② Macloureen Series

The general Series :-

$$f(x) = f(a) + \frac{f'(a)\,x}{1!} + \frac{f''(a)\,x^2}{2!} + \frac{f'''(a)\,x^3}{3!} + \dots + \frac{f^n(a)\,x^n}{n!}$$

or

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)\,x^n}{n!}$$

Ex Find the macloreen Series of the function

1. $f(x) = e^x$

$f(x) = e^x \quad ; \quad f(0) = e^0 = 1$

$f'(x) = e^x \quad ; \quad f'(0) = e^0 = 1$

$f''(x) = e^x \quad ; \quad f''(0) = e^0 = 1$

$$\therefore f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

or

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

H.w Find the macloreen series of the function :-

1. $f(x) = \sin x$

Solu

$f(0) = \sin(0) = 0$

$f'(0) = \cos(0) \Rightarrow f'(0) = \cos(0) = 1$

$f''(0) = -\sin(0) \Rightarrow f''(0) = -\sin(0) = 0$

$f^{(3)}(0) = -\cos(0) \Rightarrow f^{(3)}(0) = -1$

$f^{4}(0) = \sin(0) \Rightarrow f^{4}(0) = 0$

$$\because f(x) = f(0) + \frac{f'(0)\,x}{1!} + \frac{f''(0)\,x^2}{2!} + \dots$$

$$= 0 + \frac{x}{1!} + 0 + \frac{(-1)\,x^3}{3!} + 0 + \frac{x^5}{5!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\because \sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

or

$$\therefore \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

2. $f(x) = \dfrac{1}{x^n}$

3. $f(x) = \dfrac{1}{1-x}$; 4. $f(x) = \ln x$

2. $f(x) = \cos x$; $a = 0$

Solu

$f(0) = \cos(0) = 1$

$f'(0) = -\sin(0) = 0$

$f''(0) = -\cos(0) = -1$

$f'''(0) = \sin(0) = 0$

$f''''(0) = -\cos(0) = -1$

$$\because f(x) = f(0) + \frac{f'(0)\,x}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + 0 + \frac{(-1)x^4}{4!} + 0 + \dots + \frac{f^n(0)\,x^{2n}}{2n!}$$

or

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!}$$