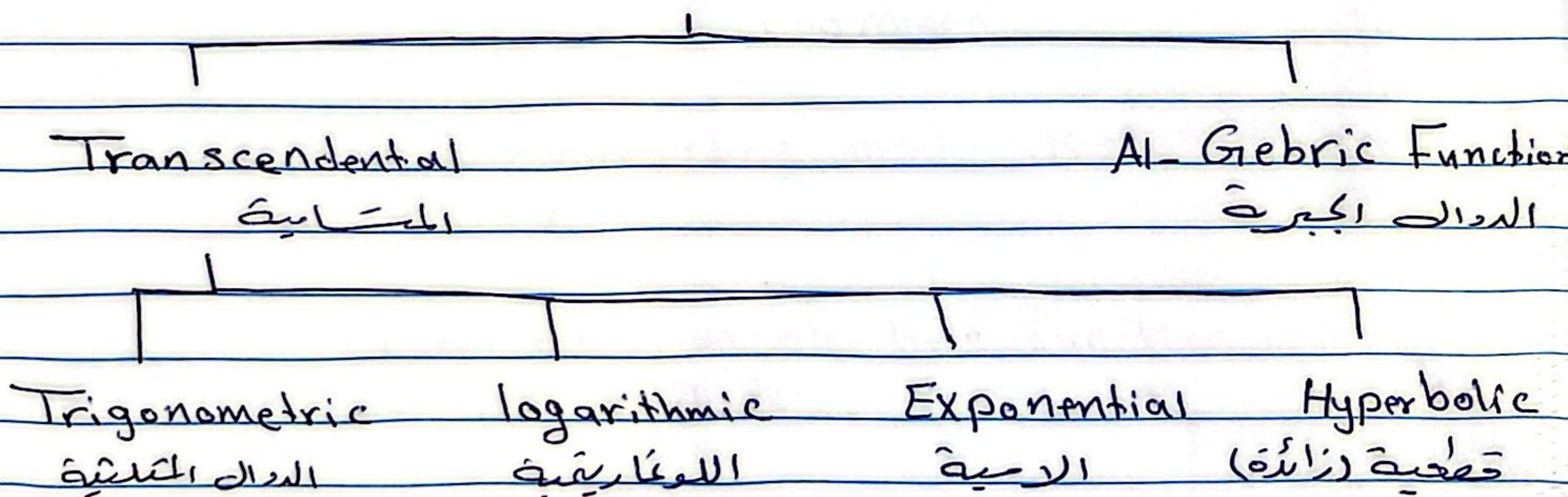
Functions

Def.: A Function of is a rule that associates a unique output with each input, is denoted by x , then the output is denoted by:

$$y = f(x)$$

Def.: If a variable y depends on a variable x is such a way that each value of x determines exactly one value of y , then we say that y is a function of x

The Functions are divide into :-



1. Al-gebric Functions Including..

Domain, Range and there Graphs

If $y = f(x)$, then the set of all possible input (x -values) is called the domain of f , and the set of output (y -values) that result when x varies over the domain is called the range of f .

المجال هو جميع قيم المتغير x التي تُولد الدالة $f(x)$.
المجال هو كل قيم المتغير y الناتج من الدالة.

١. إذا كانت الدالة متصلة حدوداً بالمنطق لا دائماً هو مجموعة الأعداد الحقيقية R .
٢. إذا كانت الدالة كسرية عند المنطق لا مجموعة الأعداد الحقيقية R على القيم التي تجعل المقام يساوي صفر.
٣. إذا كانت الدالة جذرية عند المنطق لا مجموعة الأعداد الحقيقية R على القيم التي تجعل المقدار تحت الجذر سالب.



Ex

Find the domain, range and sketch their graphs $y = 3x$

Soln This is an equation of line function, so x can take all numbers, i.e.:

$$D_y = \{x : x \in R\} = (-\infty, \infty)$$

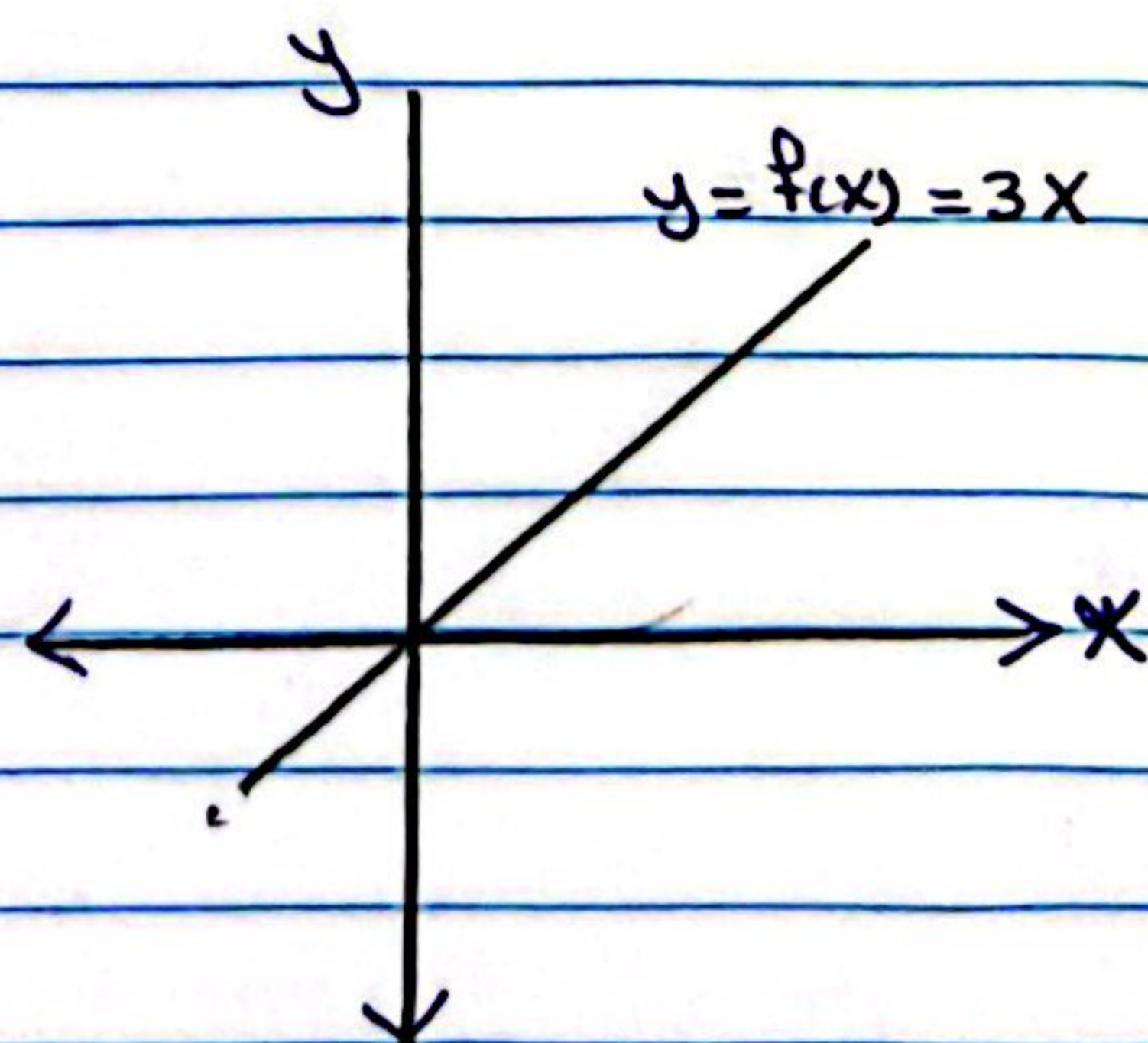
to find the range, we solve:

$$y = 3x \Rightarrow x = \frac{1}{3}y$$

Also, it is an equation of linear function, so:

$$R_y = \{y : y \in R\} = (-\infty, \infty)$$

Since $f(-x) = 3(-x) = -3x = -f(x) \Rightarrow$ odd function



Ex $f(x) = x^2$

Soly This is a polynomial function, So x can be take all numbers..

$$Dy = \{x : x \in \mathbb{R}\} = (-\infty, \infty)$$

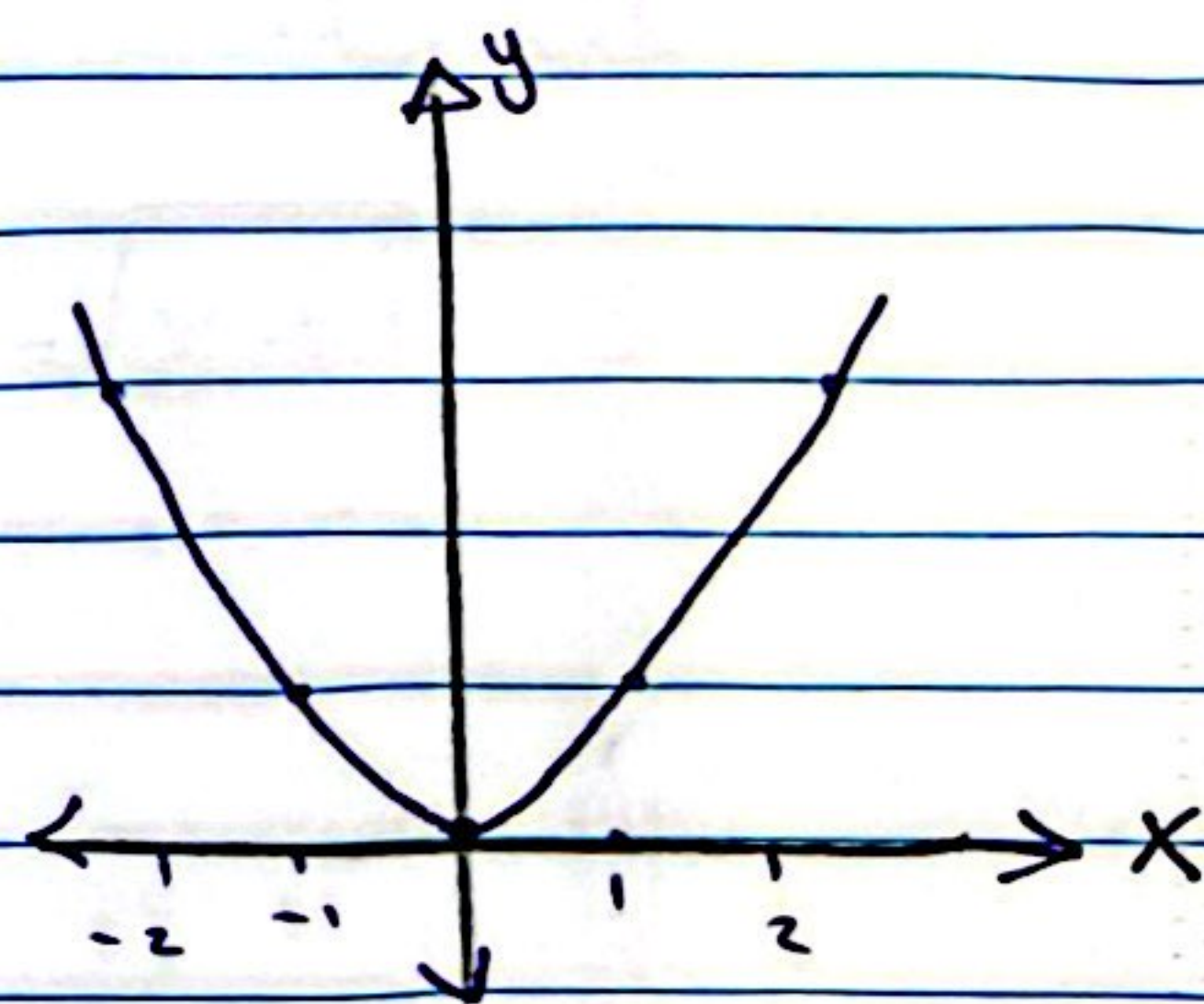
to find the range, we solve : $y = x^2 \Rightarrow x = \pm\sqrt{y}$

This is root function, so we must avoid the numbers then make the root undefined :-

x :	-3	-2	-1	0	1	2	3
y :	9	4	1	0	1	4	9

$$0 \leq \sqrt{y} \Rightarrow 0 \leq y$$

$$Ry = \{y : y \in \mathbb{R} : y \geq 0\}$$



This function is even, because :

$$f(-x) = (-x)^2 = x^2 = f(x)$$

Typ of Al-Gebric Function are:-

1. Multinomial (Polynomial): is

$$f(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \dots$$

for example

$$f(x) = x^2 + 4x + 2 \quad ; \quad f(x) = \frac{1}{2} x^3 + \frac{1}{4} x^2 + \frac{1}{3}$$

$$y = x^5 + 2$$

denominator zero

مقام صفر

$$(\sqrt[3]{x-1} - 2 = 0)^3$$

$$x-1 = 8 \Rightarrow x=9$$

$$\therefore D_y = \{x : x \in \mathbb{R} ; x \neq 9\}$$

Ex Find the domain of the function:-

$$f(x) = \frac{2x-1}{x^2-9}$$

Solu

$$x^2 - 9 = 0$$

$$x^2 = 9 \Rightarrow x = \pm 3$$

$$\therefore D_y = \{x : x \in \mathbb{R}, x \neq \pm 3\}$$

Special Kind of Functions

أنواع خاصة من الدوال

1. Fixed Function

It is a function in which the corresponding field is a constant, for example is:-

هي الدالة التي يكون فيها المجال ثابتاً

$$y = f(x) ; y = k \rightarrow \text{constant}$$

$$y = 27 ; y = 5 ; \text{ and so on}$$

2. Identity Function

It is a function in which the range is equal to the domain (Domain = Range)

هذه الدالة التي يكون فيها المدخل مساوي للخارج
 For example: (دائري ان قيم x هي نفس قيم y)
 $f(x) = x$; $f(0) = 0$, $f(1) = 1$ and so on

3. Absolute Function

$$y = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

4. Integer Function

$$y = \lfloor x \rfloor \quad \text{for example.}$$

$$y = f(3.1) = \lfloor 3.1 \rfloor = 3$$

$$f(-1.6) = \lfloor -1.6 \rfloor = -2$$

5. Odd and Even Functions

we say that $f(x)$ is even function if:-

$$f(-x) = f(x) \quad ; \quad \text{The opposite, it is odd}$$

Ex $f(x) = \frac{x^2}{x^4 + 1}$

Solu $f(-x) = \frac{(-x)^2}{(-x)^4 + 1} = \frac{x^2}{x^4 + 1} = f(x)$

$$\therefore f(-x) = f(x)$$

$f(x)$ is even function

$$\underline{\text{Ex}} \quad f(x) = \frac{1}{x} \quad ; \quad f(-x) = -\frac{1}{x}$$

$$\therefore f(x) \neq f(-x)$$

$\therefore f(x)$ is not even ; $f(x)$ is odd function

There are functions that are neither even and odd ; for example:-

$$f(x) = x^2 - 3x + 5$$

$$1. \quad f(-x) = (-x)^2 - 3(-x) + 5 = x^2 + 3x + 5 \neq f(x)$$

$$2. \quad -f(x) = -(x^2 - 3x + 5) = -x^2 + 3x - 5 \neq f(x)$$

$\therefore f(x)$ is not odd and not even

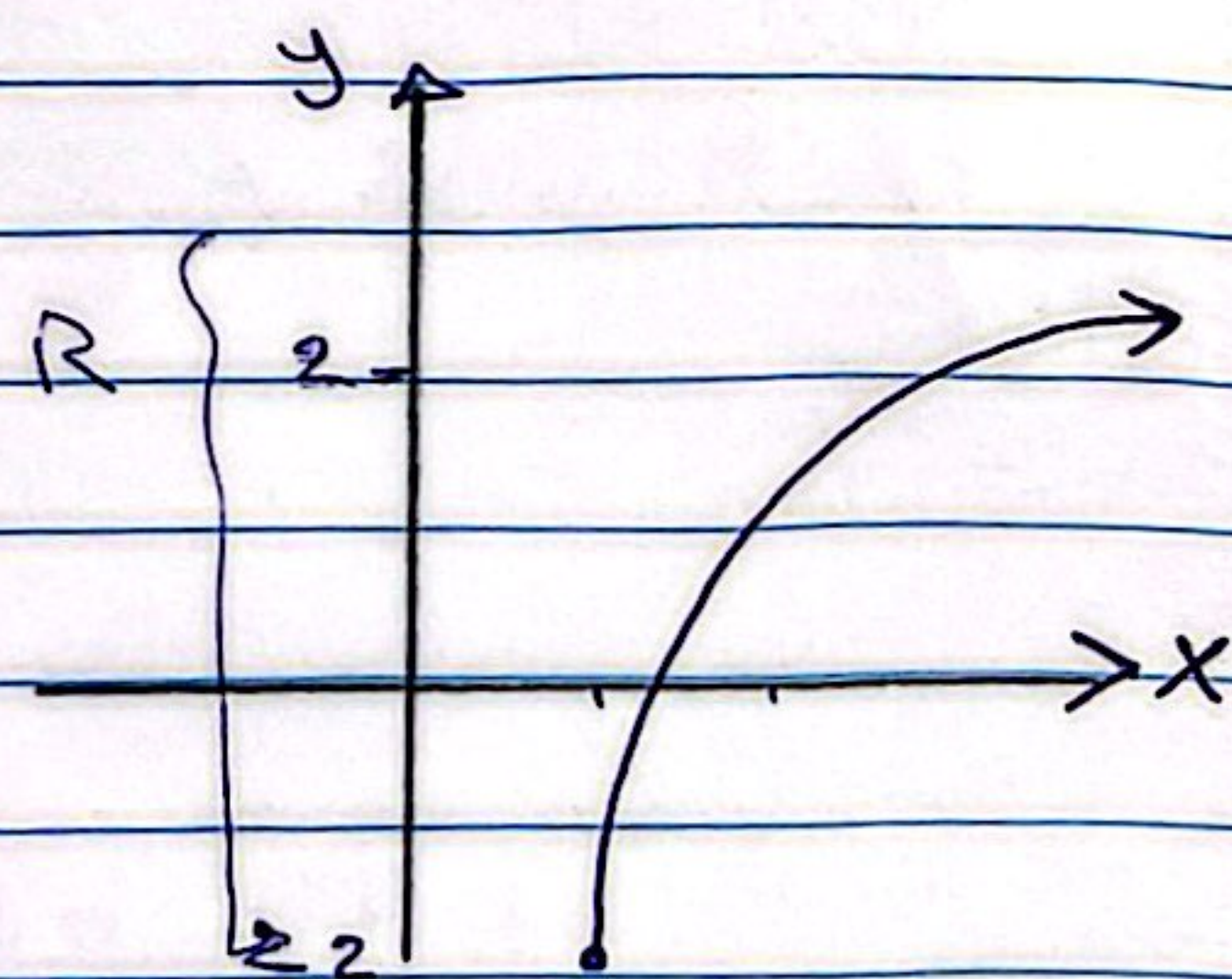
إيجاد المجال والفترة من خلال الرسم وكالاتي:-

Ex Find the range :-

$$1. \quad \text{C'sub} = [-2, \infty)$$

$$R = \{x : x \in \mathbb{R}\} = [-2, \infty)$$

$$\text{or } R = \infty > y \geq -2$$

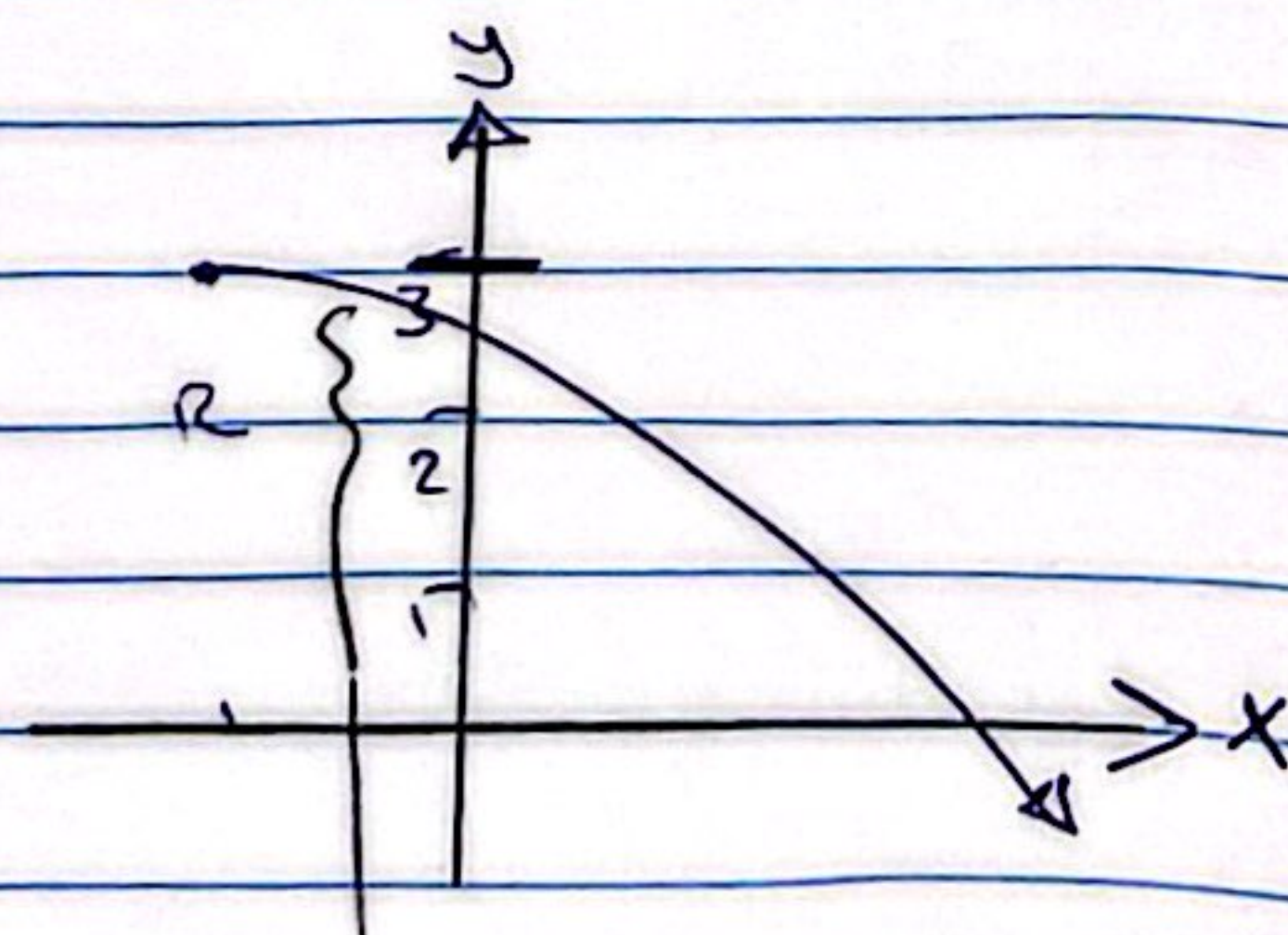


2.

$$\text{C'sub} = (-\infty, 3]$$

$$R = \{x : x \in \mathbb{R}\} = (-\infty, 3]$$

$$\text{or } R = y \leq 3$$

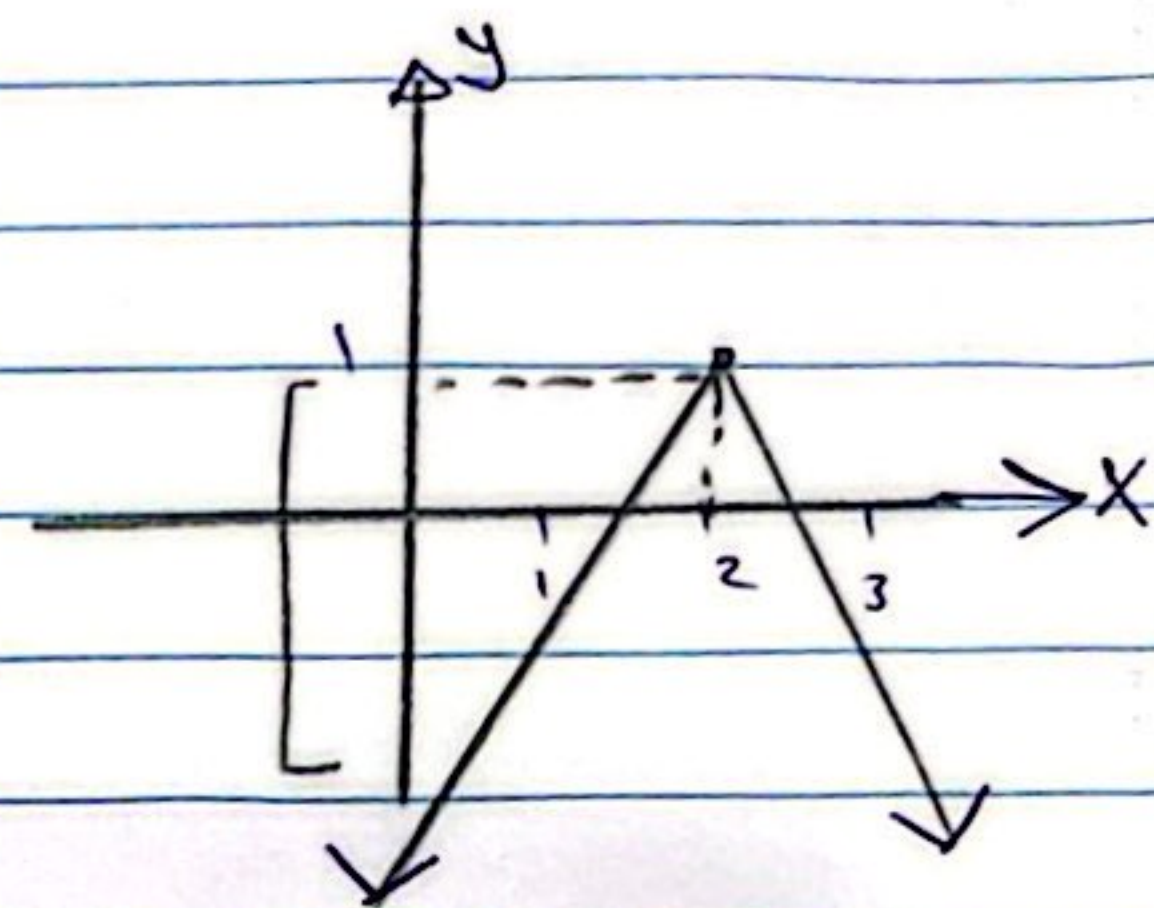


3.

$$D_{\text{subl}} = (-\infty, 1]$$

$$R = \{x : x \in \mathbb{R}\} = (-\infty, 1]$$

$$\text{or } R = y < 1$$

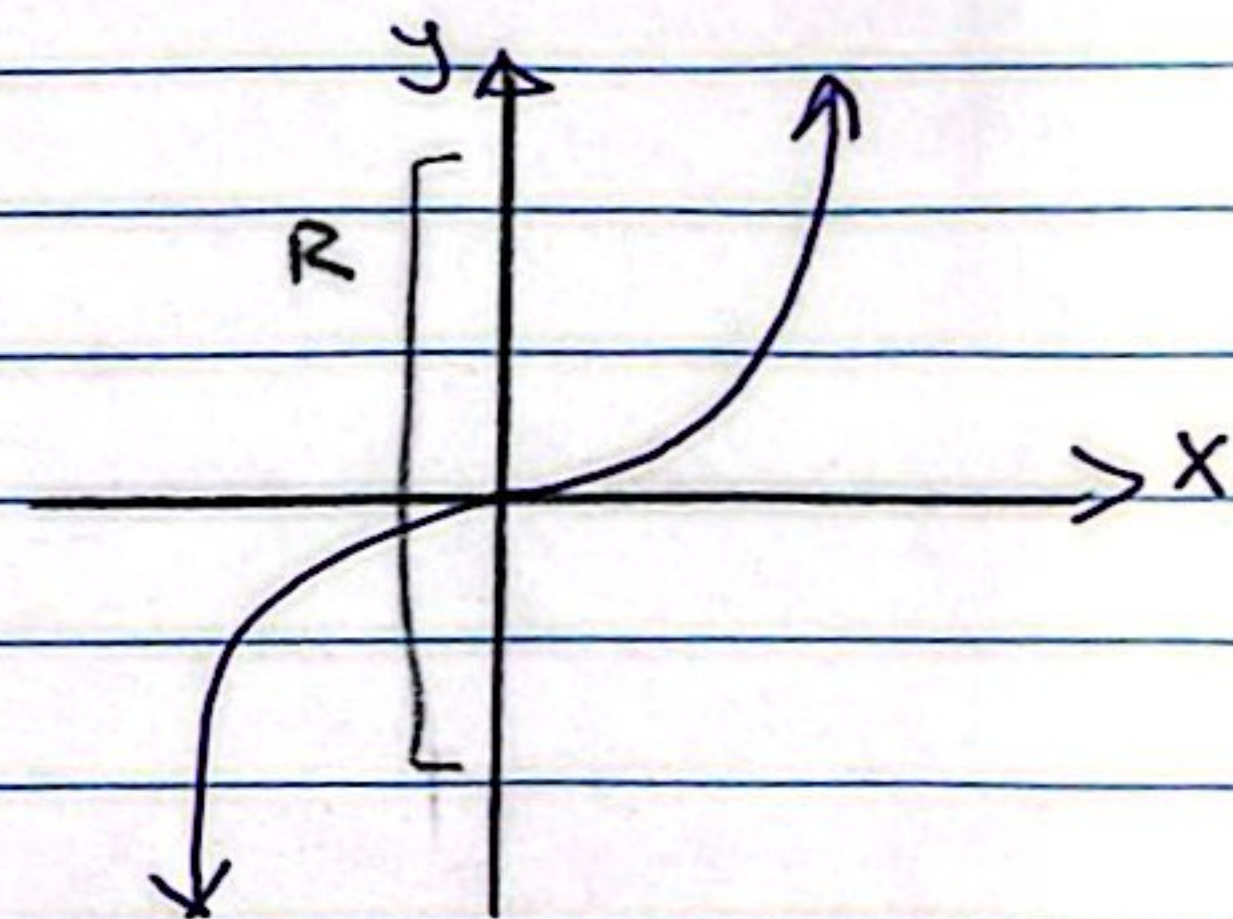


4.

$$D_{\text{subl}} = (-\infty, \infty)$$

$$R = \{x : x \in \mathbb{R}\} = (-\infty, \infty)$$

$$\text{or } y = R$$



Ex Find the Range to:

ایجاد اقلی جبراً

$$1. \quad y = \frac{x+1}{x-2} \quad ; \quad x \neq 2$$

$$D = \mathbb{R} \setminus \{2\}$$

Soly

فی X به علاوه y

$$xy - 2y = x + 1$$

$$xy - x = 2y + 1$$

$$x(y-1) = 2y+1$$

$$x = \frac{2y+1}{y-1}$$

$$; \quad y-1 \neq 0 \Rightarrow y \neq 1$$

$$R = \{y : y \in \mathbb{R} ; 1\}$$

D_{subl}