Def.:- let $x, y \in \mathbb{Q}$

if $x - y$ is positive then $x > y$.

if $x - y$ is negative then $x < y$.

if $x, y \in \mathbb{Q}$ then $x = y$ or $x > y$ or $x < y$.

Example:- Show that $x^2 = 2$ has no solution for any $x \in \mathbb{Q}$.

Proof:- let $y = \frac{a}{b} \in \mathbb{Q}$, $a, b \in \mathbb{Z}$, $b \neq 0$ s.t. $y^2 = 2$

and suppose that there is no commodivisor of a, b

لا يوجد عامل مشترك بينهم

$$y^2 = 2 \Rightarrow \left(\frac{a}{b}\right)^2 = 2 \Rightarrow \frac{a^2}{b^2} = 2 \Rightarrow a^2 = 2b^2.$$

نأخذ ثلاث احتمالات

1. if $a, b \in \mathbb{Z}_o \Rightarrow a^2 \in \mathbb{Z}_o$, $b^2 \in \mathbb{Z}_o$ but $2b^2 \in \mathbb{Z}_e$

   $\therefore$ odd = even C!

2. if $a \in \mathbb{Z}_o$ and $b \in \mathbb{Z}_e$, $a^2 \in \mathbb{Z}_o$, $b^2 \in \mathbb{Z}_e$

   $2b^2 \in \mathbb{Z}_e$ $\quad\therefore$ odd = even C!

3. if $a \in \mathbb{Z}_e$, $b \in \mathbb{Z}_o$, $b^2 \in \mathbb{Z}_o$

   $a = 2c$, $c \in \mathbb{Z}$, $\therefore a^2 = 4c^2$

   $a^2 = 2b^2 \Rightarrow 2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$

   $\therefore$ odd = even C!

$\therefore y = \frac{a}{b}$ is not root of $y^2 = 2$

$\therefore x^2 = 2$ has no solution for any $x \in \mathbb{Q}$.

* To define the relation between Z and Q.

Sol.:- let $f: Z \longrightarrow Q$, $f(n) = [n,1]$, $n \in Z$.

let $f(n_1) = f(n_2) \Rightarrow [n_1,1] = [n_2,1]$ شرح

$\Rightarrow (n_1,1) R (n_2,1)$

شرح $\Rightarrow n_1 1 = n_2 1 \Rightarrow n_1 = n_2$.

$\therefore f$ is injective.

$\because \operatorname{ran} f \neq \phi \Rightarrow f$ is not surjective.

$\therefore f$ is not bijective.

خوارزمية القسمة

Theorem:- Division Algorithm

If $a, b \in Z^+$ such that $a > b$, then there exists.

unique integers q and r such that $a = bq + r$, $0 \le r < b$.

بدون برهان

نقسم $\frac{a}{b}$

proof:- let $S = \{a - bn , n \in Z , a - bn \geqslant 0\}$

$S \neq \phi$, $S \in Z^+$

If $0 \in S \Rightarrow a - bn = 0 \Rightarrow a = bn$. اي ان a هي من مضاعفات b

If $0 \notin S \Rightarrow a - bn > 0$ and let r be the smallest element in S.

s.t. $a - bq = r \Rightarrow a = bq + r$ now to prove $r < b$

$\Rightarrow$

(28)

الخواص التامة

The set of Real Number ($\mathbb{R}$):-

الجمع والضرب معرفة على F

زوج مرتب لعمليتي الجمع والضرب

let $F \neq \emptyset$ be a set and $(+, \cdot)$ be a binary operation defined of F. Then $(F, +, \cdot)$ is said to be a field if it satisfies the following:-

1. $(a+b)+c = a+(b+c)$.
2. $a+b = b+a$.
3. $\exists\, 0 \in F$ s.t. $a+0 = 0+a = a$
4. $\forall a \in F$, $\exists -a \in F$ s.t. $a+(-a) = -a+a = 0$. معكوس جمعي
5. $(ab)c = a(bc)$.
6. $ab = ba$.
7. $\exists\, 1 \in F$ s.t. $a \cdot 1 = 1 \cdot a = a$. العنصر المحايد الضربي
8. $\forall a \in F$, $\exists a^{-1} \in F$ s.t. $aa^{-1} = a^{-1}a = 1$.
9. $(a+b)c = ac+bc$.
10. $1 \neq 0$. العنصر المحايد الجمعي $\neq$ العنصر المحايد الضربي

Note:- $F^{+} = \{x : x \geq 0\}$, $F^{-} = \{x : x < 0\}$

$$F = F^{+} \cup F^{-} \quad , \quad F^{+} \cap F^{-} = \emptyset$$

نلاحظ ان

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

جزئية

The complex numbers: $\mathbb{C}$

Definition: let $\mathbb{C} = \{(a,b) : a,b \in \mathbb{R}\} = \{a+ib, a,b \in \mathbb{R}, i^2 = -1\}$

let $z_1, z_2 \in \mathbb{C}$ s.t. $z_1 = (a_1, b_1)$, $z_2 = (a_2, b_2)$, then

$z_1 \mp z_2 = (a_1, b_1) \mp (a_2, b_2) = (a_1 \mp a_2, b_1 \mp b_2)$

$$z_1 z_2 = (a_1, b_1)(a_2, b_2) = (a_1 + b_1 i)(a_2 + b_2 i)$$
$$= (a_1 a_2 + i a_1 b_2 + i b_1 a_2 - b_1 b_2)$$
$$= (a_1 a_2 - b_1 b_2, a_1 b_2 + b_1 a_2)$$

properties of complex numbers:-

1. if $z_1 = (a_1, b_1)$, $z_2 = (a_2, b_2)$, then

   $z_1 = z_2 \Leftrightarrow a_1 = a_2, b_1 = b_2$

2. if $z_1, z_2 \in \mathbb{C}$, then $z_1 + z_2 = z_2 + z_1$, $z_1 z_2 = z_2 z_1$.

3. let $z_1, z_2, z_3 \in \mathbb{C}$, then

   $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

   $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

4. $\exists\, 0 = (0,0) \in \mathbb{C}$, s.t. $z + 0 = 0 + z = z$.

5. $\exists\, 1 = (1,0) \in \mathbb{C}$, s.t. $(1,0) z = z(1,0) = z$.

6. $\forall z \in \mathbb{C}$, $\exists -z \in \mathbb{C}$, s.t. $z + (-z) = -z + z = 0$.

7. $\forall z \in \mathbb{C}$, $\exists z^{-1} \in \mathbb{C}$, s.t. $z z^{-1} = z^{-1} z = (1,0)$

8. If $Z_1, Z_2, Z_3 \in \mathbb{C}$, then
$(Z_1+Z_2)Z_3 = Z_1Z_3 + Z_2Z_3$.

9. If $Z_1+Z_2 = Z_2+Z_3 \Rightarrow Z_1=Z_3$.

10. If $Z_1Z_2 = Z_2Z_3 \Rightarrow Z_1=Z_3$.

11. If $Z\in\mathbb{C}$, then $Z=0 \Leftrightarrow a=b=0$.

12. let $Z\in\mathbb{C}$, $r\in\mathbb{R}$, then $rZ=(ra, rb)$.

Ex.: Find Z^{-1}

Solu.:- let $Z=(a,b)$, $a,b\in\mathbb{R}$, $Z\in\mathbb{C}$

$$Z^{-1}=\frac{1}{Z}=\frac{1}{(a,b)}=\frac{1}{a+ib}\times\frac{a-ib}{a-ib}$$

$$=\frac{a-ib}{a^2+b^2}=\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right).$$

Proof (7):- $ZZ^{-1}=(1,0)$, let $Z=(a,b)$

$$ZZ^{-1}=(a,b)\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)=(a+ib)\left(\frac{a}{a^2+b^2}-\frac{ib}{a^2+b^2}\right)$$

$$=\frac{a^2}{a^2+b^2}-\cancel{\frac{iab}{a^2+b^2}}+\cancel{\frac{iab}{a^2+b^2}}+\frac{b^2}{a^2+b^2}$$

$$=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}+i0$$

$$=\frac{a^2+b^2}{a^2+b^2}+i0=1+i0=(1,0)=1.$$